Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.5-Secant/122-4.5.2.3-g-sec-^p-a+b-sec-^m-c+d-sec-^n

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [286]. This is test number [122].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (286)	0.00 (0)
Mathematica	98.60 (282)	1.40 (4)
Maple	93.36 (267)	6.64 (19)
Fricas	83.22 (238)	16.78 (48)
Giac	70.63 (202)	29.37 (84)
Mupad	66.78 (191)	33.22 (95)
Maxima	58.04 (166)	41.96 (120)
Sympy	0.35 (1)	99.65 (285)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

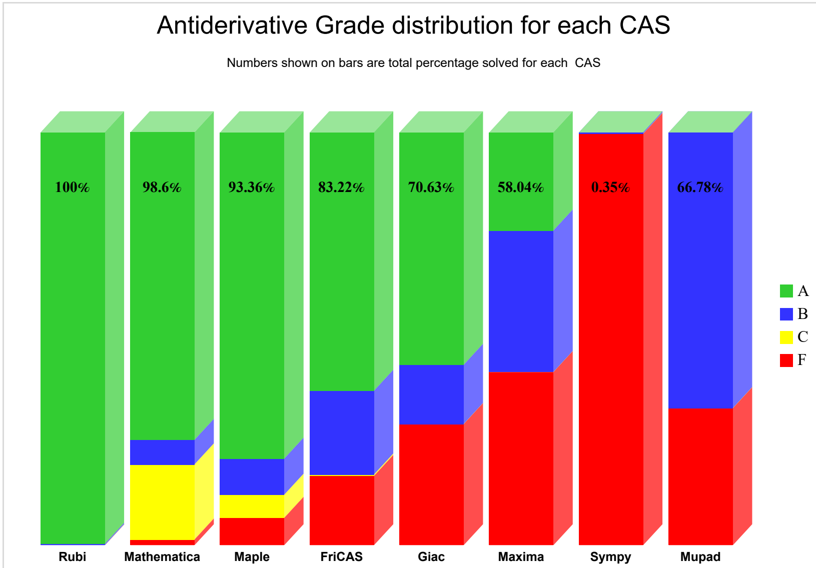
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

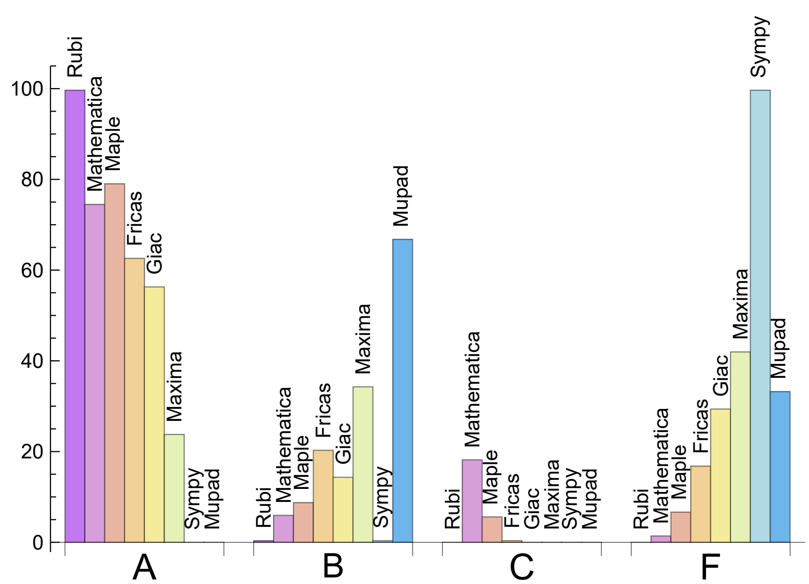
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.650	0.350	0.000	0.000
Maple	79.021	8.741	5.594	6.643
Mathematica	74.476	5.944	18.182	1.399
Fricas	62.587	20.280	0.350	16.783
Giac	56.294	14.336	0.000	29.371
Maxima	23.776	34.266	0.000	41.958
Mupad	0.000	66.783	0.000	33.217
Sympy	0.000	0.350	0.000	99.650

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Maple	19	100.00	0.00	0.00
Fricas	48	68.75	31.25	0.00
Giac	84	90.48	0.00	9.52
Mupad	95	0.00	100.00	0.00
Maxima	120	60.00	5.00	35.00
Sympy	285	85.96	14.04	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.34
Maxima	0.51
Giac	0.58
Sympy	0.85
Mathematica	2.56
Fricas	2.65
Maple	4.43
Mupad	15.97

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	51.00	3.00	51.00	3.00
Rubi	135.98	1.05	121.00	1.00
Maple	161.87	1.27	120.00	1.12
Giac	192.50	1.33	124.00	1.15
Fricas	361.42	2.60	176.50	1.72
Maxima	453.47	4.44	219.00	2.21
Mathematica	467.60	2.40	83.50	0.95
Mupad	803.47	4.72	158.00	1.46

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

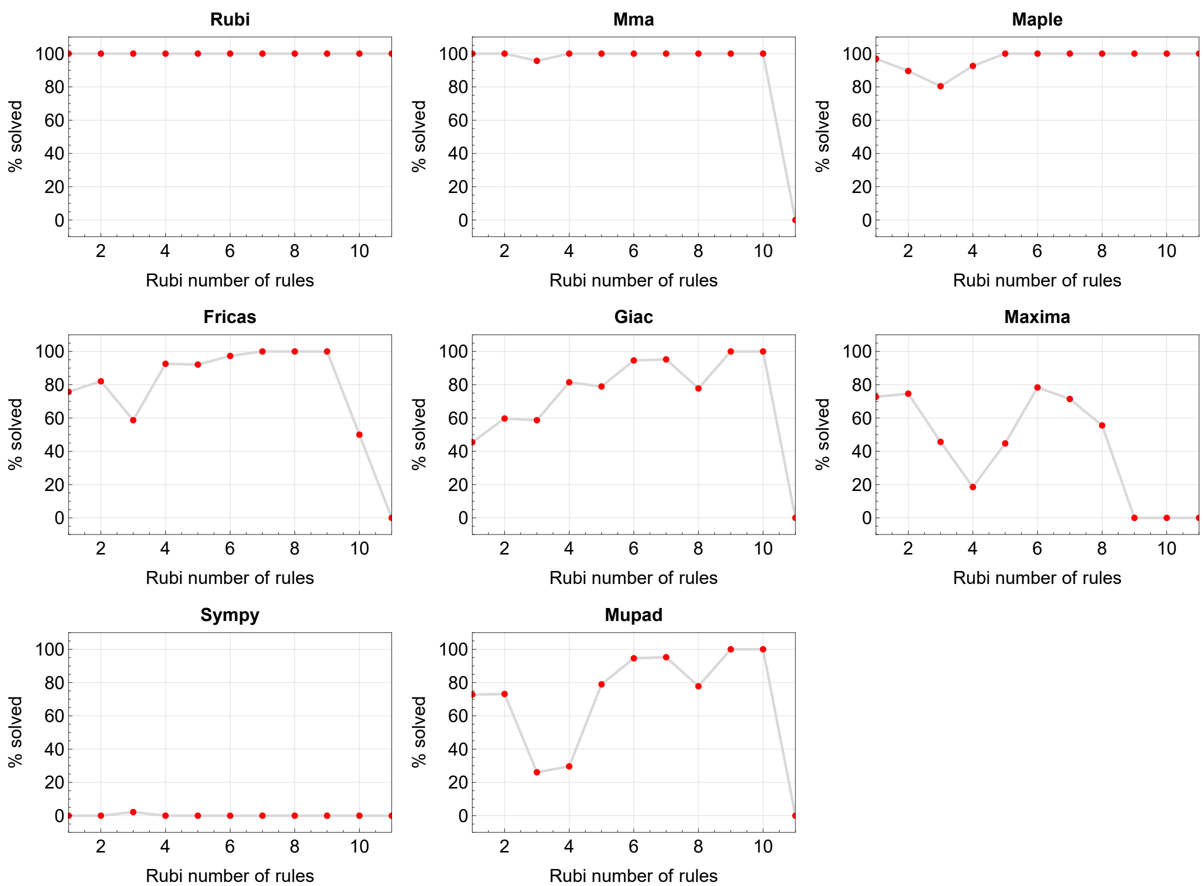


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

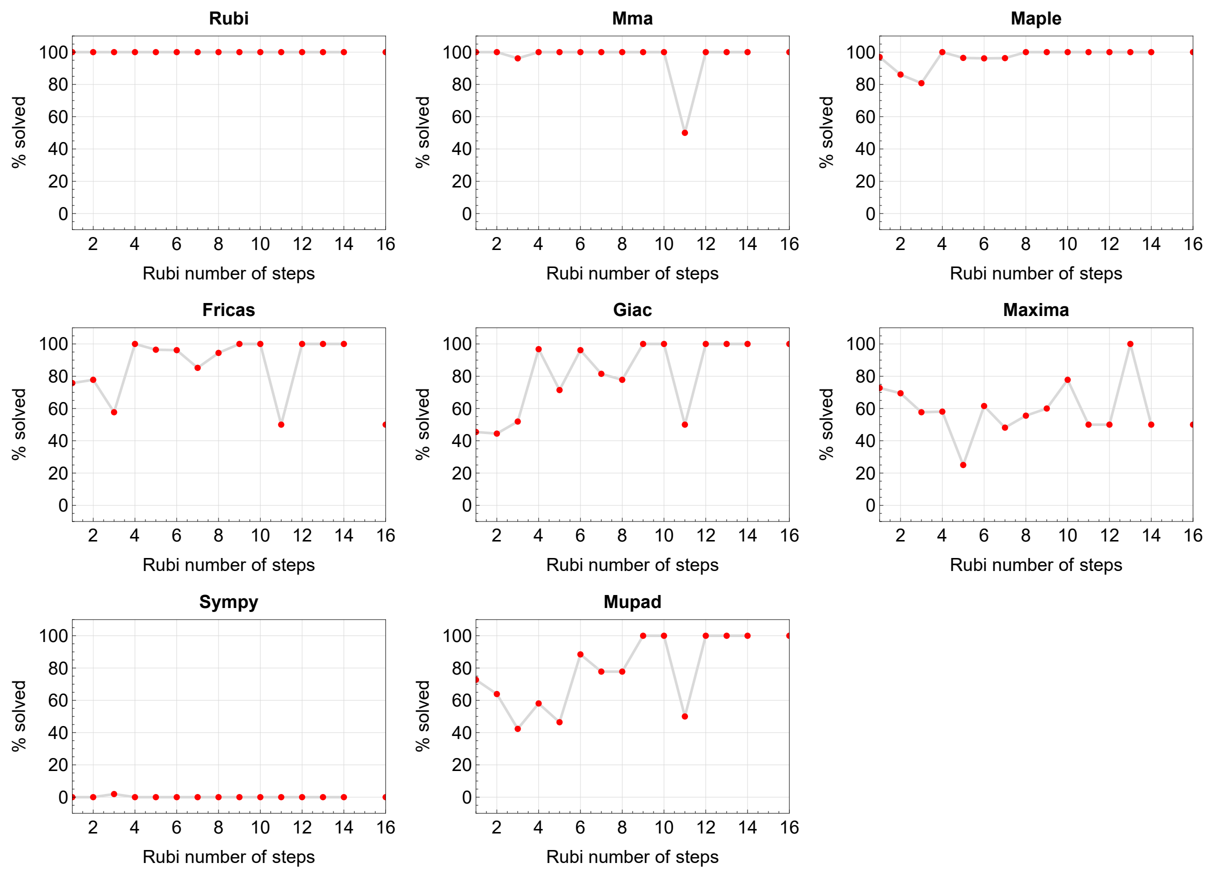


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

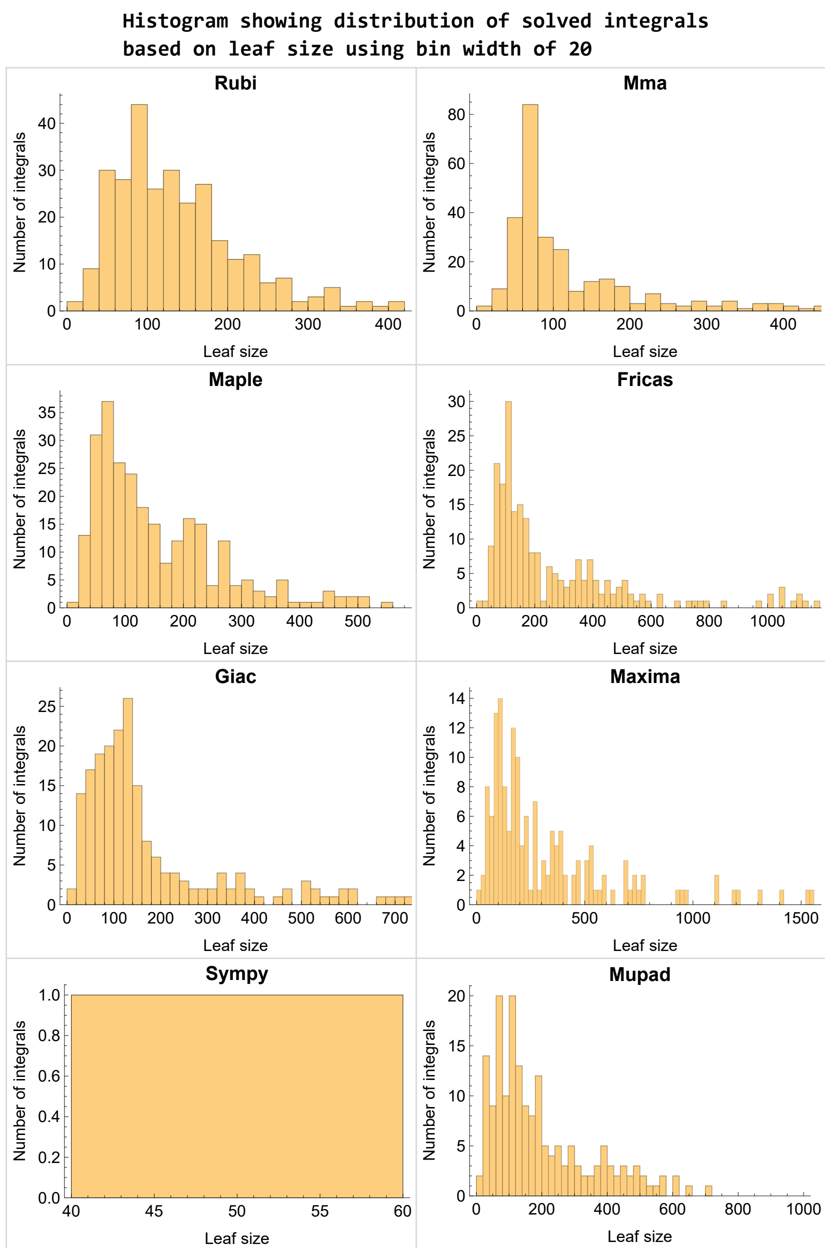


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

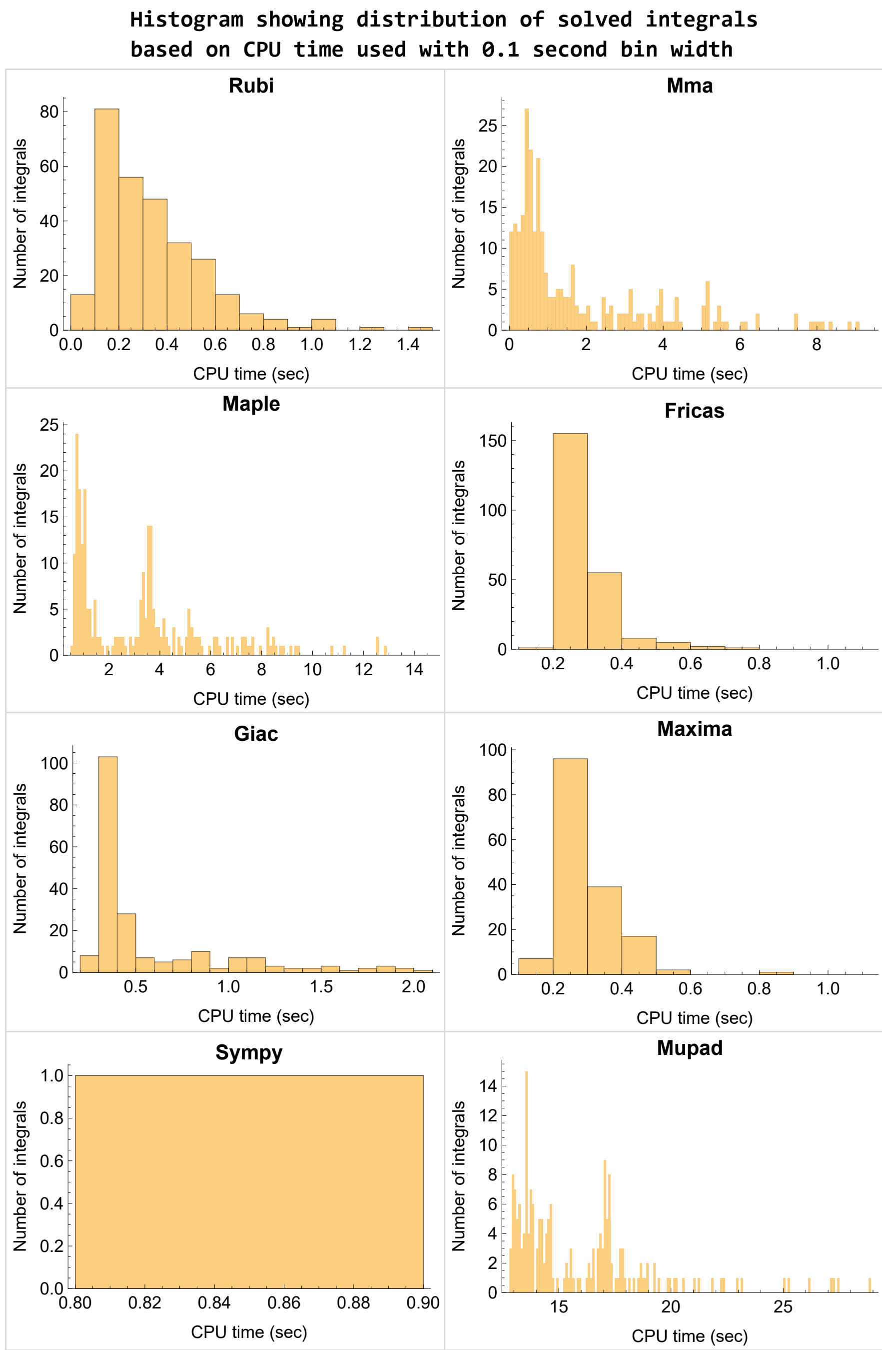


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

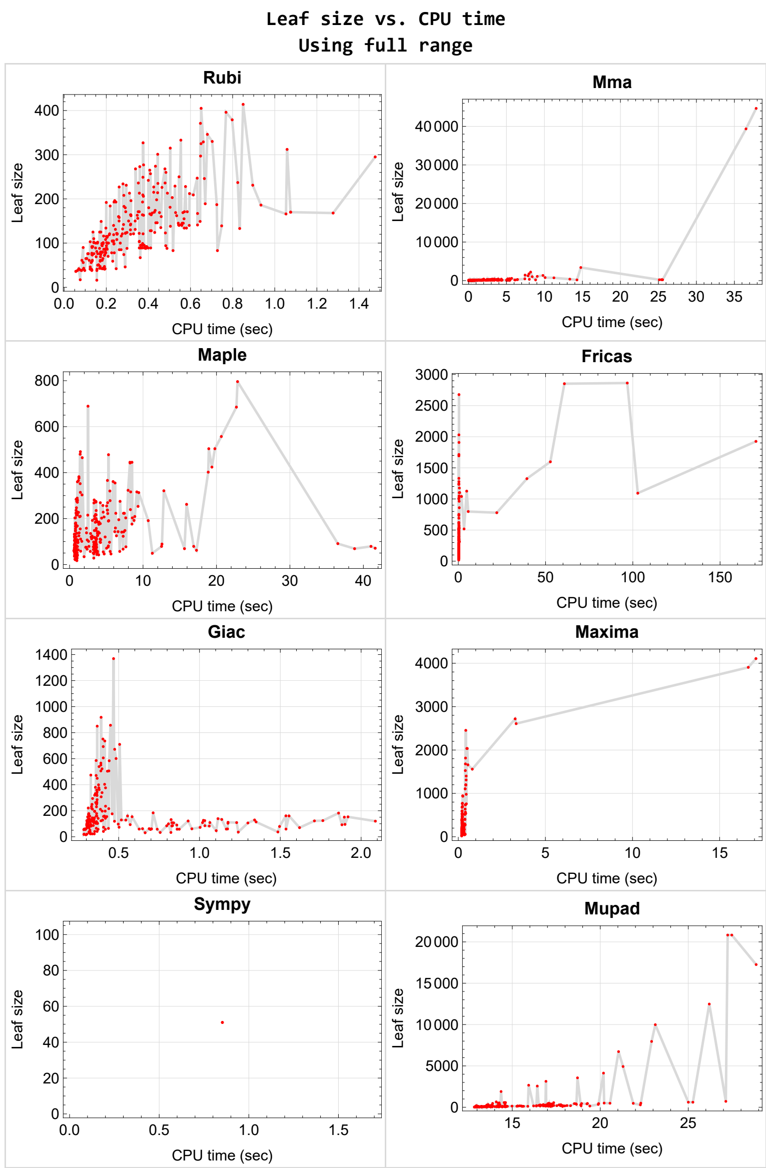


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{176, 205, 206, 207, 216, 224, 232, 233, 265, 269, 275, 277\}$

Maple $\{129, 184, 234, 235, 237, 238, 239, 240, 241, 242, 243\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

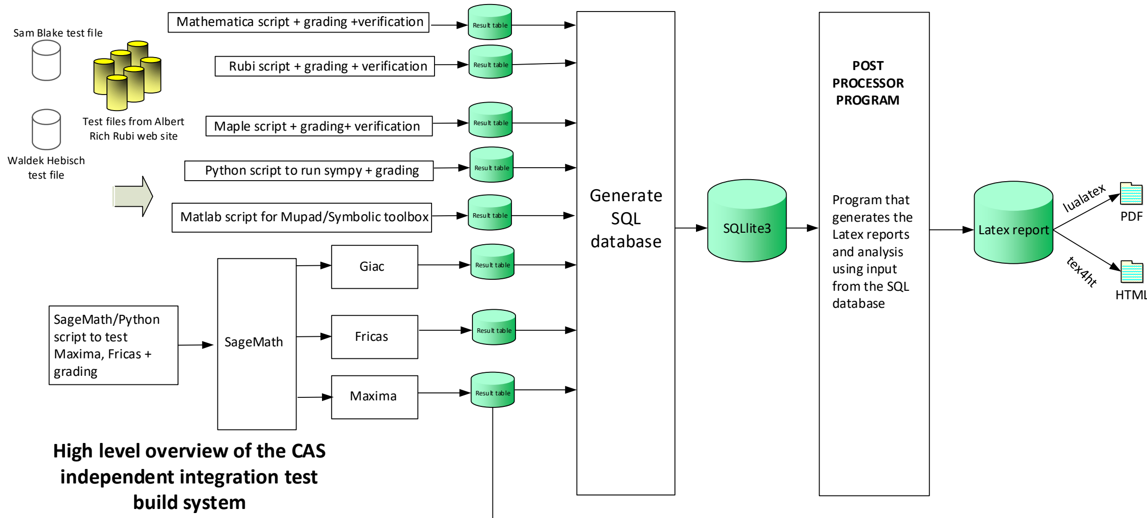
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286 }

B grade { 197 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 77, 79, 80, 81, 82, 83, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 175, 177, 178, 179, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 200, 201, 202, 203, 204, 209, 211, 217, 218, 221, 226, 227, 229, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 259, 260, 261, 262, 263, 264, 266, 267, 268, 271, 275, 276, 279, 281, 282, 283, 286 }

B grade { 3, 171, 174, 180, 181, 210, 212, 213, 219, 220, 225, 228, 252, 253, 258, 273, 285 }

C grade { 15, 27, 28, 34, 35, 36, 42, 43, 44, 52, 53, 54, 70, 76, 78, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 158, 176, 183, 197, 198, 199, 205, 206, 207, 208, 214, 215, 216, 222, 223, 224, 231, 232, 233, 265, 269, 270, 272, 277, 280, 284 }

F normal fail { 154, 155, 274, 278 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 22, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 163, 164, 168, 169, 171, 172, 173, 179, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 269, 273, 275, 276, 279, 282, 285, 286 }

B grade { 69, 70, 77, 78, 107, 110, 118, 129, 170, 178, 180, 183, 184, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 252, 281 }

C grade { 10, 12, 21, 23, 24, 25, 267, 270, 271, 272, 274, 277, 278, 280, 283, 284 }

F normal fail { 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 174, 175, 176, 177 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 113, 114, 115, 120, 121, 123, 124, 125, 132, 136, 138, 143, 144, 148, 149, 150, 156, 157, 158, 162, 163, 164, 168, 169, 171, 172, 173, 178, 179, 180, 181, 182, 185, 186, 187, 188, 189, 193, 194, 195, 196, 197, 202, 203, 204, 205, 210, 211, 213, 214, 217, 218, 220, 221, 225, 226, 227, 228, 229, 230, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 262, 286 }

B grade { 17, 30, 56, 69, 74, 82, 107, 108, 111, 112, 116, 119, 122, 126, 130, 131, 137, 141, 142, 146, 147, 170, 183, 184, 190, 191, 192, 198, 199, 200, 201, 206, 207, 208, 209, 212, 215, 216, 219, 222, 223, 224, 231, 232, 233, 234, 238, 250, 251, 252, 253, 254, 257, 259, 260, 261, 263, 285 }

C grade { 277 }

F normal fail { 110, 117, 118, 127, 128, 129, 133, 134, 135, 139, 140, 145, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 266, 267, 268, 273, 275, 276 }

F(-1) timedout fail { 258, 264, 265, 269, 270, 271, 272, 274, 278, 279, 280, 281, 282, 283, 284 }

F(-2) exception fail { }

Maxima

A grade { 2, 3, 4, 7, 8, 9, 14, 26, 38, 39, 40, 41, 47, 48, 49, 50, 51, 57, 58, 59, 60, 61, 62, 63, 86, 87, 88, 93, 94, 95, 100, 101, 109, 110, 116, 118, 126, 129, 135, 136, 140, 141, 145, 147, 156, 157, 162, 163, 164, 168, 173, 183, 185, 186, 187, 188, 195, 196, 203, 221, 229, 230, 244, 245, 246, 247, 285, 286 }

B grade { 1, 5, 6, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 52, 53, 54, 55, 56, 89, 96, 102, 103, 107, 108, 111, 112, 113, 114, 115, 117, 119, 120, 121, 122, 123, 124, 125, 127, 128, 130, 131, 132, 133, 134, 137, 138, 139, 142, 143, 146, 148, 150, 158, 169, 170, 171, 172, 178, 180, 181, 182, 193, 194, 202, 204, 210, 211, 212, 213, 217, 218, 219, 220, 225, 226, 227, 228 }

C grade { }

F normal fail { 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 82, 83, 84, 90, 91, 92, 97, 98, 99, 104, 105, 106, 151, 152, 153, 154, 155, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 179, 184, 234, 235, 236, 237, 238, 240, 241, 242, 243, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

F(-1) timedout fail { 71, 78, 79, 80, 81, 85 }

F(-2) exception fail { 144, 149, 189, 190, 191, 192, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 214, 215, 216, 222, 223, 224, 231, 232, 233, 239, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

Giac

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 168, 169, 170, 171, 172, 173, 179, 190, 195, 196, 199, 203, 204, 208, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 225, 226, 227, 229, 230, 248, 249, 255, 257, 261, 262, 263, 285, 286 }

B grade { 3, 14, 185, 186, 187, 188, 189, 191, 192, 193, 194, 197, 198, 200, 201, 202, 205, 206, 207, 209, 217, 222, 223, 224, 228, 231, 232, 233, 244, 245, 246, 247, 250, 251, 252, 253, 254, 256, 258, 259, 260 }

C grade { }

F normal fail { 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 174, 175, 178, 180, 181, 182, 184, 234, 235, 236, 237, 238, 239, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

F(-1) timedout fail { }

F(-2) exception fail { 161, 176, 177, 183, 240, 241, 242, 243 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 107, 108, 109, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 141, 146, 147, 157, 162, 163, 164, 168, 169, 170, 171, 172, 173, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 285, 286 }

C grade { }

F normal fail { }

F(-1) timedout fail { 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 104, 105, 106, 110, 117, 118, 127, 128, 129, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 165, 166, 167, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 170 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 116, 117, 118, 119, 134, 135, 136, 137, 138, 140, 141, 142, 143, 146, 147, 148, 151, 152, 153, 154, 155, 158, 159, 160, 162, 163, 164, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 285, 286 }

F(-1) timedout fail { 64, 71, 79, 86, 93, 94, 100, 101, 107, 113, 114, 115, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 139, 144, 145, 149, 150, 156, 157, 161, 165, 180, 182, 243, 278, 284 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	154	159	215	131	0	145	176
N.S.	1	1.00	1.47	1.51	2.05	1.25	0.00	1.38	1.68
time (sec)	N/A	0.228	5.132	6.241	0.242	0.286	0.000	0.337	18.912

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	144	111	133	117	0	128	146
N.S.	1	1.00	1.67	1.29	1.55	1.36	0.00	1.49	1.70
time (sec)	N/A	0.198	1.289	3.683	0.238	0.290	0.000	0.325	17.219

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	132	98	108	103	0	111	114
N.S.	1	1.00	2.16	1.61	1.77	1.69	0.00	1.82	1.87
time (sec)	N/A	0.119	0.571	4.149	0.213	0.275	0.000	0.311	15.641

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	58	68	67	0	55	77
N.S.	1	1.00	1.00	1.53	1.79	1.76	0.00	1.45	2.03
time (sec)	N/A	0.064	0.031	1.509	0.214	0.292	0.000	0.284	14.542

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	77	50	101	66	0	60	31
N.S.	1	1.00	1.83	1.19	2.40	1.57	0.00	1.43	0.74
time (sec)	N/A	0.070	0.202	0.745	0.220	0.268	0.000	0.297	13.764

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	23	21	97	51	0	20	20
N.S.	1	1.00	0.64	0.58	2.69	1.42	0.00	0.56	0.56
time (sec)	N/A	0.055	0.222	0.802	0.220	0.273	0.000	0.303	13.524

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	43	36	117	78	0	37	35
N.S.	1	1.00	0.57	0.47	1.54	1.03	0.00	0.49	0.46
time (sec)	N/A	0.112	0.399	0.832	0.232	0.258	0.000	0.316	14.247

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	55	49	177	104	0	51	61
N.S.	1	1.00	0.47	0.42	1.53	0.90	0.00	0.44	0.53
time (sec)	N/A	0.187	3.953	0.915	0.233	0.264	0.000	0.322	14.117

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	65	62	197	128	0	65	106
N.S.	1	1.00	0.41	0.39	1.25	0.81	0.00	0.41	0.67
time (sec)	N/A	0.262	5.136	0.899	0.220	0.263	0.000	0.362	13.796

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	102	209	368	177	0	197	251
N.S.	1	1.00	0.60	1.22	2.15	1.04	0.00	1.15	1.47
time (sec)	N/A	0.352	1.320	8.879	0.207	0.286	0.000	0.417	17.751

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	91	195	321	161	0	178	219
N.S.	1	1.00	0.61	1.30	2.14	1.07	0.00	1.19	1.46
time (sec)	N/A	0.306	0.893	6.697	0.215	0.294	0.000	0.363	17.241

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	82	143	227	145	0	159	187
N.S.	1	1.00	0.87	1.52	2.41	1.54	0.00	1.69	1.99
time (sec)	N/A	0.201	0.450	6.147	0.211	0.288	0.000	0.365	18.310

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	79	150	99	0	87	155
N.S.	1	1.00	1.10	1.08	2.05	1.36	0.00	1.19	2.12
time (sec)	N/A	0.134	0.038	2.635	0.212	0.274	0.000	0.321	16.965

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	96	108	103	0	111	113
N.S.	1	1.00	0.74	1.57	1.77	1.69	0.00	1.82	1.85
time (sec)	N/A	0.134	0.088	4.211	0.202	0.262	0.000	0.305	15.884

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	82	225	108	0	100	77
N.S.	1	1.00	0.88	1.11	3.04	1.46	0.00	1.35	1.04
time (sec)	N/A	0.132	0.586	0.933	0.207	0.276	0.000	0.310	13.771

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	109	65	201	128	0	84	63
N.S.	1	1.00	1.22	0.73	2.26	1.44	0.00	0.94	0.71
time (sec)	N/A	0.149	0.208	1.003	0.208	0.277	0.000	0.326	13.786

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	189	83	0	22	22
N.S.	1	1.00	0.66	0.61	4.97	2.18	0.00	0.58	0.58
time (sec)	N/A	0.098	0.140	0.876	0.216	0.279	0.000	0.325	13.543

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	47	38	270	114	0	41	37
N.S.	1	1.00	0.59	0.48	3.38	1.42	0.00	0.51	0.46
time (sec)	N/A	0.173	0.570	0.879	0.215	0.261	0.000	0.352	14.421

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	59	51	269	140	0	57	67
N.S.	1	1.00	0.49	0.42	2.22	1.16	0.00	0.47	0.55
time (sec)	N/A	0.276	0.772	1.002	0.225	0.258	0.000	0.373	14.170

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	69	64	389	168	0	73	108
N.S.	1	1.00	0.42	0.39	2.39	1.03	0.00	0.45	0.66
time (sec)	N/A	0.377	1.255	1.011	0.224	0.275	0.000	0.406	14.261

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	122	253	443	209	0	235	316
N.S.	1	1.00	0.54	1.11	1.95	0.92	0.00	1.04	1.39
time (sec)	N/A	0.425	3.241	9.323	0.227	0.295	0.000	0.419	17.264

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	111	239	408	193	0	216	284
N.S.	1	1.00	0.54	1.16	1.98	0.94	0.00	1.05	1.38
time (sec)	N/A	0.376	2.200	8.284	0.207	0.293	0.000	0.439	16.929

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	176	368	177	0	197	252
N.S.	1	1.00	0.84	1.45	3.04	1.46	0.00	1.63	2.08
time (sec)	N/A	0.216	0.917	8.492	0.213	0.284	0.000	0.398	16.854

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	125	142	244	115	0	103	220
N.S.	1	1.00	1.25	1.42	2.44	1.15	0.00	1.03	2.20
time (sec)	N/A	0.150	0.043	3.906	0.232	0.306	0.000	0.369	16.504

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	81	143	227	145	0	159	188
N.S.	1	1.00	0.86	1.52	2.41	1.54	0.00	1.69	2.00
time (sec)	N/A	0.181	0.494	6.603	0.217	0.271	0.000	0.361	17.534

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	70	111	133	117	0	128	146
N.S.	1	1.00	0.81	1.29	1.55	1.36	0.00	1.49	1.70
time (sec)	N/A	0.188	0.404	4.161	0.208	0.288	0.000	0.334	16.369

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	65	106	387	125	0	118	105
N.S.	1	1.00	0.65	1.06	3.87	1.25	0.00	1.18	1.05
time (sec)	N/A	0.158	0.574	1.111	0.207	0.279	0.000	0.338	14.682

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	65	97	349	165	0	116	93
N.S.	1	1.00	0.55	0.82	2.93	1.39	0.00	0.97	0.78
time (sec)	N/A	0.214	0.659	1.145	0.226	0.257	0.000	0.363	13.372

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	139	78	309	176	0	102	78
N.S.	1	1.00	1.05	0.59	2.34	1.33	0.00	0.77	0.59
time (sec)	N/A	0.278	0.215	1.077	0.225	0.266	0.000	0.368	13.101

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	356	111	0	22	22
N.S.	1	1.00	0.66	0.61	9.37	2.92	0.00	0.58	0.58
time (sec)	N/A	0.155	0.111	0.939	0.225	0.262	0.000	0.349	12.855

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	47	38	357	140	0	41	37
N.S.	1	1.00	0.59	0.48	4.46	1.75	0.00	0.51	0.46
time (sec)	N/A	0.196	0.350	1.045	0.233	0.262	0.000	0.406	12.926

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	59	51	518	168	0	57	67
N.S.	1	1.00	0.49	0.42	4.28	1.39	0.00	0.47	0.55
time (sec)	N/A	0.375	0.750	1.012	0.222	0.263	0.000	0.418	13.323

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	69	64	517	194	0	73	108
N.S.	1	1.00	0.43	0.40	3.19	1.20	0.00	0.45	0.67
time (sec)	N/A	0.534	5.005	1.167	0.242	0.262	0.000	0.503	13.230

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	53	138	591	153	0	132	112
N.S.	1	1.00	0.44	1.14	4.88	1.26	0.00	1.09	0.93
time (sec)	N/A	0.253	0.991	1.303	0.209	0.286	0.000	0.338	13.273

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	53	106	386	140	0	116	96
N.S.	1	1.00	0.53	1.06	3.86	1.40	0.00	1.16	0.96
time (sec)	N/A	0.211	0.478	1.104	0.199	0.270	0.000	0.326	13.594

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	62	80	224	119	0	97	77
N.S.	1	1.00	0.84	1.08	3.03	1.61	0.00	1.31	1.04
time (sec)	N/A	0.193	0.437	0.963	0.231	0.287	0.000	0.302	13.121

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	77	47	101	70	0	58	31
N.S.	1	1.00	1.88	1.15	2.46	1.71	0.00	1.41	0.76
time (sec)	N/A	0.091	0.153	0.796	0.197	0.269	0.000	0.287	13.267

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	18	18	0	18	18
N.S.	1	1.00	1.00	1.06	1.12	1.12	0.00	1.12	1.12
time (sec)	N/A	0.155	0.022	0.994	0.193	0.255	0.000	0.285	12.919

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	48	77	50	0	56	50
N.S.	1	1.00	0.90	0.81	1.31	0.85	0.00	0.95	0.85
time (sec)	N/A	0.231	0.461	0.787	0.196	0.286	0.000	0.294	13.096

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	59	97	74	0	69	63
N.S.	1	1.00	0.78	0.76	1.24	0.95	0.00	0.88	0.81
time (sec)	N/A	0.238	0.526	0.933	0.204	0.251	0.000	0.317	13.093

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	79	74	117	102	0	82	83
N.S.	1	1.00	0.66	0.62	0.98	0.85	0.00	0.68	0.69
time (sec)	N/A	0.316	1.526	0.888	0.209	0.256	0.000	0.330	13.438

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	75	155	765	210	0	156	170
N.S.	1	1.00	0.46	0.95	4.66	1.28	0.00	0.95	1.04
time (sec)	N/A	0.315	2.917	1.450	0.230	0.274	0.000	0.406	12.939

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	75	125	531	197	0	140	136
N.S.	1	1.00	0.50	0.83	3.54	1.31	0.00	0.93	0.91
time (sec)	N/A	0.261	1.964	1.267	0.212	0.291	0.000	0.349	12.991

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	75	95	341	178	0	121	104
N.S.	1	1.00	0.63	0.80	2.87	1.50	0.00	1.02	0.87
time (sec)	N/A	0.235	0.455	1.118	0.214	0.269	0.000	0.346	13.039

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	109	65	196	138	0	89	46
N.S.	1	1.00	1.24	0.74	2.23	1.57	0.00	1.01	0.52
time (sec)	N/A	0.168	0.156	0.827	0.219	0.280	0.000	0.311	13.037

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	23	21	94	53	0	20	20
N.S.	1	1.00	0.64	0.58	2.61	1.47	0.00	0.56	0.56
time (sec)	N/A	0.056	0.059	0.785	0.203	0.270	0.000	0.286	12.844

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	48	76	49	0	69	61
N.S.	1	1.00	1.00	0.81	1.29	0.83	0.00	1.17	1.03
time (sec)	N/A	0.171	0.707	0.728	0.210	0.266	0.000	0.304	12.871

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	28	31	50	0	31	28
N.S.	1	1.00	0.87	0.74	0.82	1.32	0.00	0.82	0.74
time (sec)	N/A	0.131	0.041	1.026	0.186	0.256	0.000	0.318	12.991

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	79	74	121	109	0	96	76
N.S.	1	1.00	0.99	0.92	1.51	1.36	0.00	1.20	0.95
time (sec)	N/A	0.177	0.828	0.665	0.202	0.282	0.000	0.327	13.465

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	89	87	140	120	0	109	89
N.S.	1	1.00	0.91	0.89	1.43	1.22	0.00	1.11	0.91
time (sec)	N/A	0.236	1.690	0.670	0.200	0.253	0.000	0.346	14.314

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	99	100	161	163	0	122	102
N.S.	1	1.00	0.70	0.71	1.14	1.16	0.00	0.87	0.72
time (sec)	N/A	0.315	3.868	0.851	0.214	0.269	0.000	0.391	15.563

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	75	168	935	263	0	176	193
N.S.	1	1.00	0.35	0.78	4.35	1.22	0.00	0.82	0.90
time (sec)	N/A	0.425	5.144	1.056	0.260	0.270	0.000	0.460	12.969

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	75	136	680	250	0	159	159
N.S.	1	1.00	0.39	0.70	3.52	1.30	0.00	0.82	0.82
time (sec)	N/A	0.372	3.394	0.984	0.214	0.288	0.000	0.412	12.956

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	75	108	470	231	0	141	126
N.S.	1	1.00	0.46	0.66	2.87	1.41	0.00	0.86	0.77
time (sec)	N/A	0.317	0.926	0.885	0.223	0.297	0.000	0.373	13.038

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	139	76	304	192	0	109	61
N.S.	1	1.00	1.06	0.58	2.32	1.47	0.00	0.83	0.47
time (sec)	N/A	0.243	0.187	0.725	0.211	0.272	0.000	0.355	13.042

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	23	185	82	0	22	22
N.S.	1	1.00	0.66	0.61	4.87	2.16	0.00	0.58	0.58
time (sec)	N/A	0.083	0.085	0.723	0.200	0.257	0.000	0.336	13.005

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	43	36	115	79	0	37	35
N.S.	1	1.00	0.57	0.47	1.51	1.04	0.00	0.49	0.46
time (sec)	N/A	0.115	0.196	0.659	0.204	0.262	0.000	0.320	13.132

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	59	95	76	0	87	74
N.S.	1	1.00	0.86	0.76	1.22	0.97	0.00	1.12	0.95
time (sec)	N/A	0.225	0.488	0.713	0.213	0.259	0.000	0.343	12.950

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	79	74	120	109	0	103	111
N.S.	1	1.00	0.99	0.92	1.50	1.36	0.00	1.29	1.39
time (sec)	N/A	0.182	0.733	0.703	0.205	0.258	0.000	0.359	13.239

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	50	41	41	76	0	41	38
N.S.	1	1.00	0.85	0.69	0.69	1.29	0.00	0.69	0.64
time (sec)	N/A	0.133	0.062	1.078	0.210	0.273	0.000	0.367	13.393

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	99	99	159	163	0	128	129
N.S.	1	1.00	1.00	1.00	1.61	1.65	0.00	1.29	1.30
time (sec)	N/A	0.179	3.107	0.789	0.199	0.263	0.000	0.401	14.146

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	109	110	181	190	0	142	109
N.S.	1	1.00	0.91	0.92	1.51	1.58	0.00	1.18	0.91
time (sec)	N/A	0.251	5.142	0.787	0.208	0.278	0.000	0.418	14.424

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	119	121	200	217	0	155	120
N.S.	1	1.00	0.73	0.75	1.23	1.34	0.00	0.96	0.74
time (sec)	N/A	0.317	5.141	0.889	0.216	0.253	0.000	0.426	14.679

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	70	77	0	119	0	107	483
N.S.	1	1.00	0.43	0.47	0.00	0.73	0.00	0.66	2.96
time (sec)	N/A	0.347	2.276	7.670	0.000	0.274	0.000	0.840	20.552

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	60	67	0	105	0	83	384
N.S.	1	1.00	0.49	0.55	0.00	0.86	0.00	0.68	3.15
time (sec)	N/A	0.249	0.678	7.285	0.000	0.275	0.000	0.829	17.242

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	50	55	0	82	0	56	120
N.S.	1	1.00	0.62	0.68	0.00	1.01	0.00	0.69	1.48
time (sec)	N/A	0.155	0.416	4.587	0.000	0.270	0.000	0.697	17.059

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	38	42	0	65	0	31	87
N.S.	1	1.00	0.97	1.08	0.00	1.67	0.00	0.79	2.23
time (sec)	N/A	0.077	0.170	3.961	0.000	0.264	0.000	0.664	14.442

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	78	80	0	272	0	61	0
N.S.	1	1.00	1.01	1.04	0.00	3.53	0.00	0.79	0.00
time (sec)	N/A	0.131	0.253	3.635	0.000	0.329	0.000	0.953	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	81	178	0	342	0	72	0
N.S.	1	1.00	1.07	2.34	0.00	4.50	0.00	0.95	0.00
time (sec)	N/A	0.142	0.777	4.181	0.000	0.339	0.000	1.004	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	60	269	0	405	0	104	0
N.S.	1	1.00	0.53	2.38	0.00	3.58	0.00	0.92	0.00
time (sec)	N/A	0.204	0.479	4.186	0.000	0.346	0.000	1.176	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	88	79	0	147	0	109	606
N.S.	1	1.00	0.51	0.46	0.00	0.86	0.00	0.64	3.54
time (sec)	N/A	0.585	2.477	16.900	0.000	0.288	0.000	1.019	25.263

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	78	69	0	131	0	85	503
N.S.	1	1.00	0.61	0.54	0.00	1.02	0.00	0.66	3.93
time (sec)	N/A	0.414	1.662	15.644	0.000	0.268	0.000	0.833	20.220

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	66	57	0	105	0	58	384
N.S.	1	1.00	0.78	0.67	0.00	1.24	0.00	0.68	4.52
time (sec)	N/A	0.270	1.458	5.524	0.000	0.277	0.000	0.875	17.148

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	46	0	84	0	33	93
N.S.	1	1.00	1.34	1.12	0.00	2.05	0.00	0.80	2.27
time (sec)	N/A	0.129	0.602	4.711	0.000	0.273	0.000	0.757	17.857

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	123	107	108	0	343	0	82	0
N.S.	1	1.05	0.91	0.92	0.00	2.93	0.00	0.70	0.00
time (sec)	N/A	0.292	0.563	4.243	0.000	0.316	0.000	1.061	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	124	64	120	0	372	0	109	0
N.S.	1	1.10	0.57	1.06	0.00	3.29	0.00	0.96	0.00
time (sec)	N/A	0.298	0.423	4.841	0.000	0.314	0.000	1.230	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	130	150	228	0	429	0	106	0
N.S.	1	1.11	1.28	1.95	0.00	3.67	0.00	0.91	0.00
time (sec)	N/A	0.315	1.133	4.581	0.000	0.327	0.000	1.299	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	64	320	0	517	0	139	0
N.S.	1	1.00	0.39	1.95	0.00	3.15	0.00	0.85	0.00
time (sec)	N/A	0.372	0.614	5.486	0.000	0.345	0.000	1.114	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	88	79	0	163	0	109	710
N.S.	1	1.00	0.51	0.46	0.00	0.95	0.00	0.64	4.15
time (sec)	N/A	0.571	3.768	41.034	0.000	0.290	0.000	1.064	27.136

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	78	69	0	147	0	85	607
N.S.	1	1.00	0.61	0.54	0.00	1.15	0.00	0.66	4.74
time (sec)	N/A	0.409	1.034	38.804	0.000	0.271	0.000	1.033	25.004

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	66	57	0	119	0	58	471
N.S.	1	1.00	0.78	0.67	0.00	1.40	0.00	0.68	5.54
time (sec)	N/A	0.262	0.679	6.830	0.000	0.276	0.000	0.862	21.874

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	46	0	97	0	33	375
N.S.	1	1.00	1.34	1.12	0.00	2.37	0.00	0.80	9.15
time (sec)	N/A	0.123	0.774	5.636	0.000	0.263	0.000	0.823	17.156

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	119	125	0	377	0	111	0
N.S.	1	1.00	0.73	0.76	0.00	2.30	0.00	0.68	0.00
time (sec)	N/A	0.413	0.844	6.383	0.000	0.344	0.000	1.177	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	64	141	0	432	0	124	0
N.S.	1	1.00	0.38	0.84	0.00	2.57	0.00	0.74	0.00
time (sec)	N/A	0.418	0.595	7.095	0.000	0.331	0.000	1.033	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	64	223	0	441	0	133	0
N.S.	1	1.00	0.37	1.28	0.00	2.53	0.00	0.76	0.00
time (sec)	N/A	0.432	0.615	6.293	0.000	0.318	0.000	1.138	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	72	81	163	88	0	108	164
N.S.	1	1.00	0.51	0.57	1.15	0.62	0.00	0.76	1.15
time (sec)	N/A	0.282	0.850	5.281	0.316	0.266	0.000	0.830	17.983

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	62	71	137	77	0	84	125
N.S.	1	1.00	0.57	0.66	1.27	0.71	0.00	0.78	1.16
time (sec)	N/A	0.250	0.535	5.040	0.298	0.280	0.000	0.797	15.559

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	57	110	50	0	60	77
N.S.	1	1.00	0.69	0.79	1.53	0.69	0.00	0.83	1.07
time (sec)	N/A	0.191	0.312	2.822	0.300	0.267	0.000	0.702	13.685

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	28	84	45	0	59	40
N.S.	1	1.00	0.74	0.72	2.15	1.15	0.00	1.51	1.03
time (sec)	N/A	0.132	0.114	3.295	0.303	0.275	0.000	0.627	13.157

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	52	112	0	269	0	64	0
N.S.	1	1.00	0.58	1.26	0.00	3.02	0.00	0.72	0.00
time (sec)	N/A	0.201	0.292	3.001	0.000	0.324	0.000	0.437	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	58	192	0	329	0	97	0
N.S.	1	1.00	0.48	1.57	0.00	2.70	0.00	0.80	0.00
time (sec)	N/A	0.291	0.528	3.100	0.000	0.321	0.000	0.497	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	58	268	0	401	0	128	0
N.S.	1	1.00	0.37	1.72	0.00	2.57	0.00	0.82	0.00
time (sec)	N/A	0.350	0.586	3.310	0.000	0.324	0.000	0.544	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	72	89	188	103	0	121	188
N.S.	1	1.00	0.46	0.57	1.21	0.66	0.00	0.78	1.21
time (sec)	N/A	0.569	1.505	12.579	0.305	0.270	0.000	0.930	17.736

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	64	79	163	82	0	99	136
N.S.	1	1.00	0.52	0.64	1.33	0.67	0.00	0.80	1.11
time (sec)	N/A	0.481	0.899	12.524	0.319	0.269	0.000	0.808	17.268

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	54	53	110	72	0	60	134
N.S.	1	1.00	0.61	0.60	1.24	0.81	0.00	0.67	1.51
time (sec)	N/A	0.399	0.359	3.375	0.309	0.263	0.000	0.703	16.881

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	44	109	60	0	62	94
N.S.	1	1.00	1.34	1.07	2.66	1.46	0.00	1.51	2.29
time (sec)	N/A	0.192	0.118	3.705	0.315	0.267	0.000	0.648	17.175

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	60	184	0	331	0	93	0
N.S.	1	1.00	0.43	1.33	0.00	2.40	0.00	0.67	0.00
time (sec)	N/A	0.504	0.318	3.503	0.000	0.317	0.000	0.571	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	64	200	0	369	0	129	0
N.S.	1	1.00	0.38	1.18	0.00	2.18	0.00	0.76	0.00
time (sec)	N/A	0.453	0.488	3.612	0.000	0.334	0.000	0.517	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	64	281	0	487	0	160	0
N.S.	1	1.00	0.32	1.38	0.00	2.40	0.00	0.79	0.00
time (sec)	N/A	0.513	0.798	3.350	0.000	0.334	0.000	0.555	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	74	91	214	109	0	126	492
N.S.	1	1.00	0.44	0.54	1.27	0.64	0.00	0.75	2.91
time (sec)	N/A	0.529	0.911	36.524	0.324	0.284	0.000	1.024	22.304

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	64	71	189	104	0	90	456
N.S.	1	1.00	0.47	0.53	1.40	0.77	0.00	0.67	3.38
time (sec)	N/A	0.459	1.463	41.609	0.322	0.264	0.000	0.860	19.273

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	54	73	163	88	0	58	446
N.S.	1	1.00	0.61	0.83	1.85	1.00	0.00	0.66	5.07
time (sec)	N/A	0.298	0.504	3.442	0.334	0.272	0.000	0.738	18.939

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	55	46	136	74	0	62	441
N.S.	1	1.00	1.34	1.12	3.32	1.80	0.00	1.51	10.76
time (sec)	N/A	0.126	0.117	3.775	0.316	0.261	0.000	0.689	19.906

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	60	211	0	401	0	119	0
N.S.	1	1.00	0.33	1.17	0.00	2.22	0.00	0.66	0.00
time (sec)	N/A	0.503	0.753	3.408	0.000	0.340	0.000	0.474	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	64	275	0	483	0	154	0
N.S.	1	1.00	0.30	1.30	0.00	2.28	0.00	0.73	0.00
time (sec)	N/A	0.595	0.835	3.577	0.000	0.347	0.000	0.583	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	64	272	0	461	0	184	0
N.S.	1	1.00	0.26	1.11	0.00	1.87	0.00	0.75	0.00
time (sec)	N/A	0.666	0.999	3.538	0.000	0.366	0.000	0.713	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	70	88	638	93	0	0	136
N.S.	1	1.00	1.63	2.05	14.84	2.16	0.00	0.00	3.16
time (sec)	N/A	0.169	0.801	3.337	0.392	0.261	0.000	0.000	15.320

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	74	298	78	0	0	78
N.S.	1	1.00	1.40	1.72	6.93	1.81	0.00	0.00	1.81
time (sec)	N/A	0.171	0.617	3.461	0.366	0.272	0.000	0.000	14.006

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	51	47	55	56	0	0	47
N.S.	1	1.00	1.24	1.15	1.34	1.37	0.00	0.00	1.15
time (sec)	N/A	0.156	0.323	3.299	0.342	0.274	0.000	0.000	13.506

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	98	92	0	0	0	0
N.S.	1	1.00	1.06	1.92	1.80	0.00	0.00	0.00	0.00
time (sec)	N/A	0.164	0.322	3.268	0.337	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	50	514	79	0	0	118
N.S.	1	1.00	1.00	1.19	12.24	1.88	0.00	0.00	2.81
time (sec)	N/A	0.182	0.509	3.219	0.379	0.268	0.000	0.000	14.622

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	56	67	758	106	0	0	203
N.S.	1	1.00	1.30	1.56	17.63	2.47	0.00	0.00	4.72
time (sec)	N/A	0.185	0.534	3.367	0.463	0.265	0.000	0.000	18.108

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	107	1680	112	0	0	294
N.S.	1	1.00	0.94	1.20	18.88	1.26	0.00	0.00	3.30
time (sec)	N/A	0.370	2.933	3.603	0.409	0.283	0.000	0.000	17.665

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	87	96	1105	112	0	0	195
N.S.	1	1.00	0.98	1.08	12.42	1.26	0.00	0.00	2.19
time (sec)	N/A	0.357	1.619	3.543	0.393	0.280	0.000	0.000	17.210

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	64	83	550	82	0	0	108
N.S.	1	1.00	0.72	0.93	6.18	0.92	0.00	0.00	1.21
time (sec)	N/A	0.381	0.703	3.406	0.417	0.273	0.000	0.000	14.723

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	66	54	56	76	0	0	76
N.S.	1	1.00	1.53	1.26	1.30	1.77	0.00	0.00	1.77
time (sec)	N/A	0.186	0.408	3.799	0.328	0.259	0.000	0.000	14.088

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	62	128	275	0	0	0	0
N.S.	1	1.00	0.65	1.35	2.89	0.00	0.00	0.00	0.00
time (sec)	N/A	0.363	0.412	3.830	0.389	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	70	199	122	0	0	0	0
N.S.	1	1.00	0.71	2.01	1.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	0.588	3.399	0.318	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	65	533	95	0	0	165
N.S.	1	1.00	1.00	1.55	12.69	2.26	0.00	0.00	3.93
time (sec)	N/A	0.181	0.861	3.590	0.421	0.268	0.000	0.000	16.765

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	68	77	1559	133	0	0	273
N.S.	1	1.00	0.77	0.88	17.72	1.51	0.00	0.00	3.10
time (sec)	N/A	0.387	1.694	3.227	0.804	0.269	0.000	0.000	18.890

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	68	87	2608	158	0	0	340
N.S.	1	1.00	0.74	0.95	28.35	1.72	0.00	0.00	3.70
time (sec)	N/A	0.392	4.356	3.335	3.321	0.278	0.000	0.000	19.886

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	68	97	3906	184	0	0	407
N.S.	1	1.00	0.74	1.05	42.46	2.00	0.00	0.00	4.42
time (sec)	N/A	0.391	5.381	3.577	16.645	0.280	0.000	0.000	19.241

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	92	111	2454	152	0	0	307
N.S.	1	1.00	0.69	0.83	18.31	1.13	0.00	0.00	2.29
time (sec)	N/A	0.570	4.304	1.501	0.430	0.260	0.000	0.000	17.764

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	76	101	1526	106	0	0	215
N.S.	1	1.00	0.57	0.75	11.39	0.79	0.00	0.00	1.60
time (sec)	N/A	0.581	1.229	1.665	0.404	0.272	0.000	0.000	17.358

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	88	87	1106	112	0	0	195
N.S.	1	1.00	0.99	0.98	12.43	1.26	0.00	0.00	2.19
time (sec)	N/A	0.363	0.833	3.662	0.394	0.262	0.000	0.000	16.854

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	70	58	58	93	0	0	136
N.S.	1	1.00	1.63	1.35	1.35	2.16	0.00	0.00	3.16
time (sec)	N/A	0.170	0.431	3.794	0.322	0.258	0.000	0.000	14.985

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	76	148	737	0	0	0	0
N.S.	1	1.00	0.54	1.05	5.23	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	0.369	3.606	0.412	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	78	235	2035	0	0	0	0
N.S.	1	1.00	0.54	1.62	14.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.542	0.719	3.618	0.515	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	145	81	272	169	0	0	0	0
N.S.	1	1.00	0.56	1.88	1.17	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	1.301	3.595	0.310	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	71	1815	126	0	0	199
N.S.	1	1.00	1.00	1.69	43.21	3.00	0.00	0.00	4.74
time (sec)	N/A	0.247	1.808	3.612	0.414	0.272	0.000	0.000	17.870

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	78	81	2719	166	0	0	350
N.S.	1	1.00	0.89	0.92	30.90	1.89	0.00	0.00	3.98
time (sec)	N/A	0.488	3.977	3.522	3.264	0.271	0.000	0.000	18.646

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	78	91	4108	194	0	0	419
N.S.	1	1.00	0.59	0.68	30.89	1.46	0.00	0.00	3.15
time (sec)	N/A	0.834	5.425	3.619	17.083	0.284	0.000	0.000	18.618

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	74	137	737	0	0	160	0
N.S.	1	1.00	0.53	0.99	5.30	0.00	0.00	1.15	0.00
time (sec)	N/A	0.748	0.620	3.785	0.404	0.000	0.000	1.554	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	60	116	276	0	0	129	0
N.S.	1	1.00	0.64	1.23	2.94	0.00	0.00	1.37	0.00
time (sec)	N/A	0.482	0.485	3.616	0.393	0.000	0.000	1.341	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	75	64	0	0	59	0
N.S.	1	1.00	1.00	1.50	1.28	0.00	0.00	1.18	0.00
time (sec)	N/A	0.159	0.232	3.682	0.305	0.000	0.000	1.166	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	65	44	204	0	70	0
N.S.	1	1.00	1.00	1.38	0.94	4.34	0.00	1.49	0.00
time (sec)	N/A	0.191	0.289	3.239	0.360	0.311	0.000	1.619	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	115	406	382	0	92	0
N.S.	1	1.00	0.79	1.21	4.27	4.02	0.00	0.97	0.00
time (sec)	N/A	0.372	0.486	3.594	0.378	0.341	0.000	1.881	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	83	157	1201	456	0	122	0
N.S.	1	1.00	0.59	1.12	8.58	3.26	0.00	0.87	0.00
time (sec)	N/A	0.559	0.976	3.637	0.429	0.348	0.000	1.711	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	84	192	2035	0	0	161	0
N.S.	1	1.00	0.59	1.35	14.33	0.00	0.00	1.13	0.00
time (sec)	N/A	0.556	0.635	3.681	0.490	0.000	0.000	1.535	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	70	148	99	0	0	79	0
N.S.	1	1.00	0.74	1.56	1.04	0.00	0.00	0.83	0.00
time (sec)	N/A	0.379	0.421	3.376	0.306	0.000	0.000	1.495	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	56	54	78	0	36	50
N.S.	1	1.00	1.00	1.33	1.29	1.86	0.00	0.86	1.19
time (sec)	N/A	0.186	1.116	3.572	0.312	0.265	0.000	1.240	14.626

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	60	100	397	380	0	95	0
N.S.	1	1.00	0.63	1.05	4.18	4.00	0.00	1.00	0.00
time (sec)	N/A	0.380	0.352	3.585	0.383	0.318	0.000	1.900	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	64	115	567	402	0	120	0
N.S.	1	1.00	0.62	1.11	5.45	3.87	0.00	1.15	0.00
time (sec)	N/A	0.229	0.412	3.509	0.406	0.325	0.000	2.088	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	102	216	0	544	0	153	0
N.S.	1	1.00	0.70	1.48	0.00	3.73	0.00	1.05	0.00
time (sec)	N/A	0.442	0.623	3.616	0.000	0.351	0.000	1.919	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	88	218	133	0	0	114	0
N.S.	1	1.00	0.61	1.50	0.92	0.00	0.00	0.79	0.00
time (sec)	N/A	0.560	0.722	3.541	0.299	0.000	0.000	1.350	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	51	67	98	95	0	59	119
N.S.	1	1.00	1.21	1.60	2.33	2.26	0.00	1.40	2.83
time (sec)	N/A	0.195	0.522	2.385	0.316	0.264	0.000	1.535	15.261

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	71	58	58	104	0	37	120
N.S.	1	1.00	1.65	1.35	1.35	2.42	0.00	0.86	2.79
time (sec)	N/A	0.184	1.165	3.506	0.327	0.262	0.000	1.485	14.609

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	70	138	1191	456	0	124	0
N.S.	1	1.00	0.50	0.99	8.51	3.26	0.00	0.89	0.00
time (sec)	N/A	0.594	0.350	2.307	0.433	0.342	0.000	1.763	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	102	215	0	536	0	151	0
N.S.	1	1.00	0.70	1.47	0.00	3.67	0.00	1.03	0.00
time (sec)	N/A	0.435	0.520	2.474	0.000	0.350	0.000	1.896	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	77	158	1659	482	0	182	0
N.S.	1	1.00	0.48	0.99	10.37	3.01	0.00	1.14	0.00
time (sec)	N/A	0.258	0.564	2.289	0.552	0.351	0.000	1.860	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	95	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.677	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	89	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.285	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	85	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	160	109	0	228	191	0	0	0
N.S.	1	1.00	0.68	0.00	1.42	1.19	0.00	0.00	0.00
time (sec)	N/A	0.461	0.817	0.000	0.329	0.292	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	72	0	171	112	0	0	154
N.S.	1	1.00	0.72	0.00	1.71	1.12	0.00	0.00	1.54
time (sec)	N/A	0.267	0.516	0.000	0.350	0.274	0.000	0.000	15.508

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	163	0	114	70	0	0	0
N.S.	1	1.00	3.54	0.00	2.48	1.52	0.00	0.00	0.00
time (sec)	N/A	0.124	1.697	0.000	0.328	0.276	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.152	0.366	0.000	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.174	0.459	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	104	0	156	120	0	0	290
N.S.	1	1.00	0.62	0.00	0.92	0.71	0.00	0.00	1.72
time (sec)	N/A	0.564	2.473	0.000	0.310	0.287	0.000	0.000	22.279

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	76	97	107	93	0	0	145
N.S.	1	1.00	0.73	0.93	1.03	0.89	0.00	0.00	1.39
time (sec)	N/A	0.357	1.085	5.438	0.315	0.268	0.000	0.000	19.408

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	49	76	62	72	0	0	105
N.S.	1	1.00	1.04	1.62	1.32	1.53	0.00	0.00	2.23
time (sec)	N/A	0.171	0.793	5.239	0.324	0.265	0.000	0.000	14.551

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	104	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.541	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	105	0	0	0	0	0	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	1.289	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	105	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.185	2.694	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	68	137	172	131	0	145	175
N.S.	1	1.00	0.65	1.30	1.64	1.25	0.00	1.38	1.67
time (sec)	N/A	0.235	0.228	4.052	0.231	0.269	0.000	0.349	17.843

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	57	126	160	117	0	128	146
N.S.	1	1.00	0.66	1.47	1.86	1.36	0.00	1.49	1.70
time (sec)	N/A	0.192	0.135	3.808	0.225	0.272	0.000	0.319	16.592

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	34	36	35	51	15	15
N.S.	1	1.00	1.00	2.00	2.12	2.06	3.00	0.88	0.88
time (sec)	N/A	0.076	0.013	1.974	0.230	0.249	0.853	0.297	13.273

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	154	78	194	105	0	87	71
N.S.	1	1.00	2.75	1.39	3.46	1.88	0.00	1.55	1.27
time (sec)	N/A	0.128	0.599	0.726	0.246	0.275	0.000	0.302	13.561

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	48	60	144	123	0	81	44
N.S.	1	1.00	0.69	0.86	2.06	1.76	0.00	1.16	0.63
time (sec)	N/A	0.209	0.352	0.583	0.233	0.266	0.000	0.319	13.155

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	47	36	115	78	0	37	35
N.S.	1	1.00	0.55	0.42	1.34	0.91	0.00	0.43	0.41
time (sec)	N/A	0.204	0.155	0.748	0.225	0.268	0.000	0.323	13.288

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	325	0	0	0	0	0	0
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	3.025	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	72	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.248	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	180	180	3396	0	0	0	0	0	0
N.S.	1	1.00	18.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	14.787	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	187	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.465	1.985	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	143	162	189	979	340	0	0	0
N.S.	1	1.38	1.56	1.82	9.41	3.27	0.00	0.00	0.00
time (sec)	N/A	0.316	4.330	5.285	0.455	0.343	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	116	73	107	0	260	0	132	0
N.S.	1	1.43	0.90	1.32	0.00	3.21	0.00	1.63	0.00
time (sec)	N/A	0.271	1.677	2.405	0.000	0.281	0.000	0.824	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	213	724	255	1310	462	0	0	0
N.S.	1	1.52	5.17	1.82	9.36	3.30	0.00	0.00	0.00
time (sec)	N/A	0.310	11.248	5.046	0.449	0.311	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	150	236	124	536	330	0	0	0
N.S.	1	1.29	2.03	1.07	4.62	2.84	0.00	0.00	0.00
time (sec)	N/A	0.337	3.920	3.067	0.418	0.276	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	242	328	222	1400	569	0	0	0
N.S.	1	1.35	1.83	1.24	7.82	3.18	0.00	0.00	0.00
time (sec)	N/A	0.400	3.071	5.196	0.453	0.379	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	94	101	56	255	0	0	0
N.S.	1	1.00	2.04	2.20	1.22	5.54	0.00	0.00	0.00
time (sec)	N/A	0.290	1.760	2.581	0.399	0.342	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	65	65	98	402	0	357	0	0	0
N.S.	1	1.00	1.51	6.18	0.00	5.49	0.00	0.00	0.00
time (sec)	N/A	0.184	0.592	18.878	0.000	0.423	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	153	237	379	281	0	566	361
N.S.	1	1.00	0.65	1.00	1.61	1.19	0.00	2.40	1.53
time (sec)	N/A	0.479	1.798	5.160	0.233	0.279	0.000	0.390	17.120

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	103	185	266	211	0	380	255
N.S.	1	1.00	0.60	1.08	1.56	1.23	0.00	2.22	1.49
time (sec)	N/A	0.357	0.521	4.002	0.223	0.284	0.000	0.356	17.051

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	75	118	165	150	0	232	196
N.S.	1	1.00	0.69	1.09	1.53	1.39	0.00	2.15	1.81
time (sec)	N/A	0.197	0.312	3.179	0.214	0.271	0.000	0.332	16.372

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	75	75	88	96	0	124	111
N.S.	1	1.00	1.34	1.34	1.57	1.71	0.00	2.21	1.98
time (sec)	N/A	0.091	0.027	2.195	0.207	0.267	0.000	0.303	14.433

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	107	90	0	255	0	127	195
N.S.	1	1.00	1.55	1.30	0.00	3.70	0.00	1.84	2.83
time (sec)	N/A	0.172	0.761	0.716	0.000	0.305	0.000	0.328	13.829

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	75	105	0	357	0	137	85
N.S.	1	1.00	0.95	1.33	0.00	4.52	0.00	1.73	1.08
time (sec)	N/A	0.182	0.769	0.659	0.000	0.286	0.000	0.319	13.473

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	167	178	0	736	0	263	171
N.S.	1	1.00	1.27	1.36	0.00	5.62	0.00	2.01	1.31
time (sec)	N/A	0.337	1.537	0.793	0.000	0.310	0.000	0.358	15.425

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	247	271	0	1278	0	449	321
N.S.	1	1.00	1.31	1.43	0.00	6.76	0.00	2.38	1.70
time (sec)	N/A	0.670	3.648	1.061	0.000	0.359	0.000	0.367	16.761

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	371	180	361	683	387	0	736	484
N.S.	1	1.13	0.55	1.10	2.09	1.18	0.00	2.25	1.48
time (sec)	N/A	0.647	6.186	5.925	0.219	0.278	0.000	0.414	17.084

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	277	140	269	469	294	0	506	394
N.S.	1	1.14	0.58	1.11	1.94	1.21	0.00	2.09	1.63
time (sec)	N/A	0.377	3.455	5.180	0.220	0.286	0.000	0.390	17.060

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	234	100	202	324	209	0	320	237
N.S.	1	1.33	0.57	1.15	1.84	1.19	0.00	1.82	1.35
time (sec)	N/A	0.279	1.675	4.360	0.214	0.272	0.000	0.351	17.204

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	63	120	167	138	0	178	161
N.S.	1	1.00	0.61	1.17	1.62	1.34	0.00	1.73	1.56
time (sec)	N/A	0.124	0.730	2.933	0.205	0.271	0.000	0.315	16.210

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	208	329	150	0	398	0	196	529
N.S.	1	2.19	3.46	1.58	0.00	4.19	0.00	2.06	5.57
time (sec)	N/A	0.283	2.848	0.807	0.000	0.415	0.000	0.352	14.258

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	231	312	157	0	567	0	230	2563
N.S.	1	1.97	2.67	1.34	0.00	4.85	0.00	1.97	21.91
time (sec)	N/A	0.295	2.440	0.809	0.000	0.418	0.000	0.363	16.422

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	184	249	167	0	622	0	211	158
N.S.	1	1.42	1.92	1.28	0.00	4.78	0.00	1.62	1.22
time (sec)	N/A	0.219	1.845	0.801	0.000	0.311	0.000	0.387	15.301

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	268	211	228	0	1234	0	403	286
N.S.	1	1.26	0.99	1.07	0.00	5.79	0.00	1.89	1.34
time (sec)	N/A	0.339	3.980	1.054	0.000	0.341	0.000	0.413	16.584

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	330	322	352	0	1908	0	710	438
N.S.	1	1.20	1.17	1.28	0.00	6.91	0.00	2.57	1.59
time (sec)	N/A	0.703	8.011	1.439	0.000	0.401	0.000	0.506	17.024

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	333	170	355	701	337	0	584	411
N.S.	1	1.16	0.59	1.23	2.43	1.17	0.00	2.03	1.43
time (sec)	N/A	0.554	6.094	6.190	0.242	0.282	0.000	0.441	17.051

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	273	130	257	459	245	0	376	287
N.S.	1	1.06	0.51	1.00	1.79	0.95	0.00	1.46	1.12
time (sec)	N/A	0.359	4.434	4.535	0.242	0.282	0.000	0.377	17.100

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	173	175	262	161	0	212	203
N.S.	1	1.00	1.38	1.40	2.10	1.29	0.00	1.70	1.62
time (sec)	N/A	0.169	1.044	3.983	0.227	0.289	0.000	0.338	16.880

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	153	257	419	224	0	532	0	285	1902
N.S.	1	1.68	2.74	1.46	0.00	3.48	0.00	1.86	12.43
time (sec)	N/A	0.410	3.378	0.870	0.000	0.534	0.000	0.367	14.363

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	161	274	455	216	0	859	0	317	3135
N.S.	1	1.70	2.83	1.34	0.00	5.34	0.00	1.97	19.47
time (sec)	N/A	0.434	5.178	1.076	0.000	0.563	0.000	0.366	16.914

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	188	301	393	227	0	1176	0	376	4131
N.S.	1	1.60	2.09	1.21	0.00	6.26	0.00	2.00	21.97
time (sec)	N/A	0.444	4.327	1.070	0.000	0.601	0.000	0.419	20.184

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	227	398	227	0	1012	0	307	264
N.S.	1	1.28	2.24	1.28	0.00	5.69	0.00	1.72	1.48
time (sec)	N/A	0.261	3.881	1.215	0.000	0.342	0.000	0.392	17.094

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	327	274	303	0	1714	0	601	385
N.S.	1	1.23	1.03	1.14	0.00	6.44	0.00	2.26	1.45
time (sec)	N/A	0.375	6.449	1.695	0.000	0.363	0.000	0.484	17.090

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	236	1243	275	596	297	0	344	211
N.S.	1	1.29	6.79	1.50	3.26	1.62	0.00	1.88	1.15
time (sec)	N/A	0.352	7.845	0.974	0.224	0.274	0.000	0.366	14.169

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	171	78	181	388	216	0	219	139
N.S.	1	1.46	0.67	1.55	3.32	1.85	0.00	1.87	1.19
time (sec)	N/A	0.292	1.604	0.832	0.223	0.284	0.000	0.333	13.678

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	125	237	103	223	155	0	136	85
N.S.	1	1.84	3.49	1.51	3.28	2.28	0.00	2.00	1.25
time (sec)	N/A	0.174	2.657	0.712	0.217	0.275	0.000	0.313	13.526

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	109	53	99	74	0	70	41
N.S.	1	1.00	2.53	1.23	2.30	1.72	0.00	1.63	0.95
time (sec)	N/A	0.093	0.896	0.664	0.200	0.270	0.000	0.304	13.865

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	134	160	74	0	353	0	110	110
N.S.	1	1.61	1.93	0.89	0.00	4.25	0.00	1.33	1.33
time (sec)	N/A	0.198	1.062	0.700	0.000	0.293	0.000	0.314	13.585

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	196	286	146	0	691	0	221	187
N.S.	1	1.35	1.97	1.01	0.00	4.77	0.00	1.52	1.29
time (sec)	N/A	0.317	3.139	0.795	0.000	0.321	0.000	0.312	13.544

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	207	268	1422	221	0	1331	0	362	379
N.S.	1	1.29	6.87	1.07	0.00	6.43	0.00	1.75	1.83
time (sec)	N/A	0.479	7.449	0.986	0.000	0.356	0.000	0.379	14.583

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	315	446	382	772	456	0	506	268
N.S.	1	1.22	1.73	1.48	2.99	1.77	0.00	1.96	1.04
time (sec)	N/A	0.504	7.419	1.269	0.236	0.309	0.000	0.439	13.554

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	249	310	275	536	361	0	359	193
N.S.	1	1.29	1.61	1.42	2.78	1.87	0.00	1.86	1.00
time (sec)	N/A	0.374	5.100	1.013	0.234	0.306	0.000	0.364	13.699

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	193	294	175	342	268	0	250	136
N.S.	1	1.45	2.21	1.32	2.57	2.02	0.00	1.88	1.02
time (sec)	N/A	0.246	3.648	0.843	0.249	0.284	0.000	0.364	13.521

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	149	181	84	195	155	0	158	89
N.S.	1	1.67	2.03	0.94	2.19	1.74	0.00	1.78	1.00
time (sec)	N/A	0.176	2.407	0.652	0.217	0.274	0.000	0.321	13.416

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	76	42	93	58	0	60	45
N.S.	1	1.00	1.17	0.65	1.43	0.89	0.00	0.92	0.69
time (sec)	N/A	0.103	0.797	0.694	0.212	0.267	0.000	0.288	13.581

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	183	209	122	0	598	0	249	168
N.S.	1	1.42	1.62	0.95	0.00	4.64	0.00	1.93	1.30
time (sec)	N/A	0.284	2.051	0.659	0.000	0.291	0.000	0.335	13.883

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	260	376	203	0	1242	0	474	314
N.S.	1	1.23	1.78	0.96	0.00	5.89	0.00	2.25	1.49
time (sec)	N/A	0.483	3.961	0.794	0.000	0.331	0.000	0.328	14.294

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	284	346	2220	280	0	2030	0	751	505
N.S.	1	1.22	7.82	0.99	0.00	7.15	0.00	2.64	1.78
time (sec)	N/A	0.680	8.182	1.080	0.000	0.363	0.000	0.403	14.200

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	405	1338	491	946	620	0	672	327
N.S.	1	1.12	3.69	1.35	2.61	1.71	0.00	1.85	0.90
time (sec)	N/A	0.650	9.806	1.472	0.251	0.305	0.000	0.475	13.625

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	329	439	360	689	502	0	504	252
N.S.	1	1.15	1.53	1.25	2.40	1.75	0.00	1.76	0.88
time (sec)	N/A	0.661	5.506	1.067	0.240	0.288	0.000	0.430	13.856

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	265	292	247	475	385	0	374	195
N.S.	1	1.29	1.42	1.20	2.32	1.88	0.00	1.82	0.95
time (sec)	N/A	0.407	3.859	0.936	0.230	0.280	0.000	0.377	13.540

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	193	295	117	307	248	0	259	147
N.S.	1	1.45	2.22	0.88	2.31	1.86	0.00	1.95	1.11
time (sec)	N/A	0.237	2.592	0.693	0.230	0.273	0.000	0.365	13.518

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	84	67	184	113	0	129	79
N.S.	1	1.00	0.73	0.58	1.60	0.98	0.00	1.12	0.69
time (sec)	N/A	0.201	0.793	0.639	0.220	0.265	0.000	0.332	13.752

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	135	56	115	93	0	75	66
N.S.	1	1.00	1.32	0.55	1.13	0.91	0.00	0.74	0.65
time (sec)	N/A	0.141	0.950	0.779	0.228	0.268	0.000	0.313	13.511

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	235	345	203	0	1001	0	471	228
N.S.	1	1.30	1.91	1.12	0.00	5.53	0.00	2.60	1.26
time (sec)	N/A	0.445	3.164	0.726	0.000	0.304	0.000	0.361	13.800

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	288	325	1772	284	0	1693	0	918	464
N.S.	1	1.13	6.15	0.99	0.00	5.88	0.00	3.19	1.61
time (sec)	N/A	0.650	7.962	0.893	0.000	0.341	0.000	0.391	13.882

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	368	414	1096	365	0	2677	0	1369	655
N.S.	1	1.12	2.98	0.99	0.00	7.27	0.00	3.72	1.78
time (sec)	N/A	0.850	9.054	1.208	0.000	0.395	0.000	0.468	14.089

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	61	61	102	278	0	307	0	0	0
N.S.	1	1.00	1.67	4.56	0.00	5.03	0.00	0.00	0.00
time (sec)	N/A	0.172	0.723	5.513	0.000	0.407	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	140	140	187	478	0	1048	0	0	0
N.S.	1	1.00	1.34	3.41	0.00	7.49	0.00	0.00	0.00
time (sec)	N/A	0.546	14.280	5.300	0.000	0.453	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	107	135	0	246	0	0	0
N.S.	1	1.00	1.37	1.73	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	0.198	0.790	2.860	0.000	0.328	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	171	366	0	1100	0	0	0
N.S.	1	1.00	1.21	2.60	0.00	7.80	0.00	0.00	0.00
time (sec)	N/A	0.633	0.851	5.157	0.000	0.512	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	61	61	94	424	0	343	0	0	0
N.S.	1	1.00	1.54	6.95	0.00	5.62	0.00	0.00	0.00
time (sec)	N/A	0.181	0.387	19.377	0.000	0.445	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	149	188	685	0	1126	0	0	0
N.S.	1	1.00	1.26	4.60	0.00	7.56	0.00	0.00	0.00
time (sec)	N/A	0.646	2.035	22.714	0.000	4.814	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	122	122	144	504	0	963	0	0	0
N.S.	1	1.00	1.18	4.13	0.00	7.89	0.00	0.00	0.00
time (sec)	N/A	0.356	1.433	19.783	0.000	0.488	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	124	141	504	0	1041	0	0	0
N.S.	1	1.00	1.14	4.06	0.00	8.40	0.00	0.00	0.00
time (sec)	N/A	0.430	0.717	18.953	0.000	0.572	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	167	167	198	557	0	1103	0	0	0
N.S.	1	1.00	1.19	3.34	0.00	6.60	0.00	0.00	0.00
time (sec)	N/A	0.633	1.756	20.660	0.000	0.800	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	231	231	155	796	0	1597	0	0	0
N.S.	1	1.00	0.67	3.45	0.00	6.91	0.00	0.00	0.00
time (sec)	N/A	0.896	1.489	22.869	0.000	52.752	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	201	237	379	281	0	850	555
N.S.	1	1.00	0.80	0.95	1.52	1.12	0.00	3.40	2.22
time (sec)	N/A	0.546	2.840	5.139	0.232	0.295	0.000	0.367	17.353

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	143	185	266	211	0	586	395
N.S.	1	1.00	0.79	1.03	1.48	1.17	0.00	3.26	2.19
time (sec)	N/A	0.401	0.775	3.814	0.255	0.276	0.000	0.361	17.202

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	118	165	150	0	294	227
N.S.	1	1.00	0.77	1.03	1.43	1.30	0.00	2.56	1.97
time (sec)	N/A	0.206	0.453	3.395	0.225	0.278	0.000	0.339	17.016

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	75	88	96	0	153	104
N.S.	1	1.00	1.23	1.23	1.44	1.57	0.00	2.51	1.70
time (sec)	N/A	0.084	0.018	2.275	0.229	0.291	0.000	0.308	14.561

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	316	0	127	573
N.S.	1	1.00	1.47	1.21	0.00	4.16	0.00	1.67	7.54
time (sec)	N/A	0.169	0.480	0.768	0.000	0.632	0.000	0.346	14.539

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	97	132	0	389	0	172	106
N.S.	1	1.00	0.98	1.33	0.00	3.93	0.00	1.74	1.07
time (sec)	N/A	0.186	0.427	0.770	0.000	0.306	0.000	0.313	13.878

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	172	236	0	752	0	399	250
N.S.	1	1.00	1.04	1.42	0.00	4.53	0.00	2.40	1.51
time (sec)	N/A	0.407	1.136	0.884	0.000	0.325	0.000	0.375	16.701

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	405	376	0	1238	0	693	439
N.S.	1	1.00	1.71	1.59	0.00	5.22	0.00	2.92	1.85
time (sec)	N/A	0.824	2.012	1.283	0.000	0.357	0.000	0.404	18.527

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	580	480	0	1093	0	606	9987
N.S.	1	1.00	2.35	1.94	0.00	4.43	0.00	2.45	40.43
time (sec)	N/A	0.632	5.682	1.437	0.000	102.849	0.000	0.405	23.128

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	389	289	0	779	0	339	6730
N.S.	1	1.00	2.29	1.70	0.00	4.58	0.00	1.99	39.59
time (sec)	N/A	0.428	3.112	1.038	0.000	22.065	0.000	0.397	21.040

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	135	165	0	518	0	195	3559
N.S.	1	1.00	1.31	1.60	0.00	5.03	0.00	1.89	34.55
time (sec)	N/A	0.357	2.509	0.946	0.000	3.298	0.000	0.351	18.706

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	309	0	127	571
N.S.	1	1.00	1.47	1.21	0.00	4.07	0.00	1.67	7.51
time (sec)	N/A	0.159	0.439	0.742	0.000	0.570	0.000	0.337	14.615

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	119	108	0	1040	0	522	2665
N.S.	1	1.00	0.98	0.89	0.00	8.60	0.00	4.31	22.02
time (sec)	N/A	0.330	0.444	1.025	0.000	1.855	0.000	0.397	15.932

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	229	208	0	2863	0	331	20827
N.S.	1	1.00	1.22	1.11	0.00	15.31	0.00	1.77	111.37
time (sec)	N/A	0.725	1.337	1.371	0.000	96.827	0.000	0.356	27.240

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	784	689	0	0	0	857	17256
N.S.	1	1.00	2.07	1.82	0.00	0.00	0.00	2.26	45.53
time (sec)	N/A	0.798	10.090	2.507	0.000	0.000	0.000	0.449	28.858

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	511	465	0	1925	0	551	12483
N.S.	1	1.00	1.72	1.57	0.00	6.48	0.00	1.86	42.03
time (sec)	N/A	0.648	5.400	1.725	0.000	170.550	0.000	0.390	26.195

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	362	314	0	1326	0	539	7958
N.S.	1	1.00	1.59	1.38	0.00	5.82	0.00	2.36	34.90
time (sec)	N/A	0.576	3.408	1.467	0.000	39.313	0.000	0.379	22.917

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	180	215	0	798	0	267	4926
N.S.	1	1.00	0.91	1.09	0.00	4.03	0.00	1.35	24.88
time (sec)	N/A	0.439	1.238	0.936	0.000	5.697	0.000	0.365	21.295

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	132	0	394	0	173	106
N.S.	1	1.00	0.97	1.32	0.00	3.94	0.00	1.73	1.06
time (sec)	N/A	0.206	0.436	0.648	0.000	0.280	0.000	0.315	13.753

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	176	210	0	2852	0	330	20827
N.S.	1	1.00	0.95	1.13	0.00	15.33	0.00	1.77	111.97
time (sec)	N/A	0.934	1.587	1.452	0.000	60.792	0.000	0.364	27.475

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	183	316	0	0	0	0	0
N.S.	1	1.00	0.86	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	4.103	9.154	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	44664	321	0	0	0	0	0
N.S.	1	1.00	227.88	1.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	37.864	12.835	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	233	191	0	0	0	0	0
N.S.	1	1.00	1.21	0.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	3.197	10.710	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	176	148	0	0	0	0	0
N.S.	1	1.00	1.60	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	2.645	7.537	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	176	142	0	0	0	0	0
N.S.	1	1.00	1.41	1.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.137	0.649	7.361	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	39359	262	0	0	0	0	0
N.S.	1	1.00	99.39	0.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.769	36.528	15.931	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	223	445	0	0	0	0	0
N.S.	1	1.00	1.31	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.075	25.506	8.494	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	223	0	0	0	0	0
N.S.	1	1.00	1.00	2.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.517	1.349	7.344	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	222	442	0	0	0	0	0
N.S.	1	1.00	1.32	2.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.277	25.117	8.237	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	264	123	0	0	0	0	0
N.S.	1	1.00	2.78	1.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.179	6.443	7.297	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	0	275	0	0	0	0	0
N.S.	1	1.00	0.00	0.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.477	0.000	6.872	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	375	191	0	0	0	0	0
N.S.	1	1.00	1.79	0.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.373	13.338	5.330	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	156	192	0	0	0	0	0
N.S.	1	1.00	0.73	0.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.444	5.477	8.774	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	1019	207	0	517	0	0	0
N.S.	1	1.00	4.45	0.90	0.00	2.26	0.00	0.00	0.00
time (sec)	N/A	0.526	8.387	4.700	0.000	0.118	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	0	323	0	0	0	0	0
N.S.	1	1.00	0.00	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.059	0.000	7.967	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	183	316	0	0	0	0	0
N.S.	1	1.00	0.86	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.357	0.236	8.320	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	223	445	0	0	0	0	0
N.S.	1	1.00	1.31	2.62	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.076	25.557	8.221	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	187	203	0	0	0	0	0
N.S.	1	1.00	1.83	1.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	8.833	7.645	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	165	203	0	0	0	0	0
N.S.	1	1.00	0.79	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.613	4.289	8.545	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	223	0	0	0	0	0
N.S.	1	1.00	1.00	2.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.728	1.378	7.462	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	246	313	0	0	0	0	0
N.S.	1	1.00	1.48	1.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.053	25.394	9.418	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	151	49	68	121	0	47	47
N.S.	1	1.00	2.25	0.73	1.01	1.81	0.00	0.70	0.70
time (sec)	N/A	0.361	4.031	11.267	0.218	0.312	0.000	1.104	13.517

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	175	62	88	146	0	60	60
N.S.	1	1.00	1.97	0.70	0.99	1.64	0.00	0.67	0.67
time (sec)	N/A	0.411	5.022	17.280	0.224	0.288	0.000	1.174	14.181

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [258] had the largest ratio of [.3225999999999998]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	7	1.00	30	0.233
2	A	9	6	1.00	30	0.200
3	A	6	5	1.00	30	0.167
4	A	3	3	1.00	28	0.107
5	A	2	2	1.00	30	0.067
6	A	1	1	1.00	30	0.033
7	A	2	2	1.00	30	0.067
8	A	3	2	1.00	30	0.067
9	A	4	2	1.00	30	0.067
10	A	14	7	1.00	32	0.219
11	A	11	6	1.00	32	0.188
12	A	7	5	1.00	32	0.156
13	A	4	3	1.00	32	0.094
14	A	6	5	1.00	30	0.167
15	A	5	5	1.00	32	0.156
16	A	3	2	1.00	32	0.062
17	A	1	1	1.00	32	0.031
18	A	2	2	1.00	32	0.062
19	A	3	2	1.00	32	0.062
20	A	4	2	1.00	32	0.062
21	A	16	7	1.00	32	0.219
22	A	13	6	1.00	32	0.188
23	A	8	5	1.00	32	0.156
24	A	5	3	1.00	32	0.094

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	7	5	1.00	32	0.156
26	A	9	6	1.00	30	0.200
27	A	6	6	1.00	32	0.188
28	A	6	5	1.00	32	0.156
29	A	4	2	1.00	32	0.062
30	A	1	1	1.00	32	0.031
31	A	2	2	1.00	32	0.062
32	A	3	2	1.00	32	0.062
33	A	4	2	1.00	32	0.062
34	A	10	6	1.00	32	0.188
35	A	6	6	1.00	32	0.188
36	A	5	5	1.00	32	0.156
37	A	2	2	1.00	30	0.067
38	A	3	3	1.00	32	0.094
39	A	6	4	1.00	32	0.125
40	A	10	6	1.00	32	0.188
41	A	13	7	1.00	32	0.219
42	A	11	6	1.00	32	0.188
43	A	7	6	1.00	32	0.188
44	A	6	5	1.00	32	0.156
45	A	3	2	1.00	32	0.062
46	A	1	1	1.00	30	0.033
47	A	6	4	1.00	32	0.125
48	A	3	2	1.00	32	0.062
49	A	7	5	1.00	32	0.156
50	A	10	6	1.00	32	0.188
51	A	13	7	1.00	32	0.219
52	A	12	6	1.00	32	0.188
53	A	8	6	1.00	32	0.188
54	A	7	5	1.00	32	0.156
55	A	4	2	1.00	32	0.062
56	A	1	1	1.00	32	0.031
57	A	2	2	1.00	30	0.067
58	A	10	6	1.00	32	0.188
59	A	7	5	1.00	32	0.156

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	4	3	1.00	32	0.094
61	A	7	5	1.00	32	0.156
62	A	10	6	1.00	32	0.188
63	A	13	7	1.00	32	0.219
64	A	4	2	1.00	32	0.062
65	A	3	2	1.00	32	0.062
66	A	2	2	1.00	32	0.062
67	A	1	1	1.00	32	0.031
68	A	3	3	1.00	32	0.094
69	A	3	3	1.00	32	0.094
70	A	4	4	1.00	32	0.125
71	A	4	2	1.00	34	0.059
72	A	3	2	1.00	34	0.059
73	A	2	2	1.00	34	0.059
74	A	1	1	1.00	34	0.029
75	A	4	3	1.05	34	0.088
76	A	4	4	1.10	34	0.118
77	A	4	3	1.11	34	0.088
78	A	5	4	1.00	34	0.118
79	A	4	2	1.00	34	0.059
80	A	3	2	1.00	34	0.059
81	A	2	2	1.00	34	0.059
82	A	1	1	1.00	34	0.029
83	A	5	3	1.00	34	0.088
84	A	5	4	1.00	34	0.118
85	A	5	4	1.00	34	0.118
86	A	4	3	1.00	34	0.088
87	A	3	3	1.00	34	0.088
88	A	2	2	1.00	34	0.059
89	A	1	1	1.00	34	0.029
90	A	3	3	1.00	34	0.088
91	A	4	4	1.00	34	0.118
92	A	5	4	1.00	34	0.118
93	A	4	3	1.00	34	0.088
94	A	3	2	1.00	34	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	34	0.059
96	A	1	1	1.00	34	0.029
97	A	4	3	1.00	34	0.088
98	A	5	4	1.00	34	0.118
99	A	6	4	1.00	34	0.118
100	A	4	2	1.00	34	0.059
101	A	3	2	1.00	34	0.059
102	A	2	2	1.00	34	0.059
103	A	1	1	1.00	34	0.029
104	A	5	3	1.00	34	0.088
105	A	6	4	1.00	34	0.118
106	A	7	4	1.00	34	0.118
107	A	1	1	1.00	36	0.028
108	A	1	1	1.00	36	0.028
109	A	1	1	1.00	36	0.028
110	A	1	1	1.00	36	0.028
111	A	1	1	1.00	36	0.028
112	A	1	1	1.00	36	0.028
113	A	2	2	1.00	36	0.056
114	A	2	2	1.00	36	0.056
115	A	2	2	1.00	36	0.056
116	A	1	1	1.00	36	0.028
117	A	2	2	1.00	36	0.056
118	A	2	2	1.00	36	0.056
119	A	1	1	1.00	36	0.028
120	A	2	2	1.00	36	0.056
121	A	2	2	1.00	36	0.056
122	A	2	2	1.00	36	0.056
123	A	3	2	1.00	36	0.056
124	A	3	2	1.00	36	0.056
125	A	2	2	1.00	36	0.056
126	A	1	1	1.00	36	0.028
127	A	3	2	1.00	36	0.056
128	A	3	3	1.00	36	0.083
129	A	3	2	1.00	36	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	1	1	1.00	36	0.028
131	A	2	2	1.00	36	0.056
132	A	3	2	1.00	36	0.056
133	A	3	2	1.00	36	0.056
134	A	2	2	1.00	36	0.056
135	A	1	1	1.00	36	0.028
136	A	2	2	1.00	36	0.056
137	A	3	3	1.00	36	0.083
138	A	4	3	1.00	36	0.083
139	A	3	3	1.00	36	0.083
140	A	2	2	1.00	36	0.056
141	A	1	1	1.00	36	0.028
142	A	3	3	1.00	36	0.083
143	A	3	3	1.00	36	0.083
144	A	4	4	1.00	36	0.111
145	A	3	2	1.00	36	0.056
146	A	1	1	1.00	36	0.028
147	A	1	1	1.00	36	0.028
148	A	4	3	1.00	36	0.083
149	A	4	4	1.00	36	0.111
150	A	4	3	1.00	36	0.083
151	A	3	3	1.00	32	0.094
152	A	3	3	1.00	32	0.094
153	A	3	3	1.00	30	0.100
154	A	3	3	1.00	32	0.094
155	A	3	3	1.00	32	0.094
156	A	3	2	1.00	34	0.059
157	A	2	2	1.00	34	0.059
158	A	1	1	1.00	34	0.029
159	A	2	2	1.00	34	0.059
160	A	2	2	1.00	34	0.059
161	A	2	2	1.00	34	0.059
162	A	3	2	1.00	36	0.056
163	A	2	2	1.00	36	0.056
164	A	1	1	1.00	36	0.028

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	3	1.00	34	0.088
166	A	3	3	1.00	36	0.083
167	A	3	3	1.00	36	0.083
168	A	10	7	1.00	32	0.219
169	A	7	6	1.00	32	0.188
170	A	3	3	1.00	30	0.100
171	A	5	5	1.00	32	0.156
172	A	4	4	1.00	32	0.125
173	A	3	3	1.00	32	0.094
174	A	5	3	1.00	34	0.088
175	A	2	2	1.00	32	0.062
176	A	6	4	1.00	34	0.118
177	A	7	4	1.00	34	0.118
178	A	5	5	1.38	40	0.125
179	A	4	4	1.43	36	0.111
180	A	8	8	1.52	38	0.210
181	A	4	4	1.29	40	0.100
182	A	8	8	1.35	40	0.200
183	A	3	3	1.00	38	0.079
184	A	2	2	1.00	34	0.059
185	A	8	6	1.00	29	0.207
186	A	7	6	1.00	29	0.207
187	A	6	6	1.00	29	0.207
188	A	5	5	1.00	27	0.185
189	A	5	5	1.00	29	0.172
190	A	5	5	1.00	29	0.172
191	A	6	5	1.00	29	0.172
192	A	7	5	1.00	29	0.172
193	A	9	8	1.13	31	0.258
194	A	8	7	1.14	31	0.226
195	A	8	7	1.33	31	0.226
196	A	6	6	1.00	29	0.207
197	B	8	8	2.19	31	0.258
198	A	8	8	1.97	31	0.258
199	A	5	4	1.42	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
200	A	6	5	1.26	31	0.161
201	A	8	6	1.20	31	0.194
202	A	9	7	1.16	31	0.226
203	A	9	7	1.06	31	0.226
204	A	10	6	1.00	29	0.207
205	A	9	9	1.68	31	0.290
206	A	9	9	1.70	31	0.290
207	A	9	9	1.60	31	0.290
208	A	6	4	1.28	31	0.129
209	A	7	5	1.23	31	0.161
210	A	7	7	1.29	31	0.226
211	A	6	6	1.46	31	0.194
212	A	6	6	1.84	31	0.194
213	A	3	3	1.00	29	0.103
214	A	4	4	1.61	31	0.129
215	A	6	6	1.35	31	0.194
216	A	7	7	1.29	31	0.226
217	A	8	8	1.22	31	0.258
218	A	7	7	1.29	31	0.226
219	A	6	6	1.45	31	0.194
220	A	6	6	1.67	31	0.194
221	A	2	2	1.00	29	0.069
222	A	6	6	1.42	31	0.194
223	A	7	6	1.23	31	0.194
224	A	8	7	1.22	31	0.226
225	A	9	8	1.12	31	0.258
226	A	8	7	1.15	31	0.226
227	A	7	7	1.29	31	0.226
228	A	6	6	1.45	31	0.194
229	A	4	4	1.00	31	0.129
230	A	3	3	1.00	29	0.103
231	A	7	6	1.30	31	0.194
232	A	8	6	1.13	31	0.194
233	A	9	7	1.12	31	0.226
234	A	2	2	1.00	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	5	5	1.00	35	0.143
236	A	2	2	1.00	35	0.057
237	A	5	5	1.00	37	0.135
238	A	2	2	1.00	33	0.061
239	A	5	4	1.00	39	0.103
240	A	5	5	1.00	33	0.152
241	A	5	5	1.00	35	0.143
242	A	5	4	1.00	39	0.103
243	A	8	6	1.00	39	0.154
244	A	8	6	1.00	29	0.207
245	A	7	6	1.00	29	0.207
246	A	6	6	1.00	29	0.207
247	A	5	5	1.00	27	0.185
248	A	5	5	1.00	29	0.172
249	A	5	5	1.00	29	0.172
250	A	6	5	1.00	29	0.172
251	A	7	5	1.00	29	0.172
252	A	12	8	1.00	31	0.258
253	A	10	8	1.00	31	0.258
254	A	8	7	1.00	31	0.226
255	A	5	5	1.00	29	0.172
256	A	6	4	1.00	31	0.129
257	A	7	5	1.00	31	0.161
258	A	16	10	1.00	31	0.323
259	A	14	10	1.00	31	0.323
260	A	12	9	1.00	31	0.290
261	A	10	7	1.00	31	0.226
262	A	5	5	1.00	29	0.172
263	A	7	5	1.00	31	0.161
264	A	3	3	1.00	33	0.091
265	A	1	1	1.00	35	0.029
266	A	1	1	1.00	35	0.029
267	A	1	1	1.00	35	0.029
268	A	1	1	1.00	35	0.029
269	A	3	3	1.00	37	0.081

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	7	5	1.00	39	0.128
271	A	3	3	1.00	39	0.077
272	A	8	6	1.00	39	0.154
273	A	1	1	1.00	33	0.030
274	A	11	11	1.00	39	0.282
275	A	3	3	1.00	33	0.091
276	A	3	3	1.00	35	0.086
277	A	7	7	1.00	39	0.180
278	A	11	11	1.00	39	0.282
279	A	3	3	1.00	33	0.091
280	A	7	5	1.00	39	0.128
281	A	1	1	1.00	33	0.030
282	A	3	3	1.00	35	0.086
283	A	3	3	1.00	39	0.077
284	A	7	5	1.00	39	0.128
285	A	4	2	1.00	28	0.071
286	A	4	2	1.00	28	0.071

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 30, antiderivative size = 105

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx &= \frac{7ac^4\text{arctanh}(\sin(e+fx))}{8f}-\frac{ac^4\sec(e+fx)\tan(e+fx)}{8f}\\ &\quad -\frac{3ac^4\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{4ac^4\tan^3(e+fx)}{3f}+\frac{ac^4\tan^5(e+fx)}{5f}\end{aligned}$$

```
[Out] 7/8*a*c^4*arctanh(sin(f*x+e))/f-1/8*a*c^4*sec(f*x+e)*tan(f*x+e)/f-3/4*a*c^4
*sec(f*x+e)^3*tan(f*x+e)/f+4/3*a*c^4*tan(f*x+e)^3/f+1/5*a*c^4*tan(f*x+e)^5/
f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {4043, 2691, 3855, 2687, 30, 3853, 14}

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx &= \frac{7ac^4\text{arctanh}(\sin(e+fx))}{8f}+\frac{ac^4\tan^5(e+fx)}{5f}+\frac{4ac^4\tan^3(e+fx)}{3f}\\ &\quad -\frac{3ac^4\tan(e+fx)\sec^3(e+fx)}{4f}-\frac{ac^4\tan(e+fx)\sec(e+fx)}{8f}\end{aligned}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (7*a*c^4*ArcTanh[Sin[e + f*x]])/(8*f) - (a*c^4*Sec[e + f*x]*Tan[e + f*x])/(
8*f) - (3*a*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (4*a*c^4*Tan[e + f*x]^
3)/(3*f) + (a*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\text{integral} = -\left((ac)\int\left(c^3\sec(e+fx)\tan^2(e+fx)-3c^3\sec^2(e+fx)\tan^2(e+fx)+3c^3\sec^3(e+fx)\tan^2(e+fx)-c^3\sec^4(e+fx)\tan^2(e+fx)\right)dx\right)$$

$$= -\left(\left(ac^4\right)\int\sec(e+fx)\tan^2(e+fx)\,dx\right)+\left(ac^4\right)\int\sec^4(e+fx)\tan^2(e+fx)\,dx+\left(3ac^4\right)\int\sec^2(e+fx)\tan^2(e+fx)\,dx-\left(3ac^4\right)\int\sec^3(e+fx)\tan^2(e+fx)\,dx$$

$$= -\frac{ac^4\sec(e+fx)\tan(e+fx)}{2f}-\frac{3ac^4\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{1}{2}\left(ac^4\right)\int\sec(e+fx)\,dx+\frac{1}{4}\left(3ac^4\right)\int\sec^3(e+fx)\,dx+\frac{\left(ac^4\right)\text{Subst}\left(\int x^2\left(1+x^2\right)dx,x,\tan(e+fx)\right)}{f}+\frac{\left(3ac^4\right)\text{Subst}\left(\int x^2\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{ac^4\text{arctanh}(\sin(e+fx))}{2f}-\frac{ac^4\sec(e+fx)\tan(e+fx)}{8f}-\frac{3ac^4\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{ac^4\tan^3(e+fx)}{f}+\frac{1}{8}\left(3ac^4\right)\int\sec(e+fx)\,dx+\frac{\left(ac^4\right)\text{Subst}\left(\int\left(x^2+x^4\right)dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{7ac^4\text{arctanh}(\sin(e+fx))}{8f}-\frac{ac^4\sec(e+fx)\tan(e+fx)}{8f}-\frac{3ac^4\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{4ac^4\tan^3(e+fx)}{3f}+\frac{ac^4\tan^5(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.47

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$$
$$= \frac{ac^{7/2}\left(-210\arcsin\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\sqrt{c-c\sec(e+fx)}+\sqrt{c}\sqrt{1+\sec(e+fx)}(136-121\sec(e+fx)-1\right.}{120f(-1+\sec(e+fx))\sqrt{1+}}$$

[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4,x]

[Out] (a*c^(7/2)*(-210*ArcSin[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Sqrt[c - c*Sec[e + f*x]] + Sqrt[c]*Sqrt[1 + Sec[e + f*x]]*(136 - 121*Sec[e + f*x] - 127*Sec[e + f*x]^2 + 202*Sec[e + f*x]^3 - 114*Sec[e + f*x]^4 + 24*Sec[e + f*x]^5))*Tan[e + f*x])/(120*f*(-1 + Sec[e + f*x])*Sqrt[1 + Sec[e + f*x]])

Maple [A] (verified)

Time = 6.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.51

method	result
norman	$\frac{\frac{7a\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{49a\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{6f}+\frac{224a\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}-\frac{79a\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{6f}-\frac{7a\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}-\frac{7a\,c^4\ln\left(\tan\right.}{8}$
risch	$\frac{ia\,c^4\left(15\,e^{9i(fx+e)}-360\,e^{8i(fx+e)}+390\,e^{7i(fx+e)}-960\,e^{6i(fx+e)}-400\,e^{4i(fx+e)}-390\,e^{3i(fx+e)}-320\,e^{2i(fx+e)}-15\,e^{i(fx\right.}{60f\left(1+e^{2i(fx+e)}\right)^5}$
parts	$\frac{2a\,c^4\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a\,c^4\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}-\frac{3a\,c^4\tan(fx+e)}{f}+\frac{a\,c^4\,\mathrm{se}}{}$
parallelrisch	$-\frac{13a\,c^4\left(\left(\frac{35\cos(fx+e)}{26}+\frac{35\cos(3fx+3e)}{52}+\frac{7\cos(5fx+5e)}{52}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{35\cos(fx+e)}{26}-\frac{35\cos(3fx+3e)}{52}-\frac{7\cos(5}{5}\right.\right.}{2f(\cos(5fx+5e)+5\cos(3fx+3}$
derivativedivides	$\frac{-a\,c^4\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-3a\,c^4\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+}{8}\right.}{}$
default	$\frac{-a\,c^4\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-3a\,c^4\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+}{8}\right.}{}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)

[Out] (7/4*a*c^4/f*tan(1/2*f*x+1/2*e)-49/6*a*c^4/f*tan(1/2*f*x+1/2*e)^3+224/15*a*c^4/f*tan(1/2*f*x+1/2*e)^5-79/6*a*c^4/f*tan(1/2*f*x+1/2*e)^7-7/4*a*c^4/f*tan(1/2*f*x+1/2*e)^9)/(tan(1/2*f*x+1/2*e)^2-1)^5-7/8*a*c^4/f*ln(tan(1/2*f*x+1/2*e)-1)+7/8*a*c^4/f*ln(tan(1/2*f*x+1/2*e)+1)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx = \frac{105\,ac^4\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-105\,ac^4\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-2\,\left(136\,ac^4\cos\right.}{240\,f\cos\left(fx\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(105*a*c^4*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 105*a*c^4*cos(f*x +
 e)^5*log(-sin(f*x + e) + 1) - 2*(136*a*c^4*cos(f*x + e)^4 + 15*a*c^4*cos(f
*x + e)^3 - 112*a*c^4*cos(f*x + e)^2 + 90*a*c^4*cos(f*x + e) - 24*a*c^4)*si
n(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx = ac^4\left(\int \sec\left(e+fx\right)dx+\int\left(-3\sec^2\left(e+fx\right)\right)\,dx+\int 2\sec^3\left(e+fx\right)dx + \int 2\sec^4\left(e+fx\right)dx+\int\left(-3\sec^5\left(e+fx\right)\right)\,dx+\int\sec^6\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a*c**4*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + Integ
ral(2*sec(e + f*x)**3, x) + Integral(2*sec(e + f*x)**4, x) + Integral(-3*se
c(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(95) = 190.

Time = 0.24 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.05

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx = \frac{16\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)ac^4+160\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ac^4+}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a*c^4 +
160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^4 + 45*a*c^4*(2*(3*sin(f*x + e)^3
 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
+ e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a*c^4*(2*sin(f*x + e)/(sin(f*x +
 e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*log
(sec(f*x + e) + tan(f*x + e)) - 720*a*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.38

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx$$
$$=\frac{105\,ac^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,ac^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,ac^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9+790\,ac^4\tan\cdots\right.}{\cdots}}{120\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(105*a*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a*c^4*log(abs(tan
(1/2*f*x + 1/2*e) - 1)) - 2*(105*a*c^4*tan(1/2*f*x + 1/2*e)^9 + 790*a*c^4*t
an(1/2*f*x + 1/2*e)^7 - 896*a*c^4*tan(1/2*f*x + 1/2*e)^5 + 490*a*c^4*tan(1/
2*f*x + 1/2*e)^3 - 105*a*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2
- 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 18.91 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.68

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^4\,dx=\frac{7\,a\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$
$$-\frac{\frac{7\,a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}+\frac{79\,a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{6}-\frac{224\,a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}+\frac{49\,a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{6}-\frac{7\,a\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (7*a*c^4*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((49*a*c^4*tan(e/2 + (f*x)/2)^3
)/6 - (7*a*c^4*tan(e/2 + (f*x)/2))/4 - (224*a*c^4*tan(e/2 + (f*x)/2)^5)/15
+ (79*a*c^4*tan(e/2 + (f*x)/2)^7)/6 + (7*a*c^4*tan(e/2 + (f*x)/2)^9)/4)/(f*
(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6
 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.2 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx = \frac{5ac^3\text{arctanh}(\sin(e+fx))}{8f} - \frac{3ac^3\sec(e+fx)\tan(e+fx)}{8f} - \frac{ac^3\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{2ac^3\tan^3(e+fx)}{3f}$$

```
[Out] 5/8*a*c^3*arctanh(sin(f*x+e))/f-3/8*a*c^3*sec(f*x+e)*tan(f*x+e)/f-1/4*a*c^3
*sec(f*x+e)^3*tan(f*x+e)/f+2/3*a*c^3*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4043, 2691, 3855, 2687, 30, 3853}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx = \frac{5ac^3\text{arctanh}(\sin(e+fx))}{8f} + \frac{2ac^3\tan^3(e+fx)}{3f} - \frac{ac^3\tan(e+fx)\sec^3(e+fx)}{4f} - \frac{3ac^3\tan(e+fx)\sec(e+fx)}{8f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (5*a*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a*c^3*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a*c^3*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (2*a*c^3*Tan[e + f*x]^
3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((ac)\int \left(c^2\sec(e+fx)\tan^2(e+fx) - 2c^2\sec^2(e+fx)\tan^2(e+fx) \right.\right. \\
&\qquad \left.\left. + c^2\sec^3(e+fx)\tan^2(e+fx)\right)\, dx\right) \\
&= -\left(\left(ac^3\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right) \\
&\quad - \left(ac^3\right)\int \sec^3(e+fx)\tan^2(e+fx)\,dx + \left(2ac^3\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx \\
&= -\frac{ac^3\sec(e+fx)\tan(e+fx)}{2f} - \frac{ac^3\sec^3(e+fx)\tan(e+fx)}{4f} \\
&\quad + \frac{1}{4}\left(ac^3\right)\int \sec^3(e+fx)\,dx + \frac{1}{2}\left(ac^3\right)\int \sec(e+fx)\,dx \\
&\quad + \frac{\left(2ac^3\right)\operatorname{Subst}\left(\int x^2\,dx, x, \tan(e+fx)\right)}{f} \\
&= \frac{ac^3\operatorname{arctanh}(\sin(e+fx))}{2f} - \frac{3ac^3\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad - \frac{ac^3\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{2ac^3\tan^3(e+fx)}{3f} + \frac{1}{8}\left(ac^3\right)\int \sec(e+fx)\,dx \\
&= \frac{5ac^3\operatorname{arctanh}(\sin(e+fx))}{8f} - \frac{3ac^3\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad - \frac{ac^3\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{2ac^3\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx =$$

$$-\frac{ac^{5/2}\left(30\arcsin\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\sqrt{c-c\sec(e+fx)} + \sqrt{c}\sqrt{1+\sec(e+fx)}(-16+7\sec(e+fx)+25\,\mathrm{se}}{24f(-1+\sec(e+fx))\sqrt{1+\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -1/24*(a*c^(5/2)*(30*ArcSin[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Sqr
t[c - c*Sec[e + f*x]] + Sqrt[c]*Sqrt[1 + Sec[e + f*x]]*(-16 + 7*Sec[e + f*x
] + 25*Sec[e + f*x]^2 - 22*Sec[e + f*x]^3 + 6*Sec[e + f*x]^4))*Tan[e + f*x]
)/(f*(-1 + Sec[e + f*x])*Sqrt[1 + Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-a\,c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-2a\,c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2}{f}$
default	$\frac{-a\,c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-2a\,c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-2}{f}$
parts	$\frac{a\,c^3\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{2a\,c^3\tan(fx+e)}{f}-\frac{2a\,c^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{a\,c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}\right.\right.}{}$
norman	$\frac{-\frac{5a\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{55a\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12f}-\frac{73a\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}-\frac{5a\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{5a\,c^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f}+\frac{5a}{}$
risch	$\frac{ia\,c^3\left(9\,e^{7i(fx+e)}-48\,e^{6i(fx+e)}+33\,e^{5i(fx+e)}-48\,e^{4i(fx+e)}-33\,e^{3i(fx+e)}-16\,e^{2i(fx+e)}-9\,e^{i(fx+e)}-16\right)}{12f\left(1+e^{2i(fx+e)}\right)^4}+\frac{5a\,c^3\ln(e^{i}}{8}$
parallelrisch	$-\frac{3\left(\left(\frac{10\cos(2fx+2e)}{3}+\frac{5\cos(4fx+4e)}{6}+\frac{5}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{10\cos(2fx+2e)}{3}-\frac{5\cos(4fx+4e)}{6}-\frac{5}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.\right.}{4f(3+\cos(4fx+4e)+4\cos(2fx+2e))}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^3*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+
e)+tan(f*x+e)))-2*a*c^3*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-2*a*c^3*tan(f*x+
e)+a*c^3*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$=\frac{15\,ac^3\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-15\,ac^3\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)-2\left(16\,ac^3\cos\left(f\right.\right.}{48\,f\cos\left(fx+e\right)^4}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(15*a*c^3*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a*c^3*cos(f*x + e)
^4*log(-sin(f*x + e) + 1) - 2*(16*a*c^3*cos(f*x + e)^3 + 9*a*c^3*cos(f*x +
e)^2 - 16*a*c^3*cos(f*x + e) + 6*a*c^3)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$= -ac^3\left(\int(-\sec(e+fx))\,dx + \int 2\sec^2(e+fx)\,dx + \int(-2\sec^4(e+fx))\,dx + \int \sec^5(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a*c**3*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + Inte
gral(-2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$= \frac{32\left(\tan(fx+e)^3+3\tan(fx+e)\right)ac^3+3ac^3\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log\right.}{48f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^3 + 3*a*c^3*(2*(3*sin(f*x +
e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(
f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 48*a*c^3*log(sec(f*x + e) + tan(
f*x + e)) - 96*a*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx$$
$$= \frac{15ac^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15ac^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(15ac^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7+73ac^3\tan\left(\frac{1}{2}fx\right.\right.}{\left(\tan\right.}}{24f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(15*a*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a*c^3*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a*c^3*tan(1/2*f*x + 1/2*e)^7 + 73*a*c^3*tan(1/
2*f*x + 1/2*e)^5 - 55*a*c^3*tan(1/2*f*x + 1/2*e)^3 + 15*a*c^3*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 17.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.70

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^3\,dx = \frac{5\,a\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f} - \frac{\frac{5\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}+\frac{73\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}-\frac{55\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{5\,a\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (5*a*c^3*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((5*a*c^3*tan(e/2 + (f*x)/2))/4
 - (55*a*c^3*tan(e/2 + (f*x)/2)^3)/12 + (73*a*c^3*tan(e/2 + (f*x)/2)^5)/12
+ (5*a*c^3*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2
+ (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.3 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 30, antiderivative size = 61

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = \frac{ac^2\text{arctanh}(\sin(e+fx))}{2f} - \frac{ac^2\sec(e+fx)\tan(e+fx)}{2f} + \frac{ac^2\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*a*c^2*arctanh(sin(f*x+e))/f-1/2*a*c^2*sec(f*x+e)*tan(f*x+e)/f+1/3*a*c^2
*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4043, 2691, 3855, 2687, 30}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = \frac{ac^2\text{arctanh}(\sin(e+fx))}{2f} + \frac{ac^2\tan^3(e+fx)}{3f} - \frac{ac^2\tan(e+fx)\sec(e+fx)}{2f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a*c^2*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c^2*Sec[e + f*x]*Tan[e + f*x])/(2*
f) + (a*c^2*Tan[e + f*x]^3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((ac)\int \left(c\sec(e+fx)\tan^2(e+fx)-c\sec^2(e+fx)\tan^2(e+fx)\right)\,dx\right)\\
&= -\left((ac^2)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)+(ac^2)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\\
&= -\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}(ac^2)\int \sec(e+fx)\,dx\\
&\quad +\frac{(ac^2)\,\text{Subst}\left(\int x^2\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{ac^2\text{arctanh}(\sin(e+fx))}{2f}-\frac{ac^2\sec(e+fx)\tan(e+fx)}{2f}+\frac{ac^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. 2(61) = 122.

Time = 0.57 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.16

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx$$
$$= \frac{ac^{3/2}\left(-6\arcsin\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\sqrt{c-c\sec(e+fx)}+\sqrt{c}\sqrt{1+\sec(e+fx)}(2+\sec(e+fx)-5\sec^2(e+\ldots}{6f(-1+\sec(e+fx))\sqrt{1+\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a*c^(3/2)*(-6*ArcSin[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Sqrt[c -
c*Sec[e + f*x]] + Sqrt[c]*Sqrt[1 + Sec[e + f*x]]*(2 + Sec[e + f*x] - 5*Sec[
e + f*x]^2 + 2*Sec[e + f*x]^3))*Tan[e + f*x])/(6*f*(-1 + Sec[e + f*x])*Sqrt
[1 + Sec[e + f*x]])
```

Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-a\,c^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-a\,c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,c^2\tan(fx+e)+a\,c^2\ln(\sec(\ldots}{f}$
default	$\frac{-a\,c^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-a\,c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,c^2\tan(fx+e)+a\,c^2\ln(\sec(\ldots}{f}$
risch	$\frac{ia\,c^2\left(3\,\mathrm{e}^{5i(fx+e)}-6\,\mathrm{e}^{4i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}-2\right)}{3f\left(1+\mathrm{e}^{2i(fx+e)}\right)^3}+\frac{a\,c^2\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{2f}-\frac{a\,c^2\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{2f}$
parts	$\frac{a\,c^2\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a\,c^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{a\,c^2\tan(fx+e)}{f}-\frac{a\,c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\ldots\right.}{\ldots}$
norman	$\frac{\frac{a\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{8a\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{a\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{a\,c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a\,c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$
parallelrisch	$-\frac{a\,c^2\left(\frac{3(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\sin(3fx+3e)-3\,\mathrm{s}\ldots\right.}{3f(\cos(3fx+3e)+3\cos(fx+e))}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a*c^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-a*c^2*(1/2*sec(f*x+e)*tan(f*
x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))-a*c^2*tan(f*x+e)+a*c^2*ln(sec(f*x+e)+ta
n(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.69

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = \frac{3\,ac^2\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\,ac^2\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right)-2\left(2\,ac^2\cos\left(fx+\right.\right.}{12\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*a*c^2*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a*c^2*cos(f*x + e)^3
*log(-sin(f*x + e) + 1) - 2*(2*a*c^2*cos(f*x + e)^2 + 3*a*c^2*cos(f*x + e)
- 2*a*c^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = ac^2\left(\int \sec\left(e+fx\right)dx+\int\left(-\sec^2\left(e+fx\right)\right)\,dx+\int\left(-\sec^3\left(e+fx\right)\right)\,dx+\int \sec^4\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a*c**2*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integra
l(-sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.77

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = \frac{4\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ac^2+3\,ac^2\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right.\right.}{12\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2 + 3*a*c^2*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 12*a
*c^2*log(sec(f*x + e) + tan(f*x + e)) - 12*a*c^2*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = \frac{3\,ac^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,ac^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3\,ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+8\,ac^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 \cdots}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}}{6\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*a*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a*c^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1)) - 2*(3*a*c^2*tan(1/2*f*x + 1/2*e)^5 + 8*a*c^2*tan(1/2*f*x
 + 1/2*e)^3 - 3*a*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)
/f
```

Mupad [B] (verification not implemented)

Time = 15.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.87

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^2\,dx = \frac{a\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\frac{8\,a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}-a\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a*c^2*atanh(tan(e/2 + (f*x)/2)))/f - ((8*a*c^2*tan(e/2 + (f*x)/2)^3)/3 - a
*c^2*tan(e/2 + (f*x)/2) + a*c^2*tan(e/2 + (f*x)/2)^5)/(f*(3*tan(e/2 + (f*x)
/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

3.4 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 28, antiderivative size = 38

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \frac{a\text{carctanh}(\sin(e+fx))}{2f} - \frac{ac\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*a*c*arctanh(sin(f*x+e))/f-1/2*a*c*sec(f*x+e)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4043, 2691, 3855}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \frac{a\text{carctanh}(\sin(e+fx))}{2f} - \frac{ac\tan(e+fx)\sec(e+fx)}{2f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a*c*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left((ac)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)\\ &= -\frac{ac\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}(ac)\int \sec(e+fx)\,dx\\ &= \frac{ac\,\text{arctanh}(\sin(e+fx))}{2f}-\frac{ac\sec(e+fx)\tan(e+fx)}{2f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -ac\left(-\frac{\text{arctanh}(\sin(e+fx))}{2f}+\frac{\sec(e+fx)\tan(e+fx)}{2f}\right)$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
[Out] -(a*c*(-1/2*ArcTanh[Sin[e + f*x]]/f + (Sec[e + f*x]*Tan[e + f*x])/(2*f)))
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.53

method	result	size
derivativedivides	$\frac{-ac\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+ac\ln(\sec(fx+e)+\tan(fx+e))}{f}$	58
default	$\frac{-ac\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+ac\ln(\sec(fx+e)+\tan(fx+e))}{f}$	58
parts	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{ac\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$	60
parallelrisch	$\frac{\left((-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-2\sin(fx+e)\right)ac}{2f(1+\cos(2fx+2e))}$	80
risch	$\frac{iac(e^{3i(fx+e)}-e^{i(fx+e)})}{f\left(1+e^{2i(fx+e)}\right)^2}+\frac{ac\ln(e^{i(fx+e)}+i)}{2f}-\frac{ac\ln(e^{i(fx+e)}-i)}{2f}$	84
norman	$\frac{-\frac{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{ac\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{ac\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$	91

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*(-a*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a*c*ln(
sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\frac{ac\cos(fx+e)^2\log(\sin(fx+e)+1)-ac\cos(fx+e)^2\log(-\sin(fx+e)+1)-2\,ac\sin(fx+e)}{4\,f\cos(fx+e)^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fricas
")
[Out] 1/4*(a*c*cos(f*x + e)^2*log(sin(f*x + e) + 1) - a*c*cos(f*x + e)^2*log(-sin
(f*x + e) + 1) - 2*a*c*sin(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -ac\left(\int(-\sec(e+fx))\,dx+\int\sec^3(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
[Out] -a*c*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \frac{ac\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)+4\,ac\log(\sec(fx+e)+\tan(fx+e))}{4\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxima
")
[Out] 1/4*(a*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) + 4*a*c*log(sec(f*x + e) + tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \frac{ac\log(|\sin(fx+e)+1|)-ac\log(|\sin(fx+e)-1|)+\frac{2\,ac\sin(fx+e)}{\sin(fx+e)^2-1}}{4\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] 1/4*(a*c*log(abs(sin(f*x + e) + 1)) - a*c*log(abs(sin(f*x + e) - 1)) + 2*a*
c*sin(f*x + e)/(sin(f*x + e)^2 - 1))/f
```

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\frac{a\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}-\frac{a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+a\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x)))/cos(e + f*x),x)
[Out] (a*c*atanh(tan(e/2 + (f*x)/2)))/f - (a*c*tan(e/2 + (f*x)/2)^3 + a*c*tan(e/2
 + (f*x)/2))/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

3.5 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\,\mathrm{arctanh}(\sin(e+fx))}{cf} - \frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -a*arctanh(sin(f*x+e))/c/f-2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4042, 3855}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\,\mathrm{arctanh}(\sin(e+fx))}{cf} - \frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[e + f*x]])/(c*f)) - (2*a*Tan[e + f*x])/(f*(c - c*Sec[e + f
*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
```

```
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}-\frac{a\int\sec(e+fx)\,dx}{c}\\ &= -\frac{a\text{arctanh}(\sin(e+fx))}{cf}-\frac{2a\tan(e+fx)}{f(c-c\sec(e+fx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.83

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\left(-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)}{f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x]),x]
[Out] -((a*((-2*Cot[(e + f*x)/2])/f - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f
+ Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f))/c)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{2a\left(\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}\right)}{fc}$	50
default	$\frac{2a\left(\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}\right)}{fc}$	50
parallelrisch	$\frac{a\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cf}$	67
risch	$\frac{4ia}{fc\left(e^{i(fx+e)}-1\right)}+\frac{a\ln\left(e^{i(fx+e)}-i\right)}{cf}-\frac{a\ln\left(e^{i(fx+e)}+i\right)}{cf}$	68
norman	$\frac{-\frac{2a}{cf}+\frac{2a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{a\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$	100

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*a/c*(1/tan(1/2*f*x+1/2*e)-1/2*ln(tan(1/2*f*x+1/2*e)+1)+1/2*ln(tan(1/2*f
*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-a\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-4\,a\cos\left(fx+e\right)-4\,a}{2\,cf\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] -1/2*(a*log(sin(f*x + e) + 1)*sin(f*x + e) - a*log(-sin(f*x + e) + 1)*sin(f
*x + e) - 4*a*cos(f*x + e) - 4*a)/(c*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\left(\int \frac{\sec(e+fx)}{\sec(e+fx)-1}\,dx + \int \frac{\sec^2(e+fx)}{\sec(e+fx)-1}\,dx\right)}{c}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**2
/(sec(e + f*x) - 1), x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(43) = 86$.

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{a\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c} - \frac{\cos(fx+e)+1}{c\sin(fx+e)}\right) - \frac{a(\cos(fx+e)+1)}{c\sin(fx+e)}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxima
")
[Out] -(a*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a*(cos(f*x + e)
+ 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx = -\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c} - \frac{2a}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] -(a*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - a*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/c - 2*a/(c*tan(1/2*f*x + 1/2*e)))/f
```

Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx=-\frac{2\,a\,\left(\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
[Out] -(2*a*(atanh(tan(e/2 + (f*x)/2)) - cot(e/2 + (f*x)/2)))/(c*f)
```

3.6 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx=-\frac{(a+a\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}$$

```
[Out] -1/3*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx=-\frac{\tan(e+fx)(a\sec(e+fx)+a)}{3f(c-c\sec(e+fx))^2}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral}=-\frac{(a+a\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{a\cot^3\left(\frac{1}{2}(e+fx)\right)}{3c^2f}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^2,x]
[Out] -1/3*(a*Cot[(e + f*x)/2]^3)/(c^2*f)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{a}{3f\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	21
default	$-\frac{a}{3f\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	21
parallelrisch	$-\frac{a}{3f\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	21
risch	$\frac{2ia\left(3\,e^{2i(fx+e)}+1\right)}{3f\,c^2\left(e^{i(fx+e)}-1\right)^3}$	37
norman	$\frac{\frac{a}{3cf}-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3cf}}{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	61

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] -1/3/f*a/c^2/tan(1/2*f*x+1/2*e)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = \frac{a\cos(fx+e)^2+2\,a\cos(fx+e)+a}{3\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
[Out] 1/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)/((c^2*f*cos(f*x + e) - c^2*f)
*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = \frac{a\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{\sec^2(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx\right)}{c^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(35) = 70.

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{\frac{a\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3} - \frac{a\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}}{6f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c
^2*sin(f*x + e)^3) - a*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x
 + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{a}{3c^2f\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/3*a/(c^2*f*tan(1/2*f*x + 1/2*e)^3)
```

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx = -\frac{a\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{3\,c^2\,f}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
[Out] -(a*cot(e/2 + (f*x)/2)^3)/(3*c^2*f)
```

3.7 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3} dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3} dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{15cf(c-c\sec(e+fx))^2}$$

```
[Out] -1/5*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^3-1/15*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3} dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)}{15cf(c-c\sec(e+fx))^2} - \frac{\tan(e+fx)(a\sec(e+fx)+a)}{5f(c-c\sec(e+fx))^3}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -1/5*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^3) - ((a +
 a*Sec[e + f*x])*Tan[e + f*x])/(15*c*f*(c - c*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
```

```
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3}+\frac{\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx}{5c}\\ &= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{5f(c-c\sec(e+fx))^3}-\frac{(a+a\sec(e+fx))\tan(e+fx)}{15cf(c-c\sec(e+fx))^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx=-\frac{a(-4+\sec(e+fx))(1+\sec(e+fx))\tan(e+fx)}{15c^3f(-1+\sec(e+fx))^3}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^3,x]
[Out] -1/15*(a*(-4 + Sec[e + f*x])*(1 + Sec[e + f*x])*Tan[e + f*x])/(c^3*f*(-1 +
Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

method	result	size
parallelrisch	$\frac{a\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-5\right)}{30c^{3}f}$	36
derivativedivides	$\frac{a\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}\right)}{2f\,c^{3}}$	37
default	$\frac{a\left(-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}\right)}{2f\,c^{3}}$	37
risch	$\frac{2ia\left(15\,e^{4i(fx+e)}-15\,e^{3i(fx+e)}+25\,e^{2i(fx+e)}-5\,e^{i(fx+e)}+4\right)}{15f\,c^{3}\left(e^{i(fx+e)}-1\right)^{5}}$	70
norman	$\frac{-\frac{a}{10cf}+\frac{4a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{15cf}-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{6cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}$	81

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] 1/30*a*cot(1/2*f*x+1/2*e)^3*(3*cot(1/2*f*x+1/2*e)^2-5)/c^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = \frac{4\,a\cos\left(fx+e\right)^{3}+7\,a\cos\left(fx+e\right)^{2}+2\,a\cos\left(fx+e\right)-a}{15\left(c^{3}f\cos\left(fx+e\right)^{2}-2\,c^{3}f\cos\left(fx+e\right)+c^{3}f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
[Out] 1/15*(4*a*cos(f*x + e)^3 + 7*a*cos(f*x + e)^2 + 2*a*cos(f*x + e) - a)/((c^3
*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{a\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx\right)}{c^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e +
f*x) - 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e + f*x)*
*2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{\frac{a\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}+\frac{3a\left(\frac{5\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}}{60f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
[Out] -1/60*(a*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 3*a*(5*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)
^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = -\frac{5a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3a}{30c^3f\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="giac
")
[Out] -1/30*(5*a*tan(1/2*f*x + 1/2*e)^2 - 3*a)/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

Mupad [B] (verification not implemented)

Time = 14.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx = \frac{a\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-5\right)}{30\,c^3\,f}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
[Out] (a*cot(e/2 + (f*x)/2)^3*(3*cot(e/2 + (f*x)/2)^2 - 5))/(30*c^3*f)
```

3.8 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 116

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}$$

```
[Out] -1/7*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^4-2/35*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^3-2/105*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c^2
-c^2*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105f\left(c^2-c^2\sec(e+fx)\right)^2} - \frac{2\tan(e+fx)(a\sec(e+fx)+a)}{35cf(c-c\sec(e+fx))^3} - \frac{\tan(e+fx)(a\sec(e+fx)+a)}{7f(c-c\sec(e+fx))^4}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/7*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4) - (2*(a
 + a*Sec[e + f*x])*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3) - (2*(a +
a*Sec[e + f*x])*Tan[e + f*x])/(105*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4}+\frac{2\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx}{7c}\\
&= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4}\\
&\quad-\frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}+\frac{2\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}\,dx}{35c^2}\\
&= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{7f(c-c\sec(e+fx))^4}-\frac{2(a+a\sec(e+fx))\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}\\
&\quad-\frac{2(a+a\sec(e+fx))\tan(e+fx)}{105f\left(c^2-c^2\sec(e+fx)\right)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.95 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{a(1+\sec(e+fx))\left(23-10\sec(e+fx)+2\sec^2(e+fx)\right)\tan(e+fx)}{105c^4f(-1+\sec(e+fx))^4}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/105*(a*(1 + Sec[e + f*x])*(23 - 10*Sec[e + f*x] + 2*Sec[e + f*x]^2)*Tan[
e + f*x])/(c^4*f*(-1 + Sec[e + f*x])^4)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

method	result	size
parallelrisch	$-\frac{a\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^4-42\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2+35\right)}{420c^4f}$	49
derivativedivides	$\frac{a\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}\right)}{4f\,c^4}$	50
default	$\frac{a\left(-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{2}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}\right)}{4f\,c^4}$	50
risch	$\frac{2ia\left(105\,{\rm e}^{6i(fx+e)}-210\,{\rm e}^{5i(fx+e)}+455\,{\rm e}^{4i(fx+e)}-350\,{\rm e}^{3i(fx+e)}+273\,{\rm e}^{2i(fx+e)}-56\,{\rm e}^{i(fx+e)}+23\right)}{105f\,c^4\left({\rm e}^{i(fx+e)}-1\right)^7}$	92
norman	$\frac{\frac{a}{28cf}-\frac{19a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{140cf}+\frac{11a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{60cf}-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{12cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$	101

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/420*a*cot(1/2*f*x+1/2*e)^3*(15*cot(1/2*f*x+1/2*e)^4-42*cot(1/2*f*x+1/2*e
)^2+35)/c^4/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = \frac{23\,a\cos\left(fx+e\right)^4+36\,a\cos\left(fx+e\right)^3+5\,a\cos\left(fx+e\right)^2-6\,a\cos\left(fx+e\right)+2\,a}{105\,\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-c^4f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*(23*a*cos(f*x + e)^4 + 36*a*cos(f*x + e)^3 + 5*a*cos(f*x + e)^2 - 6*a
*cos(f*x + e) + 2*a)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^
4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = \frac{a\left(\int \frac{\sec\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx+\int \frac{\sec^2\left(e+fx\right)}{\sec^4\left(e+fx\right)-4\sec^3\left(e+fx\right)+6\sec^2\left(e+fx\right)-4\sec\left(e+fx\right)+1}\,dx\right)}{c^4}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f
*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**
4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = \frac{\frac{a\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7}+\frac{3\,a\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(f}{c^4\sin(fx+e)^7}}{840\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x + e
) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = -\frac{35\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-42\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+15\,a}{420\,c^4f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/420*(35*a*tan(1/2*f*x + 1/2*e)^4 - 42*a*tan(1/2*f*x + 1/2*e)^2 + 15*a)/(
c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B] (verification not implemented)

Time = 14.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx = \frac{a\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{10\,c^4\,f}-\frac{a\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{12\,c^4\,f}-\frac{a\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^7}{28\,c^4\,f}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

```
[Out] (a*cot(e/2 + (f*x)/2)^5)/(10*c^4*f) - (a*cot(e/2 + (f*x)/2)^3)/(12*c^4*f) -
 (a*cot(e/2 + (f*x)/2)^7)/(28*c^4*f)
```

3.9 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 158

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{105c^2f(c-c\sec(e+fx))^3} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{315cf\,(c^2-c^2\sec(e+fx))^2}$$

```
[Out] -1/9*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-1/21*(a+a*sec(f*x+e))
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4-2/105*(a+a*sec(f*x+e))*tan(f*x+e)/c^2/f/
(c-c*sec(f*x+e))^3-2/315*(a+a*sec(f*x+e))*tan(f*x+e)/c/f/(c^2-c^2*sec(f*x+e
))^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used

$= \{4036, 4035\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{2\tan(e+fx)(a\sec(e+fx)+a)}{315cf\left(c^2-c^2\sec(e+fx)\right)^2} - \frac{2\tan(e+fx)(a\sec(e+fx)+a)}{105c^2f(c-c\sec(e+fx))^3} - \frac{\tan(e+fx)(a\sec(e+fx)+a)}{21cf(c-c\sec(e+fx))^4} - \frac{\tan(e+fx)(a\sec(e+fx)+a)}{9f(c-c\sec(e+fx))^5}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/9*((a + a*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) - ((a +
 a*Sec[e + f*x])*Tan[e + f*x])/(21*c*f*(c - c*Sec[e + f*x])^4) - (2*(a + a*
Sec[e + f*x])*Tan[e + f*x])/(105*c^2*f*(c - c*Sec[e + f*x])^3) - (2*(a + a*
Sec[e + f*x])*Tan[e + f*x])/(315*c*f*(c^2 - c^2*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\text{integral} = -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} + \frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^4}\,dx}{3c}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^3}\,dx}{21c^2}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{105c^2f(c-c\sec(e+fx))^3} + \frac{2\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^2}dx}{105c^3}$$

$$= -\frac{(a+a\sec(e+fx))\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))\tan(e+fx)}{21cf(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{105c^2f(c-c\sec(e+fx))^3} - \frac{2(a+a\sec(e+fx))\tan(e+fx)}{315c^3f(c-c\sec(e+fx))^2}$$

Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{a(1+\sec(e+fx))\left(-58+33\sec(e+fx)-12\sec^2(e+fx)+2\sec^3(e+fx)\right)\tan(e+fx)}{315c^5f(-1+\sec(e+fx))^5}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/315*(a*(1 + Sec[e + f*x])*(-58 + 33*Sec[e + f*x] - 12*Sec[e + f*x]^2 + 2
*Sec[e + f*x]^3)*Tan[e + f*x])/(c^5*f*(-1 + Sec[e + f*x])^5)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

method	result
parallelrisch	$\frac{a\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3\left(35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^6-135\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^4+189\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2-105\right)}{2520c^5f}$
derivativedivides	$\frac{a\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}\right)}{8fc^5}$
default	$\frac{a\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{3}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}\right)}{8fc^5}$
risch	$\frac{2ia\left(315\,e^{8i(fx+e)}-945\,e^{7i(fx+e)}+2625\,e^{6i(fx+e)}-3465\,e^{5i(fx+e)}+3843\,e^{4i(fx+e)}-2247\,e^{3i(fx+e)}+1143\,e^{2i(fx+e)}-20\right.}{315fc^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{-\frac{a}{72cf}+\frac{17a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{252cf}-\frac{9a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{70cf}+\frac{7a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{60cf}-\frac{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{24cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2520*a*cot(1/2*f*x+1/2*e)^3*(35*cot(1/2*f*x+1/2*e)^6-135*cot(1/2*f*x+1/2*
e)^4+189*cot(1/2*f*x+1/2*e)^2-105)/c^5/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = \frac{58\,a\cos(fx+e)^5+83\,a\cos(fx+e)^4+4\,a\cos(fx+e)^3-11\,a\cos(fx+e)^2+8\,a\cos(fx+e)-2\,a}{315\left(c^5f\cos(fx+e)^4-4\,c^5f\cos(fx+e)^3+6\,c^5f\cos(fx+e)^2-4\,c^5f\cos(fx+e)+c^5f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="fric
as")
```

```
[Out] 1/315*(58*a*cos(f*x + e)^5 + 83*a*cos(f*x + e)^4 + 4*a*cos(f*x + e)^3 - 11*
a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - 2*a)/((c^5*f*cos(f*x + e)^4 - 4*c^5*f
*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*si
n(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{a\left(\int \frac{\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx+\int \frac{\sec^2(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-1}\right.}{c^5}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e +
 f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e +
f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(
e + f*x)**2 + 5*sec(e + f*x) - 1), x))/c**5
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.25

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{\frac{a\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{5a\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\sin}{(\cos(f}\right.}{c^5\sin(fx+e)^9}}{5040f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="maxi
ma")
```

```
[Out] -1/5040*(a*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*x
+ e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9)
 + 5*a*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(cos(f*x
 + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) +
1)^9/(c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.41

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx = -\frac{105\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-189\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+135\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-35\,a}{2520\,c^5 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^5,x, algorithm="giac
")
```

```
[Out] -1/2520*(105*a*tan(1/2*f*x + 1/2*e)^6 - 189*a*tan(1/2*f*x + 1/2*e)^4 + 135*
a*tan(1/2*f*x + 1/2*e)^2 - 35*a)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{a\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(35\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6-135\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+189\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-1\right.}{2520\,c^5\,f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)

[Out] (a*cos(e/2 + (f*x)/2)^3*(35*cos(e/2 + (f*x)/2)^6 - 105*sin(e/2 + (f*x)/2)^6
 + 189*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 135*cos(e/2 + (f*x)/2)^4
*sin(e/2 + (f*x)/2)^2))/(2520*c^5*f*sin(e/2 + (f*x)/2)^9)
```

3.10 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$

Optimal result

Integrand size = 32, antiderivative size = 171

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx &= \frac{9a^2c^5\text{arctanh}(\sin(e+fx))}{16f}-\frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{16f}\\ &\quad-\frac{3a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f}\\ &\quad+\frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{2f}-\frac{4a^2c^5\tan^5(e+fx)}{5f}-\frac{a^2c^5\tan^7(e+fx)}{7f}\end{aligned}$$

```
[Out] 9/16*a^2*c^5*arctanh(sin(f*x+e))/f-3/16*a^2*c^5*sec(f*x+e)*tan(f*x+e)/f-3/8
*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)/f+1/4*a^2*c^5*sec(f*x+e)*tan(f*x+e)^3/f+1/
2*a^2*c^5*sec(f*x+e)^3*tan(f*x+e)^3/f-4/5*a^2*c^5*tan(f*x+e)^5/f-1/7*a^2*c^
5*tan(f*x+e)^7/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used

= {4043, 2691, 3855, 2687, 30, 3853, 14}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$=\frac{9a^2c^5\text{arctanh}(\sin(e+fx))}{16f}-\frac{a^2c^5\tan^7(e+fx)}{7f}-\frac{4a^2c^5\tan^5(e+fx)}{5f}+\frac{a^2c^5\tan^3(e+fx)\sec^3(e+fx)}{2f}-\frac{3a^2c^5\tan(e+fx)\sec^3(e+fx)}{8f}+\frac{a^2c^5\tan^3(e+fx)\sec(e+fx)}{4f}-\frac{3a^2c^5\tan(e+fx)\sec(e+fx)}{16f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (9*a^2*c^5*ArcTanh[Sin[e + f*x]])/(16*f) - (3*a^2*c^5*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (3*a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^5*Sec
[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(
2*f) - (4*a^2*c^5*Tan[e + f*x]^5)/(5*f) - (a^2*c^5*Tan[e + f*x]^7)/(7*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\text{integral} = (a^2c^2)\int \left(c^3\sec(e+fx)\tan^4(e+fx) - 3c^3\sec^2(e+fx)\tan^4(e+fx) + 3c^3\sec^3(e+fx)\tan^4(e+fx) - c^3\sec^4(e+fx)\tan^4(e+fx)\right)\,dx$$

$$= (a^2c^5)\int \sec(e+fx)\tan^4(e+fx)\,dx - (a^2c^5)\int \sec^4(e+fx)\tan^4(e+fx)\,dx - (3a^2c^5)\int \sec^2(e+fx)\tan^4(e+fx)\,dx + (3a^2c^5)\int \sec^3(e+fx)\tan^4(e+fx)\,dx$$

$$= \frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{2f} - \frac{1}{4}(3a^2c^5)\int \sec(e+fx)\tan^2(e+fx)\,dx - \frac{1}{2}(3a^2c^5)\int \sec^3(e+fx)\tan^2(e+fx)\,dx - \frac{(a^2c^5)\operatorname{Subst}\left(\int x^4(1+x^2)\,dx, x, \tan(e+fx)\right)}{f} - \frac{(3a^2c^5)\operatorname{Subst}\left(\int x^4\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{8f} - \frac{3a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{2f} - \frac{3a^2c^5\tan^5(e+fx)}{5f} + \frac{1}{8}(3a^2c^5)\int \sec(e+fx)\,dx + \frac{1}{8}(3a^2c^5)\int \sec^3(e+fx)\,dx - \frac{(a^2c^5)\operatorname{Subst}\left(\int (x^4+x^6)\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{3a^2c^5\text{arctanh}(\sin(e+fx))}{8f} - \frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{16f} - \frac{3a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{2f} - \frac{4a^2c^5\tan^5(e+fx)}{5f} - \frac{a^2c^5\tan^7(e+fx)}{7f} + \frac{1}{16}(3a^2c^5)\int\sec(e+fx)\,dx$$

$$= \frac{9a^2c^5\text{arctanh}(\sin(e+fx))}{16f} - \frac{3a^2c^5\sec(e+fx)\tan(e+fx)}{16f} - \frac{3a^2c^5\sec^3(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^5\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^5\sec^3(e+fx)\tan^3(e+fx)}{2f} - \frac{4a^2c^5\tan^5(e+fx)}{5f} - \frac{a^2c^5\tan^7(e+fx)}{7f}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

$$\int\sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = \frac{a^2c^5(10080\text{arctanh}(\sin(e+fx)) - \sec^7(e+fx)(2520\sin(e+fx) - 455\sin(2(e+fx)) - 616\sin(3(e+fx)$$
$$17920f$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*c^5*(10080*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(2520*Sin[e + f*x] -
 455*Sin[2*(e + f*x)] - 616*Sin[3*(e + f*x)] + 2380*Sin[4*(e + f*x)] - 392*
Sin[5*(e + f*x)] + 245*Sin[6*(e + f*x)] + 184*Sin[7*(e + f*x)])))/(17920*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.88 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.22

method	result
risch	$\frac{ic^5a^2\left(245\,e^{13i(fx+e)}-1680\,e^{12i(fx+e)}+2380\,e^{11i(fx+e)}-4480\,e^{10i(fx+e)}-455\,e^{9i(fx+e)}-3920\,e^{8i(fx+e)}-8960\,e^{6i(fx+\right.}{280f\left(1+e^{2i(fx+e)}\right)^7}$
norman	$\frac{\frac{9c^5a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}-\frac{15c^5a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2f}+\frac{849c^5a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{40f}-\frac{1152c^5a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{35f}+\frac{1199c^5a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{40f}+\frac{15c^5a^2}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^7}$
parallelrisch	$-\frac{9a^2c^5\left(\frac{(\cos(7fx+7e)+7\cos(5fx+5e)+21\cos(3fx+3e)+35\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{(-\cos(7fx+7e)-7\cos(5fx+5e)-21}{}\right.}{f(\cos(7fx+7e)}$
derivativedivides	$\frac{c^5a^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)+3c^5a^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\right.}{}$
default	$\frac{c^5a^2\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)+3c^5a^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\right.}{}$
parts	$\frac{c^5a^2\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{c^5a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}-\frac{3c^5a^2\tan(fx+e)}{f}-\frac{5c^5}{}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOSE)

[Out] 1/280*I*c^5*a^2*(245*exp(13*I*(f*x+e))-1680*exp(12*I*(f*x+e))+2380*exp(11*I*(f*x+e))-4480*exp(10*I*(f*x+e))-455*exp(9*I*(f*x+e))-3920*exp(8*I*(f*x+e))-8960*exp(6*I*(f*x+e))+455*exp(5*I*(f*x+e))-3248*exp(4*I*(f*x+e))-2380*exp(3*I*(f*x+e))-896*exp(2*I*(f*x+e))-245*exp(I*(f*x+e))-368)/f/(1+exp(2*I*(f*x+e)))^7+9/16*c^5*a^2/f*ln(exp(I*(f*x+e))+I)-9/16*c^5*a^2/f*ln(exp(I*(f*x+e))-I)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$=\frac{315\,a^2c^5\cos\left(fx+e\right)^7\log\left(\sin\left(fx+e\right)+1\right)-315\,a^2c^5\cos\left(fx+e\right)^7\log\left(-\sin\left(fx+e\right)+1\right)-2\left(368\,a^2c^5\right.}{}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="fricas")

[Out] 1/1120*(315*a^2*c^5*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 315*a^2*c^5*cos(f*x + e)^7*log(-sin(f*x + e) + 1) - 2*(368*a^2*c^5*cos(f*x + e)^6 + 245*a^2*c^5*cos(f*x + e)^5 - 656*a^2*c^5*cos(f*x + e)^4 + 350*a^2*c^5*cos(f*x + e)^3 + 208*a^2*c^5*cos(f*x + e)^2 - 280*a^2*c^5*cos(f*x + e) + 80*a^2*c^5)*sin(f*x + e))/(f*cos(f*x + e)^7)

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = -a^2c^5\left(\int(-\sec(e+fx))\,dx+\int 3\sec^2(e+fx)\,dx+\int(-\sec^3(e+fx))\,dx+\int(-5\sec^4(e+fx))\,dx+\int 5\sec^5(e+fx)\,dx+\int\sec^6(e+fx)\,dx+\int(-3\sec^7(e+fx))\,dx+\int\sec^8(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*c**5*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**2, x) + I
ntegral(-sec(e + f*x)**3, x) + Integral(-5*sec(e + f*x)**4, x) + Integral(5
*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x) + Integral(-3*sec(e + f
*x)**7, x) + Integral(sec(e + f*x)**8, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(157) = 314$.

Time = 0.21 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx = -\frac{96\left(5\tan(fx+e)^7+21\tan(fx+e)^5+35\tan(fx+e)^3+35\tan(fx+e)\right)a^2c^5+224\left(3\tan(fx+e)\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*
tan(f*x + e))*a^2*c^5 + 224*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(
f*x + e))*a^2*c^5 - 5600*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^5 + 105*a^
2*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x
 + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) +
1) + 15*log(sin(f*x + e) - 1)) - 1050*a^2*c^5*(2*(3*sin(f*x + e)^3 - 5*sin(
f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1)
 + 3*log(sin(f*x + e) - 1)) + 840*a^2*c^5*(2*sin(f*x + e)/(sin(f*x + e)^2 -
 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3360*a^2*c^5*log(sec
(f*x + e) + tan(f*x + e)) + 10080*a^2*c^5*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$= \frac{315\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-315\,a^2c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(315\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-2100\,}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/560*(315*a^2*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 315*a^2*c^5*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(315*a^2*c^5*tan(1/2*f*x + 1/2*e)^13 - 2100
*a^2*c^5*tan(1/2*f*x + 1/2*e)^11 - 8393*a^2*c^5*tan(1/2*f*x + 1/2*e)^9 + 92
16*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 - 5943*a^2*c^5*tan(1/2*f*x + 1/2*e)^5 + 2
100*a^2*c^5*tan(1/2*f*x + 1/2*e)^3 - 315*a^2*c^5*tan(1/2*f*x + 1/2*e))/(tan
(1/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B] (verification not implemented)

Time = 17.75 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.47

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5\,dx$$
$$= \frac{-\frac{9\,a^2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}+\frac{15\,a^2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{2}+\frac{1199\,a^2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{40}-\frac{1152\,a^2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{35}+\frac{849\,a^2\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{40}-}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-21\,\mathrm{ta}\right.}$$
$$+\frac{9\,a^2\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5)/cos(e + f*x),x)
```

```
[Out] ((849*a^2*c^5*tan(e/2 + (f*x)/2)^5)/40 - (15*a^2*c^5*tan(e/2 + (f*x)/2)^3)/
2 - (1152*a^2*c^5*tan(e/2 + (f*x)/2)^7)/35 + (1199*a^2*c^5*tan(e/2 + (f*x)/
2)^9)/40 + (15*a^2*c^5*tan(e/2 + (f*x)/2)^11)/2 - (9*a^2*c^5*tan(e/2 + (f*x
)/2)^13)/8 + (9*a^2*c^5*tan(e/2 + (f*x)/2))/8)/(f*(7*tan(e/2 + (f*x)/2)^2 -
 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^
8 + 21*tan(e/2 + (f*x)/2)^10 - 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)
^14 - 1)) + (9*a^2*c^5*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

3.11 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 32, antiderivative size = 150

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx &= \frac{7a^2c^4\operatorname{arctanh}(\sin(e+fx))}{16f}-\frac{5a^2c^4\sec(e+fx)\tan(e+fx)}{16f}\\ &\quad-\frac{a^2c^4\sec^3(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f}\\ &\quad+\frac{a^2c^4\sec^3(e+fx)\tan^3(e+fx)}{6f}-\frac{2a^2c^4\tan^5(e+fx)}{5f}\end{aligned}$$

```
[Out] 7/16*a^2*c^4*arctanh(sin(f*x+e))/f-5/16*a^2*c^4*sec(f*x+e)*tan(f*x+e)/f-1/8
*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)/f+1/4*a^2*c^4*sec(f*x+e)*tan(f*x+e)^3/f+1/
6*a^2*c^4*sec(f*x+e)^3*tan(f*x+e)^3/f-2/5*a^2*c^4*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

= {4043, 2691, 3855, 2687, 30, 3853}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{7a^2c^4\text{arctanh}(\sin(e+fx))}{16f} - \frac{2a^2c^4\tan^5(e+fx)}{5f} + \frac{a^2c^4\tan^3(e+fx)\sec^3(e+fx)}{6f} - \frac{a^2c^4\tan(e+fx)\sec^3(e+fx)}{8f} + \frac{a^2c^4\tan^3(e+fx)\sec(e+fx)}{4f} - \frac{5a^2c^4\tan(e+fx)\sec(e+fx)}{16f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (7*a^2*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^2*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) - (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x])/(8*f) + (a^2*c^4*Sec[e
 + f*x]*Tan[e + f*x]^3)/(4*f) + (a^2*c^4*Sec[e + f*x]^3*Tan[e + f*x]^3)/(6*
f) - (2*a^2*c^4*Tan[e + f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\text{integral} = (a^2c^2)\int \left(c^2\sec(e+fx)\tan^4(e+fx) - 2c^2\sec^2(e+fx)\tan^4(e+fx) + c^2\sec^3(e+fx)\tan^4(e+fx)\right)dx$$

$$= (a^2c^4)\int \sec(e+fx)\tan^4(e+fx)\,dx + (a^2c^4)\int \sec^3(e+fx)\tan^4(e+fx)\,dx - (2a^2c^4)\int \sec^2(e+fx)\tan^4(e+fx)\,dx$$

$$= \frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^4\sec^3(e+fx)\tan^3(e+fx)}{6f} - \frac{1}{2}(a^2c^4)\int \sec^3(e+fx)\tan^2(e+fx)\,dx - \frac{1}{4}(3a^2c^4)\int \sec(e+fx)\tan^2(e+fx)\,dx - \frac{(2a^2c^4)\,\text{Subst}\left(\int x^4\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{3a^2c^4\sec(e+fx)\tan(e+fx)}{8f} - \frac{a^2c^4\sec^3(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^4\sec^3(e+fx)\tan^3(e+fx)}{6f} - \frac{2a^2c^4\tan^5(e+fx)}{5f} + \frac{1}{8}(a^2c^4)\int \sec^3(e+fx)\,dx + \frac{1}{8}(3a^2c^4)\int \sec(e+fx)\,dx$$

$$= \frac{3a^2c^4\text{arctanh}(\sin(e+fx))}{8f} - \frac{5a^2c^4\sec(e+fx)\tan(e+fx)}{16f} - \frac{a^2c^4\sec^3(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^4\sec^3(e+fx)\tan^3(e+fx)}{6f} - \frac{2a^2c^4\tan^5(e+fx)}{5f} + \frac{1}{16}(a^2c^4)\int \sec(e+fx)\,dx$$

$$= \frac{7a^2c^4\text{arctanh}(\sin(e+fx))}{16f} - \frac{5a^2c^4\sec(e+fx)\tan(e+fx)}{16f} - \frac{a^2c^4\sec^3(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^4\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^2c^4\sec^3(e+fx)\tan^3(e+fx)}{6f} - \frac{2a^2c^4\tan^5(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.61

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$=\frac{a^2c^4(1680\text{arctanh}(\sin(e+fx))+\sec^6(e+fx)(330\sin(e+fx)-240\sin(2(e+fx))-445\sin(3(e+fx)}{3840f}$$

[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4,x]

[Out] (a^2*c^4*(1680*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^6*(330*Sin[e + f*x] - 2
40*Sin[2*(e + f*x)] - 445*Sin[3*(e + f*x)] + 192*Sin[4*(e + f*x)] - 135*Sin
[5*(e + f*x)] - 48*Sin[6*(e + f*x)])))/(3840*f)

Maple [A] (verified)

Time = 6.70 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.30

method	result
norman	$\frac{-\frac{7c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{119c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{24f}-\frac{231c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{20f}+\frac{281c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{20f}+\frac{119c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{24f}-\frac{7c^4a^2}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$
risch	$\frac{ic^4a^2\left(135\,e^{11i(fx+e)}-480\,e^{10i(fx+e)}+445\,e^{9i(fx+e)}-480\,e^{8i(fx+e)}-330\,e^{7i(fx+e)}-960\,e^{6i(fx+e)}+330\,e^{5i(fx+e)}-960\,e\right.}{120f\left(1+e^{2i(fx+e)}\right)^6}$
parallelrisch	$-\frac{2a^2c^4\left(\frac{7\left(5+\frac{\cos(6fx+6e)}{2}+3\cos(4fx+4e)+\frac{15\cos(2fx+2e)}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{7\left(-5-\frac{15\cos(2fx+2e)}{2}-3\cos(4fx+4e)-\text{co}\right.}{16}\right.}{f(6\cos(4fx+4e)+10+15}$
derivativedivides	$\frac{c^4a^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+2c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}\right.}{}$
default	$\frac{c^4a^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+2c^4a^2\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}\right.}{}$
parts	$\frac{c^4a^2\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{c^4a^2\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan\right.}{16}}{f}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)

[Out] (-7/8*c^4*a^2/f*tan(1/2*f*x+1/2*e)+119/24*c^4*a^2/f*tan(1/2*f*x+1/2*e)^3-23
1/20*c^4*a^2/f*tan(1/2*f*x+1/2*e)^5+281/20*c^4*a^2/f*tan(1/2*f*x+1/2*e)^7+1
19/24*c^4*a^2/f*tan(1/2*f*x+1/2*e)^9-7/8*c^4*a^2/f*tan(1/2*f*x+1/2*e)^11)/(
tan(1/2*f*x+1/2*e)^2-1)^6-7/16*c^4*a^2/f*ln(tan(1/2*f*x+1/2*e)-1)+7/16*c^4*
a^2/f*ln(tan(1/2*f*x+1/2*e)+1)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{105\,a^2c^4\cos(fx+e)^6\log(\sin(fx+e)+1) - 105\,a^2c^4\cos(fx+e)^6\log(-\sin(fx+e)+1) - 2\,(96\,a^2c^4\cos\ldots}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*a^2*c^4*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 105*a^2*c^4*cos(f
*x + e)^6*log(-sin(f*x + e) + 1) - 2*(96*a^2*c^4*cos(f*x + e)^5 + 135*a^2*c
^4*cos(f*x + e)^4 - 192*a^2*c^4*cos(f*x + e)^3 + 10*a^2*c^4*cos(f*x + e)^2
+ 96*a^2*c^4*cos(f*x + e) - 40*a^2*c^4)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx = a^2c^4\left(\int \sec(e+fx)\,dx + \int \left(-2\sec^2(e+fx)\right)dx + \int \left(-\sec^3(e+fx)\right)dx + \int 4\sec^4(e+fx)\,dx + \int \left(-\sec^5(e+fx)\right)dx + \int \left(-2\sec^6(e+fx)\right)dx + \int \sec^7(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*c**4*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + In
tegral(-sec(e + f*x)**3, x) + Integral(4*sec(e + f*x)**4, x) + Integral(-se
c(e + f*x)**5, x) + Integral(-2*sec(e + f*x)**6, x) + Integral(sec(e + f*x)
**7, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(138) = 276$.

Time = 0.22 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.14

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx =$$

$$-\frac{64\left(3\tan\left(fx+e\right)^5+10\tan\left(fx+e\right)^3+15\tan\left(fx+e\right)\right)a^2c^4-640\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^2c^4\cdots}{\cdots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] -1/480*(64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^4
 - 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^4 + 5*a^2*c^4*(2*(15*sin(f*x
 + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x
+ e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x
+ e) - 1)) - 30*a^2*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e
)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e)
- 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e)
 + 1) + log(sin(f*x + e) - 1)) - 480*a^2*c^4*log(sec(f*x + e) + tan(f*x + e
)) + 960*a^2*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$

$$= \frac{105\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,a^2c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,a^2c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}-595\,a^2\cdots\right)}{\cdots}}{240\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*a^2*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a^2*c^4*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(105*a^2*c^4*tan(1/2*f*x + 1/2*e)^11 - 595*
a^2*c^4*tan(1/2*f*x + 1/2*e)^9 - 1686*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 1386
*a^2*c^4*tan(1/2*f*x + 1/2*e)^5 - 595*a^2*c^4*tan(1/2*f*x + 1/2*e)^3 + 105*
a^2*c^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.46

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4\,dx$$
$$= \frac{-\frac{7\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}+\frac{119\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{281\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{20}-\frac{231\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{20}+\frac{119\,a^2\,c^4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24}-\frac{7\,a}{}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-6\tan\left(\right.\right.}$$
$$+\frac{7\,a^2\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] ((119*a^2*c^4*tan(e/2 + (f*x)/2)^3)/24 - (231*a^2*c^4*tan(e/2 + (f*x)/2)^5)
/20 + (281*a^2*c^4*tan(e/2 + (f*x)/2)^7)/20 + (119*a^2*c^4*tan(e/2 + (f*x)/
2)^9)/24 - (7*a^2*c^4*tan(e/2 + (f*x)/2)^11)/8 - (7*a^2*c^4*tan(e/2 + (f*x)
/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 +
 (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 +
 (f*x)/2)^12 + 1)) + (7*a^2*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f)
```

3.12 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 32, antiderivative size = 94

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \frac{3a^2c^3\operatorname{arctanh}(\sin(e+fx))}{8f} - \frac{3a^2c^3\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{a^2c^3\tan^5(e+fx)}{5f}$$

```
[Out] 3/8*a^2*c^3*arctanh(sin(f*x+e))/f-3/8*a^2*c^3*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^2*c^3*sec(f*x+e)*tan(f*x+e)^3/f-1/5*a^2*c^3*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4043, 2691, 3855, 2687, 30}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx = \frac{3a^2c^3\operatorname{arctanh}(\sin(e+fx))}{8f} - \frac{a^2c^3\tan^5(e+fx)}{5f} + \frac{a^2c^3\tan^3(e+fx)\sec(e+fx)}{4f} - \frac{3a^2c^3\tan(e+fx)\sec(e+fx)}{8f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (3*a^2*c^3*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^3*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) - (a^2*c^3*Tan[e +
 f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a^2c^2\right)\int \left(c\sec(e+fx)\tan^4(e+fx) - c\sec^2(e+fx)\tan^4(e+fx)\right)\,dx\\ &= \left(a^2c^3\right)\int \sec(e+fx)\tan^4(e+fx)\,dx - \left(a^2c^3\right)\int \sec^2(e+fx)\tan^4(e+fx)\,dx\\ &= \frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{1}{4}\left(3a^2c^3\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\\ &\quad - \frac{\left(a^2c^3\right)\text{Subst}\left(\int x^4\,dx, x, \tan(e+fx)\right)}{f}\end{aligned}$$

$$= -\frac{3a^2c^3\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{a^2c^3\tan^5(e+fx)}{5f} + \frac{1}{8}(3a^2c^3)\int \sec(e+fx)\,dx$$

$$= \frac{3a^2c^3\text{arctanh}(\sin(e+fx))}{8f} - \frac{3a^2c^3\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^3\sec(e+fx)\tan^3(e+fx)}{4f} - \frac{a^2c^3\tan^5(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= \frac{a^2c^3(120\text{arctanh}(\sin(e+fx)) - \sec^5(e+fx)(40\sin(e+fx) + 10\sin(2(e+fx)) - 20\sin(3(e+fx)) + 2}{320f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*(120*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^5*(40*Sin[e + f*x] + 10*
Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] + 25*Sin[4*(e + f*x)] + 4*Sin[5*(e +
 f*x)])))/(320*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

method	result
risch	$\frac{ia^2c^3(25\,\mathrm{e}^{9i(fx+e)}-40\,\mathrm{e}^{8i(fx+e)}+10\,\mathrm{e}^{7i(fx+e)}-80\,\mathrm{e}^{4i(fx+e)}-10\,\mathrm{e}^{3i(fx+e)}-25\,\mathrm{e}^{i(fx+e)}-8)}{20f(1+\mathrm{e}^{2i(fx+e)})^5}-\frac{3a^2c^3\ln(\mathrm{e}^{i(fx+e)}-i)}{8f}+$
parts	$\frac{a^2c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{a^2c^3\tan(fx+e)}{f}-\frac{a^2c^3\sec(fx+e)\,\mathrm{ta}}{f}$
norman	$\frac{\frac{3a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{7a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2f}+\frac{32a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5f}+\frac{7a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{2f}-\frac{3a^2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}-\frac{3a^2c^3\ln\left(\tan\left(\right.\right.}{8f}$
parallelrisch	$-\frac{\left(\left(\frac{15\cos(fx+e)}{2}+\frac{15\cos(3fx+3e)}{4}+\frac{3\cos(5fx+5e)}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{15\cos(fx+e)}{2}-\frac{15\cos(3fx+3e)}{4}-\frac{3\cos(5fx+5e)}{4}\right)\right.}{2f(\cos(5fx+5e)+5\cos(3fx+3e)}$
derivativedivides	$\frac{a^2c^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)+a^2c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}\right.}{}$
default	$\frac{a^2c^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)+a^2c^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/20*I*a^2*c^3*(25*exp(9*I*(f*x+e))-40*exp(8*I*(f*x+e))+10*exp(7*I*(f*x+e))
-80*exp(4*I*(f*x+e))-10*exp(3*I*(f*x+e))-25*exp(I*(f*x+e))-8)/f/(1+exp(2*I*
(f*x+e)))^5-3/8*a^2*c^3/f*ln(exp(I*(f*x+e))-I)+3/8*a^2*c^3/f*ln(exp(I*(f*x+
e))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$=\frac{15\,a^2c^3\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^2c^3\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)-2\left(8\,a^2c^3\cos\left(\right.\right.}{80\,f\cos\left(fx\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*a^2*c^3*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^2*c^3*cos(f*x
+ e)^5*log(-sin(f*x + e) + 1) - 2*(8*a^2*c^3*cos(f*x + e)^4 + 25*a^2*c^3*co
s(f*x + e)^3 - 16*a^2*c^3*cos(f*x + e)^2 - 10*a^2*c^3*cos(f*x + e) + 8*a^2*
c^3)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= -a^2c^3\left(\int(-\sec(e+fx))\,dx + \int\sec^2(e+fx)\,dx + \int 2\sec^3(e+fx)\,dx + \int(-2\sec^4(e+fx))\,dx + \int(-\sec^5(e+fx))\,dx + \int\sec^6(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*c**3*(Integral(-sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Int
egral(2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(-s
ec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(86) = 172$.

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.41

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx =$$
$$-\frac{16\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^2c^3-160\left(\tan(fx+e)^3+3\tan(fx+e)\right)a}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*c^3
 - 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3 + 15*a^2*c^3*(2*(3*sin(f*x
 + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(s
in(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^3*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 240*
a^2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 240*a^2*c^3*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.69

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= \frac{15\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^2c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-70\,a^2c^3\tan\left(\right.\right.}{}}{40\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/40*(15*a^2*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^2*c^3*log(abs(ta
n(1/2*f*x + 1/2*e) - 1)) - 2*(15*a^2*c^3*tan(1/2*f*x + 1/2*e)^9 - 70*a^2*c^
3*tan(1/2*f*x + 1/2*e)^7 - 128*a^2*c^3*tan(1/2*f*x + 1/2*e)^5 + 70*a^2*c^3*
tan(1/2*f*x + 1/2*e)^3 - 15*a^2*c^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/
2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 18.31 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3\,dx$$
$$= \frac{-\frac{3\,a^2\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}+\frac{7\,a^2\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2}+\frac{32\,a^2\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}-\frac{7\,a^2\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2}+\frac{3\,a^2\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$
$$+\frac{3\,a^2\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] ((32*a^2*c^3*tan(e/2 + (f*x)/2)^5)/5 - (7*a^2*c^3*tan(e/2 + (f*x)/2)^3)/2 +
 (7*a^2*c^3*tan(e/2 + (f*x)/2)^7)/2 - (3*a^2*c^3*tan(e/2 + (f*x)/2)^9)/4 +
(3*a^2*c^3*tan(e/2 + (f*x)/2))/4)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 +
 (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 +
(f*x)/2)^10 - 1)) + (3*a^2*c^3*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

3.13 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 73

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{3a^2c^2\text{arctanh}(\sin(e+fx))}{8f} - \frac{3a^2c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f}$$

```
[Out] 3/8*a^2*c^2*arctanh(sin(f*x+e))/f-3/8*a^2*c^2*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^2*c^2*sec(f*x+e)*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4043, 2691, 3855}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = \frac{3a^2c^2\text{arctanh}(\sin(e+fx))}{8f} + \frac{a^2c^2\tan^3(e+fx)\sec(e+fx)}{4f} - \frac{3a^2c^2\tan(e+fx)\sec(e+fx)}{8f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (3*a^2*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^2*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^2*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \sec(e+fx)\tan^4(e+fx)\,dx\\
&= \frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{1}{4}\left(3a^2c^2\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\\
&= -\frac{3a^2c^2\sec(e+fx)\tan(e+fx)}{8f}\\
&\quad+\frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f}+\frac{1}{8}\left(3a^2c^2\right)\int \sec(e+fx)\,dx\\
&= \frac{3a^2c^2\text{arctanh}(\sin(e+fx))}{8f}-\frac{3a^2c^2\sec(e+fx)\tan(e+fx)}{8f}+\frac{a^2c^2\sec(e+fx)\tan^3(e+fx)}{4f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx = a^2c^2\left(\frac{3\text{arctanh}(\sin(e+fx))}{8f}+\frac{3\sec(e+fx)\tan(e+fx)}{8f}-\frac{3\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{\sec(e+fx)\tan^3(e+fx)}{f}\right)$$

[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2,x]

[Out] a^2*c^2*((3*ArcTanh[Sin[e + f*x]])/(8*f) + (3*Sec[e + f*x]*Tan[e + f*x])/(8*f) - (3*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (Sec[e + f*x]*Tan[e + f*x]^3)/f)

Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

method	result
parts	$\frac{a^2c^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{a^2c^2\sec(fx+e)\tan(fx+e)}{f}$
derivativedivides	$\frac{a^2c^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-2a^2c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e}{}\right.}{f}$
default	$\frac{a^2c^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-2a^2c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e}{}\right.}{f}$
risch	$\frac{ia^2c^2\left(5\,e^{7i(fx+e)}-3\,e^{5i(fx+e)}+3\,e^{3i(fx+e)}-5\,e^{i(fx+e)}\right)}{4f\left(1+e^{2i(fx+e)}\right)^4}-\frac{3a^2c^2\ln\left(e^{i(fx+e)}-i\right)}{8f}+\frac{3a^2c^2\ln\left(e^{i(fx+e)}+i\right)}{8f}$
parallelrisch	$-\frac{3a^2\left(\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\cos(2fx+2e)-\frac{\cos(4fx+4e)}{4}-\frac{3}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f(3+\cos(4fx+4e)+4\cos(2fx+2e))}$
norman	$\frac{-\frac{3a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{11a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{4f}+\frac{11a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}-\frac{3a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{3a^2c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f}+$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] a^2*c^2/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-a^2*c^2*sec(f*x+e)*tan(f*x+e)/f

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$$
$$=\frac{3\,a^2c^2\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\,a^2c^2\cos\left(fx+e\right)^4\log\left(-\sin\left(fx+e\right)+1\right)-2\left(5\,a^2c^2\cos\left(fx\right.\right.}{16\,f\cos\left(fx+e\right)^4}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
[Out] 1/16*(3*a^2*c^2*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c^2*cos(f*x +
e)^4*log(-sin(f*x + e) + 1) - 2*(5*a^2*c^2*cos(f*x + e)^2 - 2*a^2*c^2)*sin(
f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$$
$$=a^2c^2\left(\int\sec\left(e+fx\right)dx+\int\left(-2\sec^3\left(e+fx\right)\right)\,dx+\int\sec^5\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**2,x)
[Out] a**2*c**2*(Integral(sec(e + f*x), x) + Integral(-2*sec(e + f*x)**3, x) + In
tegral(sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(67) = 134.

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.05

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx =$$
$$-\frac{a^2c^2\left(\frac{2\left(3\,\sin(fx+e)^3-5\,\sin(fx+e)\right)}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\log\left(\sin\left(fx+e\right)-1\right)\right)-8\,a^2c^2\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-}\right.}{16\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
[Out] -1/16*(a^2*c^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*s
in(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 8
*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log
(sin(f*x + e) - 1)) - 16*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$$
$$=\frac{3\,a^2c^2\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-3\,a^2c^2\log\left(\left|\sin\left(fx+e\right)-1\right|\right)+\frac{2\left(5\,a^2c^2\sin(fx+e)^3-3\,a^2c^2\sin(fx+e)\right)}{\left(\sin(fx+e)^2-1\right)^2}}{16\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/16*(3*a^2*c^2*log(abs(sin(f*x + e) + 1)) - 3*a^2*c^2*log(abs(sin(f*x + e)
 - 1)) + 2*(5*a^2*c^2*sin(f*x + e)^3 - 3*a^2*c^2*sin(f*x + e))/(sin(f*x + e
)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 16.97 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.12

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2\,dx$$
$$=\frac{-\frac{3\,a^2\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}+\frac{11\,a^2\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{4}+\frac{11\,a^2\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{4}-\frac{3\,a^2\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}+\frac{3\,a^2\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] ((11*a^2*c^2*tan(e/2 + (f*x)/2)^3)/4 + (11*a^2*c^2*tan(e/2 + (f*x)/2)^5)/4
- (3*a^2*c^2*tan(e/2 + (f*x)/2)^7)/4 - (3*a^2*c^2*tan(e/2 + (f*x)/2))/4)/(f
*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6
+ tan(e/2 + (f*x)/2)^8 + 1)) + (3*a^2*c^2*atanh(tan(e/2 + (f*x)/2)))/(4*f)
```

3.14 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 61

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\sin(e+fx))}{2f}-\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f}-\frac{a^2c\tan^3(e+fx)}{3f}$$

```
[Out] 1/2*a^2*c*arctanh(sin(f*x+e))/f-1/2*a^2*c*sec(f*x+e)*tan(f*x+e)/f-1/3*a^2*c
*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4043, 2691, 3855, 2687, 30}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\sin(e+fx))}{2f}-\frac{a^2c\tan^3(e+fx)}{3f}-\frac{a^2c\tan(e+fx)\sec(e+fx)}{2f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(2*f) - (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(2*
f) - (a^2*c*Tan[e + f*x]^3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((ac)\int \left(a\sec(e+fx)\tan^2(e+fx)+a\sec^2(e+fx)\tan^2(e+fx)\right)\,dx\right)\\
&= -\left(\left(a^2c\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^2c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\\
&= -\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\left(a^2c\right)\int \sec(e+fx)\,dx\\
&\quad -\frac{\left(a^2c\right)\text{Subst}\left(\int x^2\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a^2c\text{arctanh}(\sin(e+fx))}{2f}-\frac{a^2c\sec(e+fx)\tan(e+fx)}{2f}-\frac{a^2c\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c(3\text{arctanh}(\sin(e+fx))-3\sec(e+fx)\tan(e+fx)-2\tan^3(e+fx))}{6f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
[Out] (a^2*c*(3*ArcTanh[Sin[e + f*x]] - 3*Sec[e + f*x]*Tan[e + f*x] - 2*Tan[e + f
*x]^3))/(6*f)
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{a^2c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^2c\tan(fx+e)+a^2c\ln(\sec(fx+}{f}$
default	$\frac{a^2c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)-a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+a^2c\tan(fx+e)+a^2c\ln(\sec(fx+}{f}$
risch	$\frac{ia^2c(3\,\mathrm{e}^{5i(fx+e)}+6\,\mathrm{e}^{4i(fx+e)}-3\,\mathrm{e}^{i(fx+e)}+2)}{3f(1+\mathrm{e}^{2i(fx+e)})^3}+\frac{a^2c\ln(\mathrm{e}^{i(fx+e)}+i)}{2f}-\frac{a^2c\ln(\mathrm{e}^{i(fx+e)}-i)}{2f}$
parts	$\frac{a^2c\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{a^2c\tan(fx+e)}{f}-\frac{a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{a^2c\left(-\frac{2}{3}-\right.}{}$
norman	$\frac{\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{8a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$
parallelrisch	$\frac{a^2\left(\frac{3(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\sin(3fx+3e)-3\sin(f\right.}{3f(\cos(3fx+3e)+3\cos(fx+e))}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*(a^2*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)-a^2*c*(1/2*sec(f*x+e)*tan(f*x
+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^2*c*tan(f*x+e)+a^2*c*ln(sec(f*x+e)+tan
(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.69

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{3\,a^2c\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right) - 3\,a^2c\cos\left(fx+e\right)^3\log\left(-\sin\left(fx+e\right)+1\right) + 2\left(2\,a^2c\cos\left(fx+\right.\right.}{12\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fric
as")
[Out] 1/12*(3*a^2*c*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^3
*log(-sin(f*x + e) + 1) + 2*(2*a^2*c*cos(f*x + e)^2 - 3*a^2*c*cos(f*x + e)
- 2*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= -a^2c\left(\int\left(-\sec\left(e+fx\right)\right)\,dx + \int\left(-\sec^2\left(e+fx\right)\right)\,dx + \int\sec^3\left(e+fx\right)dx + \int\sec^4\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
[Out] -a**2*c*(Integral(-sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Integ
ral(sec(e + f*x)**3, x) + Integral(sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.77

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx =$$
$$-\frac{4\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c - 3\,a^2c\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1} - \log\left(\sin\left(fx+e\right)+1\right) + \log\left(\sin\left(fx+e\right)-\right.\right.}{12\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*sin(f*x + e)/
(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*
a^2*c*log(sec(f*x + e) + tan(f*x + e)) - 12*a^2*c*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-8\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3}}{6\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^2*c*log(abs(tan(1/2*f
*x + 1/2*e) - 1)) - 2*(3*a^2*c*tan(1/2*f*x + 1/2*e)^5 - 8*a^2*c*tan(1/2*f*x
 + 1/2*e)^3 - 3*a^2*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)
/f
```

Mupad [B] (verification not implemented)

Time = 15.88 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.85

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{-c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\frac{8\,c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+c\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}+\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{f}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (a^2*c*tan(e/2 + (f*x)/2) + (8*a^2*c*tan(e/2 + (f*x)/2)^3)/3 - a^2*c*tan(e/
2 + (f*x)/2)^5)/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e
/2 + (f*x)/2)^6 - 1)) + (a^2*c*atanh(tan(e/2 + (f*x)/2)))/f
```

3.15 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)} dx$

Optimal result

Integrand size = 32, antiderivative size = 74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)} dx = -\frac{3a^2\text{arctanh}(\sin(e+fx))}{cf} - \frac{3a^2\tan(e+fx)}{cf} - \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -3*a^2*arctanh(sin(f*x+e))/c/f-3*a^2*tan(f*x+e)/c/f-2*(a^2+a^2*sec(f*x+e))*
tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4042, 3872, 3855, 3852, 8}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)} dx = -\frac{3a^2\text{arctanh}(\sin(e+fx))}{cf} - \frac{3a^2\tan(e+fx)}{cf} - \frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (-3*a^2*ArcTanh[Sin[e + f*x]])/(c*f) - (3*a^2*Tan[e + f*x])/(c*f) - (2*(a^2
 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(3a)\int\sec(e+fx)(a+a\sec(e+fx))\,dx}{c}$$

$$= -\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(3a^2)\int\sec(e+fx)\,dx}{c} - \frac{(3a^2)\int\sec^2(e+fx)\,dx}{c}$$

$$= -\frac{3a^2\text{arctanh}(\sin(e+fx))}{cf} - \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))} + \frac{(3a^2)\,\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{cf}$$

$$= -\frac{3a^2\text{arctanh}(\sin(e+fx))}{cf} - \frac{3a^2\tan(e+fx)}{cf} - \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx$$
$$= -\frac{a^2 \operatorname{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^2\tan(e+fx)}{5cf\sqrt{2-2\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x]),x]
[Out] -1/5*(a^2*Hypergeometric2F1[3/2, 5/2, 7/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
 + f*x])^2*Tan[e + f*x])/(c*f*Sqrt[2 - 2*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{4a^2\left(\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$
default	$\frac{4a^2\left(\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{fc}$
parallelrisch	$-\frac{a^2\left(-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)-5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)+\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{cf\cos(fx+e)}$
risch	$\frac{2ia^2\left(4\,e^{2i(fx+e)}-e^{i(fx+e)}+5\right)}{fc\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}-1\right)}+\frac{3a^2\ln\left(e^{i(fx+e)}-i\right)}{cf}-\frac{3a^2\ln\left(e^{i(fx+e)}+i\right)}{cf}$
norman	$\frac{\frac{4a^2}{cf}-\frac{10a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}+\frac{6a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{3a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{cf}-\frac{3a^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{cf}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 4/f*a^2/c*(1/4/(tan(1/2*f*x+1/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1)+1/4/(tan
(1/2*f*x+1/2*e)-1)+3/4*ln(tan(1/2*f*x+1/2*e)-1)+1/tan(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{3\,a^2\cos\left(fx+e\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-3\,a^2\cos\left(fx+e\right)\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(fx+\right.}{2\,cf\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/2*(3*a^2*cos(f*x + e)*log(sin(f*x + e) + 1)*sin(f*x + e) - 3*a^2*cos(f*x
 + e)*log(-sin(f*x + e) + 1)*sin(f*x + e) - 10*a^2*cos(f*x + e)^2 - 8*a^2*c
os(f*x + e) + 2*a^2)/(c*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{a^2\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{2\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx+\int \frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)-1}\,dx\right)}{c}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(2*sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) - 1),
 x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(75) = 150.

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.04

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{a^2\left(\frac{\frac{3\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}+\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+2\,a^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+\right.}{c}\right.}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(a^2*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x + e)/(cos(f*x
 + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c) + 2*a^2
*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - a^2*(cos(f*x + e) +
 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = -\frac{\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}-\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-2\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)c}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(3*a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - 3*a^2*log(abs(tan(1/2*f*x +
1/2*e) - 1))/c - 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 - 2*a^2)/((tan(1/2*f*x + 1
/2*e)^3 - tan(1/2*f*x + 1/2*e))*c))/f
```

Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx = \frac{6\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-4\,a^2}{c\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)} - \frac{6\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] (6*a^2*tan(e/2 + (f*x)/2)^2 - 4*a^2)/(c*f*tan(e/2 + (f*x)/2)*(tan(e/2 + (f*
x)/2)^2 - 1)) - (6*a^2*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

3.16 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\text{arctanh}(\sin(e+fx))}{c^2f} - \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{2a^2\tan(e+fx)}{f\,(c^2-c^2\sec(e+fx))}$$

```
[Out] a^2*arctanh(sin(f*x+e))/c^2/f-2/3*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/f/(c-c*se
c(f*x+e))^2+2*a^2*tan(f*x+e)/f/(c^2-c^2*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4042, 3855}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\text{arctanh}(\sin(e+fx))}{c^2f} + \frac{2a^2\tan(e+fx)}{f\,(c^2-c^2\sec(e+fx))} - \frac{2\tan(e+fx)\,(a^2\sec(e+fx)+a^2)}{3f(c-c\sec(e+fx))^2}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[e + f*x]])/(c^2*f) - (2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(3*f*(c - c*Sec[e + f*x])^2) + (2*a^2*Tan[e + f*x])/(f*(c^2 - c^2*Sec[
e + f*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}-\frac{a\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx}{c}\\ &= -\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}+\frac{2a^2\tan(e+fx)}{f\,(c^2-c^2\sec(e+fx))}+\frac{a^2\int\sec(e+fx)\,dx}{c^2}\\ &= \frac{a^2\text{arctanh}(\sin(e+fx))}{c^2f}-\frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^2}+\frac{2a^2\tan(e+fx)}{f\,(c^2-c^2\sec(e+fx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.22

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\left(-\frac{4\cot\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}\right)}{c^2}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*((-4*Cot[(e + f*x)/2])/(3*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^2)
/(3*f) - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f))/c^2
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{a^{2}\left(-2\cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}+3\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)-3\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)-6\cot\left(\frac{f x}{2}+\frac{e}{2}\right)\right)}{3 c^{2} f}$
derivativedivides	$\frac{2 a^{2}\left(-\frac{\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{2}-\frac{1}{3\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}\right)}{f\,c^{2}}$
default	$\frac{2 a^{2}\left(-\frac{\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{2}-\frac{1}{3\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{f x}{2}+\frac{e}{2}\right)}\right)}{f\,c^{2}}$
risch	$\frac{8 i a^{2}\left(3\,{\mathrm e}^{i(f x+e)}-1\right)}{3 f\,c^{2}\left({\mathrm e}^{i(f x+e)}-1\right)^{3}}+\frac{a^{2}\ln\left({\mathrm e}^{i(f x+e)}+i\right)}{c^{2} f}-\frac{a^{2}\ln\left({\mathrm e}^{i(f x+e)}-i\right)}{c^{2} f}$
norman	$\frac{-\frac{2 a^{2}}{3 c f}-\frac{2 a^{2}\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{3 c f}+\frac{10 a^{2}\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{3 c f}-\frac{2 a^{2}\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{c f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} c\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{3}}+\frac{a^{2}\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)+1\right)}{c^{2} f}-\frac{a^{2}\ln\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)-1\right)}{c^{2} f}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 1/3*a^2*(-2*cot(1/2*f*x+1/2*e)^3+3*ln(tan(1/2*f*x+1/2*e)+1)-3*ln(tan(1/2*f*x+1/2*e)-1)-6*cot(1/2*f*x+1/2*e))/c^2/f

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = -\frac{8\,a^{2}\cos\left(fx+e\right)^{2}-8\,a^{2}\cos\left(fx+e\right)-3\,\left(a^{2}\cos\left(fx+e\right)-a^{2}\right)\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)+3\,(a^{2}}{6\,\left(c^{2}f\cos\left(fx+e\right)-c^{2}f\right)\sin\left(fx+e\right)}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fricas")

[Out] -1/6*(8*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) - 3*(a^2*cos(f*x + e) - a^2)*log(sin(f*x + e) + 1)*sin(f*x + e) + 3*(a^2*cos(f*x + e) - a^2)*log(-sin(f*x + e) + 1)*sin(f*x + e) - 16*a^2)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{2\sec^2(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{\sec^3(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx\right)}{c^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + In
tegral(2*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(89) = 178$.

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{a^2\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2} - \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2} - \frac{\left(\frac{9\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)+1)^3}{c^2\sin(fx+e)^3}\right) - \frac{2a^2\left(\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)(\cos(fx+e)}{c^2\sin(fx+e)^3}}{6f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(a^2*(6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3)) - 2*a^2*(3*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) + a^2*(3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x +
e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2}-\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2}-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^2\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 3*a^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/c^2 - 2*(3*a^2*tan(1/2*f*x + 1/2*e)^2 + a^2)/(c^2*tan(1/2
*f*x + 1/2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 13.79 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx = \frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^2\,f}-\frac{2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\frac{2\,a^2}{3}}{c^2\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (2*a^2*atanh(tan(e/2 + (f*x)/2)))/(c^2*f) - (2*a^2*tan(e/2 + (f*x)/2)^2 + (
2*a^2)/3)/(c^2*f*tan(e/2 + (f*x)/2)^3)
```

3.17 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx=-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}$$

```
[Out] -1/5*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx=-\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -1/5*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral}=-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2\cot^5\left(\frac{1}{2}(e+fx)\right)}{5c^3f}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^3,x]
[Out] (a^2*Cot[(e + f*x)/2]^5)/(5*c^3*f)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{a^2}{5f\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	23
default	$\frac{a^2}{5f\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	23
parallelrisch	$\frac{a^2}{5f\,c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	23
risch	$\frac{2ia^2\left(5\,e^{4i(fx+e)}+10\,e^{2i(fx+e)}+1\right)}{5f\,c^3\left(e^{i(fx+e)}-1\right)^5}$	50
norman	$\frac{\frac{a^2}{5cf}-\frac{2a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{5cf}+\frac{a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{5cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	87

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
[Out] 1/5/f*a^2/c^3/tan(1/2*f*x+1/2*e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.18

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{a^2\cos(fx+e)^3+3\,a^2\cos(fx+e)^2+3\,a^2\cos(fx+e)+a^2}{5\left(c^3f\cos(fx+e)^2-2\,c^3f\cos(fx+e)+c^3f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)/
((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^2\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{2\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{\sec^3}{\sec^3(e+fx)-3\sec^2(}\right.}{c^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) - 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)*
*3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(37) = 74$.

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx = \frac{\frac{a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5} - \frac{a^2\left(\frac{10\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5} - \frac{6a^2\left(\frac{5\sin(fx+e)^4}{(\cos(fx+e)+1)}\right.}{c^3\sin}}{60f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(a^2*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - a^2*(10*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) - 6*a^2*(5*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx=\frac{a^2}{5\,c^3f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/5*a^2/(c^3*f*tan(1/2*f*x + 1/2*e)^5)
```

Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx=\frac{a^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{5\,c^3\,f}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (a^2*cot(e/2 + (f*x)/2)^5)/(5*c^3*f)
```

3.18 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4} dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4} dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}$$

```
[Out] -1/7*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^4-1/35*(a+a*sec(f*x+e
))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4} dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{35cf(c-c\sec(e+fx))^3} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{7f(c-c\sec(e+fx))^4}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/7*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4) - ((a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*c*f*(c - c*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
```

```
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4}+\frac{\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx}{7c}\\ &= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{7f(c-c\sec(e+fx))^4}-\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{35cf(c-c\sec(e+fx))^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx=\frac{a^2(-6+\sec(e+fx))(1+\sec(e+fx))^2\tan(e+fx)}{35c^4f(-1+\sec(e+fx))^4}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^4,x]
[Out] (a^2*(-6 + Sec[e + f*x])*(1 + Sec[e + f*x])^2*Tan[e + f*x])/(35*c^4*f*(-1 +
 Sec[e + f*x])^4)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.48

method	result	size
parallelrisch	$-\frac{a^{2} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{5} \left(5 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-7\right)}{70 c^{4} f}$	38
derivativedivides	$\frac{a^{2} \left(\frac{1}{5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{2 f \,c^{4}}$	39
default	$\frac{a^{2} \left(\frac{1}{5 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{5}}-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{2 f \,c^{4}}$	39
risch	$\frac{2 i a^{2} \left(35 \,{\mathrm e}^{6 i (f x+e)}-35 \,{\mathrm e}^{5 i (f x+e)}+140 \,{\mathrm e}^{4 i (f x+e)}-70 \,{\mathrm e}^{3 i (f x+e)}+91 \,{\mathrm e}^{2 i (f x+e)}-7 \,{\mathrm e}^{i (f x+e)}+6\right)}{35 f \,c^{4} \left({\mathrm e}^{i (f x+e)}-1\right)^{7}}$	94
norman	$\frac{-\frac{a^{2}}{14 c f}+\frac{17 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{70 c f}-\frac{19 a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{70 c f}+\frac{a^{2} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{10 c f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{2} c^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}$	109

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/70*a^2*cot(1/2*f*x+1/2*e)^5*(5*cot(1/2*f*x+1/2*e)^2-7)/c^4/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4} \, dx = \frac{6 \, a^{2} \cos\left(f x + e\right)^{4} + 17 \, a^{2} \cos\left(f x + e\right)^{3} + 15 \, a^{2} \cos\left(f x + e\right)^{2} + 3 \, a^{2} \cos\left(f x + e\right) - a^{2}}{35 \, {\left(c^{4} f \cos\left(f x + e\right)^{3} - 3 \, c^{4} f \cos\left(f x + e\right)^{2} + 3 \, c^{4} f \cos\left(f x + e\right) - c^{4} f\right)} \sin\left(f x + e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/35*(6*a^2*cos(f*x + e)^4 + 17*a^2*cos(f*x + e)^3 + 15*a^2*cos(f*x + e)^2
+ 3*a^2*cos(f*x + e) - a^2)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2
 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{a^2\left(\int \frac{\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{2\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \right.}{c^4}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
+ f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(2*sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f
*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(78) = 156$.

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{\frac{2a^2\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7} + \frac{3a^2\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos}{c^4\sin(fx+e)^7}}{840f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(2*a^2*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15)*(cos(f*x
 + e) + 1)^7/(c^4*sin(f*x + e)^7) + 3*a^2*(21*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*
x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^2*(21*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*x + e) + 1)^7/(c^4*sin
(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = \frac{7\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 5\,a^2}{70\,c^4 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
[Out] 1/70*(7*a^2*tan(1/2*f*x + 1/2*e)^2 - 5*a^2)/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx = -\frac{a^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(5\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^2-7\right)}{70\,c^4\,f}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
[Out] -(a^2*cot(e/2 + (f*x)/2)^5*(5*cot(e/2 + (f*x)/2)^2 - 7))/(70*c^4*f)
```

3.19 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{315c^2f(c-c\sec(e+fx))^3}$$

```
[Out] -1/9*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-2/63*(a+a*sec(f*x+e
))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4-2/315*(a+a*sec(f*x+e))^2*tan(f*x+e)/
c^2/f/(c-c*sec(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5} dx = -\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{315c^2f(c-c\sec(e+fx))^3} - \frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{63cf(c-c\sec(e+fx))^4} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{9f(c-c\sec(e+fx))^5}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/9*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) - (2*
(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4) - (2*(
a + a*Sec[e + f*x])^2*Tan[e + f*x])/(315*c^2*f*(c - c*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5}+\frac{2\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx}{9c}\\
&= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5}\\
&\quad-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}+\frac{2\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}\,dx}{63c^2}\\
&= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{9f(c-c\sec(e+fx))^5}-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}\\
&\quad-\frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{315c^2f(c-c\sec(e+fx))^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.49

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2(1+\sec(e+fx))^2\left(47-14\sec(e+fx)+2\sec^2(e+fx)\right)\tan(e+fx)}{315c^5f(-1+\sec(e+fx))^5}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*(1 + Sec[e + f*x])^2*(47 - 14*Sec[e + f*x] + 2*Sec[e + f*x]^2)*Tan[e +
 f*x])/(315*c^5*f*(-1 + Sec[e + f*x])^5)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

method	result
parallelrisch	$\frac{a^2\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5\left(35\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^4-90\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2+63\right)}{1260c^5f}$
derivativedivides	$\frac{a^2\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{4f\,c^5}$
default	$\frac{a^2\left(\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{4f\,c^5}$
risch	$\frac{2ia^2\left(315\,e^{8i(fx+e)}-630\,e^{7i(fx+e)}+2310\,e^{6i(fx+e)}-2520\,e^{5i(fx+e)}+3402\,e^{4i(fx+e)}-1638\,e^{3i(fx+e)}+1062\,e^{2i(fx+e)}-108\right.}{315f\,c^5\left(e^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{a^2}{36cf}-\frac{8a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{63cf}+\frac{139a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{630cf}-\frac{6a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{35cf}+\frac{a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{20cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/1260*a^2*cot(1/2*f*x+1/2*e)^5*(35*cot(1/2*f*x+1/2*e)^4-90*cot(1/2*f*x+1/2
*e)^2+63)/c^5/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{47\,a^2\cos(fx+e)^5+127\,a^2\cos(fx+e)^4+101\,a^2\cos(fx+e)^3+11\,a^2\cos(fx+e)^2-8\,a^2\cos(fx+e)}{315\left(c^5f\cos(fx+e)^4-4\,c^5f\cos(fx+e)^3+6\,c^5f\cos(fx+e)^2-4\,c^5f\cos(fx+e)+c^5f\right)\sin(fx+}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/315*(47*a^2*cos(f*x + e)^5 + 127*a^2*cos(f*x + e)^4 + 101*a^2*cos(f*x + e
)^3 + 11*a^2*cos(f*x + e)^2 - 8*a^2*cos(f*x + e) + 2*a^2)/((c^5*f*cos(f*x +
 e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x +
 e) + c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^2\left(\int \frac{\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx+\int \frac{2\sec^2(e+f}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)}\right.}{c^5}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(
e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(2*sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**3/(sec(
e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 +
 5*sec(e + f*x) - 1), x))/c**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(118) = 236.

Time = 0.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.22

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx =$$
$$-\frac{\frac{a^2\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{10\,a^2\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63\,\mathrm{s}}{(\cos(}\right.}{c^5\sin(fx+e)^9}}{5040\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/5040*(a^2*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^
9) + 10*a^2*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
e) + 1)^9/(c^5*sin(f*x + e)^9) + 7*a^2*(18*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(c^5
*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{63\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-90\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+35\,a^2}{1260\,c^5f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/1260*(63*a^2*tan(1/2*f*x + 1/2*e)^4 - 90*a^2*tan(1/2*f*x + 1/2*e)^2 + 35*
a^2)/(c^5*f*tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx = \frac{a^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{20\,c^5\,f} - \frac{a^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^7}{14\,c^5\,f} + \frac{a^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{36\,c^5\,f}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (a^2*cot(e/2 + (f*x)/2)^5)/(20*c^5*f) - (a^2*cot(e/2 + (f*x)/2)^7)/(14*c^5*
f) + (a^2*cot(e/2 + (f*x)/2)^9)/(36*c^5*f)
```

3.20 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 163

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{231c^2f(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\left(c^2-c^2\sec(e+fx)\right)^3}$$

```
[Out] -1/11*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^6-1/33*(a+a*sec(f*x+
e))^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^5-2/231*(a+a*sec(f*x+e))^2*tan(f*x+e)
/c^2/f/(c-c*sec(f*x+e))^4-2/1155*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c^2-c^2*s
ec(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used

$= \{4036, 4035\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = -\frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\left(c^2-c^2\sec(e+fx)\right)^3} - \frac{2\tan(e+fx)(a\sec(e+fx)+a)^2}{231c^2f(c-c\sec(e+fx))^4} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{33cf(c-c\sec(e+fx))^5} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^2}{11f(c-c\sec(e+fx))^6}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/11*((a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^6) - ((
a + a*Sec[e + f*x])^2*Tan[e + f*x])/(33*c*f*(c - c*Sec[e + f*x])^5) - (2*(a
 + a*Sec[e + f*x])^2*Tan[e + f*x])/(231*c^2*f*(c - c*Sec[e + f*x])^4) - (2*
(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*(c^2 - c^2*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\text{integral} = -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} + \frac{3\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^5}\,dx}{11c}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^4}\,dx}{33c^2}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{231c^2f(c-c\sec(e+fx))^4} + \frac{2\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^3}dx}{231c^3}$$

$$= -\frac{(a+a\sec(e+fx))^2\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{(a+a\sec(e+fx))^2\tan(e+fx)}{33cf(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{231c^2f(c-c\sec(e+fx))^4} - \frac{2(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\left(c^2-c^2\sec(e+fx)\right)^3}$$

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.42

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = \frac{a^2(1+\sec(e+fx))^2\left(-152+61\sec(e+fx)-16\sec^2(e+fx)+2\sec^3(e+fx)\right)\tan(e+fx)}{1155c^6f(-1+\sec(e+fx))^6}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^6,x]
[Out] (a^2*(1 + Sec[e + f*x])^2*(-152 + 61*Sec[e + f*x] - 16*Sec[e + f*x]^2 + 2*S
ec[e + f*x]^3)*Tan[e + f*x])/(1155*c^6*f*(-1 + Sec[e + f*x])^6)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

method	result
parallelrisch	$-\frac{a^2\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5\left(105\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^6-385\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^4+495\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2-231\right)}{9240c^6f}$
derivativedivides	$\frac{a^2\left(\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}\right)}{8fc^6}$
default	$\frac{a^2\left(\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}\right)}{8fc^6}$
risch	$\frac{2ia^2\left(1155\,e^{10i(fx+e)}-3465\,e^{9i(fx+e)}+13860\,e^{8i(fx+e)}-23100\,e^{7i(fx+e)}+37422\,e^{6i(fx+e)}-32802\,e^{5i(fx+e)}+27060\,e^{4i(f\cdots\right.}{1155fc^6\left(e^{i(fx+e)}-1\right)^{11}}$
norman	$\frac{-\frac{a^2}{88cf}+\frac{17a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{264cf}-\frac{137a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{924cf}+\frac{73a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{420cf}-\frac{29a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{280cf}+\frac{a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{40cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/9240*a^2*cot(1/2*f*x+1/2*e)^5*(105*cot(1/2*f*x+1/2*e)^6-385*cot(1/2*f*x+
1/2*e)^4+495*cot(1/2*f*x+1/2*e)^2-231)/c^6/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = \frac{152\,a^2\cos\left(fx+e\right)^6+395\,a^2\cos\left(fx+e\right)^5+289\,a^2\cos\left(fx+e\right)^4+15\,a^2\cos\left(fx+e\right)^3-19\,a^2\cos\left(fx+e\right.}{1155\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-10\,c^6f\cos\left(fx+e\right)^2+5\,c^6f\cos\left(\right.\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/1155*(152*a^2*cos(f*x + e)^6 + 395*a^2*cos(f*x + e)^5 + 289*a^2*cos(f*x +
 e)^4 + 15*a^2*cos(f*x + e)^3 - 19*a^2*cos(f*x + e)^2 + 10*a^2*cos(f*x + e)
 - 2*a^2)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*
x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x
+ e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = \frac{a^2\left(\int \frac{\sec\left(e+fx\right)}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+15\sec^4\left(e+fx\right)-20\sec^3\left(e+fx\right)+15\sec^2\left(e+fx\right)-6\sec\left(e+fx\right)+1}\,dx+\int \frac{}{\sec^6\left(e+fx\right)-6\sec^5\left(e+fx\right)+1}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e
 + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),
 x) + Integral(2*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*
sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x)
+ 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 +
15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*
x) + 1), x))/c**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(159) = 318$.

Time = 0.22 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = \frac{\frac{a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{990\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{1386\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1155\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{3465\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-315\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}+\frac{6a^2\left(\frac{385\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{330\ldots}{(\cos(\ldots}\right.}{\ldots}}{\ldots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/110880*(a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*sin
(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1)^
10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 6*a^2*(385*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x + e) +
1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x + e) + 1)
^11/(c^6*sin(f*x + e)^11) + 5*a^2*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*x + e)^10/
(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = \frac{231\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-495\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+385\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-105\,a^2}{9240\,c^6 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/9240*(231*a^2*tan(1/2*f*x + 1/2*e)^6 - 495*a^2*tan(1/2*f*x + 1/2*e)^4 + 3
85*a^2*tan(1/2*f*x + 1/2*e)^2 - 105*a^2)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^6}\,dx = -\frac{a^2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(105\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6-385\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+495\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right.\right.}{9240\,c^6\,f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^6),x)
```

```
[Out] -(a^2*cos(e/2 + (f*x)/2)^5*(105*cos(e/2 + (f*x)/2)^6 - 231*sin(e/2 + (f*x)/
2)^6 + 495*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 385*cos(e/2 + (f*x)/
2)^4*sin(e/2 + (f*x)/2)^2))/(9240*c^6*f*sin(e/2 + (f*x)/2)^11)
```

3.21 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$

Optimal result

Integrand size = 32, antiderivative size = 227

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx &= \frac{55a^3c^6\text{arctanh}(\sin(e+fx))}{128f}-\frac{25a^3c^6\sec(e+fx)\tan(e+fx)}{128f}\\ &-\frac{15a^3c^6\sec^3(e+fx)\tan(e+fx)}{64f}+\frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f}\\ &+\frac{5a^3c^6\sec^3(e+fx)\tan^3(e+fx)}{16f}-\frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f}\\ &-\frac{3a^3c^6\sec^3(e+fx)\tan^5(e+fx)}{8f}+\frac{4a^3c^6\tan^7(e+fx)}{7f}+\frac{a^3c^6\tan^9(e+fx)}{9f}\end{aligned}$$

```
[Out] 55/128*a^3*c^6*arctanh(sin(f*x+e))/f-25/128*a^3*c^6*sec(f*x+e)*tan(f*x+e)/f
-15/64*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)/f+5/24*a^3*c^6*sec(f*x+e)*tan(f*x+e)
^3/f+5/16*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)^3/f-1/6*a^3*c^6*sec(f*x+e)*tan(f*
x+e)^5/f-3/8*a^3*c^6*sec(f*x+e)^3*tan(f*x+e)^5/f+4/7*a^3*c^6*tan(f*x+e)^7/f
+1/9*a^3*c^6*tan(f*x+e)^9/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {4043, 2691, 3855, 2687, 30, 3853, 14}

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx \\
&= \frac{55a^3c^6\text{arctanh}(\sin(e+fx))}{128f}+\frac{a^3c^6\tan^9(e+fx)}{9f}+\frac{4a^3c^6\tan^7(e+fx)}{7f} \\
&\quad -\frac{3a^3c^6\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^6\tan^3(e+fx)\sec^3(e+fx)}{16f} \\
&\quad -\frac{15a^3c^6\tan(e+fx)\sec^3(e+fx)}{64f}-\frac{a^3c^6\tan^5(e+fx)\sec(e+fx)}{6f} \\
&\quad +\frac{5a^3c^6\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{25a^3c^6\tan(e+fx)\sec(e+fx)}{128f}
\end{aligned}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

```
[Out] (55*a^3*c^6*ArcTanh[Sin[e + f*x]])/(128*f) - (25*a^3*c^6*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (15*a^3*c^6*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3
*c^6*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^6*Sec[e + f*x]^3*Tan[e
+ f*x]^3)/(16*f) - (a^3*c^6*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (3*a^3*c^6
*Sec[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (4*a^3*c^6*Tan[e + f*x]^7)/(7*f) +
(a^3*c^6*Tan[e + f*x]^9)/(9*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(a^3c^3\right)\int \left(c^3\sec(e+fx)\tan^6(e+fx)-3c^3\sec^2(e+fx)\tan^6(e+fx)\right.\right.\\ &\qquad \left.\left.+3c^3\sec^3(e+fx)\tan^6(e+fx)-c^3\sec^4(e+fx)\tan^6(e+fx)\right)\,dx\right)\\ &= -\left(\left(a^3c^6\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)+\left(a^3c^6\right)\int \sec^4(e+fx)\tan^6(e+fx)\,dx\\ &\quad +\left(3a^3c^6\right)\int \sec^2(e+fx)\tan^6(e+fx)\,dx-\left(3a^3c^6\right)\int \sec^3(e+fx)\tan^6(e+fx)\,dx\end{aligned}$$

$$= -\frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{3a^3c^6\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{1}{6}(5a^3c^6)\int\sec(e+fx)\tan^4(e+fx)\,dx + \frac{1}{8}(15a^3c^6)\int\sec^3(e+fx)\tan^4(e+fx)\,dx + \frac{(a^3c^6)\operatorname{Subst}\left(\int x^6(1+x^2)\,dx,x,\tan(e+fx)\right)}{f} + \frac{(3a^3c^6)\operatorname{Subst}\left(\int x^6\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f} + \frac{5a^3c^6\sec^3(e+fx)\tan^3(e+fx)}{16f} - \frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{3a^3c^6\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{3a^3c^6\tan^7(e+fx)}{7f} - \frac{1}{8}(5a^3c^6)\int\sec(e+fx)\tan^2(e+fx)\,dx - \frac{1}{16}(15a^3c^6)\int\sec^3(e+fx)\tan^2(e+fx)\,dx + \frac{(a^3c^6)\operatorname{Subst}\left(\int(x^6+x^8)\,dx,x,\tan(e+fx)\right)}{f}$$

$$= -\frac{5a^3c^6\sec(e+fx)\tan(e+fx)}{16f} - \frac{15a^3c^6\sec^3(e+fx)\tan(e+fx)}{64f} + \frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f} + \frac{5a^3c^6\sec^3(e+fx)\tan^3(e+fx)}{16f} - \frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{3a^3c^6\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{4a^3c^6\tan^7(e+fx)}{7f} + \frac{a^3c^6\tan^9(e+fx)}{9f} + \frac{1}{64}(15a^3c^6)\int\sec^3(e+fx)\,dx + \frac{1}{16}(5a^3c^6)\int\sec(e+fx)\,dx$$

$$= \frac{5a^3c^6\operatorname{arctanh}(\sin(e+fx))}{16f} - \frac{25a^3c^6\sec(e+fx)\tan(e+fx)}{128f} - \frac{15a^3c^6\sec^3(e+fx)\tan(e+fx)}{64f} + \frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f} + \frac{5a^3c^6\sec^3(e+fx)\tan^3(e+fx)}{16f} - \frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{3a^3c^6\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{4a^3c^6\tan^7(e+fx)}{7f} + \frac{a^3c^6\tan^9(e+fx)}{9f} + \frac{1}{128}(15a^3c^6)\int\sec(e+fx)\,dx$$

$$= \frac{55a^3c^6\text{arctanh}(\sin(e+fx))}{128f} - \frac{25a^3c^6\sec(e+fx)\tan(e+fx)}{128f} - \frac{15a^3c^6\sec^3(e+fx)\tan(e+fx)}{64f} + \frac{5a^3c^6\sec(e+fx)\tan^3(e+fx)}{24f} + \frac{5a^3c^6\sec^3(e+fx)\tan^3(e+fx)}{16f} - \frac{a^3c^6\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{3a^3c^6\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{4a^3c^6\tan^7(e+fx)}{7f} + \frac{a^3c^6\tan^9(e+fx)}{9f}$$

Mathematica [A] (verified)

Time = 3.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx = \frac{a^3c^6(443520\text{arctanh}(\sin(e+fx)) - \sec^9(e+fx)(-88704\sin(e+fx) + 88074\sin(2(e+fx)) + 37632\sin(3$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6,x]
```

```
[Out] (a^3*c^6*(443520*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^9*(-88704*Sin[e + f*x
] + 88074*Sin[2*(e + f*x)] + 37632*Sin[3*(e + f*x)] - 2142*Sin[4*(e + f*x)]
 + 2304*Sin[5*(e + f*x)] + 39858*Sin[6*(e + f*x)] - 7488*Sin[7*(e + f*x)] +
 4599*Sin[8*(e + f*x)] + 1856*Sin[9*(e + f*x)])))/(1032192*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11

method	result
risch	$\frac{ia^3c^6\left(4599\,e^{17i(fx+e)}-24192\,e^{16i(fx+e)}+39858\,e^{15i(fx+e)}-64512\,e^{14i(fx+e)}-2142\,e^{13i(fx+e)}-118272\,e^{12i(fx+e)}+8807\right.}{}$
parallelrisch	$-\frac{55a^3c^6\left(\left(\cos(9fx+9e)+9\cos(7fx+7e)+36\cos(5fx+5e)+84\cos(3fx+3e)+126\cos(fx+e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(-\text{co}\right.}{}$
derivativedivides	$\frac{-a^3c^6\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(fx+e)^2}{315}\right)\tan(fx+e)-3a^3c^6\tan(fx+e)+a^3c^6\ln(\sec(fx+\right.}{}$
default	$\frac{-a^3c^6\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(fx+e)^2}{315}\right)\tan(fx+e)-3a^3c^6\tan(fx+e)+a^3c^6\ln(\sec(fx+\right.}{}$
parts	$\frac{a^3c^6\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{a^3c^6\left(-\frac{128}{315}-\frac{\sec(fx+e)^8}{9}-\frac{8\sec(fx+e)^6}{63}-\frac{16\sec(fx+e)^4}{105}-\frac{64\sec(fx+e)^2}{315}\right)\tan(fx+e)}{f}-$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4032*I*a^3*c^6*(4599*exp(17*I*(f*x+e))-24192*exp(16*I*(f*x+e))+39858*exp(
15*I*(f*x+e))-64512*exp(14*I*(f*x+e))-2142*exp(13*I*(f*x+e))-118272*exp(12*
I*(f*x+e))+88074*exp(11*I*(f*x+e))-322560*exp(10*I*(f*x+e))-145152*exp(8*I*
(f*x+e))-88074*exp(7*I*(f*x+e))-193536*exp(6*I*(f*x+e))+2142*exp(5*I*(f*x+e
))-69120*exp(4*I*(f*x+e))-39858*exp(3*I*(f*x+e))-9216*exp(2*I*(f*x+e))-4599
*exp(I*(f*x+e))-3712)/f/(1+exp(2*I*(f*x+e)))^9-55/128*a^3*c^6/f*ln(exp(I*(f
*x+e))-I)+55/128*a^3*c^6/f*ln(exp(I*(f*x+e))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.92

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$
$$= \frac{3465\,a^3c^6\cos\left(fx+e\right)^9\log\left(\sin\left(fx+e\right)+1\right)-3465\,a^3c^6\cos\left(fx+e\right)^9\log\left(-\sin\left(fx+e\right)+1\right)-2\left(3712\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/16128*(3465*a^3*c^6*cos(f*x + e)^9*log(sin(f*x + e) + 1) - 3465*a^3*c^6*c
os(f*x + e)^9*log(-sin(f*x + e) + 1) - 2*(3712*a^3*c^6*cos(f*x + e)^8 + 459
9*a^3*c^6*cos(f*x + e)^7 - 10240*a^3*c^6*cos(f*x + e)^6 + 3066*a^3*c^6*cos(
f*x + e)^5 + 8448*a^3*c^6*cos(f*x + e)^4 - 7224*a^3*c^6*cos(f*x + e)^3 - 10
24*a^3*c^6*cos(f*x + e)^2 + 3024*a^3*c^6*cos(f*x + e) - 896*a^3*c^6)*sin(f*
x + e))/(f*cos(f*x + e)^9)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$
$$= a^3c^6\left(\int \sec\left(e+fx\right)dx + \int \left(-3\sec^2\left(e+fx\right)\right)dx + \int 8\sec^4\left(e+fx\right)dx + \int \left(-6\sec^5\left(e+fx\right)\right)dx + \int \left(-6\sec^6\left(e+fx\right)\right)dx + \int 8\sec^7\left(e+fx\right)dx + \int \left(-3\sec^9\left(e+fx\right)\right)dx + \int \sec^{10}\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**3*c**6*(Integral(sec(e + f*x), x) + Integral(-3*sec(e + f*x)**2, x) + In
tegral(8*sec(e + f*x)**4, x) + Integral(-6*sec(e + f*x)**5, x) + Integral(-
6*sec(e + f*x)**6, x) + Integral(8*sec(e + f*x)**7, x) + Integral(-3*sec(e
+ f*x)**9, x) + Integral(sec(e + f*x)**10, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(209) = 418.

Time = 0.23 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.95

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$
$$= \frac{256\left(35\,\tan\left(fx+e\right)^9+180\,\tan\left(fx+e\right)^7+378\,\tan\left(fx+e\right)^5+420\,\tan\left(fx+e\right)^3+315\,\tan\left(fx+e\right)\right)a^3}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/80640*(256*(35*tan(f*x + e)^9 + 180*tan(f*x + e)^7 + 378*tan(f*x + e)^5 +
 420*tan(f*x + e)^3 + 315*tan(f*x + e))*a^3*c^6 - 32256*(3*tan(f*x + e)^5 +
 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^6 + 215040*(tan(f*x + e)^3 + 3*
tan(f*x + e))*a^3*c^6 + 315*a^3*c^6*(2*(105*sin(f*x + e)^7 - 385*sin(f*x +
e)^5 + 511*sin(f*x + e)^3 - 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x +
 e)^6 + 6*sin(f*x + e)^4 - 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1
) + 105*log(sin(f*x + e) - 1)) - 6720*a^3*c^6*(2*(15*sin(f*x + e)^5 - 40*si
n(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(
f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 30
240*a^3*c^6*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(
f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 8064
0*a^3*c^6*log(sec(f*x + e) + tan(f*x + e)) - 241920*a^3*c^6*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx$$
$$= \frac{3465\,a^3c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3465\,a^3c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3465\,a^3c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{17}-300}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/8064*(3465*a^3*c^6*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3465*a^3*c^6*log(
abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(3465*a^3*c^6*tan(1/2*f*x + 1/2*e)^17 -
30030*a^3*c^6*tan(1/2*f*x + 1/2*e)^15 + 115038*a^3*c^6*tan(1/2*f*x + 1/2*e)
^13 + 334602*a^3*c^6*tan(1/2*f*x + 1/2*e)^11 - 360448*a^3*c^6*tan(1/2*f*x +
 1/2*e)^9 + 255222*a^3*c^6*tan(1/2*f*x + 1/2*e)^7 - 115038*a^3*c^6*tan(1/2*
f*x + 1/2*e)^5 + 30030*a^3*c^6*tan(1/2*f*x + 1/2*e)^3 - 3465*a^3*c^6*tan(1/
2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^9)/f
```

Mupad [B] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.39

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6\,dx = \frac{55\,a^3\,c^6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{64\,f} - \frac{\frac{55\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{17}}{64}-\frac{715\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{15}}{96}+\frac{913\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{32}+\frac{18589\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{224}-\frac{5632\,a^3\,c^6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\cdots}{63}\cdots}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{18}-9\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}+36\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-84\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+126\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-\cdots\right)}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6)/cos(e + f*x),x)
```

```
[Out] (55*a^3*c^6*atanh(tan(e/2 + (f*x)/2)))/(64*f) - ((715*a^3*c^6*tan(e/2 + (f*
x)/2)^3)/96 - (913*a^3*c^6*tan(e/2 + (f*x)/2)^5)/32 + (14179*a^3*c^6*tan(e/
2 + (f*x)/2)^7)/224 - (5632*a^3*c^6*tan(e/2 + (f*x)/2)^9)/63 + (18589*a^3*c
^6*tan(e/2 + (f*x)/2)^11)/224 + (913*a^3*c^6*tan(e/2 + (f*x)/2)^13)/32 - (7
15*a^3*c^6*tan(e/2 + (f*x)/2)^15)/96 + (55*a^3*c^6*tan(e/2 + (f*x)/2)^17)/6
4 - (55*a^3*c^6*tan(e/2 + (f*x)/2))/64)/(f*(9*tan(e/2 + (f*x)/2)^2 - 36*tan
(e/2 + (f*x)/2)^4 + 84*tan(e/2 + (f*x)/2)^6 - 126*tan(e/2 + (f*x)/2)^8 + 12
6*tan(e/2 + (f*x)/2)^10 - 84*tan(e/2 + (f*x)/2)^12 + 36*tan(e/2 + (f*x)/2)^
14 - 9*tan(e/2 + (f*x)/2)^16 + tan(e/2 + (f*x)/2)^18 - 1))
```

3.22 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$

Optimal result

Integrand size = 32, antiderivative size = 206

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx\\&=\frac{45a^3c^5\operatorname{arctanh}(\sin(e+fx))}{128f}-\frac{35a^3c^5\sec(e+fx)\tan(e+fx)}{128f}\\&\quad-\frac{5a^3c^5\sec^3(e+fx)\tan(e+fx)}{64f}+\frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f}\\&\quad+\frac{5a^3c^5\sec^3(e+fx)\tan^3(e+fx)}{48f}-\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f}\\&\quad-\frac{a^3c^5\sec^3(e+fx)\tan^5(e+fx)}{8f}+\frac{2a^3c^5\tan^7(e+fx)}{7f}\end{aligned}$$

```
[Out] 45/128*a^3*c^5*arctanh(sin(f*x+e))/f-35/128*a^3*c^5*sec(f*x+e)*tan(f*x+e)/f
-5/64*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)/f+5/24*a^3*c^5*sec(f*x+e)*tan(f*x+e)^
3/f+5/48*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)^3/f-1/6*a^3*c^5*sec(f*x+e)*tan(f*x
+e)^5/f-1/8*a^3*c^5*sec(f*x+e)^3*tan(f*x+e)^5/f+2/7*a^3*c^5*tan(f*x+e)^7/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4043, 2691, 3855, 2687, 30, 3853}

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx\\&=\frac{45a^3c^5\operatorname{arctanh}(\sin(e+fx))}{128f}+\frac{2a^3c^5\tan^7(e+fx)}{7f}\\&\quad-\frac{a^3c^5\tan^5(e+fx)\sec^3(e+fx)}{8f}+\frac{5a^3c^5\tan^3(e+fx)\sec^3(e+fx)}{48f}\\&\quad-\frac{5a^3c^5\tan(e+fx)\sec^3(e+fx)}{64f}-\frac{a^3c^5\tan^5(e+fx)\sec(e+fx)}{6f}\\&\quad+\frac{5a^3c^5\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{35a^3c^5\tan(e+fx)\sec(e+fx)}{128f}\end{aligned}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] (45*a^3*c^5*ArcTanh[Sin[e + f*x]])/(128*f) - (35*a^3*c^5*Sec[e + f*x]*Tan[e
 + f*x])/(128*f) - (5*a^3*c^5*Sec[e + f*x]^3*Tan[e + f*x])/(64*f) + (5*a^3*
c^5*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) + (5*a^3*c^5*Sec[e + f*x]^3*Tan[e +
 f*x]^3)/(48*f) - (a^3*c^5*Sec[e + f*x]*Tan[e + f*x]^5)/(6*f) - (a^3*c^5*Se
c[e + f*x]^3*Tan[e + f*x]^5)/(8*f) + (2*a^3*c^5*Tan[e + f*x]^7)/(7*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(a^3c^3\right)\int \left(c^2\sec(e+fx)\tan^6(e+fx)-2c^2\sec^2(e+fx)\tan^6(e+fx)+c^2\sec^3(e+fx)\tan^6(e+fx)\right)\,dx\right)\\
&= -\left(\left(a^3c^5\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)-\left(a^3c^5\right)\int \sec^3(e+fx)\tan^6(e+fx)\,dx+\left(2a^3c^5\right)\int \sec^2(e+fx)\tan^6(e+fx)\,dx\\
&= -\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f}-\frac{a^3c^5\sec^3(e+fx)\tan^5(e+fx)}{8f}+\frac{1}{8}\left(5a^3c^5\right)\int \sec^3(e+fx)\tan^4(e+fx)\,dx\\
&\quad+\frac{1}{6}\left(5a^3c^5\right)\int \sec(e+fx)\tan^4(e+fx)\,dx+\frac{\left(2a^3c^5\right)\operatorname{Subst}\left(\int x^6\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f}+\frac{5a^3c^5\sec^3(e+fx)\tan^3(e+fx)}{48f}\\
&\quad-\frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f}-\frac{a^3c^5\sec^3(e+fx)\tan^5(e+fx)}{8f}\\
&\quad+\frac{2a^3c^5\tan^7(e+fx)}{7f}-\frac{1}{16}\left(5a^3c^5\right)\int \sec^3(e+fx)\tan^2(e+fx)\,dx\\
&\quad-\frac{1}{8}\left(5a^3c^5\right)\int \sec(e+fx)\tan^2(e+fx)\,dx
\end{aligned}$$

$$= -\frac{5a^3c^5\sec(e+fx)\tan(e+fx)}{16f} - \frac{5a^3c^5\sec^3(e+fx)\tan(e+fx)}{64f} + \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f} + \frac{5a^3c^5\sec^3(e+fx)\tan^3(e+fx)}{48f} - \frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{a^3c^5\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{2a^3c^5\tan^7(e+fx)}{7f} + \frac{1}{64}(5a^3c^5)\int\sec^3(e+fx)\,dx + \frac{1}{16}(5a^3c^5)\int\sec(e+fx)\,dx$$

$$= \frac{5a^3c^5\text{arctanh}(\sin(e+fx))}{16f} - \frac{35a^3c^5\sec(e+fx)\tan(e+fx)}{128f} - \frac{5a^3c^5\sec^3(e+fx)\tan(e+fx)}{64f} + \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f} + \frac{5a^3c^5\sec^3(e+fx)\tan^3(e+fx)}{48f} - \frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{a^3c^5\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{2a^3c^5\tan^7(e+fx)}{7f} + \frac{1}{128}(5a^3c^5)\int\sec(e+fx)\,dx$$

$$= \frac{45a^3c^5\text{arctanh}(\sin(e+fx))}{128f} - \frac{35a^3c^5\sec(e+fx)\tan(e+fx)}{128f} - \frac{5a^3c^5\sec^3(e+fx)\tan(e+fx)}{64f} + \frac{5a^3c^5\sec(e+fx)\tan^3(e+fx)}{24f} + \frac{5a^3c^5\sec^3(e+fx)\tan^3(e+fx)}{48f} - \frac{a^3c^5\sec(e+fx)\tan^5(e+fx)}{6f} - \frac{a^3c^5\sec^3(e+fx)\tan^5(e+fx)}{8f} + \frac{2a^3c^5\tan^7(e+fx)}{7f}$$

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = -\frac{a^3c^5(-20160\text{arctanh}(\sin(e+fx)) + \sec^8(e+fx)(5705\sin(e+fx) - 1792\sin(2(e+fx)) + 21\sin(3(e$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/57344*(a^3*c^5*(-20160*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^8*(5705*Sin[
e + f*x] - 1792*Sin[2*(e + f*x)] + 21*Sin[3*(e + f*x)] + 1792*Sin[4*(e + f*
x)] + 2065*Sin[5*(e + f*x)] - 768*Sin[6*(e + f*x)] + 581*Sin[7*(e + f*x)] +
 128*Sin[8*(e + f*x)])))/f
```

Maple [A] (verified)

Time = 8.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.16

method	result
norman	$\frac{-\frac{45c^5a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64f}+\frac{345c^5a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{64f}-\frac{1149c^5a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{64f}+\frac{15159c^5a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{448f}-\frac{17609c^5a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{448f}-\frac{11}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^8}$
risch	$\frac{ic^5a^3(581\,\mathrm{e}^{15i(fx+e)}-1792\,\mathrm{e}^{14i(fx+e)}+2065\,\mathrm{e}^{13i(fx+e)}-1792\,\mathrm{e}^{12i(fx+e)}+21\,\mathrm{e}^{11i(fx+e)}-8960\,\mathrm{e}^{10i(fx+e)}+5705\,\mathrm{e}^{9i(fx+e}}{448f}$
parallelrisch	$-\frac{815\left(\frac{9\left(\frac{35}{2}+28\cos(2fx+2e)+14\cos(4fx+4e)+4\cos(6fx+6e)+\frac{\cos(8fx+8e)}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{163}+\frac{9\left(-\frac{35}{2}-28\cos(2fx+2e)-14\cos\right.}{}\right.}{6}$
parts	$-\frac{2a^3c^5\tan(fx+e)}{f}-\frac{a^3c^5\sec(fx+e)\tan(fx+e)}{f}-\frac{6c^5a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{6c^5a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}\right.}{}$
derivativedivides	$\frac{-c^5a^3\left(-\left(-\frac{\sec(fx+e)^7}{8}-\frac{7\sec(fx+e)^5}{48}-\frac{35\sec(fx+e)^3}{192}-\frac{35\sec(fx+e)}{128}\right)\tan(fx+e)+\frac{35\ln(\sec(fx+e)+\tan(fx+e))}{128}\right)-2c^5a^3\left(-\right.}{}$
default	$\frac{-c^5a^3\left(-\left(-\frac{\sec(fx+e)^7}{8}-\frac{7\sec(fx+e)^5}{48}-\frac{35\sec(fx+e)^3}{192}-\frac{35\sec(fx+e)}{128}\right)\tan(fx+e)+\frac{35\ln(\sec(fx+e)+\tan(fx+e))}{128}\right)-2c^5a^3\left(-\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] (-45/64*c^5*a^3/f*tan(1/2*f*x+1/2*e)+345/64*c^5*a^3/f*tan(1/2*f*x+1/2*e)^3-
1149/64*c^5*a^3/f*tan(1/2*f*x+1/2*e)^5+15159/448*c^5*a^3/f*tan(1/2*f*x+1/2*
e)^7-17609/448*c^5*a^3/f*tan(1/2*f*x+1/2*e)^9-1149/64*c^5*a^3/f*tan(1/2*f*x
+1/2*e)^11+345/64*c^5*a^3/f*tan(1/2*f*x+1/2*e)^13-45/64*c^5*a^3/f*tan(1/2*f
*x+1/2*e)^15)/(tan(1/2*f*x+1/2*e)^2-1)^8-45/128*c^5*a^3/f*ln(tan(1/2*f*x+1/
2*e)-1)+45/128*c^5*a^3/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx$$
$$=\frac{315\,a^3c^5\cos\left(fx+e\right)^8\log\left(\sin\left(fx+e\right)+1\right)-315\,a^3c^5\cos\left(fx+e\right)^8\log\left(-\sin\left(fx+e\right)+1\right)-2\left(256\,a^3c^5\,c\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/1792*(315*a^3*c^5*cos(f*x + e)^8*log(sin(f*x + e) + 1) - 315*a^3*c^5*cos(
f*x + e)^8*log(-sin(f*x + e) + 1) - 2*(256*a^3*c^5*cos(f*x + e)^7 + 581*a^3
*c^5*cos(f*x + e)^6 - 768*a^3*c^5*cos(f*x + e)^5 - 210*a^3*c^5*cos(f*x + e)
^4 + 768*a^3*c^5*cos(f*x + e)^3 - 168*a^3*c^5*cos(f*x + e)^2 - 256*a^3*c^5*
cos(f*x + e) + 112*a^3*c^5)*sin(f*x + e))/(f*cos(f*x + e)^8)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = -a^3c^5\left(\int(-\sec(e+fx))\,dx+\int 2\sec^2(e+fx)\,dx+\int 2\sec^3(e+fx)\,dx+\int\left(-6\sec^4(e+fx)\right)\,dx+\int 6\sec^6(e+fx)\,dx+\int\left(-2\sec^7(e+fx)\right)\,dx+\int\left(-2\sec^8(e+fx)\right)\,dx+\int\sec^9(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*c**5*(Integral(-sec(e + f*x), x) + Integral(2*sec(e + f*x)**2, x) + I
ntegral(2*sec(e + f*x)**3, x) + Integral(-6*sec(e + f*x)**4, x) + Integral(
6*sec(e + f*x)**6, x) + Integral(-2*sec(e + f*x)**7, x) + Integral(-2*sec(e
 + f*x)**8, x) + Integral(sec(e + f*x)**9, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(190) = 380$.

Time = 0.21 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.98

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = \frac{1536\left(5\tan(fx+e)^7+21\tan(fx+e)^5+35\tan(fx+e)^3+35\tan(fx+e)\right)a^3c^5-10752\left(3\tan(fx+\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/26880*(1536*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 3
5*tan(f*x + e))*a^3*c^5 - 10752*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*
tan(f*x + e))*a^3*c^5 + 53760*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^5 + 3
5*a^3*c^5*(2*(105*sin(f*x + e)^7 - 385*sin(f*x + e)^5 + 511*sin(f*x + e)^3
- 279*sin(f*x + e))/(sin(f*x + e)^8 - 4*sin(f*x + e)^6 + 6*sin(f*x + e)^4 -
```

```
 4*sin(f*x + e)^2 + 1) - 105*log(sin(f*x + e) + 1) + 105*log(sin(f*x + e) -
 1)) - 560*a^3*c^5*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x +
 e))/(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(si
n(f*x + e) + 1) + 15*log(sin(f*x + e) - 1)) + 13440*a^3*c^5*(2*sin(f*x + e)
/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 26
880*a^3*c^5*log(sec(f*x + e) + tan(f*x + e)) - 53760*a^3*c^5*tan(f*x + e))/
f
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.05

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = \frac{315\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-315\,a^3c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(315\,a^3c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{15}-2415\,a^3 \cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/896*(315*a^3*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 315*a^3*c^5*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(315*a^3*c^5*tan(1/2*f*x + 1/2*e)^15 - 2415
*a^3*c^5*tan(1/2*f*x + 1/2*e)^13 + 8043*a^3*c^5*tan(1/2*f*x + 1/2*e)^11 + 1
7609*a^3*c^5*tan(1/2*f*x + 1/2*e)^9 - 15159*a^3*c^5*tan(1/2*f*x + 1/2*e)^7
+ 8043*a^3*c^5*tan(1/2*f*x + 1/2*e)^5 - 2415*a^3*c^5*tan(1/2*f*x + 1/2*e)^3
 + 315*a^3*c^5*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^8)/f
```

Mupad [B] (verification not implemented)

Time = 16.93 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.38

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5\,dx = \frac{45\,a^3\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{64\,f} - \frac{\frac{45\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{15}}{64}-\frac{345\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{64}+\frac{1149\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{64}+\frac{17609\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{448}-\frac{15159\,a^3\,c^5\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right) \cdots}{448}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{16}-8\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}+28\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-56\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+70\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-\cdots\right.}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5)/cos(e + f*x),x)
```

```
[Out] (45*a^3*c^5*atanh(tan(e/2 + (f*x)/2)))/(64*f) - ((1149*a^3*c^5*tan(e/2 + (f
*x)/2)^5)/64 - (345*a^3*c^5*tan(e/2 + (f*x)/2)^3)/64 - (15159*a^3*c^5*tan(e
```

```
/2 + (f*x)/2)^7)/448 + (17609*a^3*c^5*tan(e/2 + (f*x)/2)^9)/448 + (1149*a^3
*c^5*tan(e/2 + (f*x)/2)^11)/64 - (345*a^3*c^5*tan(e/2 + (f*x)/2)^13)/64 + (
45*a^3*c^5*tan(e/2 + (f*x)/2)^15)/64 + (45*a^3*c^5*tan(e/2 + (f*x)/2))/64)/
(f*(28*tan(e/2 + (f*x)/2)^4 - 8*tan(e/2 + (f*x)/2)^2 - 56*tan(e/2 + (f*x)/2
)^6 + 70*tan(e/2 + (f*x)/2)^8 - 56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*
x)/2)^12 - 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)^16 + 1))
```

3.23 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = \frac{5a^3c^4\text{arctanh}(\sin(e+fx))}{16f} - \frac{5a^3c^4\sec(e+fx)\tan(e+fx)}{16f} + \frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f} - \frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f} + \frac{a^3c^4\tan^7(e+fx)}{7f}$$

```
[Out] 5/16*a^3*c^4*arctanh(sin(f*x+e))/f-5/16*a^3*c^4*sec(f*x+e)*tan(f*x+e)/f+5/2
4*a^3*c^4*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^3*c^4*sec(f*x+e)*tan(f*x+e)^5/f+1
/7*a^3*c^4*tan(f*x+e)^7/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4043, 2691, 3855, 2687, 30}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = \frac{5a^3c^4\text{arctanh}(\sin(e+fx))}{16f} + \frac{a^3c^4\tan^7(e+fx)}{7f} - \frac{a^3c^4\tan^5(e+fx)\sec(e+fx)}{6f} + \frac{5a^3c^4\tan^3(e+fx)\sec(e+fx)}{24f} - \frac{5a^3c^4\tan(e+fx)\sec(e+fx)}{16f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (5*a^3*c^4*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^4*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^4*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^4*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f) + (a^3*c^4*Tan[e + f*x]^7)/(7*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\text{integral} = -\left((a^3c^3)\int\left(c\sec(e+fx)\tan^6(e+fx)-c\sec^2(e+fx)\tan^6(e+fx)\right)\,dx\right)$$

$$= -\left((a^3c^4)\int\sec(e+fx)\tan^6(e+fx)\,dx\right)+(a^3c^4)\int\sec^2(e+fx)\tan^6(e+fx)\,dx$$

$$= -\frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f}+\frac{1}{6}(5a^3c^4)\int\sec(e+fx)\tan^4(e+fx)\,dx$$
$$+\frac{(a^3c^4)\,\text{Subst}\left(\int x^6\,dx,x,\tan(e+fx)\right)}{f}$$

$$= \frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f} - \frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f} + \frac{a^3c^4\tan^7(e+fx)}{7f} - \frac{1}{8}(5a^3c^4)\int \sec(e+fx)\tan^2(e+fx)\,dx$$

$$= -\frac{5a^3c^4\sec(e+fx)\tan(e+fx)}{16f} + \frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f} - \frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f} + \frac{a^3c^4\tan^7(e+fx)}{7f} + \frac{1}{16}(5a^3c^4)\int \sec(e+fx)\,dx$$

$$= \frac{5a^3c^4\text{arctanh}(\sin(e+fx))}{16f} - \frac{5a^3c^4\sec(e+fx)\tan(e+fx)}{16f} + \frac{5a^3c^4\sec(e+fx)\tan^3(e+fx)}{24f} - \frac{a^3c^4\sec(e+fx)\tan^5(e+fx)}{6f} + \frac{a^3c^4\tan^7(e+fx)}{7f}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$= \frac{a^3c^4(3360\text{arctanh}(\sin(e+fx)) - \sec^7(e+fx)(-840\sin(e+fx)+595\sin(2(e+fx))+504\sin(3(e+fx)}{10752f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*c^4*(3360*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^7*(-840*Sin[e + f*x] +
595*Sin[2*(e + f*x)] + 504*Sin[3*(e + f*x)] + 196*Sin[4*(e + f*x)] - 168*Si
n[5*(e + f*x)] + 231*Sin[6*(e + f*x)] + 24*Sin[7*(e + f*x)])))/(10752*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.49 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.45

method	result
risch	$\frac{ic^4a^3(231\,e^{13i(fx+e)}-336\,e^{12i(fx+e)}+196\,e^{11i(fx+e)}+595\,e^{9i(fx+e)}-1680\,e^{8i(fx+e)}-595\,e^{5i(fx+e)}-1008\,e^{4i(fx+e)}-1}{168f\left(1+e^{2i(fx+e)}\right)^7}$
norman	$\frac{\frac{5c^4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}-\frac{25c^4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{6f}+\frac{283c^4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{24f}-\frac{128c^4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{7f}-\frac{283c^4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{24f}+\frac{25c^4a^3\tan}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^7}$
parallelrisch	$\frac{5a^3\left(\frac{(-\cos(7fx+7e)-7\cos(5fx+5e)-21\cos(3fx+3e)-35\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{(\cos(7fx+7e)+7\cos(5fx+5e)+21\cos(3}{}\right.}{f(\cos(7fx+7e)+7\cos}$
derivativedivides	$\frac{-c^4a^3\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)-c^4a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right.\right.}{}$
default	$\frac{-c^4a^3\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)-c^4a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right.\right.}{}$
parts	$\frac{c^4a^3\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{c^4a^3\left(-\frac{16}{35}-\frac{\sec(fx+e)^6}{7}-\frac{6\sec(fx+e)^4}{35}-\frac{8\sec(fx+e)^2}{35}\right)\tan(fx+e)}{f}-\frac{c^4a^3\tan(fx-}{f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/168*I*c^4*a^3*(231*exp(13*I*(f*x+e))-336*exp(12*I*(f*x+e))+196*exp(11*I*(
f*x+e))+595*exp(9*I*(f*x+e))-1680*exp(8*I*(f*x+e))-595*exp(5*I*(f*x+e))-100
8*exp(4*I*(f*x+e))-196*exp(3*I*(f*x+e))-231*exp(I*(f*x+e))-48)/f/(1+exp(2*I
*(f*x+e)))^7-5/16*c^4*a^3/f*ln(exp(I*(f*x+e))-I)+5/16*c^4*a^3/f*ln(exp(I*(f
*x+e))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.46

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$=\frac{105\,a^3c^4\cos\left(fx+e\right)^7\log\left(\sin\left(fx+e\right)+1\right)-105\,a^3c^4\cos\left(fx+e\right)^7\log\left(-\sin\left(fx+e\right)+1\right)-2\left(48\,a^3c^4\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/672*(105*a^3*c^4*cos(f*x + e)^7*log(sin(f*x + e) + 1) - 105*a^3*c^4*cos(f
*x + e)^7*log(-sin(f*x + e) + 1) - 2*(48*a^3*c^4*cos(f*x + e)^6 + 231*a^3*c
^4*cos(f*x + e)^5 - 144*a^3*c^4*cos(f*x + e)^4 - 182*a^3*c^4*cos(f*x + e)^3
 + 144*a^3*c^4*cos(f*x + e)^2 + 56*a^3*c^4*cos(f*x + e) - 48*a^3*c^4)*sin(f
*x + e))/(f*cos(f*x + e)^7)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$= a^3c^4\bigg(\int \sec(e+fx)\,dx + \int \left(-\sec^2(e+fx)\right)\,dx + \int \left(-3\sec^3(e+fx)\right)\,dx + \int 3\sec^4(e+fx)\,dx + \int 3\sec^5(e+fx)\,dx + \int \left(-3\sec^6(e+fx)\right)\,dx + \int \left(-\sec^7(e+fx)\right)\,dx + \int \sec^8(e+fx)\,dx\bigg)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*c**4*(Integral(sec(e + f*x), x) + Integral(-sec(e + f*x)**2, x) + Inte
gral(-3*sec(e + f*x)**3, x) + Integral(3*sec(e + f*x)**4, x) + Integral(3*s
ec(e + f*x)**5, x) + Integral(-3*sec(e + f*x)**6, x) + Integral(-sec(e + f*
x)**7, x) + Integral(sec(e + f*x)**8, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(111) = 222.

Time = 0.21 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$= \frac{96\left(5\tan(fx+e)^7+21\tan(fx+e)^5+35\tan(fx+e)^3+35\tan(fx+e)\right)a^3c^4-672\left(3\tan(fx+e)^5 \cdots\right.}{\cdots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/3360*(96*(5*tan(f*x + e)^7 + 21*tan(f*x + e)^5 + 35*tan(f*x + e)^3 + 35*t
an(f*x + e))*a^3*c^4 - 672*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f
*x + e))*a^3*c^4 + 3360*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^4 + 35*a^3*
c^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x +
 e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1)
 + 15*log(sin(f*x + e) - 1)) - 630*a^3*c^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x
 + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) +
3*log(sin(f*x + e) - 1)) + 2520*a^3*c^4*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1
) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 3360*a^3*c^4*log(sec(f
*x + e) + tan(f*x + e)) - 3360*a^3*c^4*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.63

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx$$
$$= \frac{105\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-105\,a^3c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(105\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}-700\,a\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/336*(105*a^3*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 105*a^3*c^4*log(abs
(tan(1/2*f*x + 1/2*e) - 1)) - 2*(105*a^3*c^4*tan(1/2*f*x + 1/2*e)^13 - 700*
a^3*c^4*tan(1/2*f*x + 1/2*e)^11 + 1981*a^3*c^4*tan(1/2*f*x + 1/2*e)^9 + 307
2*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 1981*a^3*c^4*tan(1/2*f*x + 1/2*e)^5 + 70
0*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 - 105*a^3*c^4*tan(1/2*f*x + 1/2*e))/(tan(1
/2*f*x + 1/2*e)^2 - 1)^7)/f
```

Mupad [B] (verification not implemented)

Time = 16.85 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.08

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4\,dx = \frac{5\,a^3\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$
$$-\frac{\frac{5\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}{8}-\frac{25\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{6}+\frac{283\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{128\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{7}-\frac{283\,a^3\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{24}+}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{14}-7\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}+21\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}+35\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}-21\right.}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (5*a^3*c^4*atanh(tan(e/2 + (f*x)/2)))/(8*f) - ((25*a^3*c^4*tan(e/2 + (f*x)/
2)^3)/6 - (283*a^3*c^4*tan(e/2 + (f*x)/2)^5)/24 + (128*a^3*c^4*tan(e/2 + (f
*x)/2)^7)/7 + (283*a^3*c^4*tan(e/2 + (f*x)/2)^9)/24 - (25*a^3*c^4*tan(e/2 +
 (f*x)/2)^11)/6 + (5*a^3*c^4*tan(e/2 + (f*x)/2)^13)/8 - (5*a^3*c^4*tan(e/2
+ (f*x)/2))/8)/(f*(7*tan(e/2 + (f*x)/2)^2 - 21*tan(e/2 + (f*x)/2)^4 + 35*ta
n(e/2 + (f*x)/2)^6 - 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 - 7
*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 - 1))
```

3.24 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = \frac{5a^3c^3\text{arctanh}(\sin(e+fx))}{16f}-\frac{5a^3c^3\sec(e+fx)\tan(e+fx)}{16f}+\frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f}-\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f}$$

```
[Out] 5/16*a^3*c^3*arctanh(sin(f*x+e))/f-5/16*a^3*c^3*sec(f*x+e)*tan(f*x+e)/f+5/2
4*a^3*c^3*sec(f*x+e)*tan(f*x+e)^3/f-1/6*a^3*c^3*sec(f*x+e)*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4043, 2691, 3855}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = \frac{5a^3c^3\text{arctanh}(\sin(e+fx))}{16f}-\frac{a^3c^3\tan^5(e+fx)\sec(e+fx)}{6f}+\frac{5a^3c^3\tan^3(e+fx)\sec(e+fx)}{24f}-\frac{5a^3c^3\tan(e+fx)\sec(e+fx)}{16f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
```

```
[Out] (5*a^3*c^3*ArcTanh[Sin[e + f*x]])/(16*f) - (5*a^3*c^3*Sec[e + f*x]*Tan[e +
f*x])/(16*f) + (5*a^3*c^3*Sec[e + f*x]*Tan[e + f*x]^3)/(24*f) - (a^3*c^3*Se
c[e + f*x]*Tan[e + f*x]^5)/(6*f)
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(a^3c^3\right)\int \sec(e+fx)\tan^6(e+fx)\,dx\right)\\
&= -\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f}+\frac{1}{6}\left(5a^3c^3\right)\int \sec(e+fx)\tan^4(e+fx)\,dx\\
&= \frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f}-\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f}\\
&\quad -\frac{1}{8}\left(5a^3c^3\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\\
&= -\frac{5a^3c^3\sec(e+fx)\tan(e+fx)}{16f}+\frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f}\\
&\quad -\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f}+\frac{1}{16}\left(5a^3c^3\right)\int \sec(e+fx)\,dx\\
&= \frac{5a^3c^3\text{arctanh}(\sin(e+fx))}{16f}-\frac{5a^3c^3\sec(e+fx)\tan(e+fx)}{16f}\\
&\quad +\frac{5a^3c^3\sec(e+fx)\tan^3(e+fx)}{24f}-\frac{a^3c^3\sec(e+fx)\tan^5(e+fx)}{6f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -a^3c^3\left(-\frac{5\text{arctanh}(\sin(e+fx))}{16f}-\frac{5\sec(e+fx)\tan(e+fx)}{16f}-\frac{5\sec^3(e+fx)\tan(e+fx)}{24f}+\frac{5\sec^5(e+fx)\tan(e+fx)}{6f}-\frac{5\sec^3(e+fx)\tan^3(e+fx)}{3f}+\frac{\sec(e+fx)\tan^5(e+fx)}{f}\right)$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3,x]
[Out] -(a^3*c^3*((-5*ArcTanh[Sin[e + f*x]])/(16*f) - (5*Sec[e + f*x]*Tan[e + f*x]
)/(16*f) - (5*Sec[e + f*x]^3*Tan[e + f*x])/(24*f) + (5*Sec[e + f*x]^5*Tan[e
 + f*x])/(6*f) - (5*Sec[e + f*x]^3*Tan[e + f*x]^3)/(3*f) + (Sec[e + f*x]*Ta
n[e + f*x]^5)/f))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.91 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.42

method	result
risch	$\frac{i a^3 c^3 \left(33\,e^{11 i(fx+e)}-5\,e^{9 i(fx+e)}+90\,e^{7 i(fx+e)}-90\,e^{5 i(fx+e)}+5\,e^{3 i(fx+e)}-33\,e^{i(fx+e)}\right)}{24 f\left(1+e^{2 i(fx+e)}\right)^6}-\frac{5 c^3 a^3 \ln\left(e^{i(fx+e)}-i\right)}{16 f}+\frac{5c^3}{}$
parallelrisch	$\frac{5 a^3 c^3\left(\left(-\frac{45\cos(2fx+2e)}{2}-9\cos(4fx+4e)-\frac{3\cos(6fx+6e)}{2}-15\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(\frac{3\cos(6fx+6e)}{2}+9\cos(4fx+4e)+\frac{45 c}{}\right.\right.}{24 f(6\cos(4fx+4e)+10+15\cos(2fx+2e)+\cos(}$
derivativedivides	$\frac{-c^3a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+3c^3a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\right.\right.}{}$
default	$\frac{-c^3a^3\left(-\left(-\frac{\sec(fx+e)^5}{6}-\frac{5\sec(fx+e)^3}{24}-\frac{5\sec(fx+e)}{16}\right)\tan(fx+e)+\frac{5\ln(\sec(fx+e)+\tan(fx+e))}{16}\right)+3c^3a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\right.\right.}{}$
parts	$\frac{c^3a^3\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{3c^3a^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{3c^3a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\right.\right.}{}$
norman	$\frac{-\frac{5c^3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8f}+\frac{85c^3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{24f}-\frac{33c^3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4f}-\frac{33c^3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{85c^3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{24f}-\frac{5c^3a^3\tan(}{8f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^6}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24*I*a^3*c^3/f/(1+exp(2*I*(f*x+e)))^6*(33*exp(11*I*(f*x+e))-5*exp(9*I*(f*
x+e))+90*exp(7*I*(f*x+e))-90*exp(5*I*(f*x+e))+5*exp(3*I*(f*x+e))-33*exp(I*(
f*x+e)))-5/16*c^3*a^3/f*ln(exp(I*(f*x+e))-I)+5/16*c^3*a^3/f*ln(exp(I*(f*x+e
))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = \frac{15\,a^3c^3\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c^3\cos\left(fx+e\right)^6\log\left(-\sin\left(fx+e\right)+1\right)-2\,\left(33\,a^3c^3\cos\right.}{96\,f\cos\left(fx+e\right)^6}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/96*(15*a^3*c^3*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*a^3*c^3*cos(f*x
+ e)^6*log(-sin(f*x + e) + 1) - 2*(33*a^3*c^3*cos(f*x + e)^4 - 26*a^3*c^3*c
os(f*x + e)^2 + 8*a^3*c^3)*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx = -a^3c^3\left(\int \left(-\sec\left(e+fx\right)\right)dx+\int 3\sec^3\left(e+fx\right)dx+\int \left(-3\sec^5\left(e+fx\right)\right)dx + \int \sec^7\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*c**3*(Integral(-sec(e + f*x), x) + Integral(3*sec(e + f*x)**3, x) + I
ntegral(-3*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**7, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(92) = 184.

Time = 0.23 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.44

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$= \frac{a^3c^3\left(\frac{2\left(15\sin(fx+e)^5-40\sin(fx+e)^3+33\sin(fx+e)\right)}{\sin(fx+e)^6-3\sin(fx+e)^4+3\sin(fx+e)^2-1}-15\log\left(\sin\left(fx+e\right)+1\right)+15\log\left(\sin\left(fx+e\right)-1\right)\right)-18a}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/96*(a^3*c^3*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/
(sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
 + e) + 1) + 15*log(sin(f*x + e) - 1)) - 18*a^3*c^3*(2*(3*sin(f*x + e)^3 -
5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e
) + 1) + 3*log(sin(f*x + e) - 1)) + 72*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e
)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 96*a^3*c^3*log(
sec(f*x + e) + tan(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx$$
$$= \frac{15a^3c^3\log\left(\left|\sin\left(fx+e\right)+1\right|\right)-15a^3c^3\log\left(\left|\sin\left(fx+e\right)-1\right|\right)+\frac{2\left(33a^3c^3\sin(fx+e)^5-40a^3c^3\sin(fx+e)^3+15a^3c^3\sin\right.}{\left(\sin(fx+e)^2-1\right)^3}}{96f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/96*(15*a^3*c^3*log(abs(sin(f*x + e) + 1)) - 15*a^3*c^3*log(abs(sin(f*x +
e) - 1)) + 2*(33*a^3*c^3*sin(f*x + e)^5 - 40*a^3*c^3*sin(f*x + e)^3 + 15*a^
3*c^3*sin(f*x + e))/(sin(f*x + e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.50 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.20

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3\,dx=\frac{5\,a^3\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{8\,f}$$
$$-\frac{\frac{5\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}}{8}-\frac{85\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{9}}{24}+\frac{33\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{7}}{4}+\frac{33\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{5}}{4}-\frac{85\,a^3\,c^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}}{24}+\frac{5}{}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{12}-6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{8}-20\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{6}+15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-6\,\mathrm{ta}\right.}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (5*a^3*c^3*atanh(tan(e/2 + (f*x)/2)))/(8*f) - ((33*a^3*c^3*tan(e/2 + (f*x)/
2)^5)/4 - (85*a^3*c^3*tan(e/2 + (f*x)/2)^3)/24 + (33*a^3*c^3*tan(e/2 + (f*x
)/2)^7)/4 - (85*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (5*a^3*c^3*tan(e/2 + (f*
x)/2)^11)/8 + (5*a^3*c^3*tan(e/2 + (f*x)/2))/8)/(f*(15*tan(e/2 + (f*x)/2)^4
 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)
^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1))
```

3.25 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 94

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \frac{3a^3c^2\operatorname{arctanh}(\sin(e+fx))}{8f} - \frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^3c^2\tan^5(e+fx)}{5f}$$

```
[Out] 3/8*a^3*c^2*arctanh(sin(f*x+e))/f-3/8*a^3*c^2*sec(f*x+e)*tan(f*x+e)/f+1/4*a
^3*c^2*sec(f*x+e)*tan(f*x+e)^3/f+1/5*a^3*c^2*tan(f*x+e)^5/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4043, 2691, 3855, 2687, 30}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \frac{3a^3c^2\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{a^3c^2\tan^5(e+fx)}{5f} + \frac{a^3c^2\tan^3(e+fx)\sec(e+fx)}{4f} - \frac{3a^3c^2\tan(e+fx)\sec(e+fx)}{8f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (3*a^3*c^2*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c^2*Sec[e + f*x]*Tan[e + f
*x])/(8*f) + (a^3*c^2*Sec[e + f*x]*Tan[e + f*x]^3)/(4*f) + (a^3*c^2*Tan[e +
 f*x]^5)/(5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2c^2\right)\int \left(a\sec(e+fx)\tan^4(e+fx)+a\sec^2(e+fx)\tan^4(e+fx)\right)\,dx\\
&= \left(a^3c^2\right)\int \sec(e+fx)\tan^4(e+fx)\,dx+\left(a^3c^2\right)\int \sec^2(e+fx)\tan^4(e+fx)\,dx\\
&= \frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f}-\frac{1}{4}\left(3a^3c^2\right)\int \sec(e+fx)\tan^2(e+fx)\,dx\\
&\quad+\frac{\left(a^3c^2\right)\operatorname{Subst}\left(\int x^4\,dx,x,\tan(e+fx)\right)}{f}
\end{aligned}$$

$$= -\frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^3c^2\tan^5(e+fx)}{5f} + \frac{1}{8}(3a^3c^2)\int\sec(e+fx)\,dx$$

$$= \frac{3a^3c^2\text{arctanh}(\sin(e+fx))}{8f} - \frac{3a^3c^2\sec(e+fx)\tan(e+fx)}{8f} + \frac{a^3c^2\sec(e+fx)\tan^3(e+fx)}{4f} + \frac{a^3c^2\tan^5(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int\sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx = \frac{a^3c^2(120\text{arctanh}(\sin(e+fx))+\sec^5(e+fx)(40\sin(e+fx)-10\sin(2(e+fx))-20\sin(3(e+fx))-25}{320f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*(120*ArcTanh[Sin[e + f*x]] + Sec[e + f*x]^5*(40*Sin[e + f*x] - 10*
Sin[2*(e + f*x)] - 20*Sin[3*(e + f*x)] - 25*Sin[4*(e + f*x)] + 4*Sin[5*(e +
 f*x)])))/(320*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

method	result
risch	$\frac{ic^2a^3(25\,e^{9i(fx+e)}+40\,e^{8i(fx+e)}+10\,e^{7i(fx+e)}+80\,e^{4i(fx+e)}-10\,e^{3i(fx+e)}-25\,e^{i(fx+e)}+8)}{20f(1+e^{2i(fx+e)})^5}-\frac{3c^2a^3\ln(e^{i(fx+e)}-i)}{8f}$
parts	$\frac{c^2a^3\tan(fx+e)}{f}+\frac{c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{c^2a^3\left(-\frac{8}{15}-\frac{\sec(}{}\right.}{}$
norman	$\frac{\frac{3c^2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{7c^2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2f}-\frac{32c^2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5f}+\frac{7c^2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{2f}-\frac{3c^2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}-\frac{3c^2a^3\ln\left(\tan\right.}{}$
parallelrisch	$-\frac{a^3c^2\left(\left(\frac{15\cos(fx+e)}{2}+\frac{15\cos(3fx+3e)}{4}+\frac{3\cos(5fx+5e)}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{15\cos(fx+e)}{2}-\frac{15\cos(3fx+3e)}{4}-\frac{3\cos(5f}{4}\right.\right.}{2f(\cos(5fx+5e)+5\cos(3fx+3}$
derivativedivides	$\frac{-c^2a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)+c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+}{8}\right.}{}$
default	$\frac{-c^2a^3\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)+c^2a^3\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+}{8}\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/20*I*c^2*a^3*(25*exp(9*I*(f*x+e))+40*exp(8*I*(f*x+e))+10*exp(7*I*(f*x+e))
+80*exp(4*I*(f*x+e))-10*exp(3*I*(f*x+e))-25*exp(I*(f*x+e))+8)/f/(1+exp(2*I*
(f*x+e)))^5-3/8*c^2*a^3/f*ln(exp(I*(f*x+e))-I)+3/8*c^2*a^3/f*ln(exp(I*(f*x+
e))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$=\frac{15\,a^3c^2\cos(fx+e)^5\log(\sin(fx+e)+1)-15\,a^3c^2\cos(fx+e)^5\log(-\sin(fx+e)+1)+2\left(8\,a^3c^2\cos\right.}{80\,f\cos(f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*a^3*c^2*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c^2*cos(f*x
+ e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^3*c^2*cos(f*x + e)^4 - 25*a^3*c^2*co
s(f*x + e)^3 - 16*a^3*c^2*cos(f*x + e)^2 + 10*a^3*c^2*cos(f*x + e) + 8*a^3*
c^2)*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= a^3c^2\left(\int \sec(e+fx)\,dx + \int \sec^2(e+fx)\,dx + \int \left(-2\sec^3(e+fx)\right)\,dx + \int \left(-2\sec^4(e+fx)\right)\,dx + \int \sec^5(e+fx)\,dx + \int \sec^6(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*c**2*(Integral(sec(e + f*x), x) + Integral(sec(e + f*x)**2, x) + Integ
ral(-2*sec(e + f*x)**3, x) + Integral(-2*sec(e + f*x)**4, x) + Integral(sec
(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(86) = 172.

Time = 0.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.41

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$= \frac{16\left(3\tan(fx+e)^5 + 10\tan(fx+e)^3 + 15\tan(fx+e)\right)a^3c^2 - 160\left(\tan(fx+e)^3 + 3\tan(fx+e)\right)a^3c^2}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*(16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c^2
- 160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 - 15*a^3*c^2*(2*(3*sin(f*x
+ e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(si
n(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 120*a^3*c^2*(2*sin(f*x + e)/(s
in(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a
^3*c^2*log(sec(f*x + e) + tan(f*x + e)) + 240*a^3*c^2*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.69

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx$$
$$=\frac{15\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\,a^3c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(15\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9-70\,a^3c^2\tan}{}}{40\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/40*(15*a^3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c^2*log(abs(ta
n(1/2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c^2*tan(1/2*f*x + 1/2*e)^9 - 70*a^3*c^
2*tan(1/2*f*x + 1/2*e)^7 + 128*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 70*a^3*c^2*
tan(1/2*f*x + 1/2*e)^3 - 15*a^3*c^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/
2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 17.53 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.00

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2\,dx=\frac{3\,a^3\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$
$$-\frac{\frac{3\,a^3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{4}-\frac{7\,a^3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{2}+\frac{32\,a^3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5}+\frac{7\,a^3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{2}-\frac{3\,a^3\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (3*a^3*c^2*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((7*a^3*c^2*tan(e/2 + (f*x)/2
)^3)/2 + (32*a^3*c^2*tan(e/2 + (f*x)/2)^5)/5 - (7*a^3*c^2*tan(e/2 + (f*x)/2
)^7)/2 + (3*a^3*c^2*tan(e/2 + (f*x)/2)^9)/4 - (3*a^3*c^2*tan(e/2 + (f*x)/2)
)/4)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f
*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.26 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = \frac{5a^3c\,\text{arctanh}(\sin(e+fx))}{8f} - \frac{3a^3c\sec(e+fx)\tan(e+fx)}{8f} - \frac{a^3c\sec^3(e+fx)\tan(e+fx)}{4f} - \frac{2a^3c\tan^3(e+fx)}{3f}$$

```
[Out] 5/8*a^3*c*arctanh(sin(f*x+e))/f-3/8*a^3*c*sec(f*x+e)*tan(f*x+e)/f-1/4*a^3*c
*sec(f*x+e)^3*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4043, 2691, 3855, 2687, 30, 3853}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = \frac{5a^3c\,\text{arctanh}(\sin(e+fx))}{8f} - \frac{2a^3c\tan^3(e+fx)}{3f} - \frac{a^3c\tan(e+fx)\sec^3(e+fx)}{4f} - \frac{3a^3c\tan(e+fx)\sec(e+fx)}{8f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (5*a^3*c*ArcTanh[Sin[e + f*x]])/(8*f) - (3*a^3*c*Sec[e + f*x]*Tan[e + f*x])
/(8*f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (2*a^3*c*Tan[e + f*x]^
3)/(3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left((ac)\int \left(a^2\sec(e+fx)\tan^2(e+fx)+2a^2\sec^2(e+fx)\tan^2(e+fx)\right.\right.\\
&\qquad\left.\left.+a^2\sec^3(e+fx)\tan^2(e+fx)\right)\,dx\right)\\
&= -\left((a^3c)\int \sec(e+fx)\tan^2(e+fx)\,dx\right)\\
&\quad-(a^3c)\int \sec^3(e+fx)\tan^2(e+fx)\,dx-(2a^3c)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\\
&= -\frac{a^3c\sec(e+fx)\tan(e+fx)}{2f}-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{4f}\\
&\quad+\frac{1}{4}(a^3c)\int \sec^3(e+fx)\,dx+\frac{1}{2}(a^3c)\int \sec(e+fx)\,dx\\
&\quad-\frac{(2a^3c)\,\text{Subst}\left(\int x^2\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a^3c\text{arctanh}(\sin(e+fx))}{2f}-\frac{3a^3c\sec(e+fx)\tan(e+fx)}{8f}\\
&\quad-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{4f}-\frac{2a^3c\tan^3(e+fx)}{3f}+\frac{1}{8}(a^3c)\int \sec(e+fx)\,dx\\
&= \frac{5a^3c\text{arctanh}(\sin(e+fx))}{8f}-\frac{3a^3c\sec(e+fx)\tan(e+fx)}{8f}\\
&\quad-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{4f}-\frac{2a^3c\tan^3(e+fx)}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx\\
&=\frac{a^3c(60\text{arctanh}(\sin(e+fx))-\sec^4(e+fx)(33\sin(e+fx)+16\sin(2(e+fx))+9\sin(3(e+fx))-8\sin(}{96f}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
[Out] (a^3*c*(60*ArcTanh[Sin[e + f*x]] - Sec[e + f*x]^4*(33*Sin[e + f*x] + 16*Sin
[2*(e + f*x)] + 9*Sin[3*(e + f*x)] - 8*Sin[4*(e + f*x)])))/(96*f)
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2a}{f}$
default	$\frac{-a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+2a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2a}{f}$
parts	$\frac{a^3c\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{2a^3c\tan(fx+e)}{f}+\frac{2a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\right.\right.}{}$
norman	$\frac{-\frac{5a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{73a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12f}+\frac{55a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}-\frac{5a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{5a^3c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f}+\frac{5a^3c}{}$
risch	$\frac{ia^3c(9\,e^{7i(fx+e)}+48\,e^{6i(fx+e)}+33\,e^{5i(fx+e)}+48\,e^{4i(fx+e)}-33\,e^{3i(fx+e)}+16\,e^{2i(fx+e)}-9\,e^{i(fx+e)}+16)}{12f\left(1+e^{2i(fx+e)}\right)^4}-\frac{5a^3c\ln(e^{i(}}{8f}$
parallelrisch	$-\frac{3a^3c\left(\left(\frac{10\cos(2fx+2e)}{3}+\frac{5\cos(4fx+4e)}{6}+\frac{5}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{10\cos(2fx+2e)}{3}-\frac{5\cos(4fx+4e)}{6}-\frac{5}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.\right.}{4f(3+\cos(4fx+4e)+4\cos(2fx+2e))}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-a^3*c*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+
e)+tan(f*x+e)))+2*a^3*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2*a^3*c*tan(f*x+
e)+a^3*c*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{15\,a^3c\cos(fx+e)^4\log(\sin(fx+e)+1)-15\,a^3c\cos(fx+e)^4\log(-\sin(fx+e)+1)+2\,(16\,a^3c\cos(f}{48\,f\cos(fx+e)^4}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*a^3*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e)
^4*log(-sin(f*x + e) + 1) + 2*(16*a^3*c*cos(f*x + e)^3 - 9*a^3*c*cos(f*x +
e)^2 - 16*a^3*c*cos(f*x + e) - 6*a^3*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$= -a^3c\bigg(\int(-\sec(e+fx))\,dx + \int(-2\sec^2(e+fx))\,dx + \int 2\sec^4(e+fx)\,dx + \int \sec^5(e+fx)\,dx\bigg)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*c*(Integral(-sec(e + f*x), x) + Integral(-2*sec(e + f*x)**2, x) + Int
egral(2*sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.55

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx =$$
$$-\frac{32\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c-3a^3c\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\sin(fx+e)+1)+3\log\ldots\right.}{48f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c - 3*a^3*c*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin
(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 48*a^3*c*log(sec(f*x + e) + tan
(f*x + e)) - 96*a^3*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$= \frac{15a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(15a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7-55a^3c\tan\left(\frac{1}{2}fx\ldots\right.}{\left(\tan(\ldots\right.}}{24f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/24*(15*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c*tan(1/2*f*x + 1/2*e)^7 - 55*a^3*c*tan(1/
2*f*x + 1/2*e)^5 + 73*a^3*c*tan(1/2*f*x + 1/2*e)^3 + 15*a^3*c*tan(1/2*f*x +
 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 16.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.70

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$= \frac{5\,a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f} - \frac{\frac{5\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{55\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{73\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{5\,c\,a^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (5*a^3*c*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((5*a^3*c*tan(e/2 + (f*x)/2))/4
 + (73*a^3*c*tan(e/2 + (f*x)/2)^3)/12 - (55*a^3*c*tan(e/2 + (f*x)/2)^5)/12
+ (5*a^3*c*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2
+ (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.27 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{15a^3\text{arctanh}(\sin(e+fx))}{2cf} - \frac{10a^3\tan(e+fx)}{cf} - \frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))}$$

```
[Out] -15/2*a^3*arctanh(sin(f*x+e))/c/f-10*a^3*tan(f*x+e)/c/f-5/2*a^3*sec(f*x+e)*
tan(f*x+e)/c/f-2*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4042, 3873, 3852, 8, 4131, 3855}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{15a^3\text{arctanh}(\sin(e+fx))}{2cf} - \frac{10a^3\tan(e+fx)}{cf} - \frac{5a^3\tan(e+fx)\sec(e+fx)}{2cf} - \frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
```

```
[Out] (-15*a^3*ArcTanh[Sin[e + f*x]])/(2*c*f) - (10*a^3*Tan[e + f*x])/(c*f) - (5*
a^3*Sec[e + f*x]*Tan[e + f*x])/(2*c*f) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(c - c*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(5a)\int\sec(e+fx)(a+a\sec(e+fx))^2\,dx}{c}$$

$$= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(5a)\int\sec(e+fx)\left(a^2+a^2\sec^2(e+fx)\right)\,dx}{c} - \frac{(10a^3)\int\sec^2(e+fx)\,dx}{c}$$

$$= -\frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))} - \frac{(15a^3)\int\sec(e+fx)\,dx}{2c} + \frac{(10a^3)\,\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{cf}$$

$$= -\frac{15a^3\text{arctanh}(\sin(e+fx))}{2cf} - \frac{10a^3\tan(e+fx)}{cf} - \frac{5a^3\sec(e+fx)\tan(e+fx)}{2cf} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.57 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.65

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{7}{2},\frac{9}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^3\tan(e+fx)}{7cf\sqrt{2-2\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x]),x]
[Out] -1/7*(a^3*Hypergeometric2F1[3/2, 7/2, 9/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
 + f*x])^3*Tan[e + f*x])/(c*f*Sqrt[2 - 2*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

method	result
parallelrisch	$\frac{15\left((1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{14\left(\cos(fx+e)-\frac{12\cos(2fx+2e)}{7}-\frac{1}{}\right.}{15}\right.}{2cf(1+\cos(2fx+2e))}$
derivativedivides	$\frac{8a^3\left(-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{15}{}\right.}{fc}$
default	$\frac{8a^3\left(-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{7}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{15}{}\right.}{fc}$
risch	$\frac{ia^3\left(17\,e^{4i(fx+e)}-9\,e^{3i(fx+e)}+39\,e^{2i(fx+e)}-7\,e^{i(fx+e)}+24\right)}{fc\left(e^{i(fx+e)}-1\right)\left(1+e^{2i(fx+e)}\right)^2}+\frac{15a^3\ln\left(e^{i(fx+e)}-i\right)}{2cf}-\frac{15a^3\ln\left(e^{i(fx+e)}+i\right)}{2cf}$
norman	$\frac{-\frac{8a^3}{cf}+\frac{33a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}-\frac{40a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}+\frac{15a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{15a^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2cf}-\frac{15a^3\ln\left(\tan\left(\frac{fx}{2}+\right.\right.}{2cf}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 15/2*((1+cos(2*f*x+2*e))*ln(tan(1/2*f*x+1/2*e)-1)+(-1-cos(2*f*x+2*e))*ln(ta
n(1/2*f*x+1/2*e)+1)-14/15*(cos(f*x+e)-12/7*cos(2*f*x+2*e)-11/7)*cot(1/2*f*x
+1/2*e))*a^3/c/f/(1+cos(2*f*x+2*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx =$$

$$-\frac{15\,a^3\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)\sin\left(fx+e\right)-15\,a^3\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)\sin\left(\right.}{4\,cf\cos\left(fx+e\right)^2\sin\left(fx+\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/4*(15*a^3*cos(f*x + e)^2*log(sin(f*x + e) + 1)*sin(f*x + e) - 15*a^3*cos
(f*x + e)^2*log(-sin(f*x + e) + 1)*sin(f*x + e) - 48*a^3*cos(f*x + e)^3 - 3
4*a^3*cos(f*x + e)^2 + 16*a^3*cos(f*x + e) + 2*a^3)/(c*f*cos(f*x + e)^2*sin
(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\left(\int \frac{\sec(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{3\sec^2(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{3\sec^3(e+fx)}{\sec(e+fx)-1}\,dx+\int \frac{\sec^4(e+fx)}{\sec(e+fx)-1}\,dx\right)}{c}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*
x)**2/(sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x) - 1
), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) - 1), x))/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(97) = 194.

Time = 0.21 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.87

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{a^3\left(\frac{2\left(\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{2c\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{c\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c}\right)+6a^3\left(\frac{\frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\cdots}{\frac{c\sin(fx+e)}{\cos(fx+e)+1}-\frac{c\sin\cdots}{(\cos(\cdots}}\cdots\right.}{\cdots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(a^3*(2*(5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 1)/(c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*c*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/c - 3*log(sin(f*x + e)/(cos(f*x + e) + 1
) - 1)/c) + 6*a^3*((3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(c*sin(f*x +
 e)/(cos(f*x + e) + 1) - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + log(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
)/c) + 6*a^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c - log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/c - (cos(f*x + e) + 1)/(c*sin(f*x + e))) - 2*a^3*(
cos(f*x + e) + 1)/(c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = -\frac{\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c}-\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c}-\frac{16\,a^3}{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}-\frac{2\left(7\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-9\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2c}}{2\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/2*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c - 15*a^3*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/c - 16*a^3/(c*tan(1/2*f*x + 1/2*e)) - 2*(7*a^3*tan(1/2*f
*x + 1/2*e)^3 - 9*a^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2
*c))/f
```

Mupad [B] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c-c\sec(e+fx)}\,dx = \frac{15\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-25\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+8\,a^3}{f\,\left(c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}-\frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c\,f}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] (15*a^3*tan(e/2 + (f*x)/2)^4 - 25*a^3*tan(e/2 + (f*x)/2)^2 + 8*a^3)/(f*(c*t
an(e/2 + (f*x)/2) - 2*c*tan(e/2 + (f*x)/2)^3 + c*tan(e/2 + (f*x)/2)^5)) - (
15*a^3*atanh(tan(e/2 + (f*x)/2)))/(c*f)
```

3.28 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 119

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2} dx = \frac{5a^3\text{arctanh}(\sin(e+fx))}{c^2f} + \frac{5a^3\tan(e+fx)}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f(c^2-c^2\sec(e+fx))}$$

```
[Out] 5*a^3*arctanh(sin(f*x+e))/c^2/f+5*a^3*tan(f*x+e)/c^2/f-2/3*a*(a+a*sec(f*x+e
))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^2+10/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f
/(c^2-c^2*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4042, 3872, 3855, 3852, 8}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2} dx = \frac{5a^3\text{arctanh}(\sin(e+fx))}{c^2f} + \frac{5a^3\tan(e+fx)}{c^2f} + \frac{10\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\left(c^2-c^2\sec(e+fx)\right)} - \frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c-c\sec(e+fx))^2}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[e + f*x]])/(c^2*f) + (5*a^3*Tan[e + f*x])/(c^2*f) - (2*a
*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(c - c*Sec[e + f*x])^2) + (10*(a
^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c^2 - c^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2}-\frac{(5a)\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c-c\sec(e+fx)}\,dx}{3c}\\
&= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2}+\frac{10(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\left(c^2-c^2\sec(e+fx)\right)}\\
&\quad+\frac{(5a^2)\int\sec(e+fx)(a+a\sec(e+fx))\,dx}{c^2}\\
&= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2}+\frac{10(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\left(c^2-c^2\sec(e+fx)\right)}\\
&\quad+\frac{(5a^3)\int\sec(e+fx)\,dx}{c^2}+\frac{(5a^3)\int\sec^2(e+fx)\,dx}{c^2}
\end{aligned}$$

$$= \frac{5a^3\text{arctanh}(\sin(e+fx))}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f(c^2-c^2\sec(e+fx))} - \frac{(5a^3)\,\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{c^2f}$$

$$= \frac{5a^3\text{arctanh}(\sin(e+fx))}{c^2f} + \frac{5a^3\tan(e+fx)}{c^2f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{3f(c-c\sec(e+fx))^2} + \frac{10(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f(c^2-c^2\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.66 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{a^3\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{7}{2},\frac{9}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^3\tan(e+fx)}{14c^2f\sqrt{2-2\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^2,x]
[Out] -1/14*(a^3*Hypergeometric2F1[5/2, 7/2, 9/2, (1 + Sec[e + f*x])/2]*(1 + Sec[
e + f*x])^3*Tan[e + f*x])/(c^2*f*Sqrt[2 - 2*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{4a^3\left(-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^2}$
default	$\frac{4a^3\left(-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{f\,c^2}$
parallelrisch	$-\frac{5a^3\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+\frac{17\left(\cos(fx+e)-\frac{23\cos(2fx+2e)}{68}-\frac{29}{68}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots}{15}\right)}{f\cos(fx+e)c^2}$
risch	$-\frac{2ia^3\left(12\,e^{4i(fx+e)}-51\,e^{3i(fx+e)}+41\,e^{2i(fx+e)}-57\,e^{i(fx+e)}+23\right)}{3f\,c^2\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}-1\right)^3}-\frac{5a^3\ln\left(e^{i(fx+e)}-i\right)}{c^2f}+\frac{5a^3\ln\left(e^{i(fx+e)}+i\right)}{c^2f}$
norman	$\frac{\frac{4a^3}{3cf}+\frac{4a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{cf}-\frac{22a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{cf}+\frac{80a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{3cf}-\frac{10a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{5a^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{c^2f}+\frac{5a^3\cdots}{\cdots}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 4/f*a^3/c^2*(-1/4/(tan(1/2*f*x+1/2*e)+1)+5/4*ln(tan(1/2*f*x+1/2*e)+1)-1/4/(
tan(1/2*f*x+1/2*e)-1)-5/4*ln(tan(1/2*f*x+1/2*e)-1)-1/3/tan(1/2*f*x+1/2*e)^3
-2/tan(1/2*f*x+1/2*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{46\,a^3\cos(fx+e)^3-22\,a^3\cos(fx+e)^2-62\,a^3\cos(fx+e)+6\,a^3-15\,\left(a^3\cos(fx+e)^2-a^3\cos(fx\cdots\right.}{6\,\left(c^2f\cos(fx+e)\cdots\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(46*a^3*cos(f*x + e)^3 - 22*a^3*cos(f*x + e)^2 - 62*a^3*cos(f*x + e) +
 6*a^3 - 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(sin(f*x + e) + 1)*s
in(f*x + e) + 15*(a^3*cos(f*x + e)^2 - a^3*cos(f*x + e))*log(-sin(f*x + e)
+ 1)*sin(f*x + e))/((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{3\sec^3(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx + \int \frac{\sec^4(e+\ldots}{\sec^2(e+fx)-2\,\mathrm{s}\ldots}\right.}{c^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + In
tegral(3*sec(e + f*x)**2/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integ
ral(3*sec(e + f*x)**3/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x) + Integral
(sec(e + f*x)**4/(sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x))/c**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(117) = 234.

Time = 0.23 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.93

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = -\frac{a^3\left(\frac{\frac{14\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{27\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+1}{\frac{c^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{c^2\sin(fx+e)^5}{(\cos(fx+e)+1)^5}} - \frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2} + \frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^2}\right) - 3a^3\left(\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^2} - \ldots\right.}{\ldots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(a^3*((14*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 27*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + 1)/(c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - c^2*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5) - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
)/c^2 + 12*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^2) - 3*a^3*(6*log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/c^2 - 6*log(sin(f*x + e)/(cos(f*x + e) +
1) - 1)/c^2 - (9*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)*(cos(f*x + e) + 1
)^3/(c^2*sin(f*x + e)^3)) + 3*a^3*(3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3) - a^3*(3*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(c^2*sin(f*x + e)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{c^2}-\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{c^2}-\frac{6\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)c^2}-\frac{4\left(6\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a^3\right)}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}}{3\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^2 - 15*a^3*log(abs(tan(1/2
*f*x + 1/2*e) - 1))/c^2 - 6*a^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)
^2 - 1)*c^2) - 4*(6*a^3*tan(1/2*f*x + 1/2*e)^2 + a^3)/(c^2*tan(1/2*f*x + 1/
2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 13.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^2}\,dx = \frac{10\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^2\,f} + \frac{-10\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{20\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+\frac{4\,a^3}{3}}{c^2\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (10*a^3*atanh(tan(e/2 + (f*x)/2)))/(c^2*f) + ((20*a^3*tan(e/2 + (f*x)/2)^2)
/3 - 10*a^3*tan(e/2 + (f*x)/2)^4 + (4*a^3)/3)/(c^2*f*tan(e/2 + (f*x)/2)^3*(
tan(e/2 + (f*x)/2)^2 - 1))
```

3.29 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 132

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3} dx = -\frac{a^3\text{arctanh}(\sin(e+fx))}{c^3f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf(c-c\sec(e+fx))^2} - \frac{2a^3\tan(e+fx)}{f(c^3-c^3\sec(e+fx))}$$

```
[Out] -a^3*arctanh(sin(f*x+e))/c^3/f-2/5*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*s
ec(f*x+e))^3+2/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^2-2*a
^3*tan(f*x+e)/f/(c^3-c^3*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4042, 3855}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3} dx = -\frac{a^3\text{arctanh}(\sin(e+fx))}{c^3f} - \frac{2a^3\tan(e+fx)}{f(c^3-c^3\sec(e+fx))} + \frac{2\tan(e+fx)(a^3\sec(e+fx)+a^3)}{3cf(c-c\sec(e+fx))^2} - \frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f(c-c\sec(e+fx))^3}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Sin[e + f*x]])/(c^3*f)) - (2*a*(a + a*Sec[e + f*x])^2*Tan[e
+ f*x])/(5*f*(c - c*Sec[e + f*x])^3) + (2*(a^3 + a^3*Sec[e + f*x])*Tan[e +
f*x])/(3*c*f*(c - c*Sec[e + f*x])^2) - (2*a^3*Tan[e + f*x])/(f*(c^3 - c^3*S
ec[e + f*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} - \frac{a\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^2}\,dx}{c} \\
&= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} \\
&\quad + \frac{2(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf(c-c\sec(e+fx))^2} + \frac{a^2\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{c-c\sec(e+fx)}\,dx}{c^2} \\
&= -\frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} + \frac{2(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf(c-c\sec(e+fx))^2} \\
&\quad - \frac{2a^3\tan(e+fx)}{f(c^3-c^3\sec(e+fx))} - \frac{a^3\int\sec(e+fx)\,dx}{c^3} \\
&= -\frac{a^3\operatorname{arctanh}(\sin(e+fx))}{c^3f} - \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f(c-c\sec(e+fx))^3} \\
&\quad + \frac{2(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf(c-c\sec(e+fx))^2} - \frac{2a^3\tan(e+fx)}{f(c^3-c^3\sec(e+fx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^3\left(-\frac{26\cot\left(\frac{1}{2}(e+fx)\right)}{15f}+\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^2\left(\frac{1}{2}(e+fx)\right)}{15f}-\frac{2\cot\left(\frac{1}{2}(e+fx)\right)\csc^4\left(\frac{1}{2}(e+fx)\right)}{5f}-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\right.}{c^3}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^3,x]
```

```
[Out] -((a^3*((-26*Cot[(e + f*x)/2])/(15*f) + (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2
]^2)/(15*f) - (2*Cot[(e + f*x)/2]*Csc[(e + f*x)/2]^4)/(5*f) - Log[Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]]/f + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]/f)
)/c^3)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{2a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}\right)}{f\,c^3}$
default	$\frac{2a^3\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}\right)}{f\,c^3}$
parallelrisch	$\frac{a^3\left(6\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5+10\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+30\cot\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15c^3f}$
risch	$\frac{4ia^3\left(15\,e^{4i(fx+e)}-30\,e^{3i(fx+e)}+100\,e^{2i(fx+e)}-50\,e^{i(fx+e)}+13\right)}{15f\,c^3\left(e^{i(fx+e)}-1\right)^5}+\frac{a^3\ln\left(e^{i(fx+e)}-i\right)}{c^3f}-\frac{a^3\ln\left(e^{i(fx+e)}+i\right)}{c^3f}$
norman	$\frac{-\frac{2a^3}{5cf}+\frac{8a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{15cf}-\frac{6a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{5cf}+\frac{22a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{5cf}-\frac{16a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{3cf}+\frac{2a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{a^3\ln\left(\tan\left(\frac{fx}{2}\right.\right.}{c^3f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 2/f*a^3/c^3*(-1/2*ln(tan(1/2*f*x+1/2*e)+1)+1/5/tan(1/2*f*x+1/2*e)^5+1/3/tan
(1/2*f*x+1/2*e)^3+1/tan(1/2*f*x+1/2*e)+1/2*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.33

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{52\,a^3\cos(fx+e)^3 - 44\,a^3\cos(fx+e)^2 - 4\,a^3\cos(fx+e) + 92\,a^3 - 15\left(a^3\cos(fx+e)^2 - 2\,a^3\cos(fx+\right.}{30\left(c^3f\cos(fx+e\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(52*a^3*cos(f*x + e)^3 - 44*a^3*cos(f*x + e)^2 - 4*a^3*cos(f*x + e) +
92*a^3 - 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*log(sin(f*x + e
) + 1)*sin(f*x + e) + 15*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*lo
g(-sin(f*x + e) + 1)*sin(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x
 + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec^3(e+fx)-3\sec^2(e+fx)+3\sec(e+fx)-1}\,dx + \int \frac{3\sec^3}{\sec^3(e+fx)-3\sec^2(\right.}{c^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e
 + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 - 3*sec(e +
f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x
)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)*
*4/(sec(e + f*x)**3 - 3*sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x))/c**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(131) = 262$.

Time = 0.23 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.34

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{a^3\left(\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{105\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+3\right)(\cos(fx+e)+1)^5}{c^3\sin(fx+e)^5}\right)-\frac{3a^3\left(\frac{10\sin(fx+\cdots}{(\cos(fx+e)\cdots}\right.}{\cdots}}{\cdots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(a^3*(60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/c^3 - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 105*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*
sin(f*x + e)^5)) - 3*a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 15*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^
5) + a^3*(10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x + e)^5) + 9*a^3*(5*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(c^3*sin(f*x +
e)^5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = -\frac{\frac{15a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{c^3}-\frac{15a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{c^3}-\frac{2\left(15a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+5a^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3a^3\right)}{c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}}{15f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/15*(15*a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/c^3 - 15*a^3*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/c^3 - 2*(15*a^3*tan(1/2*f*x + 1/2*e)^4 + 5*a^3*tan(1/
2*f*x + 1/2*e)^2 + 3*a^3)/(c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^3}\,dx = \frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+\frac{2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2}{3}+\frac{2\,a^3}{5}}{c^3\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5} - \frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{c^3\,f}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^3),x)
```

```
[Out] ((2*a^3*tan(e/2 + (f*x)/2)^2)/3 + 2*a^3*tan(e/2 + (f*x)/2)^4 + (2*a^3)/5)/(
c^3*f*tan(e/2 + (f*x)/2)^5) - (2*a^3*atanh(tan(e/2 + (f*x)/2)))/(c^3*f)
```

3.30 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4} dx$

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4} dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{7f(c-c\sec(e+fx))^4}$$

```
[Out] -1/7*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4} dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{7f(c-c\sec(e+fx))^4}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
```

```
[Out] -1/7*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^4)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{7f(c-c\sec(e+fx))^4}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx=-\frac{a^3\cot^7\left(\frac{1}{2}(e+fx)\right)}{7c^4f}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^4,x]
[Out] -1/7*(a^3*Cot[(e + f*x)/2]^7)/(c^4*f)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$-\frac{a^3}{7f\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$	23
default	$-\frac{a^3}{7f\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$	23
parallelrisch	$-\frac{a^3}{7f\,c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$	23
risch	$\frac{2ia^3\left(7\,e^{6i(fx+e)}+35\,e^{4i(fx+e)}+21\,e^{2i(fx+e)}+1\right)}{7f\,c^4\left(e^{i(fx+e)}-1\right)^7}$	61
norman	$\frac{\frac{a^3}{7cf}-\frac{3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{7cf}+\frac{3a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{7cf}-\frac{a^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{7cf}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}$	109

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
[Out] -1/7/f*a^3/c^4/tan(1/2*f*x+1/2*e)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx = \frac{a^3\cos(fx+e)^4+4\,a^3\cos(fx+e)^3+6\,a^3\cos(fx+e)^2+4\,a^3\cos(fx+e)+a^3}{7\left(c^4f\cos(fx+e)^3-3\,c^4f\cos(fx+e)^2+3\,c^4f\cos(fx+e)-c^4f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + a^3)/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*
c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec^4(e+fx)-4\sec^3(e+fx)+6\sec^2(e+fx)-4\sec(e+fx)+1}\,dx + \cdots}{c^4}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e
+ f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f
*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**3/(sec(e + f*x)**4 - 4*sec(e + f*x)**3 + 6*sec(e +
 f*x)**2 - 4*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)
**4 - 4*sec(e + f*x)**3 + 6*sec(e + f*x)**2 - 4*sec(e + f*x) + 1), x))/c**4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(37) = 74$.

Time = 0.23 (sec) , antiderivative size = 356, normalized size of antiderivative = 9.37

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx =$$
$$-\frac{\frac{a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{35\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+5\right)(\cos(fx+e)+1)^7}{c^4\sin(fx+e)^7} - \frac{a^3\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{105\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-15\right)(\cos \cdots}{c^4\sin(fx+e)^7}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] -1/280*(a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5)*(cos(f*x +
e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x +
e) + 1)^6 - 15)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x + e)^7) - a^3*(21*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*
```

```
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(c^4*sin(f*x
+ e)^7) + a^3*(21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 105*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15)*(cos(f*
x + e) + 1)^7/(c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx=-\frac{a^3}{7\,c^4f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/7*a^3/(c^4*f*tan(1/2*f*x + 1/2*e)^7)
```

Mupad [B] (verification not implemented)

Time = 12.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx=-\frac{a^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^7}{7\,c^4\,f}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^4),x)
```

```
[Out] -(a^3*cot(e/2 + (f*x)/2)^7)/(7*c^4*f)
```

3.31 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5} - \frac{(a+a\sec(e+fx))^3\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}$$

```
[Out] -1/9*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^5-1/63*(a+a*sec(f*x+e
))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{63cf(c-c\sec(e+fx))^4} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{9f(c-c\sec(e+fx))^5}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/9*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^5) - ((a
 + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*c*f*(c - c*Sec[e + f*x])^4)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
```

```
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx}{9c}\\ &= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{9f(c-c\sec(e+fx))^5}-\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{63cf(c-c\sec(e+fx))^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^3(-8+\sec(e+fx))(1+\sec(e+fx))^3\tan(e+fx)}{63c^5f(-1+\sec(e+fx))^5}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^5,x]
```

```
[Out] -1/63*(a^3*(-8 + Sec[e + f*x])*(1 + Sec[e + f*x])^3*Tan[e + f*x])/(c^5*f*(-
1 + Sec[e + f*x])^5)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.48

method	result
parallelrisch	$\frac{a^{3} \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{7} \left(7 \cot\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-9\right)}{126 c^{5} f}$
derivativedivides	$\frac{a^{3} \left(\frac{1}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{2 f \,c^{5}}$
default	$\frac{a^{3} \left(\frac{1}{9 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}-\frac{1}{7 \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{7}}\right)}{2 f \,c^{5}}$
risch	$\frac{2 i a^{3} \left(63 \,{\mathrm e}^{8 i (f x+e)}-63 \,{\mathrm e}^{7 i (f x+e)}+483 \,{\mathrm e}^{6 i (f x+e)}-315 \,{\mathrm e}^{5 i (f x+e)}+693 \,{\mathrm e}^{4 i (f x+e)}-189 \,{\mathrm e}^{3 i (f x+e)}+225 \,{\mathrm e}^{2 i (f x+e)}-9 \,{\mathrm e}^{i (f x+e)}+\right.}{63 f \,c^{5} \left({\mathrm e}^{i (f x+e)}-1\right)^{9}}$
norman	$\frac{-\frac{a^{3}}{18 c f}+\frac{5 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}}{21 c f}-\frac{8 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{4}}{21 c f}+\frac{17 a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{6}}{63 c f}-\frac{a^{3} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{8}}{14 c f}}{\left(\tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{2}-1\right)^{3} c^{4} \tan\left(\frac{f x}{2}+\frac{e}{2}\right)^{9}}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/126*a^3*cot(1/2*f*x+1/2*e)^7*(7*cot(1/2*f*x+1/2*e)^2-9)/c^5/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.75

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{8\,a^3\cos(fx+e)^5+31\,a^3\cos(fx+e)^4+44\,a^3\cos(fx+e)^3+26\,a^3\cos(fx+e)^2+4\,a^3\cos(fx+e)-a^3}{63\,\left(c^5f\cos(fx+e)^4-4\,c^5f\cos(fx+e)^3+6\,c^5f\cos(fx+e)^2-4\,c^5f\cos(fx+e)+c^5f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/63*(8*a^3*cos(f*x + e)^5 + 31*a^3*cos(f*x + e)^4 + 44*a^3*cos(f*x + e)^3
+ 26*a^3*cos(f*x + e)^2 + 4*a^3*cos(f*x + e) - a^3)/((c^5*f*cos(f*x + e)^4
- 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) +
c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)-10\sec^2(e+fx)+5\sec(e+fx)-1}\,dx + \int \frac{3\sec^2(e+fx)}{\sec^5(e+fx)-5\sec^4(e+fx)+10\sec^3(e+fx)}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(
e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec
(e + f*x)**2/(sec(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10
*sec(e + f*x)**2 + 5*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**3/(se
c(e + f*x)**5 - 5*sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2
 + 5*sec(e + f*x) - 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**5 - 5*
sec(e + f*x)**4 + 10*sec(e + f*x)**3 - 10*sec(e + f*x)**2 + 5*sec(e + f*x)
- 1), x))/c**5
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(78) = 156$.

Time = 0.23 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{\frac{a^3\left(\frac{180\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{378\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{315\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-35\right)(\cos(fx+e)+1)^9}{c^5\sin(fx+e)^9}+\frac{15a^3\left(\frac{18\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{42\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{63}{(\cos}\right.}{c^5\sin(fx+e}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] -1/5040*(a^3*(180*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 378*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 315*sin(f*
x + e)^8/(cos(f*x + e) + 1)^8 - 35)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^
9) + 15*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 42*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 + 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x +
e) + 1)^9/(c^5*sin(f*x + e)^9) - 5*a^3*(18*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 42*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 63*sin(f*x + e)^8/(cos(f*x +
 e) + 1)^8 + 7)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9) + 21*a^3*(18*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 45*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 -
5)*(cos(f*x + e) + 1)^9/(c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = -\frac{9\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-7\,a^3}{126\,c^5f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] -1/126*(9*a^3*tan(1/2*f*x + 1/2*e)^2 - 7*a^3)/(c^5*f*tan(1/2*f*x + 1/2*e)^9
)
```

Mupad [B] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx = \frac{a^3\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\left(7\cot\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-9\right)}{126\,c^5\,f}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (a^3*cot(e/2 + (f*x)/2)^7*(7*cot(e/2 + (f*x)/2)^2 - 9))/(126*c^5*f)
```

3.32 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{693c^2f(c-c\sec(e+fx))^4}$$

```
[Out] -1/11*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^6-2/99*(a+a*sec(f*x+
e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^5-2/693*(a+a*sec(f*x+e))^3*tan(f*x+e)
/c^2/f/(c-c*sec(f*x+e))^4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = -\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{693c^2f(c-c\sec(e+fx))^4} - \frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{99cf(c-c\sec(e+fx))^5} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{11f(c-c\sec(e+fx))^6}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/11*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^6) - (2
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(99*c*f*(c - c*Sec[e + f*x])^5) - (2*
(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*c^2*f*(c - c*Sec[e + f*x])^4)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6}+\frac{2\int\frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx}{11c}\\
&= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6}\\
&\quad-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5}+\frac{2\int\frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}\,dx}{99c^2}\\
&= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{11f(c-c\sec(e+fx))^6}-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{99cf(c-c\sec(e+fx))^5}\\
&\quad-\frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{693c^2f(c-c\sec(e+fx))^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.49

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$$
$$= -\frac{a^3(1+\sec(e+fx))^3\left(79-18\sec(e+fx)+2\sec^2(e+fx)\right)\tan(e+fx)}{693c^6f(-1+\sec(e+fx))^6}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^6,x]
```

```
[Out] -1/693*(a^3*(1 + Sec[e + f*x])^3*(79 - 18*Sec[e + f*x] + 2*Sec[e + f*x]^2)*
Tan[e + f*x])/(c^6*f*(-1 + Sec[e + f*x])^6)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

method	result
parallelrisch	$-\frac{a^3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7\left(63\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^4-154\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2+99\right)}{2772c^6f}$
derivativedivides	$\frac{a^3\left(-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{2}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{4fc^6}$
default	$\frac{a^3\left(-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{2}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{4fc^6}$
risch	$\frac{2ia^3\left(693\,e^{10i(fx+e)}-1386\,e^{9i(fx+e)}+8085\,e^{8i(fx+e)}-10626\,e^{7i(fx+e)}+21252\,e^{6i(fx+e)}-15246\,e^{5i(fx+e)}+15444\,e^{4i(fx}\right.}{693fc^6\left(e^{i(fx+e)}-1\right)^{11}}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2772*a^3*cot(1/2*f*x+1/2*e)^7*(63*cot(1/2*f*x+1/2*e)^4-154*cot(1/2*f*x+1
/2*e)^2+99)/c^6/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = \frac{79\,a^3\cos(fx+e)^6+298\,a^3\cos(fx+e)^5+404\,a^3\cos(fx+e)^4+216\,a^3\cos(fx+e)^3+19\,a^3\cos(fx+e)}{693\left(c^6f\cos(fx+e)^5-5\,c^6f\cos(fx+e)^4+10\,c^6f\cos(fx+e)^3-10\,c^6f\cos(fx+e)^2+5\,c^6f\cos(fx\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/693*(79*a^3*cos(f*x + e)^6 + 298*a^3*cos(f*x + e)^5 + 404*a^3*cos(f*x + e
)^4 + 216*a^3*cos(f*x + e)^3 + 19*a^3*cos(f*x + e)^2 - 10*a^3*cos(f*x + e)
+ 2*a^3)/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x
 + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x +
 e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = \frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^6(e+fx)-6\sec^5(e+fx)+15\sec^4(e+fx)-20\sec^3(e+fx)+15\sec^2(e+fx)-6\sec(e+fx)+1}\,dx+\int \frac{}{\sec^6(e+fx)-6\sec^5(e+fx)+15\,\mathrm{s}}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*sec(e
 + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x) + 1),
 x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**6 - 6*sec(e + f*x)**5 + 15*
sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e + f*x)
+ 1), x) + Integral(3*sec(e + f*x)**3/(sec(e + f*x)**6 - 6*sec(e + f*x)**5
+ 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e +
f*x) + 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**6 - 6*sec(e + f*x)*
*5 + 15*sec(e + f*x)**4 - 20*sec(e + f*x)**3 + 15*sec(e + f*x)**2 - 6*sec(e
 + f*x) + 1), x))/c**6
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(118) = 236$.

Time = 0.22 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.28

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = \frac{\frac{3\,a^3\left(\frac{385\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{990\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{1386\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1155\,\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{3465\,\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-315\right)(\cos(fx+e)+1)^{11}}{c^6\sin(fx+e)^{11}}+\frac{9\,a^3\left(\frac{385\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\cdots\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/110880*(3*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 990*sin(f*x + e)
^4/(cos(f*x + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1155*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1
)^10 - 315)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) + 9*a^3*(385*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 - 330*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4
62*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^8/(cos(f*x + e)
+ 1)^8 - 1155*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 105)*(cos(f*x + e) +
1)^11/(c^6*sin(f*x + e)^11) + 5*a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 - 990*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1386*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6 - 1155*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 693*sin(f*x + e)^1
0/(cos(f*x + e) + 1)^10 - 63)*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11) -
 a^3*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 990*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 - 1386*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8 + 3465*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 + 315)
*(cos(f*x + e) + 1)^11/(c^6*sin(f*x + e)^11))/f
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx = -\frac{99\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-154\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+63\,a^3}{2772\,c^6f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] -1/2772*(99*a^3*tan(1/2*f*x + 1/2*e)^4 - 154*a^3*tan(1/2*f*x + 1/2*e)^2 + 6
3*a^3)/(c^6*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B] (verification not implemented)

Time = 13.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx$$
$$=\frac{a^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^9}{18\,c^6\,f}-\frac{a^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^7}{28\,c^6\,f}-\frac{a^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}{44\,c^6\,f}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^6),x)
[Out] (a^3*cot(e/2 + (f*x)/2)^9)/(18*c^6*f) - (a^3*cot(e/2 + (f*x)/2)^7)/(28*c^6*
f) - (a^3*cot(e/2 + (f*x)/2)^11)/(44*c^6*f)
```

3.33 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 162

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{429c^2f(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{3003c^3f(c-c\sec(e+fx))^4}$$

```
[Out] -1/13*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^7-3/143*(a+a*sec(f*x
+e))^3*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^6-2/429*(a+a*sec(f*x+e))^3*tan(f*x+e
)/c^2/f/(c-c*sec(f*x+e))^5-2/3003*(a+a*sec(f*x+e))^3*tan(f*x+e)/c^3/f/(c-c*
sec(f*x+e))^4
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used

$= \{4036, 4035\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{3003c^3f(c-c\sec(e+fx))^4} - \frac{2\tan(e+fx)(a\sec(e+fx)+a)^3}{429c^2f(c-c\sec(e+fx))^5} - \frac{3\tan(e+fx)(a\sec(e+fx)+a)^3}{143cf(c-c\sec(e+fx))^6} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^3}{13f(c-c\sec(e+fx))^7}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] -1/13*((a + a*Sec[e + f*x])^3*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^7) - (3
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(143*c*f*(c - c*Sec[e + f*x])^6) - (2
*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(429*c^2*f*(c - c*Sec[e + f*x])^5) -
(2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*c^3*f*(c - c*Sec[e + f*x])^4)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\text{integral} = -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} + \frac{3\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^6}\,dx}{13c}$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} + \frac{6\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^5}\,dx}{143c^2}$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{429c^2f(c-c\sec(e+fx))^5} + \frac{2\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^4}dx}{429c^3}$$

$$= -\frac{(a+a\sec(e+fx))^3\tan(e+fx)}{13f(c-c\sec(e+fx))^7} - \frac{3(a+a\sec(e+fx))^3\tan(e+fx)}{143cf(c-c\sec(e+fx))^6} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{429c^2f(c-c\sec(e+fx))^5} - \frac{2(a+a\sec(e+fx))^3\tan(e+fx)}{3003c^3f(c-c\sec(e+fx))^4}$$

Mathematica [A] (verified)

Time = 5.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{a^3(1+\sec(e+fx))^3\left(-310+97\sec(e+fx)-20\sec^2(e+fx)+2\sec^3(e+fx)\right)\tan(e+fx)}{3003c^7f(-1+\sec(e+fx))^7}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] -1/3003*(a^3*(1 + Sec[e + f*x])^3*(-310 + 97*Sec[e + f*x] - 20*Sec[e + f*x]
^2 + 2*Sec[e + f*x]^3)*Tan[e + f*x])/(c^7*f*(-1 + Sec[e + f*x])^7)
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

method	result
parallelrisch	$\frac{a^3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^7\left(231\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^6-819\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^4+1001\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^2-429\right)}{24024c^7f}$
derivativedivides	$\frac{a^3\left(\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{13\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{8fc^7}$
default	$\frac{a^3\left(\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}-\frac{3}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{13\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}\right)}{8fc^7}$
risch	$\frac{2ia^3\left(3003\,e^{12i(fx+e)}-9009\,e^{11i(fx+e)}+51051\,e^{10i(fx+e)}-99099\,e^{9i(fx+e)}+216216\,e^{8i(fx+e)}-246246\,e^{7i(fx+e)}+2857\right.}{3003fc^7\left(e^{i(fx+e)}\right.}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24024*a^3*cot(1/2*f*x+1/2*e)^7*(231*cot(1/2*f*x+1/2*e)^6-819*cot(1/2*f*x+
1/2*e)^4+1001*cot(1/2*f*x+1/2*e)^2-429)/c^7/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.20

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = \frac{310\,a^3\cos(fx+e)^7+1143\,a^3\cos(fx+e)^6+1492\,a^3\cos(fx+e)^5+736\,a^3\cos(fx+e)^4+34\,a^3\cos(fx-}{3003\,\left(c^7f\cos(fx+e)^6-6\,c^7f\cos(fx+e)^5+15\,c^7f\cos(fx+e)^4-20\,c^7f\cos(fx+e)^3+15\,c^7f\cos\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="fr
icas")
```

```
[Out] 1/3003*(310*a^3*cos(f*x + e)^7 + 1143*a^3*cos(f*x + e)^6 + 1492*a^3*cos(f*x
 + e)^5 + 736*a^3*cos(f*x + e)^4 + 34*a^3*cos(f*x + e)^3 - 29*a^3*cos(f*x +
 e)^2 + 12*a^3*cos(f*x + e) - 2*a^3)/((c^7*f*cos(f*x + e)^6 - 6*c^7*f*cos(f
*x + e)^5 + 15*c^7*f*cos(f*x + e)^4 - 20*c^7*f*cos(f*x + e)^3 + 15*c^7*f*co
s(f*x + e)^2 - 6*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{a^3\left(\int \frac{\sec(e+fx)}{\sec^7(e+fx)-7\sec^6(e+fx)+21\sec^5(e+fx)-35\sec^4(e+fx)+35\sec^3(e+fx)-21\sec^2(e+fx)+7\sec(e+fx)-1}\,dx+\int \frac{}{\sec^7(e+fx)-}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**7,x)
```

```
[Out] -a**3*(Integral(sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec(
e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2
+ 7*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**7 - 7
*sec(e + f*x)**6 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x
)**3 - 21*sec(e + f*x)**2 + 7*sec(e + f*x) - 1), x) + Integral(3*sec(e + f*
x)**3/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec(e + f*x)**5 - 35*sec(e
+ f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2 + 7*sec(e + f*x) - 1),
x) + Integral(sec(e + f*x)**4/(sec(e + f*x)**7 - 7*sec(e + f*x)**6 + 21*sec
(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e + f*x)**2
 + 7*sec(e + f*x) - 1), x))/c**7
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(158) = 316$.

Time = 0.24 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.19

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{\frac{a^3\left(\frac{8190\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{5005\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{25740\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{9009\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{30030\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}-\frac{45045\sin(fx+e)^{12}}{(\cos(fx+e)+1)^{12}}-3465\right)(\cos(fx+e)+1)^{13}}{c^7\sin(fx+e)^{13}}+}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="ma
xima")
```

```
[Out] -1/960960*(a^3*(8190*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5005*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 - 25740*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 9009
*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 30030*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10 - 45045*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 3465)*(cos(f*x + e)
 + 1)^13/(c^7*sin(f*x + e)^13) + 5*a^3*(1638*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 5005*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 8580*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 9009*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 6006*sin(f*x
+ e)^10/(cos(f*x + e) + 1)^10 - 3003*sin(f*x + e)^12/(cos(f*x + e) + 1)^12
- 231)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13) + 35*a^3*(468*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 715*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1287*
sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 1716*sin(f*x + e)^10/(cos(f*x + e) +
1)^10 + 1287*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - 99)*(cos(f*x + e) + 1)
^13/(c^7*sin(f*x + e)^13) + 77*a^3*(65*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 117*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^12/(cos(f*x +
e) + 1)^12 - 15)*(cos(f*x + e) + 1)^13/(c^7*sin(f*x + e)^13))/f
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx = -\frac{429\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6-1001\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+819\,a^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-231\,a^3}{24024\,c^7 f\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{13}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^7,x, algorithm="gi
ac")
```

```
[Out] -1/24024*(429*a^3*tan(1/2*f*x + 1/2*e)^6 - 1001*a^3*tan(1/2*f*x + 1/2*e)^4
+ 819*a^3*tan(1/2*f*x + 1/2*e)^2 - 231*a^3)/(c^7*f*tan(1/2*f*x + 1/2*e)^13)
```

Mupad [B] (verification not implemented)

Time = 13.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^7}\,dx$$
$$=\frac{a^3\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\left(231\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-819\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1001\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4}{24024\,c^7\,f\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{13}}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^7),x)
[Out] (a^3*cos(e/2 + (f*x)/2)^7*(231*cos(e/2 + (f*x)/2)^6 - 429*sin(e/2 + (f*x)/2
)^6 + 1001*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 819*cos(e/2 + (f*x)/
2)^4*sin(e/2 + (f*x)/2)^2))/(24024*c^7*f*sin(e/2 + (f*x)/2)^13)
```

3.34 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{35c^4\text{arctanh}(\sin(e+fx))}{2af} + \frac{28c^4\tan(e+fx)}{af} - \frac{21c^4\sec(e+fx)\tan(e+fx)}{2af} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{7c^4\tan^3(e+fx)}{3af}$$

```
[Out] -35/2*c^4*arctanh(sin(f*x+e))/a/f+28*c^4*tan(f*x+e)/a/f-21/2*c^4*sec(f*x+e)
*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))+7/3*c^
4*tan(f*x+e)^3/a/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4042, 3876, 3855, 3852, 8, 3853}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{35c^4\text{arctanh}(\sin(e+fx))}{2af} + \frac{7c^4\tan^3(e+fx)}{3af} + \frac{28c^4\tan(e+fx)}{af} - \frac{21c^4\tan(e+fx)\sec(e+fx)}{2af} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-35*c^4*ArcTanh[Sin[e + f*x]])/(2*a*f) + (28*c^4*Tan[e + f*x])/(a*f) - (21
*c^4*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^3*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x])) + (7*c^4*Tan[e + f*x]^3)/(3*a*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c)\int \sec(e+fx)(c-c\sec(e+fx))^3\,dx}{a}\\
&= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))}\\
&\quad - \frac{(7c)\int \left(c^3\sec(e+fx)-3c^3\sec^2(e+fx)+3c^3\sec^3(e+fx)-c^3\sec^4(e+fx)\right)\,dx}{a}\\
&= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(7c^4)\int \sec(e+fx)\,dx}{a}\\
&\quad + \frac{(7c^4)\int \sec^4(e+fx)\,dx}{a} + \frac{(21c^4)\int \sec^2(e+fx)\,dx}{a} - \frac{(21c^4)\int \sec^3(e+fx)\,dx}{a}\\
&= -\frac{7c^4\text{arctanh}(\sin(e+fx))}{af} - \frac{21c^4\sec(e+fx)\tan(e+fx)}{2af}\\
&\quad + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(21c^4)\int \sec(e+fx)\,dx}{2a}\\
&\quad - \frac{(7c^4)\,\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(e+fx)\right)}{af}\\
&\quad - \frac{(21c^4)\,\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{af}\\
&= -\frac{35c^4\text{arctanh}(\sin(e+fx))}{2af} + \frac{28c^4\tan(e+fx)}{af} - \frac{21c^4\sec(e+fx)\tan(e+fx)}{2af}\\
&\quad + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{7c^4\tan^3(e+fx)}{3af}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.99 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{16c^4\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}}{af}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
[Out] (-16*c^4*Cot[e + f*x]*Hypergeometric2F1[-7/2, -1/2, 1/2, (1 + Sec[e + f*x])
/2]*Sqrt[2 - 2*Sec[e + f*x]])/(a*f)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{105\left(\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\cos(fx+e)-\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{446\left(\cos(fx+e\right.}{}\right.}{2af(\cos(3fx+3e)+3\cos(fx+e))}$
derivativedivides	$\frac{16c^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{3}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{29}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{fa}$
default	$\frac{16c^4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{3}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{29}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{32}-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\right.\right.}\right.}{fa}$
risch	$\frac{ic^4\left(111\,e^{6i(fx+e)}+81\,e^{5i(fx+e)}+354\,e^{4i(fx+e)}+144\,e^{3i(fx+e)}+417\,e^{2i(fx+e)}+55\,e^{i(fx+e)}+166\right)}{3af\left(1+e^{2i(fx+e)}\right)^3\left(e^{i(fx+e)}+1\right)}+\frac{35c^4\ln\left(e^{i(fx+e)}-i\right)}{2af}$
norman	$\frac{\frac{35c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{385c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}+\frac{511c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3af}-\frac{93c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}+\frac{16c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}+\frac{35c^4\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{2af}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 105/2*((cos(f*x+e)+1/3*cos(3*f*x+3*e))*ln(tan(1/2*f*x+1/2*e)-1)+(-cos(f*x+e
)-1/3*cos(3*f*x+3*e))*ln(tan(1/2*f*x+1/2*e)+1)+446/315*(cos(f*x+e)+55/223*c
os(2*f*x+2*e)+83/223*cos(3*f*x+3*e)+59/223)*tan(1/2*f*x+1/2*e))*c^4/a/f/(co
s(3*f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.26

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4+c^4\cos\left(fx+e\right)^3\right)}{12\left(af\cos\left(fx+\right.\right.}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/12*(105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(sin(f*x + e) + 1)
- 105*(c^4*cos(f*x + e)^4 + c^4*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*
(166*c^4*cos(f*x + e)^3 + 55*c^4*cos(f*x + e)^2 - 13*c^4*cos(f*x + e) + 2*c
^4)*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$= \frac{c^4\left(\int \frac{\sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \left(-\frac{4\sec^2(e+fx)}{\sec(e+fx)+1}\right)dx + \int \frac{6\sec^3(e+fx)}{\sec(e+fx)+1}\,dx + \int \left(-\frac{4\sec^4(e+fx)}{\sec(e+fx)+1}\right)dx + \int \frac{\sec^5(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*
x)**2/(sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x) + 1
), x) + Integral(-4*sec(e + f*x)**4/(sec(e + f*x) + 1), x) + Integral(sec(e
 + f*x)**5/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(116) = 232$.

Time = 0.21 (sec) , antiderivative size = 591, normalized size of antiderivative = 4.88

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$= \frac{c^4\left(\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1} - \frac{16\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{15\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a - \frac{3a\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3a\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{a\sin(fx+e)^6}{(\cos(fx+e)+1)^6}} - \frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a} + \frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a} + \frac{6\sin(fx+e)}{a(\cos(fx+e)+1)}\right) + 12}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
[Out] 1/6*(c^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(a*(co
s(f*x + e) + 1))) + 12*c^4*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x
+ e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
 a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e)
+ 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)
/(a*(cos(f*x + e) + 1))) - 36*c^4*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(c
os(f*x + e) + 1))) - 24*c^4*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) +
 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = -\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{96\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{2\left(87\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-136\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+57\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a}}{6\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/6*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 105*c^4*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a - 96*c^4*tan(1/2*f*x + 1/2*e)/a + 2*(87*c^4*tan(1/2*
f*x + 1/2*e)^5 - 136*c^4*tan(1/2*f*x + 1/2*e)^3 + 57*c^4*tan(1/2*f*x + 1/2*
e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{16\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}-\frac{29\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{136\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+19\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^3}-\frac{35\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (16*c^4*tan(e/2 + (f*x)/2))/(a*f) - (29*c^4*tan(e/2 + (f*x)/2)^5 - (136*c^4
*tan(e/2 + (f*x)/2)^3)/3 + 19*c^4*tan(e/2 + (f*x)/2))/(a*f*(tan(e/2 + (f*x)
/2)^2 - 1)^3) - (35*c^4*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

3.35 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{15c^3\operatorname{arctanh}(\sin(e+fx))}{2af}+\frac{10c^3\tan(e+fx)}{af}-\frac{5c^3\sec(e+fx)\tan(e+fx)}{2af}+\frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] -15/2*c^3*arctanh(sin(f*x+e))/a/f+10*c^3*tan(f*x+e)/a/f-5/2*c^3*sec(f*x+e)*
tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4042, 3873, 3852, 8, 4131, 3855}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{15c^3\operatorname{arctanh}(\sin(e+fx))}{2af}+\frac{10c^3\tan(e+fx)}{af}-\frac{5c^3\tan(e+fx)\sec(e+fx)}{2af}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-15*c^3*ArcTanh[Sin[e + f*x]])/(2*a*f) + (10*c^3*Tan[e + f*x])/(a*f) - (5*
c^3*Sec[e + f*x]*Tan[e + f*x])/(2*a*f) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e
+ f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2c(c - c\sec(e + fx))^2 \tan(e + fx)}{f(a + a\sec(e + fx))} - \frac{(5c)\int \sec(e + fx)(c - c\sec(e + fx))^2\,dx}{a}$$

$$
\begin{aligned}
&= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} \\
&\quad -\frac{(5c)\int\sec(e+fx)\left(c^2+c^2\sec^2(e+fx)\right)\,dx}{a}+\frac{(10c^3)\int\sec^2(e+fx)\,dx}{a} \\
&= -\frac{5c^3\sec(e+fx)\tan(e+fx)}{2af}+\frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} \\
&\quad -\frac{(15c^3)\int\sec(e+fx)\,dx}{2a}-\frac{(10c^3)\,\mathrm{Subst}(\int 1\,dx,x,-\tan(e+fx))}{af} \\
&= -\frac{15c^3\mathrm{arctanh}(\sin(e+fx))}{2af}+\frac{10c^3\tan(e+fx)}{af} \\
&\quad -\frac{5c^3\sec(e+fx)\tan(e+fx)}{2af}+\frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.53

$$
\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{8c^3\cot(e+fx)\,\mathrm{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}}{af}
$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
[Out] (-8*c^3*Cot[e + f*x]*Hypergeometric2F1[-5/2, -1/2, 1/2, (1 + Sec[e + f*x])/
2]*Sqrt[2 - 2*Sec[e + f*x]])/(a*f)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

method	result
parallelrisch	$-\frac{15\left(\left(-1-\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(1+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-\frac{14\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{12\cos}{}\right.}{15}\right.}{2af(1+\cos(2fx+2e))}$
derivativedivides	$\frac{8c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{9}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{9}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{fa}$
default	$\frac{8c^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{9}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{9}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}\right.}{fa}$
risch	$\frac{ic^3\left(17\,e^{4i(fx+e)}+9\,e^{3i(fx+e)}+39\,e^{2i(fx+e)}+7\,e^{i(fx+e)}+24\right)}{fa\left(e^{i(fx+e)}+1\right)\left(1+e^{2i(fx+e)}\right)^2}+\frac{15c^3\ln\left(e^{i(fx+e)}-i\right)}{2af}-\frac{15c^3\ln\left(e^{i(fx+e)}+i\right)}{2af}$
norman	$\frac{-\frac{15c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{40c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}-\frac{33c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}+\frac{8c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}+\frac{15c^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2af}-\frac{15c^3\ln\left(\tan\right.}{}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -15/2*((-1-cos(2*f*x+2*e))*ln(tan(1/2*f*x+1/2*e)-1)+(1+cos(2*f*x+2*e))*ln(t
an(1/2*f*x+1/2*e)+1)-14/15*tan(1/2*f*x+1/2*e)*(cos(f*x+e)+12/7*cos(2*f*x+2*
e)+11/7))*c^3/a/f/(1+cos(2*f*x+2*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.40

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx =$$
$$-\frac{15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+c^3\cos\left(fx+e\right)^2\right)\mathrm{lo}}{4\left(af\cos\left(fx+e\right)^3+af\cos\left(\right.\right.}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/4*(15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(sin(f*x + e) + 1) -
15*(c^3*cos(f*x + e)^3 + c^3*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(24
*c^3*cos(f*x + e)^2 + 7*c^3*cos(f*x + e) - c^3)*sin(f*x + e))/(a*f*cos(f*x
+ e)^3 + a*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{c^3\left(\int \left(-\frac{\sec(e+fx)}{\sec(e+fx)+1}\right)dx + \int \frac{3\sec^2(e+fx)}{\sec(e+fx)+1}\,dx + \int \left(-\frac{3\sec^3(e+fx)}{\sec(e+fx)+1}\right)dx + \int \frac{\sec^4(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(3*sec(e + f
*x)**2/(sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f*x) +
 1), x) + Integral(sec(e + f*x)**4/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(97) = 194.

Time = 0.20 (sec) , antiderivative size = 386, normalized size of antiderivative = 3.86

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{c^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}} - \frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a} + \frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a} + \frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right) - 6c^3\left(\frac{\log\left(\frac{\sin(fx}{\cos(fx+}\right.}{a}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(c^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x + e)
+ 1))) - 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1)))
- 6*c^3*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^3*sin(f*
x + e)/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{16\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{2\left(9\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-7\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a}}{2\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/2*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 15*c^3*log(abs(tan(1/2*
f*x + 1/2*e) - 1))/a - 16*c^3*tan(1/2*f*x + 1/2*e)/a + 2*(9*c^3*tan(1/2*f*x
 + 1/2*e)^3 - 7*c^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a
))/f
```

Mupad [B] (verification not implemented)

Time = 13.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{8\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f} - \frac{9\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-7\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)^2} - \frac{15\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (8*c^3*tan(e/2 + (f*x)/2))/(a*f) - (9*c^3*tan(e/2 + (f*x)/2)^3 - 7*c^3*tan(
e/2 + (f*x)/2))/(a*f*(tan(e/2 + (f*x)/2)^2 - 1)^2) - (15*c^3*atanh(tan(e/2
+ (f*x)/2)))/(a*f)
```

3.36 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 74

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{3c^2\operatorname{arctanh}(\sin(e+fx))}{af}+\frac{3c^2\tan(e+fx)}{af}+\frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] -3*c^2*arctanh(sin(f*x+e))/a/f+3*c^2*tan(f*x+e)/a/f+2*(c^2-c^2*sec(f*x+e))*
tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4042, 3872, 3855, 3852, 8}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{3c^2\operatorname{arctanh}(\sin(e+fx))}{af}+\frac{3c^2\tan(e+fx)}{af}+\frac{2\tan(e+fx)\,(c^2-c^2\sec(e+fx))}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-3*c^2*ArcTanh[Sin[e + f*x]])/(a*f) + (3*c^2*Tan[e + f*x])/(a*f) + (2*(c^2
 - c^2*Sec[e + f*x])*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{2(c^2 - c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(3c)\int \sec(e+fx)(c-c\sec(e+fx))\,dx}{a}$$

$$= \frac{2(c^2 - c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(3c^2)\int \sec(e+fx)\,dx}{a} + \frac{(3c^2)\int \sec^2(e+fx)\,dx}{a}$$

$$= -\frac{3c^2\operatorname{arctanh}(\sin(e+fx))}{af} + \frac{2(c^2 - c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(3c^2)\operatorname{Subst}(\int 1\,dx, x, -\tan(e+fx))}{af}$$

$$= -\frac{3c^2\operatorname{arctanh}(\sin(e+fx))}{af} + \frac{3c^2\tan(e+fx)}{af} + \frac{2(c^2 - c^2\sec(e+fx))\tan(e+fx)}{f(a+a\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$$
$$=\frac{4\sqrt{2}c^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan\left(\frac{1}{2}(e+fx)\right)}{af\sqrt{1-\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
[Out] (4*Sqrt[2]*c^2*Hypergeometric2F1[-3/2, -1/2, 1/2, (1 + Sec[e + f*x])/2]*Tan
[(e + f*x)/2])/(a*f*Sqrt[1 - Sec[e + f*x]])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

method	result	s
derivativedivides	$\frac{4c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fa}$	8
default	$\frac{4c^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}\right)}{fa}$	8
parallelrisch	$\frac{c^2\left(3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af\cos(fx+e)}$	8
risch	$\frac{2ic^2\left(4\,e^{2i(fx+e)}+e^{i(fx+e)}+5\right)}{fa\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}+1\right)}+\frac{3c^2\ln\left(e^{i(fx+e)}-i\right)}{af}-\frac{3c^2\ln\left(e^{i(fx+e)}+i\right)}{af}$	1
norman	$\frac{\frac{6c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{10c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}+\frac{4c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}+\frac{3c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}-\frac{3c^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}$	1

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 4/f/a*c^2*(tan(1/2*f*x+1/2*e)-1/4/(tan(1/2*f*x+1/2*e)-1)+3/4*ln(tan(1/2*f*x
+1/2*e)-1)-1/4/(tan(1/2*f*x+1/2*e)+1)-3/4*ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c^2\cos\left(fx+e\right)^2+c^2\cos\left(fx+e\right)\right)\log\left(-\right.}{2\left(af\cos\left(fx+e\right)^2+af\cos\left(fx+e\right)\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/2*(3*(c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*(
c^2*cos(f*x + e)^2 + c^2*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(5*c^2*co
s(f*x + e) + c^2)*sin(f*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{c^2\left(\int \frac{\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{2\sec^2\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)dx+\int \frac{\sec^3\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx\right)}{a}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(-2*sec(e + f*
x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x) + 1),
 x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(75) = 150.

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.03

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{c^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+2c^2\left(\frac{\log\left(\frac{\sin(fx+}{\cos(fx+e}\right.}{a}\right.}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(c^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + 2*c^2*(
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)/(a*(
cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{\frac{3\,c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{3\,c^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}-\frac{4\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{a}+\frac{2\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a}}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 3*c^2*log(abs(tan(1/2*f*x +
1/2*e) - 1))/a - 4*c^2*tan(1/2*f*x + 1/2*e)/a + 2*c^2*tan(1/2*f*x + 1/2*e)/
((tan(1/2*f*x + 1/2*e)^2 - 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 13.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{4\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}+\frac{2\,c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)}-\frac{6\,c^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (4*c^2*tan(e/2 + (f*x)/2))/(a*f) + (2*c^2*tan(e/2 + (f*x)/2))/(f*(a - a*tan
(e/2 + (f*x)/2)^2)) - (6*c^2*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

3.37 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 41

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx=-\frac{c\,\text{arctanh}(\sin(e+fx))}{af}+\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] -c*arctanh(sin(f*x+e))/a/f+2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4042, 3855}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}-\frac{c\,\text{arctanh}(\sin(e+fx))}{af}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a*f)) + (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f
*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
```

```
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{c\int\sec(e+fx)\,dx}{a}\\ &= -\frac{c\operatorname{arctanh}(\sin(e+fx))}{af} + \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.88

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{f}\right)}{a}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f - (2*Tan[(e + f*x)/2])/f))/a)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result	size
parallelrisch	$\frac{c\left(2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{af}$	47
derivativedivides	$\frac{2c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{fa}$	48
default	$\frac{2c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{fa}$	48
risch	$\frac{4ic}{fa\left(e^{i(fx+e)}+1\right)}-\frac{c\ln\left(e^{i(fx+e)}+i\right)}{af}+\frac{c\ln\left(e^{i(fx+e)}-i\right)}{af}$	68
norman	$\frac{-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{af}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1}+\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}-\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}$	98

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] c*(2*tan(1/2*f*x+1/2*e)+ln(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*e)+1))/
a/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{(c\cos(fx+e)+c)\log(\sin(fx+e)+1)-(c\cos(fx+e)+c)\log(-\sin(fx+e)+1)-4c\sin(fx+e}{2(af\cos(fx+e)+af)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] -1/2*((c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - (c*cos(f*x + e) + c)*log
(-sin(f*x + e) + 1) - 4*c*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(\int \left(-\frac{\sec(e+fx)}{\sec(e+fx)+1}\right)dx + \int \frac{\sec^2(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**
2/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(41) = 82$.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a} - \frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right) - \frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxima
")
[Out] -(c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c*sin(f*x + e)/(
a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a} - \frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a} - \frac{2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
[Out] -(c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - c*log(abs(tan(1/2*f*x + 1/2*e) -
 1))/a - 2*c*tan(1/2*f*x + 1/2*e)/a)/f
```

Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{2\,c\,\left(\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f}$$

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
[Out] -(2*c*(atanh(tan(e/2 + (f*x)/2)) - tan(e/2 + (f*x)/2)))/(a*f)
```

3.38 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 16

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))} \, dx = \frac{\csc(e+fx)}{acf}$$

```
[Out] csc(f*x+e)/a/c/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4043, 2686, 8}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))} \, dx = \frac{\csc(e+fx)}{acf}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]
```

```
[Out] Csc[e + f*x]/(a*c*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \cot(e+fx)\csc(e+fx)\,dx}{ac}\\ &= \frac{\text{Subst}(\int 1\,dx, x, \csc(e+fx))}{acf}\\ &= \frac{\csc(e+fx)}{acf}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = \frac{\csc(e+fx)}{acf}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])),x]
[Out] Csc[e + f*x]/(a*c*f)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{\csc(fx+e)}{acf}$	17
parallelrisch	$\frac{\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)}{2acf}$	30
norman	$\frac{\frac{1}{2acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{2acf}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	47
risch	$\frac{2i\,e^{i(fx+e)}}{fac\left(e^{i(fx+e)}-1\right)\left(e^{i(fx+e)}+1\right)}$	48

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] csc(f*x+e)/a/c/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = \frac{1}{acf\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="fricas
")
[Out] 1/(a*c*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sec^2(e+fx)-1}\,dx}{ac}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x)
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**2 - 1), x)/(a*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx = \frac{1}{acf\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="maxima
")
[Out] 1/(a*c*f*sin(f*x + e))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx=\frac{1}{acf\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e)),x, algorithm="giac")
[Out] 1/(a*c*f*sin(f*x + e))
```

Mupad [B] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))}\,dx=\frac{1}{a\,c\,f\,\sin\left(e+f\,x\right)}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))),x)
[Out] 1/(a*c*f*sin(e + f*x))
```

3.39 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 59

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}$$

```
[Out] -1/3*cot(f*x+e)^3/a/c^2/f+csc(f*x+e)/a/c^2/f-1/3*csc(f*x+e)^3/a/c^2/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4043, 2686, 2687, 30}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^3(e+fx)}{3ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -1/3*Cot[e + f*x]^3/(a*c^2*f) + Csc[e + f*x]/(a*c^2*f) - Csc[e + f*x]^3/(3*
a*c^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(a\cot^3(e+fx)\csc(e+fx)+a\cot^2(e+fx)\csc^2(e+fx)\right)\,dx}{a^2c^2} \\
&= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{ac^2}+\frac{\int \cot^2(e+fx)\csc^2(e+fx)\,dx}{ac^2} \\
&= \frac{\text{Subst}\left(\int x^2\,dx,x,-\cot(e+fx)\right)}{ac^2f}-\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(e+fx)\right)}{ac^2f} \\
&= -\frac{\cot^3(e+fx)}{3ac^2f}+\frac{\csc(e+fx)}{ac^2f}-\frac{\csc^3(e+fx)}{3ac^2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx \\
&= -\frac{(-3+4\cos(e+fx)+\cos(2(e+fx)))\csc^3\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)}{24ac^2f}
\end{aligned}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^2),x]
[Out] -1/24*((-3 + 4*Cos[e + f*x] + Cos[2*(e + f*x)])*Csc[(e + f*x)/2]^3*Sec[(e +
 f*x)/2])/(a*c^2*f)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4fa\,c^{2}}$	48
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4fa\,c^{2}}$	48
parallelrisch	$\frac{3\sec\left(\frac{fx}{2}+\frac{e}{2}\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-4\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{12fa\,c^{2}}$	48
norman	$\frac{-\frac{1}{12acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{4acf}}{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}$	72
risch	$\frac{2i\left(3\,e^{3i(fx+e)}-3\,e^{2i(fx+e)}+e^{i(fx+e)}+1\right)}{3fa\,c^{2}\left(e^{i(fx+e)}-1\right)^{3}\left(e^{i(fx+e)}+1\right)}$	72

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a/c^2*(tan(1/2*f*x+1/2*e)-1/3/tan(1/2*f*x+1/2*e)^3+2/tan(1/2*f*x+1/2*
e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\cos\left(fx+e\right)^2+2\,\cos\left(fx+e\right)-2}{3\left(ac^2f\cos\left(fx+e\right)-ac^2f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*(cos(f*x + e)^2 + 2*cos(f*x + e) - 2)/((a*c^2*f*cos(f*x + e) - a*c^2*f)
*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec(e+fx)}{\sec^3(e+fx)-\sec^2(e+fx)-\sec(e+fx)+1}\,dx}{ac^2}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**3 - sec(e + f*x)**2 - sec(e + f*x) + 1
), x)/(a*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\frac{\left(\frac{6\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{ac^2\sin(fx+e)^3} + \frac{3\sin(fx+e)}{ac^2(\cos(fx+e)+1)}}{12\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*((6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a*c
^2*sin(f*x + e)^3) + 3*sin(f*x + e)/(a*c^2*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{\frac{3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^2} + \frac{6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1}{ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}}{12\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/12*(3*tan(1/2*f*x + 1/2*e)/(a*c^2) + (6*tan(1/2*f*x + 1/2*e)^2 - 1)/(a*c^
2*tan(1/2*f*x + 1/2*e)^3))/f
```

Mupad [B] (verification not implemented)

Time = 13.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^2}\,dx = \frac{3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}{12\,a\,c^2\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^2),x)
[Out] (6*tan(e/2 + (f*x)/2)^2 + 3*tan(e/2 + (f*x)/2)^4 - 1)/(12*a*c^2*f*tan(e/2 +
 (f*x)/2)^3)
```

3.40 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3} dx = \frac{2\cot^5(e+fx)}{5ac^3f} + \frac{\csc(e+fx)}{ac^3f} - \frac{\csc^3(e+fx)}{ac^3f} + \frac{2\csc^5(e+fx)}{5ac^3f}$$

```
[Out] 2/5*cot(f*x+e)^5/a/c^3/f+csc(f*x+e)/a/c^3/f-csc(f*x+e)^3/a/c^3/f+2/5*csc(f*
x+e)^5/a/c^3/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4043, 2686, 200, 2687, 30, 14}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3} dx = \frac{2\cot^5(e+fx)}{5ac^3f} + \frac{2\csc^5(e+fx)}{5ac^3f} - \frac{\csc^3(e+fx)}{ac^3f} + \frac{\csc(e+fx)}{ac^3f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] (2*Cot[e + f*x]^5)/(5*a*c^3*f) + Csc[e + f*x]/(a*c^3*f) - Csc[e + f*x]^3/(a
*c^3*f) + (2*Csc[e + f*x]^5)/(5*a*c^3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \left(a^2\cot^5(e+fx)\csc(e+fx)+2a^2\cot^4(e+fx)\csc^2(e+fx)+a^2\cot^3(e+fx)\csc^3(e+fx)\right)\,dx}{a^3c^3}$$

$$= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{ac^3} - \frac{\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{ac^3} - \frac{2\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{ac^3}$$

$$= \frac{\text{Subst}\left(\int x^2(-1+x^2)\,dx, x, \csc(e+fx)\right)}{ac^3f} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{ac^3f} - \frac{2\text{Subst}\left(\int x^4\,dx, x, -\cot(e+fx)\right)}{ac^3f}$$

$$= \frac{2\cot^5(e+fx)}{5ac^3f} + \frac{\text{Subst}\left(\int (1-2x^2+x^4)\,dx, x, \csc(e+fx)\right)}{ac^3f} + \frac{\text{Subst}\left(\int (-x^2+x^4)\,dx, x, \csc(e+fx)\right)}{ac^3f}$$

$$= \frac{2\cot^5(e+fx)}{5ac^3f} + \frac{\csc(e+fx)}{ac^3f} - \frac{\csc^3(e+fx)}{ac^3f} + \frac{2\csc^5(e+fx)}{5ac^3f}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{(5-5\cos(e+fx)+\cos(2(e+fx))+\cos(3(e+fx)))\csc^5\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{1}{2}(e+fx)\right)}{80ac^3f}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^3),x]
[Out] ((5 - 5*Cos[e + f*x] + Cos[2*(e + f*x)] + Cos[3*(e + f*x)])*Csc[(e + f*x)/2
]^5*Sec[(e + f*x)/2])/(80*a*c^3*f)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5-5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+15\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{40fa\,c^3}$	59
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8fa\,c^3}$	61
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8fa\,c^3}$	61
risch	$\frac{2i\left(5\,e^{5i(fx+e)}-10\,e^{4i(fx+e)}+10\,e^{3i(fx+e)}-3\,e^{i(fx+e)}+2\right)}{5fa\,c^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^5}$	85
norman	$\frac{\frac{1}{40acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{8acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{8acf}}{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	94

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(cot(1/2*f*x+1/2*e)^5-5*cot(1/2*f*x+1/2*e)^3+5*tan(1/2*f*x+1/2*e)+15*c
ot(1/2*f*x+1/2*e))/f/a/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{2\cos(fx+e)^3+\cos(fx+e)^2-4\cos(fx+e)+2}{5\left(ac^3f\cos(fx+e)^2-2\,ac^3f\cos(fx+e)+ac^3f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/5*(2*cos(f*x + e)^3 + cos(f*x + e)^2 - 4*cos(f*x + e) + 2)/((a*c^3*f*cos(
f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sec^4(e+fx)-2\sec^3(e+fx)+2\sec(e+fx)-1}\,dx}{ac^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**3 + 2*sec(e + f*x
) - 1), x)/(a*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = -\frac{\frac{\left(\frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{15\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-1\right)(\cos(fx+e)+1)^5}{ac^3\sin(fx+e)^5}-\frac{5\sin(fx+e)}{ac^3(\cos(fx+e)+1)}}{40\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] -1/40*((5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 15*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 1)*(cos(f*x + e) + 1)^5/(a*c^3*sin(f*x + e)^5) - 5*sin(f*x +
e)/(a*c^3*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{\frac{5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^3}+\frac{15\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1}{ac^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}}{40f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/40*(5*tan(1/2*f*x + 1/2*e)/(a*c^3) + (15*tan(1/2*f*x + 1/2*e)^4 - 5*tan(1
/2*f*x + 1/2*e)^2 + 1)/(a*c^3*tan(1/2*f*x + 1/2*e)^5))/f
```

Mupad [B] (verification not implemented)

Time = 13.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^3}\,dx = \frac{5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+15\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1}{40\,a\,c^3\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (15*tan(e/2 + (f*x)/2)^4 - 5*tan(e/2 + (f*x)/2)^2 + 5*tan(e/2 + (f*x)/2)^6
+ 1)/(40*a*c^3*f*tan(e/2 + (f*x)/2)^5)
```

3.41 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = -\frac{\cot^5(e+fx)}{5ac^4f} - \frac{4\cot^7(e+fx)}{7ac^4f} + \frac{\csc(e+fx)}{ac^4f} - \frac{2\csc^3(e+fx)}{ac^4f} + \frac{9\csc^5(e+fx)}{5ac^4f} - \frac{4\csc^7(e+fx)}{7ac^4f}$$

```
[Out] -1/5*cot(f*x+e)^5/a/c^4/f-4/7*cot(f*x+e)^7/a/c^4/f+csc(f*x+e)/a/c^4/f-2*csc
(f*x+e)^3/a/c^4/f+9/5*csc(f*x+e)^5/a/c^4/f-4/7*csc(f*x+e)^7/a/c^4/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4043, 2686, 200, 2687, 30, 276, 14}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = -\frac{4\cot^7(e+fx)}{7ac^4f} - \frac{\cot^5(e+fx)}{5ac^4f} - \frac{4\csc^7(e+fx)}{7ac^4f} + \frac{9\csc^5(e+fx)}{5ac^4f} - \frac{2\csc^3(e+fx)}{ac^4f} + \frac{\csc(e+fx)}{ac^4f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -1/5*Cot[e + f*x]^5/(a*c^4*f) - (4*Cot[e + f*x]^7)/(7*a*c^4*f) + Csc[e + f*
x]/(a*c^4*f) - (2*Csc[e + f*x]^3)/(a*c^4*f) + (9*Csc[e + f*x]^5)/(5*a*c^4*f
) - (4*Csc[e + f*x]^7)/(7*a*c^4*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

integral

$$= \frac{\int (a^3 \cot^7(e+fx)\csc(e+fx) + 3a^3\cot^6(e+fx)\csc^2(e+fx) + 3a^3\cot^5(e+fx)\csc^3(e+fx) + a^3\cot^4(}{a^4c^4}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{ac^4} + \frac{\int \cot^4(e+fx)\csc^4(e+fx)\,dx}{ac^4} + \frac{3\int \cot^6(e+fx)\csc^2(e+fx)\,dx}{ac^4} + \frac{3\int \cot^5(e+fx)\csc^3(e+fx)\,dx}{ac^4}$$

$$= -\frac{\text{Subst}\left(\int (-1+x^2)^3\,dx, x, \csc(e+fx)\right)}{ac^4f} + \frac{\text{Subst}\left(\int x^4(1+x^2)\,dx, x, -\cot(e+fx)\right)}{ac^4f} + \frac{3\text{Subst}\left(\int x^6\,dx, x, -\cot(e+fx)\right)}{ac^4f} - \frac{3\text{Subst}\left(\int x^2(-1+x^2)^2\,dx, x, \csc(e+fx)\right)}{ac^4f}$$

$$= -\frac{3\cot^7(e+fx)}{7ac^4f} - \frac{\text{Subst}\left(\int (-1+3x^2-3x^4+x^6)\,dx, x, \csc(e+fx)\right)}{ac^4f} + \frac{\text{Subst}\left(\int (x^4+x^6)\,dx, x, -\cot(e+fx)\right)}{ac^4f} - \frac{3\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \csc(e+fx)\right)}{ac^4f}$$

$$= -\frac{\cot^5(e+fx)}{5ac^4f} - \frac{4\cot^7(e+fx)}{7ac^4f} + \frac{\csc(e+fx)}{ac^4f} - \frac{2\csc^3(e+fx)}{ac^4f} + \frac{9\csc^5(e+fx)}{5ac^4f} - \frac{4\csc^7(e+fx)}{7ac^4f}$$

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{(-13+4\sec(e+fx)+20\sec^2(e+fx)-24\sec^3(e+fx)+8\sec^4(e+fx))\tan(e+fx)}{35ac^4f(-1+\sec(e+fx))^4(1+\sec(e+fx))}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] ((-13 + 4*Sec[e + f*x] + 20*Sec[e + f*x]^2 - 24*Sec[e + f*x]^3 + 8*Sec[e +
f*x]^4)*Tan[e + f*x])/(35*a*c^4*f*(-1 + Sec[e + f*x])^4*(1 + Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}}{16fa\,c^{4}}$	74
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{4}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}}{16fa\,c^{4}}$	74
parallelrisch	$\frac{-5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+28\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-70\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+140\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{560fa\,c^{4}}$	74
norman	$\frac{-\frac{1}{112acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{20acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{4acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{16acf}}{c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$	116
risch	$\frac{2i\left(35\,\mathrm{e}^{7i(fx+e)}-105\,\mathrm{e}^{6i(fx+e)}+175\,\mathrm{e}^{5i(fx+e)}-105\,\mathrm{e}^{4i(fx+e)}-7\,\mathrm{e}^{3i(fx+e)}+77\,\mathrm{e}^{2i(fx+e)}-43\,\mathrm{e}^{i(fx+e)}+13\right)}{35fa\,c^{4}\left(\mathrm{e}^{i(fx+e)}-1\right)^{7}\left(\mathrm{e}^{i(fx+e)}+1\right)}$	118

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/f/a/c^4*(tan(1/2*f*x+1/2*e)+4/5/tan(1/2*f*x+1/2*e)^5-2/tan(1/2*f*x+1/2
*e)^3+4/tan(1/2*f*x+1/2*e)-1/7/tan(1/2*f*x+1/2*e)^7)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{13\,\cos\left(fx+e\right)^{4}-4\,\cos\left(fx+e\right)^{3}-20\,\cos\left(fx+e\right)^{2}+24\,\cos\left(fx+e\right)-8}{35\,\left(ac^{4}f\cos\left(fx+e\right)^{3}-3\,ac^{4}f\cos\left(fx+e\right)^{2}+3\,ac^{4}f\cos\left(fx+e\right)-ac^{4}f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/35*(13*cos(f*x + e)^4 - 4*cos(f*x + e)^3 - 20*cos(f*x + e)^2 + 24*cos(f*x
 + e) - 8)/((a*c^4*f*cos(f*x + e)^3 - 3*a*c^4*f*cos(f*x + e)^2 + 3*a*c^4*f*
cos(f*x + e) - a*c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{\int \frac{\sec(e+fx)}{\sec^5(e+fx)-3\sec^4(e+fx)+2\sec^3(e+fx)+2\sec^2(e+fx)-3\sec(e+fx)+1}\,dx}{ac^4}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**5 - 3*sec(e + f*x)**4 + 2*sec(e + f*x)
**3 + 2*sec(e + f*x)**2 - 3*sec(e + f*x) + 1), x)/(a*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{\frac{\left(\frac{28\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{70\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{140\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{ac^4\sin(fx+e)^7}+\frac{35\sin(fx+e)}{ac^4(\cos(fx+e)+1)}}{560\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/560*((28*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x + e)^4/(cos(f*x
 + e) + 1)^4 + 140*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) +
 1)^7/(a*c^4*sin(f*x + e)^7) + 35*sin(f*x + e)/(a*c^4*(cos(f*x + e) + 1)))/
f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{\frac{35\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac^4}+\frac{140\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+28\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-5}{ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}}{560\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/560*(35*tan(1/2*f*x + 1/2*e)/(a*c^4) + (140*tan(1/2*f*x + 1/2*e)^6 - 70*t
an(1/2*f*x + 1/2*e)^4 + 28*tan(1/2*f*x + 1/2*e)^2 - 5)/(a*c^4*tan(1/2*f*x +
 1/2*e)^7))/f
```

Mupad [B] (verification not implemented)

Time = 13.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^4}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{16\,a\,c^4\,f}+\frac{\frac{{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^6}{4}-\frac{{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4}{8}+\frac{{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2}{20}-\frac{1}{112}}{a\,c^4\,f\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^7}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^4),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(16*a*c^4*f) + (tan(e/2 + (f*x)/2)^2/20 - tan(e/2 + (f*x
)/2)^4/8 + tan(e/2 + (f*x)/2)^6/4 - 1/112)/(a*c^4*f*tan(e/2 + (f*x)/2)^7)
```

3.42 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 164

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{105c^5\operatorname{arctanh}(\sin(e+fx))}{2a^2f} - \frac{84c^5\tan(e+fx)}{a^2f} + \frac{63c^5\sec(e+fx)\tan(e+fx)}{2a^2f} - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{7c^5\tan^3(e+fx)}{a^2f}$$

```
[Out] 105/2*c^5*arctanh(sin(f*x+e))/a^2/f-84*c^5*tan(f*x+e)/a^2/f+63/2*c^5*sec(f*
x+e)*tan(f*x+e)/a^2/f-6*c^2*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a^2+a^2*sec(f*
x+e))+2/3*c*(c-c*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-7*c^5*tan(f*
x+e)^3/a^2/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

$= \{4042, 3876, 3855, 3852, 8, 3853\}$

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{105c^5\text{arctanh}(\sin(e+fx))}{2a^2f} - \frac{7c^5\tan^3(e+fx)}{a^2f} - \frac{84c^5\tan(e+fx)}{a^2f} + \frac{63c^5\tan(e+fx)\sec(e+fx)}{2a^2f} - \frac{6c^2\tan(e+fx)(c-c\sec(e+fx))^3}{f\,(a^2\sec(e+fx)+a^2)} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (105*c^5*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (84*c^5*Tan[e + f*x])/(a^2*f) +
 (63*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^2*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^4
*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (7*c^5*Tan[e + f*x]^3)/(a^2*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
```

```
GtQ[m, 0] && RationalQ[n]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(3c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx}{a} \\
&= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&\quad + \frac{(21c^2)\int\sec(e+fx)(c-c\sec(e+fx))^3\,dx}{a^2} \\
&= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&\quad + \frac{(21c^2)\int\left(c^3\sec(e+fx)-3c^3\sec^2(e+fx)+3c^3\sec^3(e+fx)-c^3\sec^4(e+fx)\right)\,dx}{a^2} \\
&= -\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&\quad + \frac{(21c^5)\int\sec(e+fx)\,dx}{a^2} - \frac{(21c^5)\int\sec^4(e+fx)\,dx}{a^2} \\
&\quad - \frac{(63c^5)\int\sec^2(e+fx)\,dx}{a^2} + \frac{(63c^5)\int\sec^3(e+fx)\,dx}{a^2} \\
&= \frac{21c^5\text{arctanh}(\sin(e+fx))}{a^2f} + \frac{63c^5\sec(e+fx)\tan(e+fx)}{2a^2f} \\
&\quad - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&\quad + \frac{(63c^5)\int\sec(e+fx)\,dx}{2a^2} + \frac{(21c^5)\,\text{Subst}\left(\int\left(1+x^2\right)\,dx,x,-\tan(e+fx)\right)}{a^2f} \\
&\quad + \frac{(63c^5)\,\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{a^2f}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{105c^5\operatorname{arctanh}(\sin(e+fx))}{2a^2f} - \frac{84c^5\tan(e+fx)}{a^2f} \\
&\quad + \frac{63c^5\sec(e+fx)\tan(e+fx)}{2a^2f} - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))} \\
&\quad + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{7c^5\tan^3(e+fx)}{a^2f}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

$$
\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{32c^5\,\text{Hypergeometric2F1}\left(-\frac{9}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{3a^2f(-1+\sec(e+fx))(1+\sec(e+fx))^2}
$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
[Out] (-32*c^5*Hypergeometric2F1[-9/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[2 -
 2*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*
x])^2)
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{16c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{55}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{105\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\cdots\right.\right.}{32}\cdots\right.}{f\,a^2}$
default	$\frac{16c^5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{55}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{105\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\cdots\right.\right.}{32}\cdots\right.}{f\,a^2}$
parallelrisch	$-\frac{1969\left(\frac{630(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{1969}+\frac{630(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{1969}+\tan\left(\frac{fx}{2}+\frac{e}{2}\cdots\right.\right.}{12f\,a^2(\cos(3fx+3e)+3\cos(fx\cdots}$
risch	$-\frac{ic^5\left(309\,e^{8i(fx+e)}+969\,e^{7i(fx+e)}+1693\,e^{6i(fx+e)}+3027\,e^{5i(fx+e)}+2901\,e^{4i(fx+e)}+3247\,e^{3i(fx+e)}+1995\,e^{2i(fx+e)}+11\cdots\right.}{3a^2f\left(1+e^{2i(fx+e)}\right)^3\left(e^{i(fx+e)}+1\right)^3}$
norman	$\frac{\frac{105c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{490c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}+\frac{896c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}-\frac{790c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}+\frac{965c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}-\frac{112c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}\cdots}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5a}$

[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 16/f*c^5/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-4*tan(1/2*f*x+1/2*e)+1/48/(tan(1/2*f*x+1/2*e)+1)^3-1/4/(tan(1/2*f*x+1/2*e)+1)^2+55/32/(tan(1/2*f*x+1/2*e)+1)+105/32*ln(tan(1/2*f*x+1/2*e)+1)+1/48/(tan(1/2*f*x+1/2*e)-1)^3+1/4/(tan(1/2*f*x+1/2*e)-1)^2+55/32/(tan(1/2*f*x+1/2*e)-1)-105/32*ln(tan(1/2*f*x+1/2*e)-1))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.28

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{315\left(c^5\cos\left(fx+e\right)^5+2\,c^5\cos\left(fx+e\right)^4+c^5\cos\left(fx+e\right)^3\right)\log\left(\sin\left(fx+e\right)+1\right)-315\left(c^5\cos\left(fx+e\right)^5\right.}{}$$

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/12*(315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^5*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 2*c^5*cos(f*x + e)^4 + c^5*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) - 2*(494*c^5*cos(f*x + e)^4 + 679*c^5*cos(f*x + e)^3 + 102*c^5*cos(f*x + e)^2 - 17*c^5*cos(f*x + e) + 2*c^5)*sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f*cos(f*x + e)^3)

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^5\left(\int\left(-\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{5\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{10\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\right.}{a^2}$$

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)

[Out] -c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(5*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(10*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(-5*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(160) = 320.

Time = 0.23 (sec) , antiderivative size = 765, normalized size of antiderivative = 4.66

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
 a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x + e)
+ 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(cos(
f*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) +
 5*c^5*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1)
+ 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10*c^5*((15*s
in(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 -
 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 10*c^5*((9*sin(f*x + e)/(cos(f*x + e
) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos
(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) +
 5*c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 - c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} - \frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} + \frac{2\left(165\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-280\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+123\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^2}}{6\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(315*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 315*c^5*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^2 + 2*(165*c^5*tan(1/2*f*x + 1/2*e)^5 - 280*c^5*tan
(1/2*f*x + 1/2*e)^3 + 123*c^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^
2 - 1)^3*a^2) - 32*(a^4*c^5*tan(1/2*f*x + 1/2*e)^3 + 12*a^4*c^5*tan(1/2*f*x
 + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{55\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{280\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+41\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)} - \frac{64\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} - \frac{16\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} + \frac{105\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (55*c^5*tan(e/2 + (f*x)/2)^5 - (280*c^5*tan(e/2 + (f*x)/2)^3)/3 + 41*c^5*ta
n(e/2 + (f*x)/2))/(f*(3*a^2*tan(e/2 + (f*x)/2)^2 - 3*a^2*tan(e/2 + (f*x)/2)
^4 + a^2*tan(e/2 + (f*x)/2)^6 - a^2)) - (64*c^5*tan(e/2 + (f*x)/2))/(a^2*f)
 - (16*c^5*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) + (105*c^5*atanh(tan(e/2 + (f*x)
/2)))/(a^2*f)
```

3.43 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 150

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{35c^4\text{arctanh}(\sin(e+fx))}{2a^2f} - \frac{70c^4\tan(e+fx)}{3a^2f} + \frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}$$

```
[Out] 35/2*c^4*arctanh(sin(f*x+e))/a^2/f-70/3*c^4*tan(f*x+e)/a^2/f+35/6*c^4*sec(f
*x+e)*tan(f*x+e)/a^2/f+2/3*c*(c-c*sec(f*x+e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e
))^2-14/3*(c^2-c^2*sec(f*x+e))^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4042, 3873, 3852, 8, 4131, 3855}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{35c^4\text{arctanh}(\sin(e+fx))}{2a^2f} - \frac{70c^4\tan(e+fx)}{3a^2f} + \frac{35c^4\tan(e+fx)\sec(e+fx)}{6a^2f} - \frac{14\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{3f\left(a^2\sec(e+fx)+a^2\right)} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (35*c^4*ArcTanh[Sin[e + f*x]])/(2*a^2*f) - (70*c^4*Tan[e + f*x])/(3*a^2*f)
+ (35*c^4*Sec[e + f*x]*Tan[e + f*x])/(6*a^2*f) + (2*c*(c - c*Sec[e + f*x])^
3*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (14*(c^2 - c^2*Sec[e + f*x])
^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(7c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx}{3a} \\
&= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} \\
&\quad + \frac{(35c^2)\int\sec(e+fx)(c-c\sec(e+fx))^2\,dx}{3a^2} \\
&= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} \\
&\quad + \frac{(35c^2)\int\sec(e+fx)\left(c^2+c^2\sec^2(e+fx)\right)\,dx}{3a^2} - \frac{(70c^4)\int\sec^2(e+fx)\,dx}{3a^2} \\
&= \frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&\quad - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(35c^4)\int\sec(e+fx)\,dx}{2a^2} \\
&\quad + \frac{(70c^4)\,\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{3a^2f} \\
&= \frac{35c^4\text{arctanh}(\sin(e+fx))}{2a^2f} - \frac{70c^4\tan(e+fx)}{3a^2f} + \frac{35c^4\sec(e+fx)\tan(e+fx)}{6a^2f} \\
&\quad + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{14\left(c^2-c^2\sec(e+fx)\right)^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.96 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.50

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{16c^4\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{3a^2f(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-16*c^4*Hypergeometric2F1[-7/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[2 -
 2*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*
x])^2)
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{8c^4\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{13}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}\right.}{f\,a^2}$
default	$\frac{8c^4\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}+\frac{13}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{35\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{16}-\frac{1}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}\right.}{f\,a^2}$
parallelrisch	$-\frac{51\left(\frac{35(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{51}+\frac{35(-1-\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{51}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\cos(f\cdots\right.\right.}{2f\,a^2(1+\cos(2fx+2e))}$
risch	$-\frac{ic^4\left(99\,e^{6i(fx+e)}+333\,e^{5i(fx+e)}+434\,e^{4i(fx+e)}+714\,e^{3i(fx+e)}+487\,e^{2i(fx+e)}+393\,e^{i(fx+e)}+164\right)}{3f\,a^2\left(1+e^{2i(fx+e)}\right)^2\left(e^{i(fx+e)}+1\right)^3}+\frac{35c^4\ln\left(e^{i(fx+\cdots}\right.}{2a^2f}$
norman	$\frac{-\frac{35c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{385c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{511c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3af}+\frac{93c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}-\frac{40c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}-\frac{8c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4a}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*c^4/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3-3*tan(1/2*f*x+1/2*e)+1/16/(tan(1/2*f
*x+1/2*e)-1)^2+13/16/(tan(1/2*f*x+1/2*e)-1)-35/16*ln(tan(1/2*f*x+1/2*e)-1)-
1/16/(tan(1/2*f*x+1/2*e)+1)^2+13/16/(tan(1/2*f*x+1/2*e)+1)+35/16*ln(tan(1/2
*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{105\left(c^4\cos\left(fx+e\right)^4+2\,c^4\cos\left(fx+e\right)^3+c^4\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-105\left(c^4\cos\left(fx+e\right)^4\right.}{12\left(a^2f\cos\right.}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^4*cos(f*x + e)^2)*
log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 2*c^4*cos(f*x + e)^3 + c^
4*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*(164*c^4*cos(f*x + e)^3 + 229*
c^4*cos(f*x + e)^2 + 30*c^4*cos(f*x + e) - 3*c^4)*sin(f*x + e))/(a^2*f*cos(
f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{c^4\left(\int \frac{\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx+\int\frac{6\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{}{\sec^2}\right.\right.}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-4*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(6*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integra
l(-4*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integral(
sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(142) = 284$.

Time = 0.21 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.54

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^4\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 4*c^4*((15
*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2
 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/(c
os(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^4*((9*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(co
s(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2)
+ 4*c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{105c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2}-\frac{105c^4\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2}+\frac{6\left(13c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-11c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^2}-\frac{16\left(a^4c^4\tan\left(\frac{1}{2}fx\right.\right.}{}}{6f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 105*c^4*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^2 + 6*(13*c^4*tan(1/2*f*x + 1/2*e)^3 - 11*c^4*tan(1
/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^2) - 16*(a^4*c^4*tan(1/2
*f*x + 1/2*e)^3 + 9*a^4*c^4*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{13\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-11\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^2\right)} - \frac{24\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} - \frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} + \frac{35\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (13*c^4*tan(e/2 + (f*x)/2)^3 - 11*c^4*tan(e/2 + (f*x)/2))/(f*(a^2*tan(e/2 +
 (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) - (24*c^4*tan(e/2 + (f*x)/
2))/(a^2*f) - (8*c^4*tan(e/2 + (f*x)/2)^3)/(3*a^2*f) + (35*c^4*atanh(tan(e/
2 + (f*x)/2)))/(a^2*f)
```

3.44 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 119

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{5c^3\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{5c^3\tan(e+fx)}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10(c^3-c^3\sec(e+fx))\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}$$

```
[Out] 5*c^3*arctanh(sin(f*x+e))/a^2/f-5*c^3*tan(f*x+e)/a^2/f+2/3*c*(c-c*sec(f*x+e
))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-10/3*(c^3-c^3*sec(f*x+e))*tan(f*x+e)/f
/(a^2+a^2*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4042, 3872, 3855, 3852, 8}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{5c^3\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{5c^3\tan(e+fx)}{a^2f} - \frac{10\tan(e+fx)\,(c^3-c^3\sec(e+fx))}{3f\,(a^2\sec(e+fx)+a^2)} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (5*c^3*ArcTanh[Sin[e + f*x]])/(a^2*f) - (5*c^3*Tan[e + f*x])/(a^2*f) + (2*c
*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (10*(c
^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{(5c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx}{3a}\\
&= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{10(c^3-c^3\sec(e+fx))\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}\\
&\quad+\frac{(5c^2)\int\sec(e+fx)(c-c\sec(e+fx))\,dx}{a^2}\\
&= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{10(c^3-c^3\sec(e+fx))\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}\\
&\quad+\frac{(5c^3)\int\sec(e+fx)\,dx}{a^2}-\frac{(5c^3)\int\sec^2(e+fx)\,dx}{a^2}
\end{aligned}$$

$$= \frac{5c^3\text{arctanh}(\sin(e+fx))}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10(c^3-c^3\sec(e+fx))\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(5c^3)\,\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{a^2f}$$

$$= \frac{5c^3\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{5c^3\tan(e+fx)}{a^2f} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{10(c^3-c^3\sec(e+fx))\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{8c^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{2},-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{3a^2f(-1+\sec(e+fx))(1+\sec(e+fx))^2}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-8*c^3*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 + Sec[e + f*x])/2]*Sqrt[2 -
2*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x
])^2)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{4c^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f\,a^2}$
default	$\frac{4c^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+4}+\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}+\frac{1}{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-4}-\frac{5\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}\right)}{f\,a^2}$
parallelrisch	$-\frac{5\left(\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+\frac{17\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(\cos(fx+e)+\frac{23\cos(2fx+2e)}{68}+\frac{29}{68}\right)\mathrm{se}}{15}\right.}{f\,a^2\cos(fx+e)}$
risch	$-\frac{2ic^3\left(12\,\mathrm{e}^{4i(fx+e)}+51\,\mathrm{e}^{3i(fx+e)}+41\,\mathrm{e}^{2i(fx+e)}+57\,\mathrm{e}^{i(fx+e)}+23\right)}{3f\,a^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3\left(1+\mathrm{e}^{2i(fx+e)}\right)}+\frac{5c^3\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{a^2f}-\frac{5c^3\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{a^2f}$
norman	$\frac{\frac{10c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{80c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}+\frac{22c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}-\frac{4c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}-\frac{4c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3a}-\frac{5c^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}{a^2f}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 4/f/a^2*c^3*(-1/3*tan(1/2*f*x+1/2*e)^3-2*tan(1/2*f*x+1/2*e)+1/4/(tan(1/2*f*
x+1/2*e)+1)+5/4*ln(tan(1/2*f*x+1/2*e)+1)+1/4/(tan(1/2*f*x+1/2*e)-1)-5/4*ln(
tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$
$$=\frac{15\left(c^3\cos\left(fx+e\right)^3+2\,c^3\cos\left(fx+e\right)^2+c^3\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx+e\right)^3+2\right.}{6\left(a^2f\cos\left(fx+e\right)^3+2\right.}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos(f*x + e))*log(
sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 2*c^3*cos(f*x + e)^2 + c^3*cos
(f*x + e))*log(-sin(f*x + e) + 1) - 2*(23*c^3*cos(f*x + e)^2 + 34*c^3*cos(f
*x + e) + 3*c^3)*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)
^2 + a^2*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^3\left(\int\left(-\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{3\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int\left(-\frac{3\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\right.}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(3*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(-3*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(117) = 234$.

Time = 0.21 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.87

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}+\frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right)+}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(c^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x +
e) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 3*c^3*((9*sin(f*x +
e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(si
n(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) +
 1) - 1)/a^2) + 3*c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(
cos(f*x + e) + 1)^3)/a^2 - c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} - \frac{15\,c^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} + \frac{6\,c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2} - \frac{4\left(a^4c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+6\,a^4c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{a^6}}{3\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 15*c^3*log(abs(tan(1/2
*f*x + 1/2*e) - 1))/a^2 + 6*c^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)
^2 - 1)*a^2) - 4*(a^4*c^3*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^3*tan(1/2*f*x +
1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{10\,c^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f} - \frac{4\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f} - \frac{8\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^2\,f} + \frac{2\,c^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}$$

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (10*c^3*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (4*c^3*tan(e/2 + (f*x)/2)^3)/(
3*a^2*f) - (8*c^3*tan(e/2 + (f*x)/2))/(a^2*f) + (2*c^3*tan(e/2 + (f*x)/2))/
(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2))
```

3.45 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 88

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{2c^2\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] c^2*arctanh(sin(f*x+e))/a^2/f-2*c^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+2/3*(
c^2-c^2*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4042, 3855}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\text{arctanh}(\sin(e+fx))}{a^2f} - \frac{2c^2\tan(e+fx)}{f\left(a^2\sec(e+fx)+a^2\right)} + \frac{2\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (c^2*ArcTanh[Sin[e + f*x]])/(a^2*f) - (2*c^2*Tan[e + f*x])/(f*(a^2 + a^2*Se
c[e + f*x])) + (2*(c^2 - c^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e
+ f*x])^2)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{c\int\frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{a}\\&=-\frac{2c^2\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))}+\frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{c^2\int\sec(e+fx)\,dx}{a^2}\\&=\frac{c^2\text{arctanh}(\sin(e+fx))}{a^2f}-\frac{2c^2\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))}+\frac{2(c^2-c^2\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx=\frac{c^2\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{4\tan\left(\frac{1}{2}(e+fx)\right)}{3f}-\frac{2\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{3f}\right)}{a^2}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
[Out] (c^2*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]/f - (4*Tan[(e + f*x)/2])/(3*f) - (2*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/(3*f)))/a^2
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{2c^{2}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^{2}}$
default	$\frac{2c^{2}\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^{2}}$
parallelrisch	$\frac{c^{2}\left(-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^{2}f}$
risch	$-\frac{8ic^{2}\left(3\,\mathrm{e}^{i(fx+e)}+1\right)}{3f\,a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}+\frac{c^{2}\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{a^{2}f}-\frac{c^{2}\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{a^{2}f}$
norman	$\frac{-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{10c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{3af}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}a}+\frac{c^{2}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{a^{2}f}-\frac{c^{2}\ln\left(\tan\left(\frac{f}{}\right.\right.}{a^{2}}$

[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 2/f/a^2*c^2*(-1/3*tan(1/2*f*x+1/2*e)^3-tan(1/2*f*x+1/2*e)-1/2*ln(tan(1/2*f*x+1/2*e)-1)+1/2*ln(tan(1/2*f*x+1/2*e)+1))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.57

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{2}}{(a+a\sec(e+fx))^{2}}\,dx$$
$$=\frac{3\left(c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\cos\left(fx+e\right)+c^{2}\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c^{2}\cos\left(fx+e\right)^{2}+2\,c^{2}\cos\left(fx+e\right)\right.}{6\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)\right.}$$

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fricas")

[Out] 1/6*(3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(sin(f*x + e) + 1) - 3*(c^2*cos(f*x + e)^2 + 2*c^2*cos(f*x + e) + c^2)*log(-sin(f*x + e) + 1) - 8*(c^2*cos(f*x + e) + 2*c^2)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{c^2\left(\int \frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int\left(-\frac{2\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(-2*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(87) = 174$.

Time = 0.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.23

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{c^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right) + \frac{2c^2\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2} - \frac{c^2(\cdots}{\cdots}}{6f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
[Out] -1/6*(c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 2*c^2*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/(cos
(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{3c^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} - \frac{3c^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} - \frac{2\left(a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6}}{3f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*c^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 3*c^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/a^2 - 2*(a^4*c^2*tan(1/2*f*x + 1/2*e)^3 + 3*a^4*c^2*tan(1
/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{2\,c^2\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\right)}{3\,a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] -(2*c^2*(3*tan(e/2 + (f*x)/2) - 3*atanh(tan(e/2 + (f*x)/2)) + tan(e/2 + (f*
x)/2)^3))/(3*a^2*f)
```

3.46 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\frac{(c-c\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] 1/3*(c-c*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {4035}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\frac{\tan(e+fx)(c-c\sec(e+fx))}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral}=\frac{(c-c\sec(e+fx))\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=-\frac{c\tan^3\left(\frac{1}{2}(e+fx)\right)}{3a^2f}$$

[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]

[Out] -1/3*(c*Tan[(e + f*x)/2]^3)/(a^2*f)

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f\,a^2}$	21
default	$-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f\,a^2}$	21
parallelrisch	$-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f\,a^2}$	21
risch	$\frac{2ic\left(3\,\mathrm{e}^{2i(fx+e)}+1\right)}{3f\,a^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3}$	37
norman	$\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3af}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$	61

[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] -1/3/f/a^2*c*tan(1/2*f*x+1/2*e)^3

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\frac{(c\cos(fx+e)-c)\sin(fx+e)}{3\left(a^2f\cos(fx+e)^2+2\,a^2f\cos(fx+e)+a^2f\right)}$$

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fric as")

[Out] 1/3*(c*cos(f*x + e) - c)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f *x + e) + a^2*f)

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\int\left(-\frac{\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Int
egral(sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(35) = 70$.

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.61

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/6*(c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{3a^2f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/3*c*tan(1/2*f*x + 1/2*e)^3/(a^2*f)
```

Mupad [B] (verification not implemented)

Time = 12.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=-\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
[Out] -(c*tan(e/2 + (f*x)/2)^3)/(3*a^2*f)
```

3.47 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))} dx$

Optimal result

Integrand size = 32, antiderivative size = 59

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))} dx = \frac{\cot^3(e+fx)}{3a^2cf} + \frac{\csc(e+fx)}{a^2cf} - \frac{\csc^3(e+fx)}{3a^2cf}$$

```
[Out] 1/3*cot(f*x+e)^3/a^2/c/f+csc(f*x+e)/a^2/c/f-1/3*csc(f*x+e)^3/a^2/c/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4043, 2686, 2687, 30}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))} dx = \frac{\cot^3(e+fx)}{3a^2cf} - \frac{\csc^3(e+fx)}{3a^2cf} + \frac{\csc(e+fx)}{a^2cf}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
```

```
[Out] Cot[e + f*x]^3/(3*a^2*c*f) + Csc[e + f*x]/(a^2*c*f) - Csc[e + f*x]^3/(3*a^2
*c*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \left(c\cot^3(e+fx)\csc(e+fx) - c\cot^2(e+fx)\csc^2(e+fx)\right) dx}{a^2c^2} \\
&= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2c} - \frac{\int \cot^2(e+fx)\csc^2(e+fx)\,dx}{a^2c} \\
&= -\frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(e+fx)\right)}{a^2cf} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(e+fx)\right)}{a^2cf} \\
&= \frac{\cot^3(e+fx)}{3a^2cf} + \frac{\csc(e+fx)}{a^2cf} - \frac{\csc^3(e+fx)}{3a^2cf}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx \\
&= \frac{(-1+2\sec(e+fx)+2\sec^2(e+fx))\tan(e+fx)}{3a^2cf(-1+\sec(e+fx))(1+\sec(e+fx))^2}
\end{aligned}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])),x]
[Out] ((-1 + 2*Sec[e + f*x] + 2*Sec[e + f*x]^2)*Tan[e + f*x])/(3*a^2*c*f*(-1 + Se
c[e + f*x])*(1 + Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	48
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{4f\,a^{2}c}$	48
parallelrisch	$\frac{-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{12a^{2}cf}$	48
norman	$\frac{\frac{1}{4acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{2acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{12acf}}{a\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	72
risch	$\frac{2i\left(3\,\mathrm{e}^{3i(fx+e)}+3\,\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}-1\right)}{3f\,a^{2}c\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}\left(\mathrm{e}^{i(fx+e)}-1\right)}$	72

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a^2/c*(-1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+1/tan(1/2*f*x+1
/2*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\cos\left(fx+e\right)^2-2\,\cos\left(fx+e\right)-2}{3\left(a^2cf\cos\left(fx+e\right)+a^2cf\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/3*(cos(f*x + e)^2 - 2*cos(f*x + e) - 2)/((a^2*c*f*cos(f*x + e) + a^2*c*f
)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sec^3(e+fx)+\sec^2(e+fx)-\sec(e+fx)-1}\,dx}{a^2c}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**3 + sec(e + f*x)**2 - sec(e + f*x) -
1), x)/(a**2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\frac{\frac{6\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2c}+\frac{3(\cos(fx+e)+1)}{a^2c\sin(fx+e)}}{12f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*((6*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1
)^3)/(a^2*c) + 3*(cos(f*x + e) + 1)/(a^2*c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = \frac{\frac{3}{a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}-\frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-6a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6c^3}}{12f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/12*(3/(a^2*c*tan(1/2*f*x + 1/2*e)) - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 6*
a^4*c^2*tan(1/2*f*x + 1/2*e))/(a^6*c^3))/f
```

Mupad [B] (verification not implemented)

Time = 12.87 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))}\,dx = -\frac{4\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4-8\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1}{12\,a^2\,c\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))),x)
```

```
[Out] -(4*cos(e/2 + (f*x)/2)^4 - 8*cos(e/2 + (f*x)/2)^2 + 1)/(12*a^2*c*f*cos(e/2
+ (f*x)/2)^3*sin(e/2 + (f*x)/2))
```

3.48 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2} \, dx = \frac{\csc(e+fx)}{a^2c^2f} - \frac{\csc^3(e+fx)}{3a^2c^2f}$$

```
[Out] csc(f*x+e)/a^2/c^2/f-1/3*csc(f*x+e)^3/a^2/c^2/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4043, 2686}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2} \, dx = \frac{\csc(e+fx)}{a^2c^2f} - \frac{\csc^3(e+fx)}{3a^2c^2f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] Csc[e + f*x]/(a^2*c^2*f) - Csc[e + f*x]^3/(3*a^2*c^2*f)
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
```

```
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \cot^3(e+fx)\csc(e+fx)\,dx}{a^2c^2}\\ &= -\frac{\text{Subst}\left(\int (-1+x^2)\,dx, x, \csc(e+fx)\right)}{a^2c^2f}\\ &= \frac{\csc(e+fx)}{a^2c^2f} - \frac{\csc^3(e+fx)}{3a^2c^2f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\frac{\csc(e+fx)}{f} - \frac{\csc^3(e+fx)}{3f}}{a^2c^2}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^2),x]
[Out] (Csc[e + f*x]/f - Csc[e + f*x]^3/(3*f))/(a^2*c^2)
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{-\frac{\csc(fx+e)^3}{3}+\csc(fx+e)}{a^2c^2f}$	28
parallelrisch	$\frac{-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+9\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{24f\,a^2c^2}$	61
risch	$\frac{2i\left(3\,e^{5i(fx+e)}-2\,e^{3i(fx+e)}+3\,e^{i(fx+e)}\right)}{3f\,a^2c^2\left(e^{i(fx+e)}+1\right)^3\left(e^{i(fx+e)}-1\right)^3}$	73
norman	$\frac{-\frac{1}{24acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{8acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{24acf}}{ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	97

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
[Out] 1/a^2/c^2/f*(-1/3*csc(f*x+e)^3+csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{3\cos\left(fx+e\right)^2-2}{3\left(a^2c^2f\cos\left(fx+e\right)^2-a^2c^2f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3*(3*cos(f*x + e)^2 - 2)/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x
+ e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec(e+fx)}{\sec^4(e+fx)-2\sec^2(e+fx)+1}\,dx}{a^2c^2}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**4 - 2*sec(e + f*x)**2 + 1), x)/(a**2*c
**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{3\sin\left(fx+e\right)^2-1}{3\,a^2c^2f\sin\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{3\,\sin\left(fx+e\right)^2-1}{3\,a^2c^2f\sin\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
[Out] 1/3*(3*sin(f*x + e)^2 - 1)/(a^2*c^2*f*sin(f*x + e)^3)
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^2}\,dx = \frac{\sin\left(e+f\,x\right)^2-\frac{1}{3}}{a^2\,c^2\,f\,\sin\left(e+f\,x\right)^3}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^2),x)
[Out] (sin(e + f*x)^2 - 1/3)/(a^2*c^2*f*sin(e + f*x)^3)
```

3.49 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}$$

```
[Out] 1/5*cot(f*x+e)^5/a^2/c^3/f+csc(f*x+e)/a^2/c^3/f-2/3*csc(f*x+e)^3/a^2/c^3/f+
1/5*csc(f*x+e)^5/a^2/c^3/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4043, 2686, 200, 2687, 30}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Cot[e + f*x]^5/(5*a^2*c^3*f) + Csc[e + f*x]/(a^2*c^3*f) - (2*Csc[e + f*x]^3
)/(3*a^2*c^3*f) + Csc[e + f*x]^5/(5*a^2*c^3*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \left(a\cot^5(e+fx)\csc(e+fx)+a\cot^4(e+fx)\csc^2(e+fx)\right)\,dx}{a^3c^3}\\
&= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^2c^3}-\frac{\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^2c^3}\\
&= -\frac{\text{Subst}\left(\int x^4\,dx,x,-\cot(e+fx)\right)}{a^2c^3f}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\csc(e+fx)\right)}{a^2c^3f}\\
&= \frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(e+fx)\right)}{a^2c^3f}\\
&= \frac{\cot^5(e+fx)}{5a^2c^3f}+\frac{\csc(e+fx)}{a^2c^3f}-\frac{2\csc^3(e+fx)}{3a^2c^3f}+\frac{\csc^5(e+fx)}{5a^2c^3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{(3+12\sec(e+fx)-12\sec^2(e+fx)-8\sec^3(e+fx)+8\sec^4(e+fx))\tan(e+fx)}{15a^2c^3f(-1+\sec(e+fx))^3(1+\sec(e+fx))^2}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] ((3 + 12*Sec[e + f*x] - 12*Sec[e + f*x]^2 - 8*Sec[e + f*x]^3 + 8*Sec[e + f*
x]^4)*Tan[e + f*x])/(15*a^2*c^3*f*(-1 + Sec[e + f*x])^3*(1 + Sec[e + f*x])^
2)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^5-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-20\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+60\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+90\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{240f\,a^2c^3}$	74
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,a^2c^3}$	76
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{4}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{6}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{16f\,a^2c^3}$	76
risch	$\frac{2i\left(15\,\mathrm{e}^{7i(fx+e)}-15\,\mathrm{e}^{6i(fx+e)}-5\,\mathrm{e}^{5i(fx+e)}+25\,\mathrm{e}^{4i(fx+e)}+13\,\mathrm{e}^{3i(fx+e)}-21\,\mathrm{e}^{2i(fx+e)}+9\,\mathrm{e}^{i(fx+e)}+3\right)}{15f\,a^2c^3\left(\mathrm{e}^{i(fx+e)}+1\right)^3\left(\mathrm{e}^{i(fx+e)}-1\right)^5}$	118
norman	$\frac{\frac{1}{80acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{12acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{4acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{48acf}}{a\,c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	119

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/240*(3*cot(1/2*f*x+1/2*e)^5-5*tan(1/2*f*x+1/2*e)^3-20*cot(1/2*f*x+1/2*e)^
3+60*tan(1/2*f*x+1/2*e)+90*cot(1/2*f*x+1/2*e))/f/a^2/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{3\cos\left(fx+e\right)^4+12\cos\left(fx+e\right)^3-12\cos\left(fx+e\right)^2-8\cos\left(fx+e\right)+8}{15\left(a^2c^3f\cos\left(fx+e\right)^3-a^2c^3f\cos\left(fx+e\right)^2-a^2c^3f\cos\left(fx+e\right)+a^2c^3f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(3*cos(f*x + e)^4 + 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 - 8*cos(f*x
+ e) + 8)/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f
*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sec^5(e+fx)-\sec^4(e+fx)-2\sec^3(e+fx)+2\sec^2(e+fx)+\sec(e+fx)-1}\,dx}{a^2c^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**3,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**5 - sec(e + f*x)**4 - 2*sec(e + f*x)*
*3 + 2*sec(e + f*x)**2 + sec(e + f*x) - 1), x)/(a**2*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\frac{5\left(\frac{12\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^3}-\frac{\left(\frac{20\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{90\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-3\right)(\cos(fx+e)+1)^5}{a^2c^3\sin(fx+e)^5}}{240\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/240*(5*(12*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/(a^2*c^3) - (20*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 90*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 - 3)*(cos(f*x + e) + 1)^5/(a^2*c^3*sin(f*x + e)^
5))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{\frac{90\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-20\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+3}{a^2c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}-\frac{5\left(a^4c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-12a^4c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6c^9}}{240f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/240*((90*tan(1/2*f*x + 1/2*e)^4 - 20*tan(1/2*f*x + 1/2*e)^2 + 3)/(a^2*c^3
*tan(1/2*f*x + 1/2*e)^5) - 5*(a^4*c^6*tan(1/2*f*x + 1/2*e)^3 - 12*a^4*c^6*t
an(1/2*f*x + 1/2*e))/(a^6*c^9))/f
```

Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^3}\,dx = \frac{-5\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+60\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+90\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-20\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3}{240\,a^2\,c^3\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^3),x)
```

```
[Out] (90*tan(e/2 + (f*x)/2)^4 - 20*tan(e/2 + (f*x)/2)^2 + 60*tan(e/2 + (f*x)/2)^
6 - 5*tan(e/2 + (f*x)/2)^8 + 3)/(240*a^2*c^3*f*tan(e/2 + (f*x)/2)^5)
```

3.50 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 98

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{2\cot^7(e+fx)}{7a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}$$

```
[Out] -2/7*cot(f*x+e)^7/a^2/c^4/f+csc(f*x+e)/a^2/c^4/f-4/3*csc(f*x+e)^3/a^2/c^4/f
+csc(f*x+e)^5/a^2/c^4/f-2/7*csc(f*x+e)^7/a^2/c^4/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4043, 2686, 200, 2687, 30, 276}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = -\frac{2\cot^7(e+fx)}{7a^2c^4f}-\frac{2\csc^7(e+fx)}{7a^2c^4f}+\frac{\csc^5(e+fx)}{a^2c^4f}-\frac{4\csc^3(e+fx)}{3a^2c^4f}+\frac{\csc(e+fx)}{a^2c^4f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] (-2*Cot[e + f*x]^7)/(7*a^2*c^4*f) + Csc[e + f*x]/(a^2*c^4*f) - (4*Csc[e + f
*x]^3)/(3*a^2*c^4*f) + Csc[e + f*x]^5/(a^2*c^4*f) - (2*Csc[e + f*x]^7)/(7*a
^2*c^4*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

integral

$$= \frac{\int \left(a^2 \cot^7(e+fx)\csc(e+fx) + 2a^2\cot^6(e+fx)\csc^2(e+fx) + a^2\cot^5(e+fx)\csc^3(e+fx)\right)\,dx}{a^4c^4}$$

$$= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^2c^4} + \frac{\int \cot^5(e+fx)\csc^3(e+fx)\,dx}{a^2c^4} + \frac{2\int \cot^6(e+fx)\csc^2(e+fx)\,dx}{a^2c^4}$$

$$= -\frac{\text{Subst}\left(\int x^2(-1+x^2)^2\,dx, x, \csc(e+fx)\right)}{a^2c^4f} - \frac{\text{Subst}\left(\int (-1+x^2)^3\,dx, x, \csc(e+fx)\right)}{a^2c^4f} + \frac{2\text{Subst}\left(\int x^6\,dx, x, -\cot(e+fx)\right)}{a^2c^4f}$$

$$= -\frac{2\cot^7(e+fx)}{7a^2c^4f} - \frac{\text{Subst}\left(\int (-1+3x^2-3x^4+x^6)\,dx, x, \csc(e+fx)\right)}{a^2c^4f} - \frac{\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \csc(e+fx)\right)}{a^2c^4f}$$

$$= -\frac{2\cot^7(e+fx)}{7a^2c^4f} + \frac{\csc(e+fx)}{a^2c^4f} - \frac{4\csc^3(e+fx)}{3a^2c^4f} + \frac{\csc^5(e+fx)}{a^2c^4f} - \frac{2\csc^7(e+fx)}{7a^2c^4f}$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{(-6-9\sec(e+fx)+24\sec^2(e+fx)-4\sec^3(e+fx)-16\sec^4(e+fx)+8\sec^5(e+fx))\tan(e+fx)}{21a^2c^4f(-1+\sec(e+fx))^4(1+\sec(e+fx))^2}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] ((-6 - 9*Sec[e + f*x] + 24*Sec[e + f*x]^2 - 4*Sec[e + f*x]^3 - 16*Sec[e + f
*x]^4 + 8*Sec[e + f*x]^5)*Tan[e + f*x])/(21*a^2*c^4*f*(-1 + Sec[e + f*x])^4
*(1 + Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{10}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{10}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{32f\,c^{4}a^{2}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{10}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{10}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}-\frac{1}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}}{32f\,c^{4}a^{2}}$
parallelrisch	$\frac{-3\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+21\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-70\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+105\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+210\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{672f\,a^{2}c^{4}}$
risch	$\frac{2i\left(21\,e^{9i(fx+e)}-42\,e^{8i(fx+e)}+28\,e^{7i(fx+e)}+56\,e^{6i(fx+e)}-42\,e^{5i(fx+e)}-28\,e^{4i(fx+e)}+76\,e^{3i(fx+e)}-24\,e^{2i(fx+e)}-3\,e^{i(\ldots}\right.}{21f\,c^{4}a^{2}\left(e^{i(fx+e)}-1\right)^{7}\left(e^{i(fx+e)}+1\right)^{3}}$
norman	$\frac{-\frac{1}{224acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{32acf}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{48acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{16acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{32acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{96acf}}{a\,c^{3}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/32/f/c^4/a^2*(-1/3*tan(1/2*f*x+1/2*e)^3+5*tan(1/2*f*x+1/2*e)+10/tan(1/2*f
*x+1/2*e)-10/3/tan(1/2*f*x+1/2*e)^3-1/7/tan(1/2*f*x+1/2*e)^7+1/tan(1/2*f*x+
1/2*e)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx$$
$$=\frac{6\cos(fx+e)^5+9\cos(fx+e)^4-24\cos(fx+e)^3+4\cos(fx+e)^2+16\cos(fx+e)-8}{21\left(a^2c^4f\cos(fx+e)^4-2\,a^2c^4f\cos(fx+e)^3+2\,a^2c^4f\cos(fx+e)-a^2c^4f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/21*(6*cos(f*x + e)^5 + 9*cos(f*x + e)^4 - 24*cos(f*x + e)^3 + 4*cos(f*x +
 e)^2 + 16*cos(f*x + e) - 8)/((a^2*c^4*f*cos(f*x + e)^4 - 2*a^2*c^4*f*cos(f
*x + e)^3 + 2*a^2*c^4*f*cos(f*x + e) - a^2*c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{\sec(e+fx)}{\sec^6(e+fx)-2\sec^5(e+fx)-\sec^4(e+fx)+4\sec^3(e+fx)-\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{a^2c^4}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**4,x)
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**6 - 2*sec(e + f*x)**5 - sec(e + f*x)**
4 + 4*sec(e + f*x)**3 - sec(e + f*x)**2 - 2*sec(e + f*x) + 1), x)/(a**2*c**
4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.43

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\frac{7\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^4}+\frac{\left(\frac{21\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{70\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{210\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-3\right)(\cos(fx+e)+1)^7}{a^2c^4\sin(fx+e)^7}}{672\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
[Out] 1/672*(7*(15*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/(a^2*c^4) + (21*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 70*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 210*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3)*(
cos(f*x + e) + 1)^7/(a^2*c^4*sin(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.11

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{\frac{210\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-70\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+21\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3}{a^2c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}-\frac{7\left(a^4c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-15a^4c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{a^6c^{12}}}{672\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/672*((210*tan(1/2*f*x + 1/2*e)^6 - 70*tan(1/2*f*x + 1/2*e)^4 + 21*tan(1/2
*f*x + 1/2*e)^2 - 3)/(a^2*c^4*tan(1/2*f*x + 1/2*e)^7) - 7*(a^4*c^8*tan(1/2*
f*x + 1/2*e)^3 - 15*a^4*c^8*tan(1/2*f*x + 1/2*e))/(a^6*c^12))/f
```

Mupad [B] (verification not implemented)

Time = 14.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^4}\,dx = \frac{-7\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+105\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}+210\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}-70\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}-3}{672\,a^2\,c^4\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^4),x)
```

```
[Out] (21*tan(e/2 + (f*x)/2)^2 - 70*tan(e/2 + (f*x)/2)^4 + 210*tan(e/2 + (f*x)/2)
^6 + 105*tan(e/2 + (f*x)/2)^8 - 7*tan(e/2 + (f*x)/2)^10 - 3)/(672*a^2*c^4*f
*tan(e/2 + (f*x)/2)^7)
```

3.51 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 141

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}$$

```
[Out] 1/7*cot(f*x+e)^7/a^2/c^5/f+4/9*cot(f*x+e)^9/a^2/c^5/f+csc(f*x+e)/a^2/c^5/f-
7/3*csc(f*x+e)^3/a^2/c^5/f+3*csc(f*x+e)^5/a^2/c^5/f-13/7*csc(f*x+e)^7/a^2/c
^5/f+4/9*csc(f*x+e)^9/a^2/c^5/f
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4043, 2686, 200, 2687, 30, 276, 14}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] Cot[e + f*x]^7/(7*a^2*c^5*f) + (4*Cot[e + f*x]^9)/(9*a^2*c^5*f) + Csc[e + f
*x]/(a^2*c^5*f) - (7*Csc[e + f*x]^3)/(3*a^2*c^5*f) + (3*Csc[e + f*x]^5)/(a^
2*c^5*f) - (13*Csc[e + f*x]^7)/(7*a^2*c^5*f) + (4*Csc[e + f*x]^9)/(9*a^2*c^
5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
```

```
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

integral =

$$-\frac{\int (a^3\cot^9(e+fx)\csc(e+fx)+3a^3\cot^8(e+fx)\csc^2(e+fx)+3a^3\cot^7(e+fx)\csc^3(e+fx)+a^3\cot^6}{a^5c^5}$$

$$=-\frac{\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^2c^5}-\frac{\int \cot^6(e+fx)\csc^4(e+fx)\,dx}{a^2c^5}-\frac{3\int \cot^8(e+fx)\csc^2(e+fx)\,dx}{a^2c^5}-\frac{3\int \cot^7(e+fx)\csc^3(e+fx)\,dx}{a^2c^5}$$

$$=\frac{\text{Subst}\left(\int (-1+x^2)^4\,dx,x,\csc(e+fx)\right)}{a^2c^5f}-\frac{\text{Subst}\left(\int x^6(1+x^2)\,dx,x,-\cot(e+fx)\right)}{a^2c^5f}-\frac{3\text{Subst}\left(\int x^8\,dx,x,-\cot(e+fx)\right)}{a^2c^5f}+\frac{3\text{Subst}\left(\int x^2(-1+x^2)^3\,dx,x,\csc(e+fx)\right)}{a^2c^5f}$$

$$=\frac{\cot^9(e+fx)}{3a^2c^5f}+\frac{\text{Subst}\left(\int (1-4x^2+6x^4-4x^6+x^8)\,dx,x,\csc(e+fx)\right)}{a^2c^5f}-\frac{\text{Subst}\left(\int (x^6+x^8)\,dx,x,-\cot(e+fx)\right)}{a^2c^5f}+\frac{3\text{Subst}\left(\int (-x^2+3x^4-3x^6+x^8)\,dx,x,\csc(e+fx)\right)}{a^2c^5f}$$

$$=\frac{\cot^7(e+fx)}{7a^2c^5f}+\frac{4\cot^9(e+fx)}{9a^2c^5f}+\frac{\csc(e+fx)}{a^2c^5f}-\frac{7\csc^3(e+fx)}{3a^2c^5f}+\frac{3\csc^5(e+fx)}{a^2c^5f}-\frac{13\csc^7(e+fx)}{7a^2c^5f}+\frac{4\csc^9(e+fx)}{9a^2c^5f}$$

Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx=\frac{(19+6\sec(e+fx)-66\sec^2(e+fx)+56\sec^3(e+fx)+24\sec^4(e+fx)-48\sec^5(e+fx)+16\sec^6(e+}{63a^2c^5f(-1+\sec(e+fx))^5(1+\sec(e+fx))^2}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] ((19 + 6*Sec[e + f*x] - 66*Sec[e + f*x]^2 + 56*Sec[e + f*x]^3 + 24*Sec[e +
f*x]^4 - 48*Sec[e + f*x]^5 + 16*Sec[e + f*x]^6)*Tan[e + f*x])/(63*a^2*c^5*f
*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{7\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}-54\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}+189\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-420\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+378\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+945\cot(}{4032f\,a^{2}c^{5}}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{20}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{6}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{15}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}}{64f\,c^{5}a^{2}}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{20}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}+\frac{3}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}-\frac{6}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}+\frac{15}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}}{64f\,c^{5}a^{2}}$
risch	$\frac{2i(63\,e^{11i(fx+e)}-189\,e^{10i(fx+e)}+273\,e^{9i(fx+e)}+63\,e^{8i(fx+e)}-378\,e^{7i(fx+e)}+294\,e^{6i(fx+e)}+306\,e^{5i(fx+e)}-450\,e^{4i(fx}}{63f\,c^{5}a^{2}\left(e^{i(fx+e)}-1\right)^{9}\left(e^{i(fx+e)}+1\right)^{3}}$
norman	$\frac{\frac{1}{576acf}-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{224acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{64acf}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{48acf}+\frac{15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{8}}{64acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{32acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{192acf}}{a\,c^{4}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4032*(7*cot(1/2*f*x+1/2*e)^9-54*cot(1/2*f*x+1/2*e)^7+189*cot(1/2*f*x+1/2*
e)^5-21*tan(1/2*f*x+1/2*e)^3-420*cot(1/2*f*x+1/2*e)^3+378*tan(1/2*f*x+1/2*e
)+945*cot(1/2*f*x+1/2*e))/f/a^2/c^5
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{19\cos\left(fx+e\right)^{6}+6\cos\left(fx+e\right)^{5}-66\cos\left(fx+e\right)^{4}+56\cos\left(fx+e\right)^{3}+24\cos\left(fx+e\right)^{2}-}{63\left(a^{2}c^{5}f\cos\left(fx+e\right)^{5}-3\,a^{2}c^{5}f\cos\left(fx+e\right)^{4}+2\,a^{2}c^{5}f\cos\left(fx+e\right)^{3}+2\,a^{2}c^{5}f\cos\left(fx+e\right)^{2}-3\,a^{2}c^{5}f\,c}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/63*(19*cos(f*x + e)^6 + 6*cos(f*x + e)^5 - 66*cos(f*x + e)^4 + 56*cos(f*x
 + e)^3 + 24*cos(f*x + e)^2 - 48*cos(f*x + e) + 16)/((a^2*c^5*f*cos(f*x + e
)^5 - 3*a^2*c^5*f*cos(f*x + e)^4 + 2*a^2*c^5*f*cos(f*x + e)^3 + 2*a^2*c^5*f
*cos(f*x + e)^2 - 3*a^2*c^5*f*cos(f*x + e) + a^2*c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sec^7(e+fx)-3\sec^6(e+fx)+\sec^5(e+fx)+5\sec^4(e+fx)-5\sec^3(e+fx)-\sec^2(e+fx)+3\sec(e+fx)-1}\,dx}{a^2c^5}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**5,x)

[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**7 - 3*sec(e + f*x)**6 + sec(e + f*x)**5 + 5*sec(e + f*x)**4 - 5*sec(e + f*x)**3 - sec(e + f*x)**2 + 3*sec(e + f*x) - 1), x)/(a**2*c**5)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\frac{21\left(\frac{18\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2c^5}-\frac{\left(\frac{54\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{189\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{420\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{945\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-7\right)(\cos(fx+e)+1)^9}{a^2c^5\sin(fx+e)^9}}{4032\,f}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="maxima")

[Out] 1/4032*(21*(18*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^2*c^5) - (54*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 420*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 945*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 7)*(cos(f*x + e) + 1)^9/(a^2*c^5*sin(f*x + e)^9))/f

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{\frac{945\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-420\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+189\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-54\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+7}{a^2c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}-\frac{21\left(a^4c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-18a^4c^{10}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}{a^6c^{15}}}{4032\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/4032*((945*tan(1/2*f*x + 1/2*e)^8 - 420*tan(1/2*f*x + 1/2*e)^6 + 189*tan(
1/2*f*x + 1/2*e)^4 - 54*tan(1/2*f*x + 1/2*e)^2 + 7)/(a^2*c^5*tan(1/2*f*x +
1/2*e)^9) - 21*(a^4*c^10*tan(1/2*f*x + 1/2*e)^3 - 18*a^4*c^10*tan(1/2*f*x +
 1/2*e))/(a^6*c^15))/f
```

Mupad [B] (verification not implemented)

Time = 15.56 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.72

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^5}\,dx = \frac{-21\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{12}+378\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{10}+945\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{8}-420\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+189\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}-5}{4032\,a^2\,c^5\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{9}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^5),x)
```

```
[Out] (189*tan(e/2 + (f*x)/2)^4 - 54*tan(e/2 + (f*x)/2)^2 - 420*tan(e/2 + (f*x)/2
)^6 + 945*tan(e/2 + (f*x)/2)^8 + 378*tan(e/2 + (f*x)/2)^10 - 21*tan(e/2 + (
f*x)/2)^12 + 7)/(4032*a^2*c^5*f*tan(e/2 + (f*x)/2)^9)
```

3.52 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 215

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = -\frac{231c^6\operatorname{arctanh}(\sin(e+fx))}{2a^3f}+\frac{924c^6\tan(e+fx)}{5a^3f} - \frac{693c^6\sec(e+fx)\tan(e+fx)}{10a^3f} - \frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{66\left(c^2-c^2\sec(e+fx)\right)^3\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} + \frac{77c^6\tan^3(e+fx)}{5a^3f}$$

```
[Out] -231/2*c^6*arctanh(sin(f*x+e))/a^3/f+924/5*c^6*tan(f*x+e)/a^3/f-693/10*c^6*
sec(f*x+e)*tan(f*x+e)/a^3/f-22/15*c^2*(c-c*sec(f*x+e))^4*tan(f*x+e)/a/f/(a+
a*sec(f*x+e))^2+2/5*c*(c-c*sec(f*x+e))^5*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+66
/5*(c^2-c^2*sec(f*x+e))^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+77/5*c^6*tan(f*
x+e)^3/a^3/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4042, 3876, 3855, 3852, 8, 3853}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = -\frac{231c^6\text{arctanh}(\sin(e+fx))}{2a^3f} + \frac{77c^6\tan^3(e+fx)}{5a^3f} + \frac{924c^6\tan(e+fx)}{5a^3f} - \frac{693c^6\tan(e+fx)\sec(e+fx)}{10a^3f} + \frac{66\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^3}{5f\left(a^3\sec(e+fx)+a^3\right)} - \frac{22c^2\tan(e+fx)(c-c\sec(e+fx))^4}{15af(a\sec(e+fx)+a)^2} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-231*c^6*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (924*c^6*Tan[e + f*x])/(5*a^3*
f) - (693*c^6*Sec[e + f*x]*Tan[e + f*x])/(10*a^3*f) - (22*c^2*(c - c*Sec[e
+ f*x])^4*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e
 + f*x])^5*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (66*(c^2 - c^2*Sec[
e + f*x])^3*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) + (77*c^6*Tan[e +
f*x]^3)/(5*a^3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{(11c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx}{5a}\\
&=-\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\\
&\quad+\frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(33c^2)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx}{5a^2}\\
&=-\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad+\frac{66\left(c^2-c^2\sec(e+fx)\right)^3\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}-\frac{(231c^3)\int\sec(e+fx)(c-c\sec(e+fx))^3\,dx}{5a^3}\\
&=-\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad+\frac{66\left(c^2-c^2\sec(e+fx)\right)^3\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}\\
&\quad-\frac{(231c^3)\int\left(c^3\sec(e+fx)-3c^3\sec^2(e+fx)+3c^3\sec^3(e+fx)-c^3\sec^4(e+fx)\right)dx}{5a^3}
\end{aligned}$$

$$= -\frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{66(c^2-c^2\sec(e+fx))^3\tan(e+fx)}{5f(a^3+a^3\sec(e+fx))} - \frac{(231c^6)\int\sec(e+fx)\,dx}{5a^3} + \frac{(231c^6)\int\sec^4(e+fx)\,dx}{5a^3} + \frac{(693c^6)\int\sec^2(e+fx)\,dx}{5a^3} - \frac{(693c^6)\int\sec^3(e+fx)\,dx}{5a^3}$$

$$= -\frac{231c^6\text{arctanh}(\sin(e+fx))}{5a^3f} - \frac{693c^6\sec(e+fx)\tan(e+fx)}{10a^3f} - \frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{66(c^2-c^2\sec(e+fx))^3\tan(e+fx)}{5f(a^3+a^3\sec(e+fx))} - \frac{(693c^6)\int\sec(e+fx)\,dx}{10a^3} - \frac{(231c^6)\,\text{Subst}\left(\int(1+x^2)\,dx,x,-\tan(e+fx)\right)}{5a^3f} - \frac{(693c^6)\,\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{5a^3f}$$

$$= -\frac{231c^6\text{arctanh}(\sin(e+fx))}{2a^3f} + \frac{924c^6\tan(e+fx)}{5a^3f} - \frac{693c^6\sec(e+fx)\tan(e+fx)}{10a^3f} - \frac{22c^2(c-c\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{66(c^2-c^2\sec(e+fx))^3\tan(e+fx)}{5f(a^3+a^3\sec(e+fx))} + \frac{77c^6\tan^3(e+fx)}{5a^3f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.35

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = -\frac{64c^6\,\text{Hypergeometric2F1}\left(-\frac{11}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{5a^3f(-1+\sec(e+fx))(1+\sec(e+fx))^3}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
[Out] (-64*c^6*Hypergeometric2F1[-11/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[2
- 2*Sec[e + f*x]]*Tan[e + f*x])/(5*a^3*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f
*x])^3)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{16c^6\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{5}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{89}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\right.}{f\,a^3}$
default	$\frac{16c^6\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{48\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{5}{16\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{89}{32\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\right.}{f\,a^3}$
parallelrisch	$\frac{2723c^6\left(\frac{3960(\cos(3fx+3e)+3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{389}+\frac{3960(-\cos(3fx+3e)-3\cos(fx+e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{389}+\tan\left(\frac{fx}{2}+\right.\right.}{240f\,a^3(\cos(3fx+3e}$
risch	$\frac{ic^6\left(3495\,e^{10i(fx+e)}+17205\,e^{9i(fx+e)}+44480\,e^{8i(fx+e)}+79450\,e^{7i(fx+e)}+120176\,e^{6i(fx+e)}+130340\,e^{5i(fx+e)}+127498\,e^{4}\right.}{15f\,a^3\left(1+e^{2i(fx+e)}\right)^3\left(e^{i(fx+e)}+1\right)^5}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/f*c^6/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+4/3*tan(1/2*f*x+1/2*e)^3+10*tan(1/2*
f*x+1/2*e)-1/48/(tan(1/2*f*x+1/2*e)+1)^3+5/16/(tan(1/2*f*x+1/2*e)+1)^2-89/3
2/(tan(1/2*f*x+1/2*e)+1)-231/32*ln(tan(1/2*f*x+1/2*e)+1)-1/48/(tan(1/2*f*x+
1/2*e)-1)^3-5/16/(tan(1/2*f*x+1/2*e)-1)^2-89/32/(tan(1/2*f*x+1/2*e)-1)+231/
32*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx =$$
$$-\frac{3465\left(c^6\cos(fx+e)^6+3c^6\cos(fx+e)^5+3c^6\cos(fx+e)^4+c^6\cos(fx+e)^3\right)\log(\sin(fx+e)+1)-}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/60*(3465*(c^6*cos(f*x + e)^6 + 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)
^4 + c^6*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 3465*(c^6*cos(f*x + e)^6 +
 3*c^6*cos(f*x + e)^5 + 3*c^6*cos(f*x + e)^4 + c^6*cos(f*x + e)^3)*log(-sin
(f*x + e) + 1) - 2*(5446*c^6*cos(f*x + e)^5 + 12843*c^6*cos(f*x + e)^4 + 87
11*c^6*cos(f*x + e)^3 + 815*c^6*cos(f*x + e)^2 - 105*c^6*cos(f*x + e) + 10*
c^6)*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3*a^3*f
*cos(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^6\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int\left(-\frac{6\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx + \int \frac{15}{\sec^3(e+fx)+3\sec\cdots}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**6*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-6*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*sec(e + f*x)**3/(sec(e + f*
x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-20*sec(e +
f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + In
tegral(15*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
f*x) + 1), x) + Integral(-6*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e + f*
x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**7/(sec(e + f*x)**3
 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(204) = 408$.

Time = 0.26 (sec) , antiderivative size = 935, normalized size of antiderivative = 4.35

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*x +
e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 690
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^6*(60*(5*sin(f*x + e)/
(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 - 2*a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/
a^3) + 45*c^6*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1
```

```
)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x
 + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6
0*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a^3) + 20*c^6*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*
sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x +
e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1
) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e)
 + 1)^5)/a^3 + c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 18*c^6*
(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = -\frac{\frac{3465\,c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} - \frac{3465\,c^6\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} + \frac{10\left(267\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-472\,c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+213\,c^6\tan\left(\frac{1}{2}\,f\right.}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^3a^3}}{30\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(3465*c^6*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 3465*c^6*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/a^3 + 10*(267*c^6*tan(1/2*f*x + 1/2*e)^5 - 472*c^
6*tan(1/2*f*x + 1/2*e)^3 + 213*c^6*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/
2*e)^2 - 1)^3*a^3) - 32*(3*a^12*c^6*tan(1/2*f*x + 1/2*e)^5 + 20*a^12*c^6*ta
n(1/2*f*x + 1/2*e)^3 + 150*a^12*c^6*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 12.97 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{160\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}$$
$$- \frac{89\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5-\frac{472\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+71\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$
$$+ \frac{64\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f}+\frac{16\,c^6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}-\frac{231\,c^6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}$$

```
[In] int((c - c/cos(e + f*x))^6/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (160*c^6*tan(e/2 + (f*x)/2))/(a^3*f) - (89*c^6*tan(e/2 + (f*x)/2)^5 - (472*
c^6*tan(e/2 + (f*x)/2)^3)/3 + 71*c^6*tan(e/2 + (f*x)/2))/(f*(3*a^3*tan(e/2
+ (f*x)/2)^2 - 3*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^6 - a^3)
) + (64*c^6*tan(e/2 + (f*x)/2)^3)/(3*a^3*f) + (16*c^6*tan(e/2 + (f*x)/2)^5)
/(5*a^3*f) - (231*c^6*atanh(tan(e/2 + (f*x)/2)))/(a^3*f)
```

3.53 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 193

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{63c^5\operatorname{arctanh}(\sin(e+fx))}{2a^3f}+\frac{42c^5\tan(e+fx)}{a^3f} - \frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f} - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{42c(c^2-c^2\sec(e+fx))^2\tan(e+fx)}{5f(a^3+a^3\sec(e+fx))}$$

```
[Out] -63/2*c^5*arctanh(sin(f*x+e))/a^3/f+42*c^5*tan(f*x+e)/a^3/f-21/2*c^5*sec(f*
x+e)*tan(f*x+e)/a^3/f-6/5*c^2*(c-c*sec(f*x+e))^3*tan(f*x+e)/a/f/(a+a*sec(f*
x+e))^2+2/5*c*(c-c*sec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+42/5*c*(c^
2-c^2*sec(f*x+e))^2*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used

$= \{4042, 3873, 3852, 8, 4131, 3855\}$

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{63c^5\operatorname{arctanh}(\sin(e+fx))}{2a^3f} + \frac{42c^5\tan(e+fx)}{a^3f} - \frac{21c^5\tan(e+fx)\sec(e+fx)}{2a^3f} + \frac{42c\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{5f\left(a^3\sec(e+fx)+a^3\right)} - \frac{6c^2\tan(e+fx)(c-c\sec(e+fx))^3}{5af(a\sec(e+fx)+a)^2} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-63*c^5*ArcTanh[Sin[e + f*x]])/(2*a^3*f) + (42*c^5*Tan[e + f*x])/(a^3*f) -
 (21*c^5*Sec[e + f*x]*Tan[e + f*x])/(2*a^3*f) - (6*c^2*(c - c*Sec[e + f*x])
^3*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*(c - c*Sec[e + f*x])
^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (42*c*(c^2 - c^2*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
```

```
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{(9c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx}{5a}\\
&=-\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2}\\
&\quad+\frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(21c^2)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx}{5a^2}\\
&=\frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}-\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2}\\
&\quad+\frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{(21c^3)\int\sec(e+fx)(c-c\sec(e+fx))^2\,dx}{a^3}\\
&=\frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}-\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2}\\
&\quad+\frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad-\frac{(21c^3)\int\sec(e+fx)\left(c^2+c^2\sec^2(e+fx)\right)\,dx}{a^3}+\frac{(42c^5)\int\sec^2(e+fx)\,dx}{a^3}\\
&=-\frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f}+\frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)}\\
&\quad-\frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2}+\frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad-\frac{(63c^5)\int\sec(e+fx)\,dx}{2a^3}-\frac{(42c^5)\,\text{Subst}\left(\int 1\,dx,x,-\tan(e+fx)\right)}{a^3f}
\end{aligned}$$

$$
= -\frac{63c^5\operatorname{arctanh}(\sin(e+fx))}{2a^3f} + \frac{42c^5\tan(e+fx)}{a^3f} - \frac{21c^5\sec(e+fx)\tan(e+fx)}{2a^3f} + \frac{42c^3(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a^3+a^3\sec(e+fx))} - \frac{6c^2(c-c\sec(e+fx))^3\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.39

$$
\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{32c^5\,\text{Hypergeometric2F1}\left(-\frac{9}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{5a^3f(-1+\sec(e+fx))(1+\sec(e+fx))^3}
$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-32*c^5*Hypergeometric2F1[-9/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[2 -
 2*Sec[e + f*x]]*Tan[e + f*x])/(5*a^3*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*
x])^3)
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{8c^5 (\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}+\tan(\frac{fx}{2}+\frac{e}{2})^3+6\tan(\frac{fx}{2}+\frac{e}{2})+\frac{1}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}-\frac{17}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)}-\frac{63\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{16} -}{f\,a^3}$
default	$\frac{8c^5 (\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}+\tan(\frac{fx}{2}+\frac{e}{2})^3+6\tan(\frac{fx}{2}+\frac{e}{2})+\frac{1}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)^2}-\frac{17}{16(\tan(\frac{fx}{2}+\frac{e}{2})+1)}-\frac{63\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{16} -}{f\,a^3}$
parallelrisch	$\frac{3749 (\frac{2520(1+\cos(2fx+2e))\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{3749}+\frac{2520(-1-\cos(2fx+2e))\ln(\tan(\frac{fx}{2}+\frac{e}{2})+1)}{3749}+\tan(\frac{fx}{2}+\frac{e}{2})\sec(\frac{fx}{2}+\frac{e}{2})^4 (\mathrm{co}}{80f\,a^3(1+\cos(2fx+2e))}$
risch	$\frac{ic^5 (325\,e^{8i(fx+e)}+1545\,e^{7i(fx+e)}+3805\,e^{6i(fx+e)}+5545\,e^{5i(fx+e)}+7351\,e^{4i(fx+e)}+6115\,e^{3i(fx+e)}+4407\,e^{2i(fx+e)}+2}{5f\,a^3(e^{i(fx+e)}+1)^5(1+e^{2i(fx+e)})^2}$
norman	$\frac{-\frac{63c^5\tan(\frac{fx}{2}+\frac{e}{2})}{af}+\frac{294c^5\tan(\frac{fx}{2}+\frac{e}{2})^3}{af}-\frac{2688c^5\tan(\frac{fx}{2}+\frac{e}{2})^5}{5af}+\frac{474c^5\tan(\frac{fx}{2}+\frac{e}{2})^7}{af}-\frac{193c^5\tan(\frac{fx}{2}+\frac{e}{2})^9}{af}+\frac{24c^5\tan(\frac{fx}{2}+\frac{e}{2}}{af}}{(\tan(\frac{fx}{2}+\frac{e}{2})^2-1)^5a^2}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*c^5/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+tan(1/2*f*x+1/2*e)^3+6*tan(1/2*f*x+1/
2*e)+1/16/(tan(1/2*f*x+1/2*e)+1)^2-17/16/(tan(1/2*f*x+1/2*e)+1)-63/16*ln(ta
n(1/2*f*x+1/2*e)+1)-1/16/(tan(1/2*f*x+1/2*e)-1)^2-17/16/(tan(1/2*f*x+1/2*e)
-1)+63/16*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.30

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{315\left(c^5\cos\left(fx+e\right)^5+3\,c^5\cos\left(fx+e\right)^4+3\,c^5\cos\left(fx+e\right)^3+c^5\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/20*(315*(c^5*cos(f*x + e)^5 + 3*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^
3 + c^5*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 315*(c^5*cos(f*x + e)^5 + 3
*c^5*cos(f*x + e)^4 + 3*c^5*cos(f*x + e)^3 + c^5*cos(f*x + e)^2)*log(-sin(f
*x + e) + 1) - 2*(496*c^5*cos(f*x + e)^4 + 1163*c^5*cos(f*x + e)^3 + 801*c^
5*cos(f*x + e)^2 + 65*c^5*cos(f*x + e) - 5*c^5)*sin(f*x + e))/(a^3*f*cos(f*
x + e)^5 + 3*a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x
+ e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^5\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{5\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{}{\sec^3\left(e+fx\right)}\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**5*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(5*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-10*sec(e + f*x)**3/(sec(e +
f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*sec(e +
 f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + I
```

```
ntegral(-5*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(186) = 372$.

Time = 0.21 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.52

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^5\left(\frac{60\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^3-\frac{2a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{465\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(f\cdots}{\cos(fx\cdots}\right.}{a^3}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^5*(40*sin(f*x + e)/((a^3 - a^3*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(
cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^
3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*((105*sin(f*x
 + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^5*(15*
sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^5*(15*sin(f*x + e)/(cos(f*x
+ e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5)/a^3 - 15*c^5*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx =$$
$$-\frac{\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{315\,c^5\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{10\left(17\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-15\,c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)^2a^3}-\frac{16\left(a^{12}c^5\tan\left(\frac{1}{2}\right.\right.}{}}{10\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/10*(315*c^5*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 315*c^5*log(abs(tan
(1/2*f*x + 1/2*e) - 1))/a^3 + 10*(17*c^5*tan(1/2*f*x + 1/2*e)^3 - 15*c^5*ta
n(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a^3) - 16*(a^12*c^5*tan
(1/2*f*x + 1/2*e)^5 + 5*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c^5*tan(1
/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 12.96 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{48\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f} - \frac{17\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3-15\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-2\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a^3\right)} + \frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{a^3\,f} + \frac{8\,c^5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f} - \frac{63\,c^5\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}$$

```
[In] int((c - c/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (48*c^5*tan(e/2 + (f*x)/2))/(a^3*f) - (17*c^5*tan(e/2 + (f*x)/2)^3 - 15*c^5
*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (f*x)/2)^4 - 2*a^3*tan(e/2 + (f*x)/2
)^2 + a^3)) + (8*c^5*tan(e/2 + (f*x)/2)^3)/(a^3*f) + (8*c^5*tan(e/2 + (f*x)
/2)^5)/(5*a^3*f) - (63*c^5*atanh(tan(e/2 + (f*x)/2)))/(a^3*f)
```

3.54 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 164

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{7c^4\text{arctanh}(\sin(e+fx))}{a^3f}+\frac{7c^4\tan(e+fx)}{a^3f}+\frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{14(c^2-c^2\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{14(c^4-c^4\sec(e+fx))\tan(e+fx)}{3f\,(a^3+a^3\sec(e+fx))}$$

```
[Out] -7*c^4*arctanh(sin(f*x+e))/a^3/f+7*c^4*tan(f*x+e)/a^3/f+2/5*c*(c-c*sec(f*x+
e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^3-14/15*(c^2-c^2*sec(f*x+e))^2*tan(f*x+
e)/a/f/(a+a*sec(f*x+e))^2+14/3*(c^4-c^4*sec(f*x+e))*tan(f*x+e)/f/(a^3+a^3*s
ec(f*x+e))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used

$= \{4042, 3872, 3855, 3852, 8\}$

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{7c^4\text{arctanh}(\sin(e+fx))}{a^3f}+\frac{7c^4\tan(e+fx)}{a^3f}+\frac{14\tan(e+fx)\left(c^4-c^4\sec(e+fx)\right)}{3f\left(a^3\sec(e+fx)+a^3\right)}-\frac{14\tan(e+fx)\left(c^2-c^2\sec(e+fx)\right)^2}{15af(a\sec(e+fx)+a)^2}+\frac{2c\tan(e+fx)(c-c\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-7*c^4*ArcTanh[Sin[e + f*x]])/(a^3*f) + (7*c^4*Tan[e + f*x])/(a^3*f) + (2*
c*(c - c*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (14*(
c^2 - c^2*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + (
14*(c^4 - c^4*Sec[e + f*x])*Tan[e + f*x])/(3*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
```

```
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(7c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(7c^2)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx}{3a^2}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{14(c^4-c^4\sec(e+fx))\tan(e+fx)}{3f\left(a^3+a^3\sec(e+fx)\right)} - \frac{(7c^3)\int\sec(e+fx)(c-c\sec(e+fx))\,dx}{a^3}$$

$$= \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{14(c^4-c^4\sec(e+fx))\tan(e+fx)}{3f\left(a^3+a^3\sec(e+fx)\right)} - \frac{(7c^4)\int\sec(e+fx)\,dx}{a^3} + \frac{(7c^4)\int\sec^2(e+fx)\,dx}{a^3}$$

$$= -\frac{7c^4\text{arctanh}(\sin(e+fx))}{a^3f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{14(c^4-c^4\sec(e+fx))\tan(e+fx)}{3f\left(a^3+a^3\sec(e+fx)\right)} - \frac{(7c^4)\,\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{a^3f}$$

$$= -\frac{7c^4\text{arctanh}(\sin(e+fx))}{a^3f} + \frac{7c^4\tan(e+fx)}{a^3f} + \frac{2c(c-c\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{14(c^2-c^2\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{14(c^4-c^4\sec(e+fx))\tan(e+fx)}{3f\left(a^3+a^3\sec(e+fx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.93 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{16c^4\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{2},-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\sqrt{2-2\sec(e+fx)}\tan(e+fx)}{5a^3f(-1+\sec(e+fx))(1+\sec(e+fx))^3}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-16*c^4*Hypergeometric2F1[-7/2, -5/2, -3/2, (1 + Sec[e + f*x])/2]*Sqrt[2 -
 2*Sec[e + f*x]]*Tan[e + f*x])/(5*a^3*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*
x])^3)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{4c^4\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{7\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{7\ln(\text{ta}}{}\right.}{f\,a^3}$
default	$\frac{4c^4\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{7\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4}-\frac{1}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{7\ln(\text{ta}}{}\right.}{f\,a^3}$
parallelrisch	$\frac{1609c^4\left(\frac{1680\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)}{1609}-\frac{1680\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)}{1609}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^4\left(\cos(fx+e)+\frac{76}{}\right.\right.}{240f\,a^3\cos(fx+e)}$
risch	$\frac{2ic^4\left(120\,e^{6i(fx+e)}+495\,e^{5i(fx+e)}+1235\,e^{4i(fx+e)}+1270\,e^{3i(fx+e)}+1342\,e^{2i(fx+e)}+715\,e^{i(fx+e)}+167\right)}{15f\,a^3\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}+1\right)^5}-\frac{7c^4\ln(e^{i(fx+}}{a^3f}$
norman	$\frac{\frac{14c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{154c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}+\frac{1022c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15af}-\frac{186c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{5af}+\frac{92c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{15af}-\frac{8c^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{15af}+}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4a^2}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 4/f*c^4/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+2/3*tan(1/2*f*x+1/2*e)^3+3*tan(1/2*f*
x+1/2*e)-1/4/(tan(1/2*f*x+1/2*e)+1)-7/4*ln(tan(1/2*f*x+1/2*e)+1)-1/4/(tan(1
/2*f*x+1/2*e)-1)+7/4*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.41

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{105\left(c^4\cos\left(fx+e\right)^4+3\,c^4\cos\left(fx+e\right)^3+3\,c^4\cos\left(fx+e\right)^2+c^4\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(105*(c^4*cos(f*x + e)^4 + 3*c^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^
2 + c^4*cos(f*x + e))*log(sin(f*x + e) + 1) - 105*(c^4*cos(f*x + e)^4 + 3*c
^4*cos(f*x + e)^3 + 3*c^4*cos(f*x + e)^2 + c^4*cos(f*x + e))*log(-sin(f*x +
 e) + 1) - 2*(167*c^4*cos(f*x + e)^3 + 381*c^4*cos(f*x + e)^2 + 277*c^4*cos
(f*x + e) + 15*c^4)*sin(f*x + e))/(a^3*f*cos(f*x + e)^4 + 3*a^3*f*cos(f*x +
 e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{c^4\left(\int \frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{4\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{6}{\sec^3\left(e+fx\right)+3\sec}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**4*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-4*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*sec(e + f*x)**3/(sec(e + f*x
)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-4*sec(e + f*
x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(161) = 322$.

Time = 0.22 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.87

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{3\,c^4\left(\frac{40\,\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}+\frac{\frac{85\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\,\log\left(\frac{\sin(fx+}{\cos(fx+e}\right.}{a^3}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
[Out] 1/60*(3*c^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^3) + 4*c^4*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^3) + 6*c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) +
10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1
)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 12*c^4*(5*s
in(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/
f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{105\,c^4\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}+\frac{30\,c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3}-\frac{4\left(3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+10\,a^{12}c^4\tan}{a^1}}{15\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
[Out] -1/15*(105*c^4*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 105*c^4*log(abs(tan
(1/2*f*x + 1/2*e) - 1))/a^3 + 30*c^4*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1
/2*e)^2 - 1)*a^3) - 4*(3*a^12*c^4*tan(1/2*f*x + 1/2*e)^5 + 10*a^12*c^4*tan(
1/2*f*x + 1/2*e)^3 + 45*a^12*c^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 13.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.77

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{12\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a^3\,f}+\frac{8\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3\,a^3\,f}+\frac{4\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}-\frac{14\,c^4\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^3\,f}-\frac{2\,c^4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)}$$

```
[In] int((c - c/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (12*c^4*tan(e/2 + (f*x)/2))/(a^3*f) + (8*c^4*tan(e/2 + (f*x)/2)^3)/(3*a^3*f
) + (4*c^4*tan(e/2 + (f*x)/2)^5)/(5*a^3*f) - (14*c^4*atanh(tan(e/2 + (f*x)/
2)))/(a^3*f) - (2*c^4*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2 + (f*x)/2)^2 - a^
3))
```

3.55 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 131

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3} dx = -\frac{c^3\text{arctanh}(\sin(e+fx))}{a^3f} + \frac{2c^3\tan(e+fx)}{f(a^3+a^3\sec(e+fx))} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{2(c^3-c^3\sec(e+fx))\tan(e+fx)}{3af(a+a\sec(e+fx))^2}$$

```
[Out] -c^3*arctanh(sin(f*x+e))/a^3/f+2*c^3*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+2/5*
c*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3-2/3*(c^3-c^3*sec(f*x+e
))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4042, 3855}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3} dx = -\frac{c^3\text{arctanh}(\sin(e+fx))}{a^3f} + \frac{2c^3\tan(e+fx)}{f(a^3\sec(e+fx)+a^3)} - \frac{2\tan(e+fx)(c^3-c^3\sec(e+fx))}{3af(a\sec(e+fx)+a)^2} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*ArcTanh[Sin[e + f*x]])/(a^3*f)) + (2*c^3*Tan[e + f*x])/(f*(a^3 + a^3
*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec
[e + f*x])^3) - (2*(c^3 - c^3*Sec[e + f*x])*Tan[e + f*x])/(3*a*f*(a + a*Sec
[e + f*x])^2)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{c\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx}{a} \\
&= \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} \\
&\quad - \frac{2(c^3-c^3\sec(e+fx))\tan(e+fx)}{3af(a+a\sec(e+fx))^2} + \frac{c^2\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{a^2} \\
&= \frac{2c^3\tan(e+fx)}{f\left(a^3+a^3\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} \\
&\quad - \frac{2(c^3-c^3\sec(e+fx))\tan(e+fx)}{3af(a+a\sec(e+fx))^2} - \frac{c^3\int \sec(e+fx)\,dx}{a^3} \\
&= -\frac{c^3\operatorname{arctanh}(\sin(e+fx))}{a^3f} + \frac{2c^3\tan(e+fx)}{f\left(a^3+a^3\sec(e+fx)\right)} \\
&\quad + \frac{2c(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{2(c^3-c^3\sec(e+fx))\tan(e+fx)}{3af(a+a\sec(e+fx))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(-\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{26\tan\left(\frac{1}{2}(e+fx)\right)}{15f}+\frac{2\sec^2\left(\frac{1}{2}(e+fx)\right)\tan\left(\frac{1}{2}(e+fx)\right)}{15f}\right.}{a^3}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] -((c^3*(-(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]/f) + Log[Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]]/f - (26*Tan[(e + f*x)/2])/(15*f) + (2*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/(15*f) - (2*Sec[(e + f*x)/2]^4*Tan[(e + f*x)/2])/(5*f
)))/a^3)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{2c^3\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^3}$
default	$\frac{2c^3\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}\right)}{f\,a^3}$
parallelrisch	$\frac{c^3\left(6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3+15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-15\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+30\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{15a^3 f}$
risch	$\frac{4ic^3\left(15\,e^{4i(fx+e)}+30\,e^{3i(fx+e)}+100\,e^{2i(fx+e)}+50\,e^{i(fx+e)}+13\right)}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}-\frac{c^3\ln\left(e^{i(fx+e)}+i\right)}{a^3 f}+\frac{c^3\ln\left(e^{i(fx+e)}-i\right)}{a^3 f}$
norman	$\frac{-\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{16c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{22c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5af}+\frac{6c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{5af}-\frac{8c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{15af}+\frac{2c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{5af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3 a^2}+\frac{c^3}{}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 2/f*c^3/a^3*(1/5*tan(1/2*f*x+1/2*e)^5+1/3*tan(1/2*f*x+1/2*e)^3+tan(1/2*f*x+
1/2*e)+1/2*ln(tan(1/2*f*x+1/2*e)-1)-1/2*ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{15\left(c^3\cos\left(fx+e\right)^3+3\,c^3\cos\left(fx+e\right)^2+3\,c^3\cos\left(fx+e\right)+c^3\right)\log\left(\sin\left(fx+e\right)+1\right)-15\left(c^3\cos\left(fx\right.\right.}{30\left(a^3f\cos\left(fx+\right.\right.}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2 + 3*c^3*cos(f*x + e) +
 c^3)*log(sin(f*x + e) + 1) - 15*(c^3*cos(f*x + e)^3 + 3*c^3*cos(f*x + e)^2
 + 3*c^3*cos(f*x + e) + c^3)*log(-sin(f*x + e) + 1) - 4*(13*c^3*cos(f*x + e
)^2 + 24*c^3*cos(f*x + e) + 23*c^3)*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3
*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^3\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\right)dx+\int\frac{3\sec^2\left(e+fx\right)}{\sec^3\left(e+fx\right)+3\sec^2\left(e+fx\right)+3\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{}{\sec^3\left(e+f\right.}\right.\right.}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c**3*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(3*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
 f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-3*sec(e + f*x)**3/(sec(e + f
*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x
)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(129) = 258.

Time = 0.21 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.32

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{c^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)+\frac{3c^3\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(f}{(\cos(fx+}\right.}{a^3}}{60f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x
 + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
- 1)/a^3) + 3*c^3*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(
cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^3*(15*
sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^3*(5*sin(f*x + e)/(cos(f*x
 + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = -\frac{\frac{15c^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^3}-\frac{15c^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^3}-\frac{2\left(3a^{12}c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+5a^{12}c^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+15a^{12}c^3\tan\left(\frac{1}{2}fx\right.\right.}{a^{15}}}{15f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/15*(15*c^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 15*c^3*log(abs(tan(1
/2*f*x + 1/2*e) - 1))/a^3 - 2*(3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 + 5*a^12*c
^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 13.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.47

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{2\,c^3\,\left(15\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-15\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\right)}{15\,a^3\,f}$$

```
[In] int((c - c/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
[Out] (2*c^3*(15*tan(e/2 + (f*x)/2) - 15*atanh(tan(e/2 + (f*x)/2)) + 5*tan(e/2 +
(f*x)/2)^3 + 3*tan(e/2 + (f*x)/2)^5))/(15*a^3*f)
```

3.56 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 38

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

```
[Out] 1/5*(c-c*sec(f*x+e))^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {4035}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan(e+fx)(c-c\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - c*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(c-c\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{c^2\tan^5\left(\frac{1}{2}(e+fx)\right)}{5a^3f}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
[Out] (c^2*Tan[(e + f*x)/2]^5)/(5*a^3*f)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5f\,a^3}$	23
default	$\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5f\,a^3}$	23
parallelrisch	$\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5f\,a^3}$	23
risch	$\frac{2ic^2\left(5\,e^{4i(fx+e)}+10\,e^{2i(fx+e)}+1\right)}{5f\,a^3\left(e^{i(fx+e)}+1\right)^5}$	50
norman	$\frac{\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5af}-\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{5af}+\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{5af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2a^2}$	87

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
[Out] 1/5/f*c^2/a^3*tan(1/2*f*x+1/2*e)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\left(c^2\cos\left(fx+e\right)^2-2\,c^2\cos\left(fx+e\right)+c^2\right)\sin\left(fx+e\right)}{5\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/5*(c^2*cos(f*x + e)^2 - 2*c^2*cos(f*x + e) + c^2)*sin(f*x + e)/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{c^2\left(\int \frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int\left(-\frac{2\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int \frac{\sec}{\sec^3(e+fx)+3\sec^2}\right.}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] c**2*(Integral(sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(-2*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e +
f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)*
*3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(37) = 74$.

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 4.87

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{6c^2\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1}\right.}{a^3}}{60f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(c^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*x
+ e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 6*c^2*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5}{5\,a^3f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
[Out] 1/5*c^2*tan(1/2*f*x + 1/2*e)^5/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 13.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{c^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{5\,a^3\,f}$$

```
[In] int((c - c/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
[Out] (c^2*tan(e/2 + (f*x)/2)^5)/(5*a^3*f)
```

3.57 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(c-c\sec(e+fx))\tan(e+fx)}{15af(a+a\sec(e+fx))^2}$$

```
[Out] 1/5*(c-c*sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(c-c*sec(f*x+e))*
tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan(e+fx)(c-c\sec(e+fx))}{15af(a\sec(e+fx)+a)^2} + \frac{\tan(e+fx)(c-c\sec(e+fx))}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - c*Sec[e + f*x])*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - c*
Sec[e + f*x])*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
```

```
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{\int\frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx}{5a}\\ &= \frac{(c-c\sec(e+fx))\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(c-c\sec(e+fx))\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx=-\frac{c(-1+\sec(e+fx))(4+\sec(e+fx))\tan(e+fx)}{15a^3f(1+\sec(e+fx))^3}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
[Out] -1/15*(c*(-1 + Sec[e + f*x])*(4 + Sec[e + f*x])*Tan[e + f*x])/(a^3*f*(1 + S
ec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

method	result	size
parallelrisch	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-5\right)}{30a^{3}f}$	36
derivativedivides	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}\right)}{2f\,a^{3}}$	37
default	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}\right)}{2f\,a^{3}}$	37
risch	$\frac{2ic\left(15\,\mathrm{e}^{4i(fx+e)}+15\,\mathrm{e}^{3i(fx+e)}+25\,\mathrm{e}^{2i(fx+e)}+5\,\mathrm{e}^{i(fx+e)}+4\right)}{15f\,a^{3}\left(\mathrm{e}^{i(fx+e)}+1\right)^{5}}$	70
norman	$\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{6af}-\frac{4c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{15af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{10af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)a^{2}}$	81

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*c*tan(1/2*f*x+1/2*e)^3*(3*tan(1/2*f*x+1/2*e)^2-5)/a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\left(4\,c\cos\left(fx+e\right)^{2}-3\,c\cos\left(fx+e\right)-c\right)\sin\left(fx+e\right)}{15\,\left(a^{3}f\cos\left(fx+e\right)^{3}+3\,a^{3}f\cos\left(fx+e\right)^{2}+3\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*(4*c*cos(f*x + e)^2 - 3*c*cos(f*x + e) - c)*sin(f*x + e)/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{c\left(\int\left(-\frac{\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx\right)}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
[Out] -c*(Integral(-sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e +
 f*x) + 1), x) + Integral(sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{3c\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxi
ma")
[Out] 1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-5c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{30a^3f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac
")
[Out] 1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 - 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 13.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-5\right)}{30\,a^3\,f}$$

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
[Out] (c*tan(e/2 + (f*x)/2)^3*(3*tan(e/2 + (f*x)/2)^2 - 5))/(30*a^3*f)
```

3.58 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{\csc(e+fx)}{a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}$$

```
[Out] -2/5*cot(f*x+e)^5/a^3/c/f+csc(f*x+e)/a^3/c/f-csc(f*x+e)^3/a^3/c/f+2/5*csc(f
*x+e)^5/a^3/c/f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4043, 2686, 200, 2687, 30, 14}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{2\cot^5(e+fx)}{5a^3cf}+\frac{2\csc^5(e+fx)}{5a^3cf}-\frac{\csc^3(e+fx)}{a^3cf}+\frac{\csc(e+fx)}{a^3cf}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
```

```
[Out] (-2*Cot[e + f*x]^5)/(5*a^3*c*f) + Csc[e + f*x]/(a^3*c*f) - Csc[e + f*x]^3/(
a^3*c*f) + (2*Csc[e + f*x]^5)/(5*a^3*c*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\int \left(c^2\cot^5(e+fx)\csc(e+fx)-2c^2\cot^4(e+fx)\csc^2(e+fx)+c^2\cot^3(e+fx)\csc^3(e+fx)\right)\,dx}{a^3c^3}\\ &= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c}-\frac{\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{a^3c}\\ &\quad+\frac{2\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^3c}\end{aligned}$$

$$= \frac{\text{Subst}\left(\int x^2(-1+x^2)\,dx,x,\csc(e+fx)\right)}{a^3cf} + \frac{\text{Subst}\left(\int (-1+x^2)^2\,dx,x,\csc(e+fx)\right)}{a^3cf} + \frac{2\text{Subst}\left(\int x^4\,dx,x,-\cot(e+fx)\right)}{a^3cf}$$

$$= -\frac{2\cot^5(e+fx)}{5a^3cf} + \frac{\text{Subst}\left(\int (1-2x^2+x^4)\,dx,x,\csc(e+fx)\right)}{a^3cf} + \frac{\text{Subst}\left(\int (-x^2+x^4)\,dx,x,\csc(e+fx)\right)}{a^3cf}$$

$$= -\frac{2\cot^5(e+fx)}{5a^3cf} + \frac{\csc(e+fx)}{a^3cf} - \frac{\csc^3(e+fx)}{a^3cf} + \frac{2\csc^5(e+fx)}{5a^3cf}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{(-2+\sec(e+fx)+4\sec^2(e+fx)+2\sec^3(e+fx))\tan(e+fx)}{5a^3cf(-1+\sec(e+fx))(1+\sec(e+fx))^3}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])),x]
[Out] ((-2 + Sec[e + f*x] + 4*Sec[e + f*x]^2 + 2*Sec[e + f*x]^3)*Tan[e + f*x])/(5
*a^3*c*f*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.76

method	result	size
parallelrisch	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}-5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)+15\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{40f\,a^{3}c}$	59
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8f\,a^{3}c}$	61
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}}{8f\,a^{3}c}$	61
risch	$\frac{2i\left(5\,{\mathrm e}^{5i(fx+e)}+10\,{\mathrm e}^{4i(fx+e)}+10\,{\mathrm e}^{3i(fx+e)}-3\,{\mathrm e}^{i(fx+e)}-2\right)}{5f\,a^{3}c\left({\mathrm e}^{i(fx+e)}+1\right)^{5}\left({\mathrm e}^{i(fx+e)}-1\right)}$	85
norman	$\frac{\frac{1}{8acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{4}}{8acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{6}}{40acf}}{a^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}$	94

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*(tan(1/2*f*x+1/2*e)^5-5*tan(1/2*f*x+1/2*e)^3+5*cot(1/2*f*x+1/2*e)+15*t
an(1/2*f*x+1/2*e))/f/a^3/c
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{2\,\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{2}-4\,\cos\left(fx+e\right)-2}{5\,\left(a^{3}cf\cos\left(fx+e\right)^{2}+2\,a^{3}cf\cos\left(fx+e\right)+a^{3}cf\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/5*(2*cos(f*x + e)^3 - cos(f*x + e)^2 - 4*cos(f*x + e) - 2)/((a^3*c*f*cos
(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sec^4(e+fx)+2\sec^3(e+fx)-2\sec(e+fx)-1}\,dx}{a^3c}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**4 + 2*sec(e + f*x)**3 - 2*sec(e + f*x
) - 1), x)/(a**3*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c}+\frac{5(\cos(fx+e)+1)}{a^3c\sin(fx+e)}}{40f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/40*((15*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x + e)
+ 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c) + 5*(cos(f*x + e) + 1
)/(a^3*c*sin(f*x + e)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx = \frac{\frac{5}{a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}+\frac{a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-5a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+15a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^5}}{40f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/40*(5/(a^3*c*tan(1/2*f*x + 1/2*e)) + (a^12*c^4*tan(1/2*f*x + 1/2*e)^5 - 5
*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e))/(a^15*
c^5))/f
```

Mupad [B] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))}\,dx$$
$$= -\frac{16\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6-28\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4+8\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}{40\,a^3\,c\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))),x)
```

```
[Out] -(8*cos(e/2 + (f*x)/2)^2 - 28*cos(e/2 + (f*x)/2)^4 + 16*cos(e/2 + (f*x)/2)^
6 - 1)/(40*a^3*c*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2))
```

3.59 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 80

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}$$

```
[Out] -1/5*cot(f*x+e)^5/a^3/c^2/f+csc(f*x+e)/a^3/c^2/f-2/3*csc(f*x+e)^3/a^3/c^2/f
+1/5*csc(f*x+e)^5/a^3/c^2/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4043, 2686, 200, 2687, 30}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]
```

```
[Out] -1/5*Cot[e + f*x]^5/(a^3*c^2*f) + Csc[e + f*x]/(a^3*c^2*f) - (2*Csc[e + f*x
]^3)/(3*a^3*c^2*f) + Csc[e + f*x]^5/(5*a^3*c^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \left(c\cot^5(e+fx)\csc(e+fx)-c\cot^4(e+fx)\csc^2(e+fx)\right)\,dx}{a^3c^3}\\
&= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c^2}+\frac{\int \cot^4(e+fx)\csc^2(e+fx)\,dx}{a^3c^2}\\
&= \frac{\text{Subst}\left(\int x^4\,dx,x,-\cot(e+fx)\right)}{a^3c^2f}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\csc(e+fx)\right)}{a^3c^2f}\\
&= -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(e+fx)\right)}{a^3c^2f}\\
&= -\frac{\cot^5(e+fx)}{5a^3c^2f}+\frac{\csc(e+fx)}{a^3c^2f}-\frac{2\csc^3(e+fx)}{3a^3c^2f}+\frac{\csc^5(e+fx)}{5a^3c^2f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{(3-12\sec(e+fx)-12\sec^2(e+fx)+8\sec^3(e+fx)+8\sec^4(e+fx))\tan(e+fx)}{15a^3c^2f(-1+\sec(e+fx))^2(1+\sec(e+fx))^3}$$

[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^2),x]

[Out] ((3 - 12*Sec[e + f*x] - 12*Sec[e + f*x]^2 + 8*Sec[e + f*x]^3 + 8*Sec[e + f*x]^4)*Tan[e + f*x])/(15*a^3*c^2*f*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x])^3)

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-20\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3-5\cot\left(\frac{fx}{2}+\frac{e}{2}\right)^3+90\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+60\cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{240f\,a^3c^2}$	74
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}}{16f\,c^2a^3}$	76
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{4}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}}{16f\,c^2a^3}$	76
risch	$\frac{2i\left(15\,e^{7i(fx+e)}+15\,e^{6i(fx+e)}-5\,e^{5i(fx+e)}-25\,e^{4i(fx+e)}+13\,e^{3i(fx+e)}+21\,e^{2i(fx+e)}+9\,e^{i(fx+e)}-3\right)}{15f\,c^2a^3\left(e^{i(fx+e)}+1\right)^5\left(e^{i(fx+e)}-1\right)^3}$	118
norman	$\frac{-\frac{1}{48acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{4acf}+\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{8acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{12acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{80acf}}{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}$	119

[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 1/240*(3*tan(1/2*f*x+1/2*e)^5-20*tan(1/2*f*x+1/2*e)^3-5*cot(1/2*f*x+1/2*e)^3+90*tan(1/2*f*x+1/2*e)+60*cot(1/2*f*x+1/2*e))/f/a^3/c^2

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = -\frac{3\cos\left(fx+e\right)^4-12\cos\left(fx+e\right)^3-12\cos\left(fx+e\right)^2+8\cos\left(fx+e\right)+8}{15\left(a^3c^2f\cos\left(fx+e\right)^3+a^3c^2f\cos\left(fx+e\right)^2-a^3c^2f\cos\left(fx+e\right)-a^3c^2f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/15*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^3 - 12*cos(f*x + e)^2 + 8*cos(f*x
 + e) + 8)/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*
f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{\sec^5\left(e+fx\right)+\sec^4\left(e+fx\right)-2\sec^3\left(e+fx\right)-2\sec^2\left(e+fx\right)+\sec\left(e+fx\right)+1}\,dx}{a^3c^2}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**5 + sec(e + f*x)**4 - 2*sec(e + f*x)**
3 - 2*sec(e + f*x)**2 + sec(e + f*x) + 1), x)/(a**3*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.50

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\frac{\frac{90\sin(fx+e)}{\cos(fx+e)+1}-\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3c^2}+\frac{5\left(\frac{12\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)(\cos(fx+e)+1)^3}{a^3c^2\sin(fx+e)^3}}{240\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*((90*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^2) + 5*(12*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 - 1)*(cos(f*x + e) + 1)^3/(a^3*c^2*sin(f*x + e
)^3))/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{\frac{5\left(12\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)}{a^3c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}+\frac{3a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-20a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+90a^{12}c^8\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^{15}c^{10}}}{240f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/240*(5*(12*tan(1/2*f*x + 1/2*e)^2 - 1)/(a^3*c^2*tan(1/2*f*x + 1/2*e)^3) +
 (3*a^12*c^8*tan(1/2*f*x + 1/2*e)^5 - 20*a^12*c^8*tan(1/2*f*x + 1/2*e)^3 +
90*a^12*c^8*tan(1/2*f*x + 1/2*e))/(a^15*c^10))/f
```

Mupad [B] (verification not implemented)

Time = 13.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.39

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^2}\,dx = \frac{48\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^8-192\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6+168\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4-32\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3}{240\,a^3\,c^2\,f\,\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)-\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^2),x)
```

```
[Out] (168*cos(e/2 + (f*x)/2)^4 - 32*cos(e/2 + (f*x)/2)^2 - 192*cos(e/2 + (f*x)/2
)^6 + 48*cos(e/2 + (f*x)/2)^8 + 3)/(240*a^3*c^2*f*(cos(e/2 + (f*x)/2)^5*sin
(e/2 + (f*x)/2) - cos(e/2 + (f*x)/2)^7*sin(e/2 + (f*x)/2)))
```

3.60 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 59

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\csc(e+fx)}{a^3c^3f} - \frac{2\csc^3(e+fx)}{3a^3c^3f} + \frac{\csc^5(e+fx)}{5a^3c^3f}$$

```
[Out] csc(f*x+e)/a^3/c^3/f-2/3*csc(f*x+e)^3/a^3/c^3/f+1/5*csc(f*x+e)^5/a^3/c^3/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4043, 2686, 200}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\csc^5(e+fx)}{5a^3c^3f} - \frac{2\csc^3(e+fx)}{3a^3c^3f} + \frac{\csc(e+fx)}{a^3c^3f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
```

```
[Out] Csc[e + f*x]/(a^3*c^3*f) - (2*Csc[e + f*x]^3)/(3*a^3*c^3*f) + Csc[e + f*x]^
5/(5*a^3*c^3*f)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \cot^5(e+fx)\csc(e+fx)\,dx}{a^3c^3}\\
&= \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{a^3c^3f}\\
&= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(e+fx)\right)}{a^3c^3f}\\
&= \frac{\csc(e+fx)}{a^3c^3f}-\frac{2\csc^3(e+fx)}{3a^3c^3f}+\frac{\csc^5(e+fx)}{5a^3c^3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{-\frac{\csc(e+fx)}{f}+\frac{2\csc^3(e+fx)}{3f}-\frac{\csc^5(e+fx)}{5f}}{a^3c^3}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^3),x]
[Out] -((-(Csc[e + f*x]/f) + (2*Csc[e + f*x]^3)/(3*f) - Csc[e + f*x]^5/(5*f))/(a^
3*c^3))
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

method	result	size
default	$-\frac{-\frac{\csc(fx+e)^5}{5}+\frac{2\csc(fx+e)^3}{3}-\csc(fx+e)}{c^3a^3f}$	41
parallelrisch	$\frac{(15\cos(4fx+4e)-20\cos(2fx+2e)+29)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^5\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{3840f\,c^3a^3}$	58
risch	$\frac{2i\left(15\,\mathrm{e}^{9i(fx+e)}-20\,\mathrm{e}^{7i(fx+e)}+58\,\mathrm{e}^{5i(fx+e)}-20\,\mathrm{e}^{3i(fx+e)}+15\,\mathrm{e}^{i(fx+e)}\right)}{15f\,c^3a^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(\mathrm{e}^{i(fx+e)}-1\right)^5}$	95
norman	$\frac{\frac{1}{160acf}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{96acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{16acf}+\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{16acf}-\frac{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{96acf}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{160acf}}{a^2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}$	141

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/c^3/a^3/f*(-1/5*csc(f*x+e)^5+2/3*csc(f*x+e)^3-csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{15\,\cos\left(fx+e\right)^4-20\,\cos\left(fx+e\right)^2+8}{15\left(a^3c^3f\cos\left(fx+e\right)^4-2\,a^3c^3f\cos\left(fx+e\right)^2+a^3c^3f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)/((a^3*c^3*f*cos(f*x + e)^4
 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = -\frac{\int \frac{\sec(e+fx)}{\sec^6(e+fx)-3\sec^4(e+fx)+3\sec^2(e+fx)-1}\,dx}{a^3c^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**3,x)
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**6 - 3*sec(e + f*x)**4 + 3*sec(e + f*x
)**2 - 1), x)/(a**3*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{15\,\sin(fx+e)^4-10\,\sin(fx+e)^2+3}{15\,a^3c^3f\sin(fx+e)^5}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="ma
xima")
[Out] 1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{15\,\sin(fx+e)^4-10\,\sin(fx+e)^2+3}{15\,a^3c^3f\sin(fx+e)^5}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^3,x, algorithm="gi
ac")
[Out] 1/15*(15*sin(f*x + e)^4 - 10*sin(f*x + e)^2 + 3)/(a^3*c^3*f*sin(f*x + e)^5)
```

Mupad [B] (verification not implemented)

Time = 13.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^3}\,dx = \frac{\sin\left(e+f\,x\right)^4-\frac{2\sin\left(e+f\,x\right)^2}{3}+\frac{1}{5}}{a^3\,c^3\,f\,\sin\left(e+f\,x\right)^5}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^3),x)
[Out] (sin(e + f*x)^4 - (2*sin(e + f*x)^2)/3 + 1/5)/(a^3*c^3*f*sin(e + f*x)^5)
```

3.61 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 99

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\cot^7(e+fx)}{7a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}$$

```
[Out] -1/7*cot(f*x+e)^7/a^3/c^4/f+csc(f*x+e)/a^3/c^4/f-csc(f*x+e)^3/a^3/c^4/f+3/5
*csc(f*x+e)^5/a^3/c^4/f-1/7*csc(f*x+e)^7/a^3/c^4/f
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {4043, 2686, 200, 2687, 30}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = -\frac{\cot^7(e+fx)}{7a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] -1/7*Cot[e + f*x]^7/(a^3*c^4*f) + Csc[e + f*x]/(a^3*c^4*f) - Csc[e + f*x]^3
/(a^3*c^4*f) + (3*Csc[e + f*x]^5)/(5*a^3*c^4*f) - Csc[e + f*x]^7/(7*a^3*c^4
*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \left(a\cot^7(e+fx)\csc(e+fx)+a\cot^6(e+fx)\csc^2(e+fx)\right)\,dx}{a^4c^4}\\
&= \frac{\int \cot^7(e+fx)\csc(e+fx)\,dx}{a^3c^4}+\frac{\int \cot^6(e+fx)\csc^2(e+fx)\,dx}{a^3c^4}\\
&= \frac{\operatorname{Subst}\left(\int x^6\,dx,x,-\cot(e+fx)\right)}{a^3c^4f}-\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\csc(e+fx)\right)}{a^3c^4f}\\
&= -\frac{\cot^7(e+fx)}{7a^3c^4f}-\frac{\operatorname{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\csc(e+fx)\right)}{a^3c^4f}\\
&= -\frac{\cot^7(e+fx)}{7a^3c^4f}+\frac{\csc(e+fx)}{a^3c^4f}-\frac{\csc^3(e+fx)}{a^3c^4f}+\frac{3\csc^5(e+fx)}{5a^3c^4f}-\frac{\csc^7(e+fx)}{7a^3c^4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx$$
$$= \frac{(-5-30\sec(e+fx)+30\sec^2(e+fx)+40\sec^3(e+fx)-40\sec^4(e+fx)-16\sec^5(e+fx)+16\sec^6(}{35a^3c^4f(-1+\sec(e+fx))^4(1+\sec(e+fx))^3}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^4),x]
```

```
[Out] ((-5 - 30*Sec[e + f*x] + 30*Sec[e + f*x]^2 + 40*Sec[e + f*x]^3 - 40*Sec[e +
 f*x]^4 - 16*Sec[e + f*x]^5 + 16*Sec[e + f*x]^6)*Tan[e + f*x])/(35*a^3*c^4*
f*(-1 + Sec[e + f*x])^4*(1 + Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{(-90\cos(4fx+4e)+5\cos(6fx+6e)+152\cos(fx+e)+60\cos(5fx+5e)-182-20\cos(3fx+3e)+235\cos(2fx+2e))\sec(}{71680f\,a^3c^4}$
derivativedivides	$\frac{\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}-2\tan(\frac{fx}{2}+\frac{e}{2})^3+15\tan(\frac{fx}{2}+\frac{e}{2})-\frac{1}{7\tan(\frac{fx}{2}+\frac{e}{2})^7}+\frac{20}{\tan(\frac{fx}{2}+\frac{e}{2})}+\frac{6}{5\tan(\frac{fx}{2}+\frac{e}{2})^5}-\frac{5}{\tan(\frac{fx}{2}+\frac{e}{2})^3}}{64f\,c^4a^3}$
default	$\frac{\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}-2\tan(\frac{fx}{2}+\frac{e}{2})^3+15\tan(\frac{fx}{2}+\frac{e}{2})-\frac{1}{7\tan(\frac{fx}{2}+\frac{e}{2})^7}+\frac{20}{\tan(\frac{fx}{2}+\frac{e}{2})}+\frac{6}{5\tan(\frac{fx}{2}+\frac{e}{2})^5}-\frac{5}{\tan(\frac{fx}{2}+\frac{e}{2})^3}}{64f\,c^4a^3}$
risch	$\frac{2i(35\,e^{11i(fx+e)}-35\,e^{10i(fx+e)}-35\,e^{9i(fx+e)}+105\,e^{8i(fx+e)}+126\,e^{7i(fx+e)}-182\,e^{6i(fx+e)}+26\,e^{5i(fx+e)}+130\,e^{4i(fx+e}}{35f\,c^4a^3(e^{i(fx+e)}+1)^5(e^{i(fx+e)}-1)^7}$
norman	$\frac{-\frac{1}{448acf}+\frac{3\tan(\frac{fx}{2}+\frac{e}{2})^2}{160acf}-\frac{5\tan(\frac{fx}{2}+\frac{e}{2})^4}{64acf}+\frac{5\tan(\frac{fx}{2}+\frac{e}{2})^6}{16acf}+\frac{15\tan(\frac{fx}{2}+\frac{e}{2})^8}{64acf}-\frac{\tan(\frac{fx}{2}+\frac{e}{2})^{10}}{32acf}+\frac{\tan(\frac{fx}{2}+\frac{e}{2})^{12}}{320acf}}{a^2c^3\tan(\frac{fx}{2}+\frac{e}{2})^7}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/71680*(-90*cos(4*f*x+4*e)+5*cos(6*f*x+6*e)+152*cos(f*x+e)+60*cos(5*f*x+5
*e)-182-20*cos(3*f*x+3*e)+235*cos(2*f*x+2*e))*sec(1/2*f*x+1/2*e)^5*csc(1/2*
f*x+1/2*e)^7/f/a^3/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.65

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{5\,\cos\left(fx+e\right)^6+30\,\cos\left(fx+e\right)^5-30\,\cos\left(fx+e\right)^4-40\,\cos\left(fx+e\right)^3+40\,\cos\left(fx+e\right)^2+16}{35\,\left(a^3c^4f\cos\left(fx+e\right)^5-a^3c^4f\cos\left(fx+e\right)^4-2\,a^3c^4f\cos\left(fx+e\right)^3+2\,a^3c^4f\cos\left(fx+e\right)^2+a^3c^4f\cos\left(f\right.\right.}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/35*(5*cos(f*x + e)^6 + 30*cos(f*x + e)^5 - 30*cos(f*x + e)^4 - 40*cos(f*x
 + e)^3 + 40*cos(f*x + e)^2 + 16*cos(f*x + e) - 16)/((a^3*c^4*f*cos(f*x + e
)^5 - a^3*c^4*f*cos(f*x + e)^4 - 2*a^3*c^4*f*cos(f*x + e)^3 + 2*a^3*c^4*f*c
os(f*x + e)^2 + a^3*c^4*f*cos(f*x + e) - a^3*c^4*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{\sec^7\left(e+fx\right)-\sec^6\left(e+fx\right)-3\sec^5\left(e+fx\right)+3\sec^4\left(e+fx\right)+3\sec^3\left(e+fx\right)-3\sec^2\left(e+fx\right)-\sec\left(e+fx\right)+1}\,dx}{a^3c^4}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**4,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**7 - sec(e + f*x)**6 - 3*sec(e + f*x)**
5 + 3*sec(e + f*x)**4 + 3*sec(e + f*x)**3 - 3*sec(e + f*x)**2 - sec(e + f*x
) + 1), x)/(a**3*c**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.61

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\frac{7\left(\frac{75\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^4}+\frac{\left(\frac{42\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{175\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{700\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-5\right)(\cos(fx+e)+1)^7}{a^3c^4\sin(fx+e)^7}}{2240\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/2240*(7*(75*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^4) + (42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 700
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 5)*(cos(f*x + e) + 1)^7/(a^3*c^4*sin
(f*x + e)^7))/f
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\frac{700\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-175\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+42\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-5}{a^3c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^7}+\frac{7\left(a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-10a^{12}c^{16}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+75a^{12}c^{16}\tan\left(\frac{1}{2}\right.\right.}{a^{15}c^{20}}}{2240f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/2240*((700*tan(1/2*f*x + 1/2*e)^6 - 175*tan(1/2*f*x + 1/2*e)^4 + 42*tan(1
/2*f*x + 1/2*e)^2 - 5)/(a^3*c^4*tan(1/2*f*x + 1/2*e)^7) + 7*(a^12*c^16*tan(
1/2*f*x + 1/2*e)^5 - 10*a^12*c^16*tan(1/2*f*x + 1/2*e)^3 + 75*a^12*c^16*tan
(1/2*f*x + 1/2*e))/(a^15*c^20))/f
```

Mupad [B] (verification not implemented)

Time = 14.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^4}\,dx = \frac{\left(2\sin\left(\frac{e}{4}+\frac{fx}{4}\right)^2-1\right)\left(\frac{235\sin(e+fx)^2}{16}-\frac{45\sin(2e+2fx)^2}{8}+\frac{19\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{2}+\frac{5\sin(3e+3fx)^2}{16}-\frac{5\sin\left(\frac{3e}{2}+\frac{3fx}{2}\right)^2}{4}+\frac{15\ldots}{}\right.}{2240a^3c^4f\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^7\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)^3}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^4),x)
```

```
[Out] ((2*sin(e/4 + (f*x)/4)^2 - 1)*((19*sin(e/2 + (f*x)/2)^2)/2 - (45*sin(2*e +
2*f*x)^2)/8 + (5*sin(3*e + 3*f*x)^2)/16 - (5*sin((3*e)/2 + (3*f*x)/2)^2)/4
+ (15*sin((5*e)/2 + (5*f*x)/2)^2)/4 + (235*sin(e + f*x)^2)/16 - 5))/(2240*a
^3*c^4*f*sin(e/2 + (f*x)/2)^7*(sin(e/2 + (f*x)/2)^2 - 1)^3)
```

3.62 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}$$

```
[Out] 2/9*cot(f*x+e)^9/a^3/c^5/f+csc(f*x+e)/a^3/c^5/f-5/3*csc(f*x+e)^3/a^3/c^5/f+
9/5*csc(f*x+e)^5/a^3/c^5/f-csc(f*x+e)^7/a^3/c^5/f+2/9*csc(f*x+e)^9/a^3/c^5/
f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4043, 2686, 200, 2687, 30, 276}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{2\cot^9(e+fx)}{9a^3c^5f}+\frac{2\csc^9(e+fx)}{9a^3c^5f}-\frac{\csc^7(e+fx)}{a^3c^5f}+\frac{9\csc^5(e+fx)}{5a^3c^5f}-\frac{5\csc^3(e+fx)}{3a^3c^5f}+\frac{\csc(e+fx)}{a^3c^5f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
```

```
[Out] (2*Cot[e + f*x]^9)/(9*a^3*c^5*f) + Csc[e + f*x]/(a^3*c^5*f) - (5*Csc[e + f*
x]^3)/(3*a^3*c^5*f) + (9*Csc[e + f*x]^5)/(5*a^3*c^5*f) - Csc[e + f*x]^7/(a^
3*c^5*f) + (2*Csc[e + f*x]^9)/(9*a^3*c^5*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

integral =

$$-\frac{\int \left(a^2\cot^9(e+fx)\csc(e+fx)+2a^2\cot^8(e+fx)\csc^2(e+fx)+a^2\cot^7(e+fx)\csc^3(e+fx)\right)\,dx}{a^5c^5}$$

$$= -\frac{\int \cot^9(e+fx)\csc(e+fx)\,dx}{a^3c^5} - \frac{\int \cot^7(e+fx)\csc^3(e+fx)\,dx}{a^3c^5} - \frac{2\int \cot^8(e+fx)\csc^2(e+fx)\,dx}{a^3c^5}$$

$$= \frac{\text{Subst}\left(\int x^2(-1+x^2)^3\,dx, x, \csc(e+fx)\right)}{a^3c^5f} + \frac{\text{Subst}\left(\int (-1+x^2)^4\,dx, x, \csc(e+fx)\right)}{a^3c^5f} - \frac{2\text{Subst}\left(\int x^8\,dx, x, -\cot(e+fx)\right)}{a^3c^5f}$$

$$= \frac{2\cot^9(e+fx)}{9a^3c^5f} + \frac{\text{Subst}\left(\int (1-4x^2+6x^4-4x^6+x^8)\,dx, x, \csc(e+fx)\right)}{a^3c^5f} + \frac{\text{Subst}\left(\int (-x^2+3x^4-3x^6+x^8)\,dx, x, \csc(e+fx)\right)}{a^3c^5f}$$

$$= \frac{2\cot^9(e+fx)}{9a^3c^5f} + \frac{\csc(e+fx)}{a^3c^5f} - \frac{5\csc^3(e+fx)}{3a^3c^5f} + \frac{9\csc^5(e+fx)}{5a^3c^5f} - \frac{\csc^7(e+fx)}{a^3c^5f} + \frac{2\csc^9(e+fx)}{9a^3c^5f}$$

Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{(10+25\sec(e+fx)-60\sec^2(e+fx)-10\sec^3(e+fx)+80\sec^4(e+fx)-24\sec^5(e+fx)-32\sec^6(e+}{45a^3c^5f(-1+\sec(e+fx))^5(1+\sec(e+fx))^3}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^5),x]
[Out] ((10 + 25*Sec[e + f*x] - 60*Sec[e + f*x]^2 - 10*Sec[e + f*x]^3 + 80*Sec[e +
 f*x]^4 - 24*Sec[e + f*x]^5 - 32*Sec[e + f*x]^6 + 16*Sec[e + f*x]^7)*Tan[e
+ f*x])/(45*a^3*c^5*f*(-1 + Sec[e + f*x])^5*(1 + Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
parallelrisch	$-\frac{(-5\cos(7fx+7e)-110\cos(4fx+4e)-25\cos(6fx+6e)+129\cos(fx+e)+85\cos(5fx+5e)-145\cos(3fx+3e)+169\cos(}{184320f\,a^3c^5}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{21}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{35}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{35}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{9\tan(}}{128f\,c^5a^3}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{21}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}+\frac{35}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}-\frac{35}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}-\frac{1}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{9\tan(}}{128f\,c^5a^3}$
risch	$\frac{2i\left(45\,\mathrm{e}^{13i(fx+e)}-90\,\mathrm{e}^{12i(fx+e)}+30\,\mathrm{e}^{11i(fx+e)}+240\,\mathrm{e}^{10i(fx+e)}-69\,\mathrm{e}^{9i(fx+e)}-354\,\mathrm{e}^{8i(fx+e)}+516\,\mathrm{e}^{7i(fx+e)}+96\,\mathrm{e}^{6i(fx+}}{45f\,c^5a^3\left(\mathrm{e}^{i(fx+e)}+1\right)^5\left(\mathrm{e}^{i(fx+e)}-1\right)^9}$
norman	$\frac{\frac{1}{1152acf}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{128acf}+\frac{21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4}{640acf}-\frac{35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^6}{384acf}+\frac{35\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^8}{128acf}+\frac{21\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{10}}{128acf}-\frac{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{12}}{384acf}+\frac{\tan(}{6}}{c^4a^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/184320*(-5*cos(7*f*x+7*e)-110*cos(4*f*x+4*e)-25*cos(6*f*x+6*e)+129*cos(f
*x+e)+85*cos(5*f*x+5*e)-145*cos(3*f*x+3*e)+169*cos(2*f*x+2*e)-258)*sec(1/2*
f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^9/f/a^3/c^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.58

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$=\frac{10\,\cos\left(fx+e\right)^7+25\,\cos\left(fx+e\right)^6-60\,\cos\left(fx+e\right)^5-10\,\cos\left(fx+e\right)^4+80\,\cos\left(fx+e\right)^3}{45\left(a^3c^5f\cos\left(fx+e\right)^6-2\,a^3c^5f\cos\left(fx+e\right)^5-a^3c^5f\cos\left(fx+e\right)^4+4\,a^3c^5f\cos\left(fx+e\right)^3-a^3c^5f\cos\right.}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] 1/45*(10*cos(f*x + e)^7 + 25*cos(f*x + e)^6 - 60*cos(f*x + e)^5 - 10*cos(f*
x + e)^4 + 80*cos(f*x + e)^3 - 24*cos(f*x + e)^2 - 32*cos(f*x + e) + 16)/((
a^3*c^5*f*cos(f*x + e)^6 - 2*a^3*c^5*f*cos(f*x + e)^5 - a^3*c^5*f*cos(f*x +
 e)^4 + 4*a^3*c^5*f*cos(f*x + e)^3 - a^3*c^5*f*cos(f*x + e)^2 - 2*a^3*c^5*f
*cos(f*x + e) + a^3*c^5*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= -\frac{\int \frac{\sec(e+fx)}{\sec^8(e+fx)-2\sec^7(e+fx)-2\sec^6(e+fx)+6\sec^5(e+fx)-6\sec^3(e+fx)+2\sec^2(e+fx)+2\sec(e+fx)-1}\,dx}{a^3c^5}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**5,x)
```

```
[Out] -Integral(sec(e + f*x)/(sec(e + f*x)**8 - 2*sec(e + f*x)**7 - 2*sec(e + f*x
)**6 + 6*sec(e + f*x)**5 - 6*sec(e + f*x)**3 + 2*sec(e + f*x)**2 + 2*sec(e
+ f*x) - 1), x)/(a**3*c**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{3\left(\frac{315\sin(fx+e)}{\cos(fx+e)+1}-\frac{35\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^5}-\frac{\left(\frac{45\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{189\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{525\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{1575\sin(fx+e)^8}{(\cos(fx+e)+1)^8}-5\right)(\cos(fx+e)+1)^9}{a^3c^5\sin(fx+e)^9}}{5760\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] 1/5760*(3*(315*sin(f*x + e)/(cos(f*x + e) + 1) - 35*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^5) - (45*sin(f
*x + e)^2/(cos(f*x + e) + 1)^2 - 189*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
525*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1575*sin(f*x + e)^8/(cos(f*x + e)
 + 1)^8 - 5)*(cos(f*x + e) + 1)^9/(a^3*c^5*sin(f*x + e)^9))/f
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{1575\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^8-525\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6+189\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-45\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+5}{a^3c^5\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9}+\frac{3\left(3a^{12}c^{20}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-35a^{12}c^{20}\tan\left(\frac{1}{2}fx+\right.\right.}{a^{15}c^{25}}}{5760\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/5760*((1575*tan(1/2*f*x + 1/2*e)^8 - 525*tan(1/2*f*x + 1/2*e)^6 + 189*tan
(1/2*f*x + 1/2*e)^4 - 45*tan(1/2*f*x + 1/2*e)^2 + 5)/(a^3*c^5*tan(1/2*f*x +
 1/2*e)^9) + 3*(3*a^12*c^20*tan(1/2*f*x + 1/2*e)^5 - 35*a^12*c^20*tan(1/2*f
*x + 1/2*e)^3 + 315*a^12*c^20*tan(1/2*f*x + 1/2*e))/(a^15*c^25))/f
```

Mupad [B] (verification not implemented)

Time = 14.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^5}\,dx = \frac{\frac{145\cos(3e+3fx)}{32}-\frac{169\cos(2e+2fx)}{32}-\frac{129\cos(e+fx)}{32}+\frac{55\cos(4e+4fx)}{16}-\frac{85\cos(5e+5fx)}{32}+\frac{25\cos(6e+6fx)}{32}+\frac{5\cos(7e+7}{32}}{5760\,a^3\,c^5\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^9}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^5),x)
```

```
[Out] ((145*cos(3*e + 3*f*x))/32 - (169*cos(2*e + 2*f*x))/32 - (129*cos(e + f*x))
/32 + (55*cos(4*e + 4*f*x))/16 - (85*cos(5*e + 5*f*x))/32 + (25*cos(6*e + 6
*f*x))/32 + (5*cos(7*e + 7*f*x))/32 + 129/16)/(5760*a^3*c^5*f*cos(e/2 + (f*
x)/2)^5*sin(e/2 + (f*x)/2)^9)
```

3.63 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 162

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = -\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\csc(e+fx)}{a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}$$

```
[Out] -1/9*cot(f*x+e)^9/a^3/c^6/f-4/11*cot(f*x+e)^11/a^3/c^6/f+csc(f*x+e)/a^3/c^6
/f-8/3*csc(f*x+e)^3/a^3/c^6/f+22/5*csc(f*x+e)^5/a^3/c^6/f-4*csc(f*x+e)^7/a^
3/c^6/f+17/9*csc(f*x+e)^9/a^3/c^6/f-4/11*csc(f*x+e)^11/a^3/c^6/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4043, 2686, 200, 2687, 30, 276, 14}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = -\frac{4\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}+\frac{\csc(e+fx)}{a^3c^6f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
```

```
[Out] -1/9*Cot[e + f*x]^9/(a^3*c^6*f) - (4*Cot[e + f*x]^11)/(11*a^3*c^6*f) + Csc[
e + f*x]/(a^3*c^6*f) - (8*Csc[e + f*x]^3)/(3*a^3*c^6*f) + (22*Csc[e + f*x]^
5)/(5*a^3*c^6*f) - (4*Csc[e + f*x]^7)/(a^3*c^6*f) + (17*Csc[e + f*x]^9)/(9*
a^3*c^6*f) - (4*Csc[e + f*x]^11)/(11*a^3*c^6*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4043

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, I
nt[ExpandTrig[csc[e + f*x]*cot[e + f*x]^(2*m), (c + d*csc[e + f*x])^(n - m)
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ
```

```
[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0] && GtQ[m*n, 0]
```

Rubi steps

integral

$$= \frac{\int (a^3\cot^{11}(e+fx)\csc(e+fx)+3a^3\cot^{10}(e+fx)\csc^2(e+fx)+3a^3\cot^9(e+fx)\csc^3(e+fx)+a^3\cot^8}{a^6c^6}$$

$$= \frac{\int \cot^{11}(e+fx)\csc(e+fx)\,dx}{a^3c^6}+\frac{\int \cot^8(e+fx)\csc^4(e+fx)\,dx}{a^3c^6}+\frac{3\int \cot^{10}(e+fx)\csc^2(e+fx)\,dx}{a^3c^6}+\frac{3\int \cot^9(e+fx)\csc^3(e+fx)\,dx}{a^3c^6}$$

$$= -\frac{\text{Subst}\left(\int (-1+x^2)^5\,dx,x,\csc(e+fx)\right)}{a^3c^6f}+\frac{\text{Subst}\left(\int x^8(1+x^2)\,dx,x,-\cot(e+fx)\right)}{a^3c^6f}+\frac{3\text{Subst}\left(\int x^{10}\,dx,x,-\cot(e+fx)\right)}{a^3c^6f}-\frac{3\text{Subst}\left(\int x^2(-1+x^2)^4\,dx,x,\csc(e+fx)\right)}{a^3c^6f}$$

$$= -\frac{3\cot^{11}(e+fx)}{11a^3c^6f}-\frac{\text{Subst}\left(\int (-1+5x^2-10x^4+10x^6-5x^8+x^{10})\,dx,x,\csc(e+fx)\right)}{a^3c^6f}+\frac{\text{Subst}\left(\int (x^8+x^{10})\,dx,x,-\cot(e+fx)\right)}{a^3c^6f}-\frac{3\text{Subst}\left(\int (x^2-4x^4+6x^6-4x^8+x^{10})\,dx,x,\csc(e+fx)\right)}{a^3c^6f}$$

$$= -\frac{\cot^9(e+fx)}{9a^3c^6f}-\frac{4\cot^{11}(e+fx)}{11a^3c^6f}+\frac{\csc(e+fx)}{a^3c^6f}-\frac{8\csc^3(e+fx)}{3a^3c^6f}+\frac{22\csc^5(e+fx)}{5a^3c^6f}-\frac{4\csc^7(e+fx)}{a^3c^6f}+\frac{17\csc^9(e+fx)}{9a^3c^6f}-\frac{4\csc^{11}(e+fx)}{11a^3c^6f}$$

Mathematica [A] (verified)

Time = 5.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = \frac{(-125-120\sec(e+fx)+680\sec^2(e+fx)-400\sec^3(e+fx)-720\sec^4(e+fx)+832\sec^5(e+fx)+6}{495a^3c^6f(-1+\sec(e+fx))^6(1+\sec(e+f}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^6),x]
[Out] ((-125 - 120*Sec[e + f*x] + 680*Sec[e + f*x]^2 - 400*Sec[e + f*x]^3 - 720*S
ec[e + f*x]^4 + 832*Sec[e + f*x]^5 + 64*Sec[e + f*x]^6 - 384*Sec[e + f*x]^7
 + 128*Sec[e + f*x]^8)*Tan[e + f*x])/(495*a^3*c^6*f*(-1 + Sec[e + f*x])^6*(
1 + Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

method	result
parallelrisch	$-\frac{(240\cos(7fx+7e)-1300\cos(4fx+4e)-1720\cos(6fx+6e)+9680\cos(fx+e)+4880\cos(5fx+5e)-5584\cos(3fx+3e)+}{16220160fa^3c^6}$
derivativedivides	$\frac{\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}-\frac{8\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+28\tan(\frac{fx}{2}+\frac{e}{2})-\frac{1}{11\tan(\frac{fx}{2}+\frac{e}{2})^{11}}+\frac{8}{9\tan(\frac{fx}{2}+\frac{e}{2})^9}+\frac{56}{5\tan(\frac{fx}{2}+\frac{e}{2})^5}-\frac{70}{3\tan(\frac{fx}{2}+\frac{e}{2})^3}+}{256fa^3c^6}$
default	$\frac{\frac{\tan(\frac{fx}{2}+\frac{e}{2})^5}{5}-\frac{8\tan(\frac{fx}{2}+\frac{e}{2})^3}{3}+28\tan(\frac{fx}{2}+\frac{e}{2})-\frac{1}{11\tan(\frac{fx}{2}+\frac{e}{2})^{11}}+\frac{8}{9\tan(\frac{fx}{2}+\frac{e}{2})^9}+\frac{56}{5\tan(\frac{fx}{2}+\frac{e}{2})^5}-\frac{70}{3\tan(\frac{fx}{2}+\frac{e}{2})^3}+}{256fa^3c^6}$
risch	$\frac{2i(495\,e^{15i(fx+e)}-1485\,e^{14i(fx+e)}+1815\,e^{13i(fx+e)}+2475\,e^{12i(fx+e)}-4917\,e^{11i(fx+e)}-33\,e^{10i(fx+e)}+11715\,e^{9i(fx+e)}}{495fa^3c^6(e^{i}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x,method=_RETURNVERBOS
E)
```

```
[Out] -1/16220160*(240*cos(7*f*x+7*e)-1300*cos(4*f*x+4*e)-1720*cos(6*f*x+6*e)+968
0*cos(f*x+e)+4880*cos(5*f*x+5*e)-5584*cos(3*f*x+3*e)+8184*cos(2*f*x+2*e)-87
45+125*cos(8*f*x+8*e))*sec(1/2*f*x+1/2*e)^5*csc(1/2*f*x+1/2*e)^11/f/a^3/c^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{125\cos(fx+e)^8+120\cos(fx+e)^7-680\cos(fx+e)^6+400\cos(fx+e)^5+720\cos(fx+}{495\,(a^3c^6f\cos(fx+e)^7-3\,a^3c^6f\cos(fx+e)^6+a^3c^6f\cos(fx+e)^5+5\,a^3c^6f\cos(fx+e)^4-5\,a^3c^6f\,c}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="fr
icas")
```

```
[Out] 1/495*(125*cos(f*x + e)^8 + 120*cos(f*x + e)^7 - 680*cos(f*x + e)^6 + 400*c
os(f*x + e)^5 + 720*cos(f*x + e)^4 - 832*cos(f*x + e)^3 - 64*cos(f*x + e)^2
 + 384*cos(f*x + e) - 128)/((a^3*c^6*f*cos(f*x + e)^7 - 3*a^3*c^6*f*cos(f*x
 + e)^6 + a^3*c^6*f*cos(f*x + e)^5 + 5*a^3*c^6*f*cos(f*x + e)^4 - 5*a^3*c^6
*f*cos(f*x + e)^3 - a^3*c^6*f*cos(f*x + e)^2 + 3*a^3*c^6*f*cos(f*x + e) - a
^3*c^6*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{\int \frac{\sec(e+fx)}{\sec^9(e+fx)-3\sec^8(e+fx)+8\sec^6(e+fx)-6\sec^5(e+fx)-6\sec^4(e+fx)+8\sec^3(e+fx)-3\sec(e+fx)+1}\,dx}{a^3c^6}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**6,x)
```

```
[Out] Integral(sec(e + f*x)/(sec(e + f*x)**9 - 3*sec(e + f*x)**8 + 8*sec(e + f*x)
**6 - 6*sec(e + f*x)**5 - 6*sec(e + f*x)**4 + 8*sec(e + f*x)**3 - 3*sec(e +
 f*x) + 1), x)/(a**3*c**6)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.23

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{\frac{33\left(\frac{420\sin(fx+e)}{\cos(fx+e)+1}-\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3c^6}+\frac{\left(\frac{440\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{1980\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5544\sin(fx+e)^6}{(\cos(fx+e)+1)^6}-\frac{11550\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{27720\sin(fx+e)^{1}}{(\cos(fx+e)+1)^{10}}\right.}{a^3c^6\sin(fx+e)^{11}}}{126720\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="ma
xima")
```

```
[Out] 1/126720*(33*(420*sin(f*x + e)/(cos(f*x + e) + 1) - 40*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3*c^6) + (440*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1980*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 + 5544*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 11550*sin(f*x + e)^8/(cos(f
*x + e) + 1)^8 + 27720*sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - 45)*(cos(f*x
 + e) + 1)^11/(a^3*c^6*sin(f*x + e)^11))/f
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx$$
$$= \frac{\frac{27720\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{10}-11550\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{8}+5544\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}-1980\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+440\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-45}{a^3c^6\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{11}}+\frac{33\left(3a^{12}c^{24}\tan\left(\frac{1}{2}\right.\right.}{}}{126720\,f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^6,x, algorithm="gi
ac")
```

```
[Out] 1/126720*((27720*tan(1/2*f*x + 1/2*e)^10 - 11550*tan(1/2*f*x + 1/2*e)^8 + 5
544*tan(1/2*f*x + 1/2*e)^6 - 1980*tan(1/2*f*x + 1/2*e)^4 + 440*tan(1/2*f*x
+ 1/2*e)^2 - 45)/(a^3*c^6*tan(1/2*f*x + 1/2*e)^11) + 33*(3*a^12*c^24*tan(1/
2*f*x + 1/2*e)^5 - 40*a^12*c^24*tan(1/2*f*x + 1/2*e)^3 + 420*a^12*c^24*tan(
1/2*f*x + 1/2*e))/(a^15*c^30))/f
```

Mupad [B] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^6}\,dx = -\frac{\frac{605\cos(e+fx)}{8}+\frac{1023\cos(2e+2fx)}{16}-\frac{349\cos(3e+3fx)}{8}-\frac{325\cos(4e+4fx)}{32}+\frac{305\cos(5e+5fx)}{8}-\frac{215\cos(6e+6fx)}{16}+15\,\mathrm{c}}{126720\,a^3\,c^6\,f\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^6),x)
```

```
[Out] -((605*cos(e + f*x))/8 + (1023*cos(2*e + 2*f*x))/16 - (349*cos(3*e + 3*f*x)
)/8 - (325*cos(4*e + 4*f*x))/32 + (305*cos(5*e + 5*f*x))/8 - (215*cos(6*e +
 6*f*x))/16 + (15*cos(7*e + 7*f*x))/8 + (125*cos(8*e + 8*f*x))/128 - 8745/1
28)/(126720*a^3*c^6*f*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^11)
```

3.64 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 163

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = -\frac{256c^4(a+a\sec(e+fx))\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{105f} - \frac{8c^2(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{21f} - \frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{9f}$$

```
[Out] -8/21*c^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/9*c*(a+a*s
ec(f*x+e))*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/315*c^4*(a+a*sec(f*x+e))
*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/105*c^3*(a+a*sec(f*x+e))*(c-c*sec(f
*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = -\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{105f} - \frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{21f} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}{9f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x]
]) - (64*c^3*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(1
05*f) - (8*c^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(21*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x
])/(9*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{9f} \\
&\quad+\frac{1}{3}(4c)\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx \\
&= -\frac{8c^2(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{21f} \\
&\quad-\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{9f} \\
&\quad+\frac{1}{21}\left(32c^2\right)\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx \\
&= -\frac{64c^3(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{105f} \\
&\quad-\frac{8c^2(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{21f} \\
&\quad-\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{9f} \\
&\quad+\frac{1}{105}\left(128c^3\right)\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{256c^4(a+a\sec(e+fx))\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} \\
&\quad-\frac{64c^3(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{105f} \\
&\quad-\frac{8c^2(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{21f} \\
&\quad-\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{9f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.43

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = \frac{2ac^4(1+\sec(e+fx))\left(-319+321\sec(e+fx)-165\sec^2(e+fx)+35\sec^3(e+fx)\right)\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(7/2),x]
[Out] (2*a*c^4*(1 + Sec[e + f*x])*(-319 + 321*Sec[e + f*x] - 165*Sec[e + f*x]^2 +
 35*Sec[e + f*x]^3)*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 7.67 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.47

method	result
default	$\frac{2 a \,c^{3} \left(319 \cos \left(f x +e \right)^{3}-321 \cos \left(f x +e \right)^{2}+165 \cos \left(f x +e \right)-35\right) \sqrt{-c \left(\sec \left(f x +e \right)-1\right)}\, \left(\cos \left(f x +e \right)+1\right)^{2} \sec \left(f x +e \right)^{4} \csc \left(f x +e \right)}{315 f}$
parts	$-\frac{2 a \left(\sec \left(f x +e \right)-1\right)^{3} \left(177 \cos \left(f x +e \right)^{3}-71 \cos \left(f x +e \right)^{2}+27 \cos \left(f x +e \right)-5\right) c^{3} \sqrt{-c \left(\sec \left(f x +e \right)-1\right)}\, \left(\cos \left(f x +e \right)+1\right) \csc \left(f x +e \right)}{35 f \left(\cos \left(f x +e \right)-1\right)^{3}}+\frac{2 a (}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/315*a*c^3/f*(319*cos(f*x+e)^3-321*cos(f*x+e)^2+165*cos(f*x+e)-35)*(-c*(se
c(f*x+e)-1))^(1/2)*(cos(f*x+e)+1)^2*sec(f*x+e)^4*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = \frac{2\left(319\,ac^{3}\cos\left(fx+e\right)^{5}+317\,ac^{3}\cos\left(fx+e\right)^{4}-158\,ac^{3}\cos\left(fx+e\right)^{3}-26\,ac^{3}\cos\left(fx+e\right)^{2}}{315\,f\cos\left(fx+e\right)^{4}\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
fricas")
```

```
[Out] 2/315*(319*a*c^3*cos(f*x + e)^5 + 317*a*c^3*cos(f*x + e)^4 - 158*a*c^3*cos(
f*x + e)^3 - 26*a*c^3*cos(f*x + e)^2 + 95*a*c^3*cos(f*x + e) - 35*a*c^3)*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = \int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{7}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
maxima")
```

```
[Out] -2/315*(315*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*(5*(a*c^3*f*cos(2*f*x + 2*e)^4 + a*c^3*f*sin(2*f*x + 2*e)^4 + 4*a
*c^3*f*cos(2*f*x + 2*e)^3 + 6*a*c^3*f*cos(2*f*x + 2*e)^2 + 4*a*c^3*f*cos(2*
f*x + 2*e) + a*c^3*f + 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + 2*a*c^3*f*cos(2*f*x
+ 2*e) + a*c^3*f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x
 + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*
f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f
*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*
f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f
*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*
f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f
*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x +
 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*
x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)))/((cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4*cos(2*f*x + 2*e)^5
+ (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2*f*x + 2*e)^4 + 6*co
s(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x +
 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^
2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(
cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*
e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^3 +
(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4
+ sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x
+ 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4 + 8*cos(2*f*x + 2*e)
^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*
```

(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4*cos(2*f*x + 2*e)^5 + (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4 + 8*cos(2*f*x + 2*e)^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + 2

```
*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x +
2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^4 + 4
*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*
f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4 + 4*co
s(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x
 + 2*e))*sin(4*f*x + 4*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1))^2), x) - 14*(a*c^3*f*cos(2*f*x + 2*e)^4 + a*c^3*f*sin(2*f*x + 2*e)
^4 + 4*a*c^3*f*cos(2*f*x + 2*e)^3 + 6*a*c^3*f*cos(2*f*x + 2*e)^2 + 4*a*c^3*
f*cos(2*f*x + 2*e) + a*c^3*f + 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + 2*a*c^3*f*co
s(2*f*x + 2*e) + a*c^3*f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*c
os(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2
+ sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
 sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) -
 cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*s
in(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) +
2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2
*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos
(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4*cos(2*f*x +
 2*e)^5 + (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2*f*x + 2*e)^
4 + 6*cos(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos
(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x
 + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)
^2 + 4*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*
(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*
f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2
*e)^3 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2
*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2
*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x +
 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4 + 8*cos(2*f*
x + 2*e)^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
)^2 + 2*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 + 4*cos(2
*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos(2*f*x
+ 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^4 + s
in(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*
```

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f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*
f*x + 6*e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 +
4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos
(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e)
^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x
 + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
)^3 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2
*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2
*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e
)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)
*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4
 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*si
n(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4*cos(2*f*x
+ 2*e)^5 + (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2*f*x + 2*e)
^4 + 6*cos(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*co
s(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e
)^2 + 4*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2
*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2
*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*cos(2*f*x +
2*e)^3 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 +
2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(
2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x
+ 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*c
os(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4 + 8*cos(2*f
*x + 2*e)^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e)^2 + 2*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 + 4*cos(
2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos(2*f*x
 + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^4 +
sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x +
2*e) + 1)*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6
*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 +
 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*co
s(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*
x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e)^3 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 +
2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(
2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*
e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1
```

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)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^
4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*s
in(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1))^2), x) + 8*(a*c^3*f*cos(2*f*x + 2*e)^4 + a*c^3*f*sin(2*f*x
 + 2*e)^4 + 4*a*c^3*f*cos(2*f*x + 2*e)^3 + 6*a*c^3*f*cos(2*f*x + 2*e)^2 + 4
*a*c^3*f*cos(2*f*x + 2*e) + a*c^3*f + 2*(a*c^3*f*cos(2*f*x + 2*e)^2 + 2*a*c
^3*f*cos(2*f*x + 2*e) + a*c^3*f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x +
 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x +
2*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x +
6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2
*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4
*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*
e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4*cos(
2*f*x + 2*e)^5 + (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2*f*x
+ 2*e)^4 + 6*cos(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4
+ 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*si
n(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x
 + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)
^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6
*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*cos(2*
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e
)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 +
6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(
2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2
 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4 + 8*c
os(2*f*x + 2*e)^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 +
4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*cos
(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e
)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))
*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*
e)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3
```

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+ 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x
 + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(si
n(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e
)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 +
6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*
x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*
e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 + 2*(cos
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2*f*x +
 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e)
+ 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(7/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4*cos
(2*f*x + 2*e)^5 + (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2*f*x
 + 2*e)^4 + 6*cos(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4
 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*
x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e
)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 +
6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*cos(2
*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*
e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 +
 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos
(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^
2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4 + 8*
cos(2*f*x + 2*e)^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^4 +
 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 + 2*co
s(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*
e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2
*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e)
)*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2
*e)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3
 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos(2*f*
x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(s
in(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*
e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 +
 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f
*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2
*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 + 2*(co
s(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2*f*x
+ 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e)
 + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(7/2*arctan2(sin(2*f*x + 2*e),
```

```
 cos(2*f*x + 2*e) + 1))^2), x) + (a*c^3*f*cos(2*f*x + 2*e)^4 + a*c^3*f*sin(
2*f*x + 2*e)^4 + 4*a*c^3*f*cos(2*f*x + 2*e)^3 + 6*a*c^3*f*cos(2*f*x + 2*e)^
2 + 4*a*c^3*f*cos(2*f*x + 2*e) + a*c^3*f + 2*(a*c^3*f*cos(2*f*x + 2*e)^2 +
2*a*c^3*f*cos(2*f*x + 2*e) + a*c^3*f)*sin(2*f*x + 2*e)^2)*integrate((cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*
f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f
*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*
f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f
*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*
x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*
x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x
 + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 + 4
*cos(2*f*x + 2*e)^5 + (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(2
*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*
e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(
6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x +
 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^
2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*c
os(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x
+ 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)
^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*
(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2
*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4
+ 8*cos(2*f*x + 2*e)^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin
(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)
^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 +
2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x
+ 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*c
os(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x +
2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x
 + 2*e)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*
e)^3 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos(
2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 +
2*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x
```

```
+ 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)
^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos
(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x
 + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 + 2
*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2*
f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x +
2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(7/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^6 + sin(2*f*x + 2*e)^6 +
4*cos(2*f*x + 2*e)^5 + (3*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 2)*sin(
2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2
*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos
(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x
+ 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)
^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 4*
cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x
 + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4
*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x +
2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (3*cos(2*f*x + 2*e)^4
 + 8*cos(2*f*x + 2*e)^3 + 8*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*si
n(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e
)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^3 +
 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x
 + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*
cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)^2 + cos(2*f*x +
 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^5 + cos(2*f*x + 2*e)*sin(2*f*
x + 2*e)^4 + 4*cos(2*f*x + 2*e)^4 + 6*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2
*e)^3 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*sin(2*f*x + 2*e)^2 + 4*cos
(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 +
 2*(sin(2*f*x + 2*e)^5 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*si
n(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + 4*cos(2*f*x
 + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (co
s(2*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*
x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^5 +
2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^3 + (cos(2
*f*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x +
 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(7/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1))^2), x))*sqrt(c) + (3*(105*a*c^3*sin(8*f*x + 8*
e) + 560*a*c^3*sin(6*f*x + 6*e) + 378*a*c^3*sin(4*f*x + 4*e) + 216*a*c^3*si
n(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
(315*a*c^3*cos(8*f*x + 8*e) + 1680*a*c^3*cos(6*f*x + 6*e) + 1134*a*c^3*cos(
```

```
4*f*x + 4*e) + 648*a*c^3*cos(2*f*x + 2*e) + 319*a*c^3)*sin(9/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(c))/((f*cos(2*f*x + 2*e)^4 + f*s
in(2*f*x + 2*e)^4 + 4*f*cos(2*f*x + 2*e)^3 + 6*f*cos(2*f*x + 2*e)^2 + 2*(f*
cos(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*sin(2*f*x + 2*e)^2 + 4*f*cos
(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.84 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx = \frac{32\sqrt{2}\left(105\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^3c^2+189\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^3+135\left(c\tan\left(\frac{1}{2}fx\right.\right.\right.}{315\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(7/2),x, algorithm="
giac")
```

```
[Out] 32/315*sqrt(2)*(105*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^2 + 189*(c*tan(1/2*f
*x + 1/2*e)^2 - c)^2*c^3 + 135*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 35*c^5)
*a*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.96

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{7/2}\,dx=\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a\,c^3\,2i}{f}+\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,638i}{315\,f}\right)}{e^{e\,1i+f\,x\,1i}-1}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a\,c^3\,32i}{9\,f}+\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,32i}{9\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^4}+\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a\,c^3\,96i}{7\,f}+\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,32i}{63\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^3}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a\,c^3\,64i}{5\,f}-\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,736i}{105\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\,{\left(e^{e\,2i+f\,x\,2i}+1\right)}^2}+\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,\left(\frac{a\,c^3\,8i}{3\,f}-\frac{a\,c^3\,e^{e\,1i+f\,x\,1i}\,1256i}{315\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\,\left(e^{e\,2i+f\,x\,2i}+1\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*2i)/
f + (a*c^3*exp(e*1i + f*x*1i)*638i)/(315*f)))/(exp(e*1i + f*x*1i) - 1) - ((
c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*32i)/(
9*f) + (a*c^3*exp(e*1i + f*x*1i)*32i)/(9*f)))/((exp(e*1i + f*x*1i) - 1)*(ex
p(e*2i + f*x*2i) + 1)^4) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a*c^3*96i)/(7*f) + (a*c^3*exp(e*1i + f*x*1i)*32i)/(63*f)))
/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^3*64i)/(5*f) - (a*c^3*ex
p(e*1i + f*x*1i)*736i)/(105*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*
2i) + 1)^2) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2
)*((a*c^3*8i)/(3*f) - (a*c^3*exp(e*1i + f*x*1i)*1256i)/(315*f)))/((exp(e*1i
 + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
```

3.65 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 122

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3(a+a\sec(e+fx))\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{35f} - \frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{7f}$$

```
[Out] -2/7*c*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/105*c^3*(a+a
*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/35*c^2*(a+a*sec(f*x+e))
*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used

= {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)}{105f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{35f} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}{7f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])*Tan[e + f*x])/(105*f*Sqrt[c - c*Sec[e + f*x]]
) - (16*c^2*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(35
*f) - (2*c*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(7
*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{7f} + \frac{1}{7}(8c)\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx$$

$$= -\frac{16c^2(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{35f} - \frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{7f} + \frac{1}{35}(32c^2)\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$$

$$= -\frac{64c^3(a+a\sec(e+fx))\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{35f} - \frac{2c(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{7f}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.49

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2ac^3(1+\sec(e+fx))\left(71-54\sec(e+fx)+15\sec^2(e+fx)\right)\tan(e+fx)}{105f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2),x]
[Out] (-2*a*c^3*(1 + Sec[e + f*x])*(71 - 54*Sec[e + f*x] + 15*Sec[e + f*x]^2)*Tan
[e + f*x])/(105*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.55

method	result
default	$\frac{2a\,c^2\left(71\cos(fx+e)^2-54\cos(fx+e)+15\right)\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)^2\sec(fx+e)^3\csc(fx+e)}{105f}$
parts	$\frac{2a(\sec(fx+e)-1)^2\left(43\cos(fx+e)^2-14\cos(fx+e)+3\right)c^2\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)\csc(fx+e)}{15f(\cos(fx+e)-1)^2} - \frac{2a\left(46\cos(fx+e)^3-23\cos\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERB
OSE)
[Out] 2/105*a*c^2/f*(71*cos(f*x+e)^2-54*cos(f*x+e)+15)*(-c*(sec(f*x+e)-1))^(1/2)*
(cos(f*x+e)+1)^2*sec(f*x+e)^3*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.86

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = \frac{2\left(71\,ac^2\cos\left(fx+e\right)^4+88\,ac^2\cos\left(fx+e\right)^3-22\,ac^2\cos\left(fx+e\right)^2-24\,ac^2\cos\left(fx+e\right)+15}{105\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] 2/105*(71*a*c^2*cos(f*x + e)^4 + 88*a*c^2*cos(f*x + e)^3 - 22*a*c^2*cos(f*x
 + e)^2 - 24*a*c^2*cos(f*x + e) + 15*a*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = a\left(\int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx + \int\left(-c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)\right)dx + \int\left(-c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)\right)dx + \int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^4\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-c**
2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-c**2*sqrt(-c*se
c(e + f*x) + c)*sec(e + f*x)**3, x) + Integral(c**2*sqrt(-c*sec(e + f*x) +
c)*sec(e + f*x)**4, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx=\int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm=" maxima")

[Out] -2/105*(105*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4)*(3*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2*e) + a*c^2*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*

(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*
x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*
e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6
e) + 4(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/4)), x) - 5*(a*c^2*f*cos(2*f*x + 2*e)^2 + a*c^2*f*sin(2*f*
x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2*e) + a*c^2*f)*integrate((((cos(6*f*x +
6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x +
2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x +
2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x +
4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x +
2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x +
6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2
*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4
*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*
e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6
e)^2 + 4(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (
2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(
2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2
*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3
+ cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x +
2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x +
4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin
(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +

```
2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 +
 (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f
*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)
^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2
*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2
*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2
*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*
f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 +
 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4
*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) + (a*c^2*f*cos(2*f*x + 2*e)^2 + a*
c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2*e) + a*c^2*f)*integrate(
(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
+ cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*
e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*
e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e
)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*c
os(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e
)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)
*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2
*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*
sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2
)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x +
2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*
x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x +
4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2
*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2
+ cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x +
```

```
2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*
f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^
2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*si
n(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*
f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*
(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*
f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) + (a*c^2*f*cos(2*f*x
 + 2*e)^2 + a*c^2*f*sin(2*f*x + 2*e)^2 + 2*a*c^2*f*cos(2*f*x + 2*e) + a*c^2
*f)*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos
(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*
cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*c
os(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e)*cos(2*
f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin
(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(
2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4
+ sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x
 + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2
*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
```

```
)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6
*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2
*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*
cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2
*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)
^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f
*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f
*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)
^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x + 2*e)^3 + (
cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x +
4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x))*sqrt(
c) + (7*(15*a*c^2*sin(6*f*x + 6*e) + 35*a*c^2*sin(4*f*x + 4*e) + 13*a*c^2*s
in(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) -
 (105*a*c^2*cos(6*f*x + 6*e) + 245*a*c^2*cos(4*f*x + 4*e) + 91*a*c^2*cos(2*
f*x + 2*e) + 71*a*c^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)))*sqrt(c))/((f*cos(2*f*x + 2*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*f*cos(2*f*
x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
 + 1)^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.83 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.68

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx=\frac{16\sqrt{2}\left(35\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^2+42\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^3+15c^4\right)ac^2}{105\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{7}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] 16/105*sqrt(2)*(35*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 + 42*(c*tan(1/2*f*x
 + 1/2*e)^2 - c)*c^3 + 15*c^4)*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*
f)
```

Mupad [B] (verification not implemented)

Time = 17.24 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.15

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}\,dx = \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^2\,2\mathrm{i}}{f}+\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,142\mathrm{i}}{105\,f}\right)}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1} + \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^2\,16\mathrm{i}}{7\,f}-\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,16\mathrm{i}}{7\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3} - \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^2\,8\mathrm{i}}{5\,f}-\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,184\mathrm{i}}{35\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} - \frac{\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{a\,c^2\,4\mathrm{i}}{3\,f}+\frac{a\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,244\mathrm{i}}{105\,f}\right)}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*2i)/
f + (a*c^2*exp(e*1i + f*x*1i)*142i)/(105*f)))/(exp(e*1i + f*x*1i) - 1) + ((
c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*16i)/(
7*f) - (a*c^2*exp(e*1i + f*x*1i)*16i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(ex
p(e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a*c^2*8i)/(5*f) - (a*c^2*exp(e*1i + f*x*1i)*184i)/(35*f)))
/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) - ((c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a*c^2*4i)/(3*f) + (a*c^2*exp
(e*1i + f*x*1i)*244i)/(105*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2
i) + 1))
```

3.66 $\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 81

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2(a+a\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f}$$

```
[Out] -8/15*c^2*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/5*c*(a+a*s
ec(f*x+e))*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)}{15f\sqrt{c-c\sec(e+fx)}} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}{5f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f*x]])
- (2*c*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f} \\
&\quad+\frac{1}{5}(4c)\int\sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{8c^2(a+a\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}}-\frac{2c(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int\sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx=\frac{2ac^2(1+\sec(e+fx))(-7+3\sec(e+fx))\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a*c^2*(1 + Sec[e + f*x])*(-7 + 3*Sec[e + f*x])*Tan[e + f*x])/(15*f*Sqrt[
c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

method	result
default	$\frac{2ac(7\cos(fx+e)-3)\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)^2\sec(fx+e)^2\csc(fx+e)}{15f}$
parts	$-\frac{2a(\sec(fx+e)-1)(5\cos(fx+e)-1)c\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)\csc(fx+e)}{3f(\cos(fx+e)-1)}+\frac{2a\left(6\cos(fx+e)^2-3\cos(fx+e)+1\right)(\sec(fx\cdots}{}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/15*a*c/f*(7*cos(f*x+e)-3)*(-c*(sec(f*x+e)-1))^(1/2)*(cos(f*x+e)+1)^2*sec(f*x+e)^2*csc(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx=\frac{2\left(7\,ac\cos\left(fx+e\right)^3+11\,ac\cos\left(fx+e\right)^2+ac\cos\left(fx+e\right)-3\,ac\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{15\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2/15*(7*a*c*cos(f*x + e)^3 + 11*a*c*cos(f*x + e)^2 + a*c*cos(f*x + e) - 3*a*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx=a\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx+\int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)\right)dx\right)$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(3/2),x)

[Out] a*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3, x))

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{3}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] -2/15*(15*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/4)*((a*c*f*cos(2*f*x + 2*e)^2 + a*c*f*sin(2*f*x + 2*e)^2 + 2*a*c*f*cos
(2*f*x + 2*e) + a*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos
(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(
2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*co
s(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(
6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2
*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(
4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*
f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f
*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*
f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos
(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*
e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e
))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*
f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*f*x +
2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*si
n(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2
+ (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*
```

```
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*
cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
 + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) +
cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1))^2), x) - 2*(a*c*f*cos(2*f*x + 2*e)^2 + a*c*f*sin(2*f*x + 2*e)^2 + 2*a*c
*f*cos(2*f*x + 2*e) + a*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) +
 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e
)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)
^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e
)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)
*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e
)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)
*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (
cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + c
os(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))
/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*
cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
 + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) +
cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(sin(2*
```

```
f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*
e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^
2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + co
s(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos
(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4
*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(
sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))^2), x) + (a*c*f*cos(2*f*x + 2*e)^2 + a*c*f*sin(2*f*x + 2*e)^2 + 2
*a*c*f*cos(2*f*x + 2*e) + a*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(6*f*x + 6*e)*cos(2*f*x + 2*
e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x +
 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x +
 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x +
6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x +
 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x +
4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - (cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
 + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4
*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2
+ 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(
2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2
*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e
```

```
) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + si
n(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 4*(si
n(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x
+ 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 4*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4
*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 4*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3
+ cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 +
 cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 4*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2
*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x
 + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 4*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(
2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))^2), x))*sqrt(c) + (5*(3*a*c*sin(4*f*x + 4*e) + 2*a*c*sin(2*f*
x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (15*a*
c*cos(4*f*x + 4*e) + 10*a*c*cos(2*f*x + 2*e) + 7*a*c)*sin(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(c))/((f*cos(2*f*x + 2*e)^2 + f*si
n(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = \frac{8\sqrt{2}\left(5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^3+3c^4\right)a}{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] 8/15*sqrt(2)*(5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3 + 3*c^4)*a/((c*tan(1/2*f
*x + 1/2*e)^2 - c)^(5/2)*f)
```

Mupad [B] (verification not implemented)

Time = 17.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48

$$\int \sec(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2\,a\,c\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^3\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(7+7\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-6\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\right)}{15\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] -(2*a*c*(exp(e*1i + f*x*1i)*1i + 1i)^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp
(e*1i + f*x*1i)/2))^(1/2)*(7*exp(e*2i + f*x*2i) - 6*exp(e*1i + f*x*1i) + 7)
)/(15*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

3.67 $\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 39

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/3*c*(a+a*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c\tan(e+fx)(a\sec(e+fx)+a)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
[Out] (-2*c*(a + a*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{2c(a+a\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2ac(1+\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]],x]
[Out] (-2*a*c*(1 + Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{2a(\cos(fx+e)+1)^2\sqrt{-c(\sec(fx+e)-1)}\,\sec(fx+e)\csc(fx+e)}{3f}$	42
parts	$-\frac{2a\sqrt{-c(\sec(fx+e)-1)}\,\sin(fx+e)}{f(\cos(fx+e)-1)}-\frac{2a\sqrt{-c(\sec(fx+e)-1)}\,(-\sec(fx+e)\csc(fx+e)+2\cot(fx+e)+\csc(fx+e))}{3f}$	85

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
[Out] 2/3*a/f*(cos(f*x+e)+1)^2*(-c*(sec(f*x+e)-1))^(1/2)*sec(f*x+e)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\left(a\cos\left(fx+e\right)^2+2\,a\cos\left(fx+e\right)+a\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
[Out] 2/3*(a*cos(f*x + e)^2 + 2*a*cos(f*x + e) + a)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$$
$$=a\left(\int \sqrt{-c\sec{(e+fx)}+c}\sec{(e+fx)}\,dx+\int \sqrt{-c\sec{(e+fx)}+c}\sec^2{(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(sqrt(-c*s
ec(e + f*x) + c)*sec(e + f*x)**2, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int (a\sec{(fx+e)}+a)\sqrt{-c\sec{(fx+e)}+c}\sec{(fx+e)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] 2/3*(3*(a*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x +
4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2
*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(6*f*x + 6
*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*
e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4
*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*
e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(6*f*x + 6*e
)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)
^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(2*cos(4*
f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 4*co
s(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2
 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x +
 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(
2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2
+ (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x +
 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(
```

```
2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e)
 + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x
 + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/4)), x) - a*f*integrate((((cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*
cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*s
in(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*s
in(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(6*f*x + 6*e)*si
n(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 2*cos(2*f*x + 2*e)*s
in(4*f*x + 4*e) - cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 2*cos(4*f*x + 4*e)*si
n(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos
(6*f*x + 6*e)*cos(2*f*x + 2*e) + 2*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(
2*f*x + 2*e)^2 + sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 2*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((
(2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + cos(6*f*x + 6
*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*
f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))^2 + (2*(2*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) +
 cos(6*f*x + 6*e)^2 + 4*cos(4*f*x + 4*e)^2 + 4*cos(4*f*x + 4*e)*cos(2*f*x +
 2*e) + cos(2*f*x + 2*e)^2 + 2*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(
6*f*x + 6*e) + sin(6*f*x + 6*e)^2 + 4*sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*
e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/4)), x))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(3/4)*sqrt(c) - (3*a*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1))*sin(2*f*x + 2*e) - (3*a*cos(2*f*x + 2*e) + a)*sin
(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(c))/((cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4)*f)
```

Giac [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx = \frac{4\sqrt{2}ac^2}{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c-c*sec(f*x+e))^(1/2),x, algorithm="
```

```
giac")
```

```
[Out] 4/3*sqrt(2)*a*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*f)
```

Mupad [B] (verification not implemented)

Time = 14.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

$$\int \sec(e+fx)(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\,a\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(2\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{3\,f\,\left(8\,\cos\left(2\,e+2\,f\,x\right)-12\,\cos\left(e+f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+7\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a*(c - c/cos(e + f*x))^(1/2)*(2*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(3
*f*(8*cos(2*e + 2*f*x) - 12*cos(e + f*x) - 4*cos(3*e + 3*f*x) + cos(4*e + 4
*f*x) + 7))
```

3.68 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 77

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)/
f/c^(1/2)+2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4041, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[2]*a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x
]])])/(Sqrt[c]*f) + (2*a*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx \\
&= \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} - \frac{(4a)\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{f} \\
&= -\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{2a\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$
\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a\left(-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\sec(e+fx)}}{\sqrt{2}}\right) + \sqrt{1+\sec(e+fx)}\right)\tan(e+fx)}{f\sqrt{1+\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/Sqrt[c - c*Sec[e + f*x]],x]
[Out] (2*a*(-(Sqrt[2]*ArcTanh[Sqrt[1 + Sec[e + f*x]]/Sqrt[2]]) + Sqrt[1 + Sec[e +
 f*x]])*Tan[e + f*x])/(f*Sqrt[1 + Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

method	result
default	$\frac{a\sqrt{2}\left(-2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)+\sqrt{2}\right)\tan(fx+e)}{f\sqrt{-c(\sec(fx+e)-1)}}$
parts	$\frac{a\sqrt{2}\,\sin(fx+e)\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f(\cos(fx+e)+1)\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}+\frac{a\sqrt{2}\left(-\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)+\sqrt{2}\right)\tan(fx+e)}{f\sqrt{-c(\sec(fx+e)-1)}}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] a/f*2^(1/2)*(-2*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*arctan(1/2*2^(1/2)/(-cos
(f*x+e)/(cos(f*x+e)+1))^(1/2))+2^(1/2))/(-c*(sec(f*x+e)-1))^(1/2)*tan(f*x+e
)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \left[\frac{\sqrt{2}ac\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)-2\,(a\cos\right.$$

$$cf\sin(fx+e)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [(sqrt(2)*a*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*sin
(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(a*cos(f*x +
 e) + a)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e)), 2*(sq
rt(2)*a*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(
f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - (a*cos(f*x + e) + a)*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = a\left(\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx + \int \frac{\sec^2(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)
[Out] a*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f
*x)**2/sqrt(-c*sec(e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
maxima")
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 0.95 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2\,a\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{2}}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}\right)}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
giac")
[Out] 2*a*(sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c) + s
qrt(2)/sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{a+\frac{a}{\cos(e+f\,x)}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.69 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] 1/2*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(3/2)
/f*2^(1/2)-a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4042, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f}-\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(Sqrt
[2]*c^(3/2)*f) - (a*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{a\int\frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c} \\
&= -\frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} + \frac{a\text{Subst}\left(\int\frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{cf} \\
&= \frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}c^{3/2}f} - \frac{a\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{a\left(\cot\left(\frac{1}{2}(e+fx)\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{1+\sec(e+fx)}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{2}}\right)}{cf\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(Cot[(e + f*x)/2] + (ArcTanh[Sqrt[1 + Sec[e + f*x]]/Sqrt[2]]*Sqrt[1 + Se
c[e + f*x]]*Tan[(e + f*x)/2])/Sqrt[2]))/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(66) = 132.

Time = 4.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.34

method	result
default	$-\frac{a\sqrt{2}\left(\arctan\left(\frac{1}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)(1-\cos(fx+e))^2\csc(fx+e)-\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\sin(fx+e)\right)}{2cf\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}$
parts	$-\frac{a\sqrt{2}\,(1-\cos(fx+e))\left(\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}-\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,(1-\cos(fx+e))^2\csc(fx+e)^2-\arctan\right.}{4f\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{3}{2}}\left((1-\cos(fx+e))^2\csc(fx+e\right.}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2*a/c/f*2^(1/2)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*cs
c(f*x+e)^2)^(1/2)/(1-cos(f*x+e))/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(a
rctan(1/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(1-cos(f*x+e))^2*csc(f*x+e
)-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*sin(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(66) = 132.

Time = 0.34 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.50

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = \left[\frac{\sqrt{2}(ac\cos(fx+e)-ac)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)}{\cos(fx}}}{(\cos(fx+e)-}\right.}{4\,(c^2}\right.$$

$$-\frac{\frac{\sqrt{2}(ac\cos(fx+e)-ac)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}}-2\left(a\cos(fx+e)^2+a\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\left(c^2f\cos(fx+e)-c^2f\right)\sin(fx+e)}\right]$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*(a*c*cos(f*x + e) - a*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos(f*x +
 e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) +
```

```
(3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f
*x + e) + 4*(a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e)), -1/2*(sqrt(2)*(a
*c*cos(f*x + e) - a*c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sqrt(c) - 2*(a*cos(f*x
 + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*
cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = a\left(\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx + \int \frac{\sec^2(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
[Out] a*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqr
t(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**2/(-c*sqrt(-c*sec(e +
f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}\right)a}{2c^2f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)) +
sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/tan(1/2*f*x + 1/2*e)^2)*a/(c^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{a+\frac{a}{\cos(e+f\,x)}}{\cos\left(e+f\,x\right)\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.70 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 113

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} - \frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}$$

```
[Out] 1/16*a*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(5/2
)/f*2^(1/2)-1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+1/8*a*tan(f*x+e)/c/f/
(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4042, 3881, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} - \frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*Sq
rt[2]*c^(5/2)*f) - (a*Tan[e + f*x])/(2*f*(c - c*Sec[e + f*x])^(5/2)) + (a*T
an[e + f*x])/(8*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{a\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c} \\
&= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{16c^2} \\
&= -\frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}} + \frac{a\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{8c^2f} \\
&= \frac{a\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}c^{5/2}f} - \frac{a\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a\tan(e+fx)}{8cf(c-c\sec(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))\tan(e+fx)}{12c^2f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/12*(a*Hypergeometric2F1[3/2, 3, 5/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e +
f*x])*Tan[e + f*x])/(c^2*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(94) = 188.

Time = 4.19 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.38

method	result
default	$-\frac{a\sqrt{2}\left(\arctan\left(\frac{1}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)(1-\cos(fx+e))^4\csc(fx+e)-\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(1-\cos(fx+e))^2}{16c^2f\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(}$
parts	$\frac{a\sqrt{2}\,(1-\cos(fx+e))\left(\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}(1-\cos(fx+e))^2\csc(fx+e)^2-\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(1-\cos}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/16*a/c^2/f*2^(1/2)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
*csc(f*x+e)^2)^(1/2)/(1-cos(f*x+e))^3/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*(arctan(1/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(1-cos(f*x+e))^4*csc(
f*x+e)-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(1-cos(f*x+e))^2*sin(f*x+e)+
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(1-cos(f*x+e))^4*csc(f*x+e)-2*((1-c
os(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*sin(f*x+e)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.58

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{\sqrt{2}\left(a\cos(fx+e)^2-2\,a\cos(fx+e)+a\right)\sqrt{-c}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+\right.}{}\right.}{}\right.$$

$$-\frac{\sqrt{2}\left(a\cos(fx+e)^2-2\,a\cos(fx+e)+a\right)\sqrt{c}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)-2\left(3\,a\cos(fx\right.}{16\left(c^3f\cos(fx+e)^2-2\,c^3f\cos(fx+e)+c^3f\right)\sin(fx}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/32*(sqrt(2)*(a*cos(f*x + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(-c)*log(-(2*
sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e)) - (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*si
n(f*x + e)))*sin(f*x + e) - 4*(3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*
cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)
^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), -1/16*(sqrt(2)*(a*cos(f*x
 + e)^2 - 2*a*cos(f*x + e) + a)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(
3*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3
*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = a\left(\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+}}\right.$$

$$+\int \frac{\sec^2(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 -
2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) +
 c)), x) + Integral(sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e +
 f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec
(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 1.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{2}\left(a\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}ac-\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,ac^2}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4}\right)}{16\,c^3f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/16*sqrt(2)*(a*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))
 + ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a*c - sqrt(c*tan(1/2*f*x + 1/2*e)^
2 - c)*a*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))/(c^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{a+\frac{a}{\cos(e+f\,x)}}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.71 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 171

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = -\frac{256c^4(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{231f} - \frac{8c^2(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{33f} - \frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{11f}$$

```
[Out] -8/33*c^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/11*c*(a+
a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/1155*c^4*(a+a*sec(f
*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/231*c^3*(a+a*sec(f*x+e))^2*
(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = -\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^2}{1155f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{231f} - \frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{33f} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}{11f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(1155*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(231*f) - (8*c^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(33*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)*Ta
n[e + f*x])/(11*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{11f} \\
&\quad+\frac{1}{11}(12c)\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx \\
&= -\frac{8c^2(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{33f} \\
&\quad-\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{11f} \\
&\quad+\frac{1}{33}\left(32c^2\right)\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx \\
&= -\frac{64c^3(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{231f} \\
&\quad-\frac{8c^2(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{33f} \\
&\quad-\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{11f} \\
&\quad+\frac{1}{231}\left(128c^3\right)\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{256c^4(a+a\sec(e+fx))^2\tan(e+fx)}{1155f\sqrt{c-c\sec(e+fx)}} \\
&\quad-\frac{64c^3(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{231f} \\
&\quad-\frac{8c^2(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{33f} \\
&\quad-\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{11f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

$$
\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = \frac{2a^2c^3\cos^4\left(\frac{1}{2}(e+fx)\right)(-1930+3419\cos(e+fx)-1510\cos(2(e+fx))+533\cos(3(e+fx))}{1155f}
$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(7/2),x]
[Out] (2*a^2*c^3*Cos[(e + f*x)/2]^4*(-1930 + 3419*Cos[e + f*x] - 1510*Cos[2*(e +
f*x)] + 533*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Se
c[e + f*x]])/(1155*f)
```

Maple [A] (verified)

Time = 16.90 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.46

method	result
default	$\frac{2a^{2}c^{3}\left(533\cos\left(fx+e\right)^{3}-755\cos\left(fx+e\right)^{2}+455\cos\left(fx+e\right)-105\right)\sqrt{-c\left(\sec\left(fx+e\right)-1\right)}\,\left(\cos\left(fx+e\right)+1\right)^{3}\sec\left(fx+e\right)^{5}\csc\left(fx+e\right)}{1155f}$
parts	$-\frac{2a^{2}\left(\sec\left(fx+e\right)-1\right)^{3}\left(177\cos\left(fx+e\right)^{3}-71\cos\left(fx+e\right)^{2}+27\cos\left(fx+e\right)-5\right)c^{3}\sqrt{-c\left(\sec\left(fx+e\right)-1\right)}\,\left(\cos\left(fx+e\right)+1\right)\csc\left(fx+e\right)}{35f\left(\cos\left(fx+e\right)-1\right)^{3}}-\frac{2a}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/1155*a^2*c^3/f*(533*cos(f*x+e)^3-755*cos(f*x+e)^2+455*cos(f*x+e)-105)*(-c
*(sec(f*x+e)-1))^(1/2)*(cos(f*x+e)+1)^3*sec(f*x+e)^5*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.86

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = \frac{2\left(533\,a^{2}c^{3}\cos\left(fx+e\right)^{6}+844\,a^{2}c^{3}\cos\left(fx+e\right)^{5}-211\,a^{2}c^{3}\cos\left(fx+e\right)^{4}-472\,a^{2}c^{3}\cos\left(fx\right.\right.}{1155\,f\cos\left(fx+e\right)^{5}\sin}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] 2/1155*(533*a^2*c^3*cos(f*x + e)^6 + 844*a^2*c^3*cos(f*x + e)^5 - 211*a^2*c
^3*cos(f*x + e)^4 - 472*a^2*c^3*cos(f*x + e)^3 + 295*a^2*c^3*cos(f*x + e)^2
 + 140*a^2*c^3*cos(f*x + e) - 105*a^2*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*
x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.64

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx =$$
$$-\frac{64\sqrt{2}\left(231\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^3c^3+495\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^4+385\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right.\right.}{1155\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{11}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] -64/1155*sqrt(2)*(231*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^3 + 495*(c*tan(1/2
*f*x + 1/2*e)^2 - c)^2*c^4 + 385*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 105*c
^6)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11/2)*f)
```

Mupad [B] (verification not implemented)

Time = 25.26 (sec) , antiderivative size = 606, normalized size of antiderivative = 3.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{7/2}\,dx = \frac{\left(\frac{a^2\,c^3\,2\mathrm{i}}{f}+\frac{a^2\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1066\mathrm{i}}{1155\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1} + \frac{\left(\frac{a^2\,c^3\,64\mathrm{i}}{11\,f}-\frac{a^2\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,64\mathrm{i}}{11\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^5} - \frac{\left(\frac{a^2\,c^3\,32\mathrm{i}}{3\,f}-\frac{a^2\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,608\mathrm{i}}{33\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4} - \frac{\left(\frac{a^2\,c^3\,4\mathrm{i}}{f}+\frac{a^2\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2932\mathrm{i}}{1155\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} + \frac{\left(\frac{a^2\,c^3\,16\mathrm{i}}{5\,f}+\frac{a^2\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,4272\mathrm{i}}{385\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} + \frac{\left(\frac{a^2\,c^3\,32\mathrm{i}}{7\,f}-\frac{a^2\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,4640\mathrm{i}}{231\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] (((a^2*c^3*2i)/f + (a^2*c^3*exp(e*1i + f*x*1i)*1066i)/(1155*f))*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1
) + (((a^2*c^3*64i)/(11*f) - (a^2*c^3*exp(e*1i + f*x*1i)*64i)/(11*f))*(c -
c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*
1i) - 1)*(exp(e*2i + f*x*2i) + 1)^5) - (((a^2*c^3*32i)/(3*f) - (a^2*c^3*exp
(e*1i + f*x*1i)*608i)/(33*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^4) - (
((a^2*c^3*4i)/f + (a^2*c^3*exp(e*1i + f*x*1i)*2932i)/(1155*f))*(c - c/(exp(
- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1
)*(exp(e*2i + f*x*2i) + 1)) + (((a^2*c^3*16i)/(5*f) + (a^2*c^3*exp(e*1i + f
*x*1i)*4272i)/(385*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^2*c
^3*32i)/(7*f) - (a^2*c^3*exp(e*1i + f*x*1i)*4640i)/(231*f))*(c - c/(exp(- e
*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(
exp(e*2i + f*x*2i) + 1)^3)
```

3.72 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 128

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3(a+a\sec(e+fx))^2\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{63f} - \frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{9f}$$

```
[Out] -2/9*c*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/315*c^3*(a
+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/63*c^2*(a+a*sec(f*x
+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used

$= \{4040, 4038\}$

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^2}{315f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{63f} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}{9f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(315*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(63*f) - (2*c*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(9*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{9f} + \frac{1}{9}(8c)\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx$$

$$= -\frac{16c^2(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{63f} - \frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{9f} + \frac{1}{63}(32c^2)\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$$

$$= -\frac{64c^3(a+a\sec(e+fx))^2\tan(e+fx)}{315f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{63f} - \frac{2c(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{9f}$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = \frac{4a^2c^2\cos^4\left(\frac{1}{2}(e+fx)\right)(177-220\cos(e+fx)+107\cos(2(e+fx)))\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+}{315f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*a^2*c^2*Cos[(e + f*x)/2]^4*(177 - 220*Cos[e + f*x] + 107*Cos[2*(e + f*x)
])*Cot[(e + f*x)/2]*Sec[e + f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(315*f)
```

Maple [A] (verified)

Time = 15.64 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.54

method	result
default	$\frac{2a^2c^2\left(107\cos(fx+e)^2-110\cos(fx+e)+35\right)\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)^3\sec(fx+e)^4\csc(fx+e)}{315f}$
parts	$\frac{2a^2(\sec(fx+e)-1)^2\left(43\cos(fx+e)^2-14\cos(fx+e)+3\right)c^2\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)\csc(fx+e)}{15f(\cos(fx+e)-1)^2} + \frac{2a^2\left(584\cos(fx+e)^4-29\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/315*a^2*c^2/f*(107*cos(f*x+e)^2-110*cos(f*x+e)+35)*(-c*(sec(f*x+e)-1))^(1
/2)*(cos(f*x+e)+1)^3*sec(f*x+e)^4*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx=\frac{2\left(107\,a^2c^2\cos\left(fx+e\right)^5+211\,a^2c^2\cos\left(fx+e\right)^4+26\,a^2c^2\cos\left(fx+e\right)^3-118\,a^2c^2\cos\left(fx+\right.\right.}{315\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2/315*(107*a^2*c^2*cos(f*x + e)^5 + 211*a^2*c^2*cos(f*x + e)^4 + 26*a^2*c^2
*cos(f*x + e)^3 - 118*a^2*c^2*cos(f*x + e)^2 - 5*a^2*c^2*cos(f*x + e) + 35*
a^2*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x
+ e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx=a^2\left(\int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx+\int\left(-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)\right)dx+\int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^5\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(-
2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3, x) + Integral(c**2*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x)**5, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx=\int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm ="maxima")

[Out] -2/315*(315*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*((a^2*c^2*f*cos(2*f*x + 2*e)^4 + a^2*c^2*f*sin(2*f*x + 2*e)^4 + 4*a^2*c^2*f*cos(2*f*x + 2*e)^3 + 6*a^2*c^2*f*cos(2*f*x + 2*e)^2 + 4*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f + 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*sin(2*f*x + 2*e)^2)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +

```
2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
 + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*
f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*
f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e
)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*
x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + si
n(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x +
 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(
2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2
*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*
f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f
*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e)
+ cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (c
os(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8
*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 +
(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x +
 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 4*(
a^2*c^2*f*cos(2*f*x + 2*e)^4 + a^2*c^2*f*sin(2*f*x + 2*e)^4 + 4*a^2*c^2*f*c
os(2*f*x + 2*e)^3 + 6*a^2*c^2*f*cos(2*f*x + 2*e)^2 + 4*a^2*c^2*f*cos(2*f*x
+ 2*e) + a^2*c^2*f + 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*
```

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x + 2*e) + a^2*c^2*f)*sin(2*f*x + 2*e)^2)*integrate((((cos(8*f*x + 8*e)*cos
(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*co
s(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3
*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
sin(2*f*x + 2*e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
(cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) +
3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3
*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*s
in(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) +
3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) -
 cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) -
3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*co
s(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
 sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) +
3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2
 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos
(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x +
6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)
^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*c
os(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos
(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2
 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos
(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) +
 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin
(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 +
 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4
*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*
```

```
e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*c
os(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x
+ 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)
*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(
2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*s
in(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*c
os(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x
+ 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)
^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin
(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) + 6*(a^2*c^2*f*co
s(2*f*x + 2*e)^4 + a^2*c^2*f*sin(2*f*x + 2*e)^4 + 4*a^2*c^2*f*cos(2*f*x + 2
*e)^3 + 6*a^2*c^2*f*cos(2*f*x + 2*e)^2 + 4*a^2*c^2*f*cos(2*f*x + 2*e) + a^2
*c^2*f + 2*(a^2*c^2*f*cos(2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a
^2*c^2*f)*sin(2*f*x + 2*e)^2)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e
) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*
e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x +
 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x +
 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x
+ 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x +
 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x
+ 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x +
 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x
+ 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*
e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x +
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8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)))/(((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e
)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x
+ 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6
e) + 3(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e
) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e
) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*
x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + c
os(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e
)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)
+ (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x
+ 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8
e)^2 + 9(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9
*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f
*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x +
2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e)
)*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2
e)^2 + 3(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6

e) + 6(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2
*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e
)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x
+ 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 4*(a^2*c^2*f*cos(2*f*x + 2*
e)^4 + a^2*c^2*f*sin(2*f*x + 2*e)^4 + 4*a^2*c^2*f*cos(2*f*x + 2*e)^3 + 6*a^
2*c^2*f*cos(2*f*x + 2*e)^2 + 4*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f + 2*(
a^2*c^2*f*cos(2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*si
n(2*f*x + 2*e)^2)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*
f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f
*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*
f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(
3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*
f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*
f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6
*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*
f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2
*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*
f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4
*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*
f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((c
os(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x +
4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 +
(2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos
(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x +

4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2
*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*
f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 +
 cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2
*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x +
6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x
 + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 +
 sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*
(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f
*x + 2*e))*sin(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x
+ 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2
+ 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(
4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x
+ 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*
x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x
+ 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
+ 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos
(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^
2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x
+ 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x +
4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(
6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/4)), x) + (a^2*c^2*f*cos(2*f*x + 2*e)^4 + a^2*c^2
*f*sin(2*f*x + 2*e)^4 + 4*a^2*c^2*f*cos(2*f*x + 2*e)^3 + 6*a^2*c^2*f*cos(2*
f*x + 2*e)^2 + 4*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f + 2*(a^2*c^2*f*cos(
2*f*x + 2*e)^2 + 2*a^2*c^2*f*cos(2*f*x + 2*e) + a^2*c^2*f)*sin(2*f*x + 2*e)

```
^2)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos
(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3
*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3
*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) -
cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3
*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (
(cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) +
3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3
*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos
(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*co
s(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3
*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) +
sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*s
in(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((cos(2*f*x + 2*e
)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos
(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^
3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2
*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3
+ cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 +
 cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2
*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x
 + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4
*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8
*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2
*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin
(4*f*x + 4*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 +
 (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x
```

```
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
 + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2
 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(
cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x
 + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(co
s(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos
(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^
3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x
+ 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x
 + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*
f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(
2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/4)), x))*sqrt(c) + (3*(105*a^2*c^2*sin(8*f*x + 8*e) + 140*a^2*c
^2*sin(6*f*x + 6*e) + 294*a^2*c^2*sin(4*f*x + 4*e) + 108*a^2*c^2*sin(2*f*x
+ 2*e))*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (315*a^2
*c^2*cos(8*f*x + 8*e) + 420*a^2*c^2*cos(6*f*x + 6*e) + 882*a^2*c^2*cos(4*f*
x + 4*e) + 324*a^2*c^2*cos(2*f*x + 2*e) + 107*a^2*c^2)*sin(9/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(c))/((f*cos(2*f*x + 2*e)^4 + f*s
in(2*f*x + 2*e)^4 + 4*f*cos(2*f*x + 2*e)^3 + 6*f*cos(2*f*x + 2*e)^2 + 2*(f*
cos(2*f*x + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*sin(2*f*x + 2*e)^2 + 4*f*cos
(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
 2*e) + 1)^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.83 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = -\frac{32\sqrt{2}\left(63\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^3+90\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^4+35\,c^5\right)a^2c^2}{315\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
[Out] -32/315*sqrt(2)*(63*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^3 + 90*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^4 + 35*c^5)*a^2*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(9/
2)*f)
```

Mupad [B] (verification not implemented)

Time = 20.22 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.93

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}\,dx = \frac{\left(\frac{a^2\,c^2\,2\mathrm{i}}{f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,214\mathrm{i}}{315\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1} + \frac{\left(\frac{a^2\,c^2\,32\mathrm{i}}{9\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,32\mathrm{i}}{9\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4} - \frac{\left(\frac{a^2\,c^2\,64\mathrm{i}}{7\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,320\mathrm{i}}{63\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3} + \frac{\left(\frac{a^2\,c^2\,48\mathrm{i}}{5\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,368\mathrm{i}}{105\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} - \frac{\left(\frac{a^2\,c^2\,16\mathrm{i}}{3\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,208\mathrm{i}}{315\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
[Out] (((a^2*c^2*2i)/f + (a^2*c^2*exp(e*1i + f*x*1i)*214i)/(315*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1)
```

```
+ (((a^2*c^2*32i)/(9*f) + (a^2*c^2*exp(e*1i + f*x*1i)*32i)/(9*f))*(c - c/(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i)
- 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^2*c^2*64i)/(7*f) + (a^2*c^2*exp(e*1
i + f*x*1i)*320i)/(63*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3) + (((a^
2*c^2*48i)/(5*f) + (a^2*c^2*exp(e*1i + f*x*1i)*368i)/(105*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)
*(exp(e*2i + f*x*2i) + 1)^2) - (((a^2*c^2*16i)/(3*f) + (a^2*c^2*exp(e*1i +
f*x*1i)*208i)/(315*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1))
```

3.73 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 85

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2(a+a\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{7f}$$

```
[Out] -8/35*c^2*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/7*c*(a+a
*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^2}{35f\sqrt{c-c\sec(e+fx)}} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}{7f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(35*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(7*f
)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2c(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{7f} \\
&\quad+\frac{1}{7}(4c)\int\sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{8c^2(a+a\sec(e+fx))^2\tan(e+fx)}{35f\sqrt{c-c\sec(e+fx)}} \\
&\quad-\frac{2c(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{7f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$
\int\sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx=\frac{8a^2c\cos^4\left(\frac{1}{2}(e+fx)\right)(-5+9\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{35f}
$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (8*a^2*c*Cos[(e + f*x)/2]^4*(-5 + 9*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e +
f*x]^3*Sqrt[c - c*Sec[e + f*x]])/(35*f)
```

Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result
default	$\frac{2a^2c(9\cos(fx+e)-5)\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)^3\sec(fx+e)^3\csc(fx+e)}{35f}$
parts	$-\frac{2a^2(\sec(fx+e)-1)(5\cos(fx+e)-1)c\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)\csc(fx+e)}{3f(\cos(fx+e)-1)} - \frac{2a^2\left(104\cos(fx+e)^3-52\cos(fx+e)^2+39\right.}{}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/35*a^2*c/f*(9*cos(f*x+e)-5)*(-c*(sec(f*x+e)-1))^(1/2)*(cos(f*x+e)+1)^3*sec(f*x+e)^3*csc(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\left(9\,a^2c\cos\left(fx+e\right)^4+22\,a^2c\cos\left(fx+e\right)^3+12\,a^2c\cos\left(fx+e\right)^2-6\,a^2c\cos\left(fx+e\right)-5\,a^2\right.}{35\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2/35*(9*a^2*c*cos(f*x + e)^4 + 22*a^2*c*cos(f*x + e)^3 + 12*a^2*c*cos(f*x + e)^2 - 6*a^2*c*cos(f*x + e) - 5*a^2*c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = a^2\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx + \int c\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)dx + \int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)\right)dx + \int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^4\left(e+fx\right)\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(c*sq
rt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-c*sqrt(-c*sec(e + f
*x) + c)*sec(e + f*x)**3, x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e
+ f*x)**4, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c - c\sec(e+fx))^{3/2}\,dx = \int (a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^{\frac{3}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] 2/35*(35*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(3/4)*((a^2*c*f*cos(2*f*x + 2*e)^2 + a^2*c*f*sin(2*f*x + 2*e)^2 + 2*a^2*c*
f*cos(2*f*x + 2*e) + a^2*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e)
+ 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e)
 + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6
*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*
e)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2
*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x +
2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6
*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x +
2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8
*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x +
4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e)
 + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8
*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x +
4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2
 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos
(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2
```

+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos
(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x +
4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2
+ 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)
+ 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos
(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) +
6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x
+ 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) +
2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x +
2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(
2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3
+ 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (
cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x +
6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f
*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)
^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*c
os(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x
+ 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*s
in(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*
e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*c
os(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)
^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*c
os(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e)
+ 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(s
in(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2
*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3
+ 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin
(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2
*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos

```
(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x) - 3*(a^2*c*f*cos(2*f*x + 2*e)^2
 + a^2*c*f*sin(2*f*x + 2*e)^2 + 2*a^2*c*f*cos(2*f*x + 2*e) + a^2*c*f)*integ
rate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*
e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x +
 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x
+ 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x +
 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x
+ 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x
+ 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x
 + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x
+ 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*
e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2
*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x
+ 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*
f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e
)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*si
n(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^
2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^
2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x
+ 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x +
 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + co
s(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
+ 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*co
s(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
```

```
2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e
))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x
+ 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 +
2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 +
sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x +
2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(
2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*
cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*
e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*
f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e
)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*
x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(
4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*
x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*
x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)
)*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
)^2), x) + 3*(a^2*c*f*cos(2*f*x + 2*e)^2 + a^2*c*f*sin(2*f*x + 2*e)^2 + 2*a
^2*c*f*cos(2*f*x + 2*e) + a^2*c*f)*integrate((cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x +
2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x +
 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*
x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x
 + 2*e)^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*
x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f
*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*
x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f
*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*
x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f
*x + 4*e)*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - (cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x +
```

2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*
x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f
*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2
+ sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6
e)^2 + 9(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9
*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f
*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2
e)^2 + 2(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x +
6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8
e) + 6(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(
2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4
*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*
f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 +
cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2
e)^3 + 3(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e
) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f
*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2 + (cos(2*f*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x +
8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x +
2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 +
9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6
*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x
+ 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*
e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x +
2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +

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 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x +
 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 +
 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2
*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2), x) - (a^2*c*f*cos(2*f*x + 2*e
)^2 + a^2*c*f*sin(2*f*x + 2*e)^2 + 2*a^2*c*f*cos(2*f*x + 2*e) + a^2*c*f)*in
tegrate((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^
(1/4)*(((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x +
 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*
x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f
*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f
*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*
x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f
*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f
*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*
f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f
*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*x + 8*e)*cos(2*f*x +
 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x
+ 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f
*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*
x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((cos(2*f*x + 2*e)^4 + sin
(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2 + 2*cos(2*f*x +
2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x + 2*e)^3 + cos(2
*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*
e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f
```

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*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x + 2*e)^2 + cos(2*f*
x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2*f*x + 2*e)*sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(4*f*x + 4*e) +
 cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) + 3*(cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (co
s(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(8*f*x + 8*
e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x
 + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2*f*x + 2*e)^3 + (
cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(4*f*x +
4*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (cos(2*f
*x + 2*e)^4 + sin(2*f*x + 2*e)^4 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
 + 2*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)^2 + 9*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e)^2
 + 2*cos(2*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)*sin(8*f*x + 8*e)^2 + 9*(cos(2*f*x + 2*e)^2 + sin(2*f*x +
2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e)^2 + 9*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e)^2 + (2*cos
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e)^2 + 2*(cos(2*f*x
 + 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + s
in(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 3*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) +
 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 6*(cos(2*f*x +
 2*e)^3 + cos(2*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 3*(cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 2*cos(2*f*x +
2*e)^2 + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 6*(cos(2*f*x + 2*e)^3 + cos(2
*f*x + 2*e)*sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)^2 + cos(2*f*x + 2*e))*c
os(4*f*x + 4*e) + cos(2*f*x + 2*e)^2 + 2*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(6*f*x + 6*e) +
 3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4
*f*x + 4*e) + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e
))*sin(8*f*x + 8*e) + 6*(sin(2*f*x + 2*e)^3 + 3*(cos(2*f*x + 2*e)^2 + sin(2
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(4*f*x + 4*e) + (cos(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 6*(sin(2
*f*x + 2*e)^3 + (cos(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*sin(2*f*x + 2
*e))*sin(4*f*x + 4*e))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1))^2), x))*sqrt(c) - (7*(5*a^2*c*sin(6*f*x + 6*e) + 5*a^2*c*sin(4*f*x + 4
*e) + 7*a^2*c*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)) - (35*a^2*c*cos(6*f*x + 6*e) + 35*a^2*c*cos(4*f*x + 4*e) + 49
*a^2*c*cos(2*f*x + 2*e) + 9*a^2*c)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1)))*sqrt(c))/((f*cos(2*f*x + 2*e)^2 + f*sin(2*f*x + 2*e)^2 +
2*f*cos(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
```

```
2*f*x + 2*e) + 1)^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.87 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = -\frac{16\sqrt{2}\left(7\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^4+5c^5\right)a^2}{35\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{7}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -16/35*sqrt(2)*(7*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4 + 5*c^5)*a^2/((c*tan(1
/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

Mupad [B] (verification not implemented)

Time = 17.15 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.52

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}\,dx = \frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^2\,c\,2i}{f}+\frac{a^2\,c\,e^{e\,1i+f\,x\,1i}\,18i}{35\,f}\right)}{e^{e\,1i+f\,x\,1i}-1} - \frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^2\,c\,16i}{7\,f}-\frac{a^2\,c\,e^{e\,1i+f\,x\,1i}\,16i}{7\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^3} - \frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^2\,c\,4i}{f}-\frac{a^2\,c\,e^{e\,1i+f\,x\,1i}\,44i}{35\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)} + \frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^2\,c\,24i}{5\,f}-\frac{a^2\,c\,e^{e\,1i+f\,x\,1i}\,72i}{35\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^2}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*2i)/
f + (a^2*c*exp(e*1i + f*x*1i)*18i)/(35*f)))/(exp(e*1i + f*x*1i) - 1) - ((c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*16i)/(7*
f) - (a^2*c*exp(e*1i + f*x*1i)*16i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(
```

```
e*2i + f*x*2i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*((a^2*c*4i)/f - (a^2*c*exp(e*1i + f*x*1i)*44i)/(35*f)))/((exp(
e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i - f*x*1
i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^2*c*24i)/(5*f) - (a^2*c*exp(e*1i +
f*x*1i)*72i)/(35*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

3.74 $\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/5*c*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{2c(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = \frac{8a^2\cos^4\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^2(e+fx)\sqrt{c-c\sec(e+fx)}}{5f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]],x]
[Out] (8*a^2*Cos[(e + f*x)/2]^4*Cot[(e + f*x)/2]*Sec[e + f*x]^2*Sqrt[c - c*Sec[e
+ f*x]])/(5*f)
```

Maple [A] (verified)

Time = 4.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

method	result
default	$\frac{2a^2(\cos(fx+e)+1)^3\sqrt{-c(\sec(fx+e)-1)}\,\sec(fx+e)^2\csc(fx+e)}{5f}$
parts	$-\frac{2a^2\sqrt{-c(\sec(fx+e)-1)}\,\sin(fx+e)}{f(\cos(fx+e)-1)}+\frac{2a^2\sqrt{-c(\sec(fx+e)-1)}\,\left(3+8\cos(fx+e)^3+4\cos(fx+e)^2-\cos(fx+e)\right)\sec(fx+e)^2\csc(fx+}{15f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
[Out] 2/5*a^2/f*(cos(f*x+e)+1)^3*(-c*(sec(f*x+e)-1))^(1/2)*sec(f*x+e)^2*csc(f*x+e
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(37) = 74.

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\left(a^2\cos\left(fx+e\right)^3+3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)+a^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{5\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/5*(a^2*cos(f*x + e)^3 + 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$$
$$=a^2\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx+\int2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)dx\right.$$
$$\left.+\int\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int\left(a\sec\left(fx+e\right)+a\right)^2\sqrt{-c\sec\left(fx+e\right)+c}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] 2/5*(5*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/4)*(3*(a^2*f*cos(2*f*x + 2*e)^2 + a^2*f*sin(2*f*x + 2*e)^2 + 2*a^2*f*cos(
2*f*x + 2*e) + a^2*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*co
s(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos
(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*si
n(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*
cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*si
n(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*s
in(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*si
n(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*s
in(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*si
n(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*s
in(2*f*x + 2*e))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (co
s(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*c
os(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*si
n(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/
(((2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x
 + 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*co
s(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x
+ 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*si
n(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 +
6*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x +
6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2
*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 +
 (2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x
+ 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos
(6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x +
 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin
(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6
*(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6
*e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*
f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(
cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x
) - 2*(a^2*f*cos(2*f*x + 2*e)^2 + a^2*f*sin(2*f*x + 2*e)^2 + 2*a^2*f*cos(2*
f*x + 2*e) + a^2*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(
6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2
*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(
2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*co
s(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(
8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin
(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(
```

```
2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin
(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(
2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(
8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos
(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(
2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/((
(2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x +
 8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(
6*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x +
4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(
4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*
(3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*
e)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f
*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (
2*(3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x +
8*e) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6
*f*x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4
*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4
*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(
3*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e
)^2 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*
x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x)
- (a^2*f*cos(2*f*x + 2*e)^2 + a^2*f*sin(2*f*x + 2*e)^2 + 2*a^2*f*cos(2*f*x
+ 2*e) + a^2*f)*integrate((((cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*
x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
 + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*
x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(8*f*
x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f
*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*
x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) - ((cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 3*cos(2*f*x + 2*e)*sin(6*f
*x + 6*e) + 3*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(8*f*x + 8*e)*sin(2*f*
x + 2*e) - 3*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 3*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(8*f*
x + 8*e)*cos(2*f*x + 2*e) + 3*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 3*cos(4*f
*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(8*f*x + 8*e)*sin(2*f*
x + 2*e) + 3*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 3*sin(4*f*x + 4*e)*sin(2*f
```

```
*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(
3*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e
) + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*
x + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)
*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*
x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*s
in(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2
 + 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(3
*cos(6*f*x + 6*e) + 3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e)
 + cos(8*f*x + 8*e)^2 + 6*(3*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x
 + 6*e) + 9*cos(6*f*x + 6*e)^2 + 9*cos(4*f*x + 4*e)^2 + 6*cos(4*f*x + 4*e)*
cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(3*sin(6*f*x + 6*e) + 3*sin(4*f*x
 + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 6*(3*si
n(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 9*sin(6*f*x + 6*e)^2
+ 9*sin(4*f*x + 4*e)^2 + 6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x +
2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*
f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x))*sqr
t(c) - (5*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (5*a^2*cos(4*f*x + 4*e) + 10*a^2*
cos(2*f*x + 2*e) + a^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)))*sqrt(c))/((f*cos(2*f*x + 2*e)^2 + f*sin(2*f*x + 2*e)^2 + 2*f*cos(2*f
*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/4))
```

Giac [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx = -\frac{8\sqrt{2}a^2c^3}{5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] -8/5*sqrt(2)*a^2*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*f)
```

Mupad [B] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.27

$$\int \sec(e+fx)(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2\,a^2\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}+1{}\mathrm{i}\right)^5\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}}{5\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)^2}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a^2*(exp(e*1i + f*x*1i)*1i + 1i)^5*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(
e*1i + f*x*1i)/2))^(1/2))/(5*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i)
 + 1)^2)
```

3.75 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 117

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{4\sqrt{2}a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{16a^2\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{2a^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3cf}$$

```
[Out] -4*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/f/c^(1/2)+16/3*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/3*a^2*(c-c*sec(f
*x+e))^(1/2)*tan(f*x+e)/c/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4041, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{4\sqrt{2}a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-4*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (4*a^2*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx \\
&= \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (4a^2)\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx \\
&= \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{(8a^2)\operatorname{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{f} \\
&= -\frac{4\sqrt{2}a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{4a^2\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{2(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a^2\left(-6\sqrt{2}\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)+\sqrt{a(1+\sec(e+fx))}(7+\sec(e+fx))\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (2*a^2*(-6*Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt
[a])] + Sqrt[a*(1 + Sec[e + f*x])]*(7 + Sec[e + f*x]))*Tan[e + f*x])/(3*f*S
qrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 4.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{2}\sqrt{2}\left(12\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\tan(fx+e)-7\sqrt{2}\,\tan(fx+e)-\sqrt{2}\,\tan(fx+e)\sec(fx+e)\right)}{3f\sqrt{-c(\sec(fx+e)-1)}}$
parts	$\frac{a^{2}\sqrt{2}\,\sin(fx+e)\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f(\cos(fx+e)+1)\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}+\frac{a^{2}\sqrt{2}\,(-3\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\tan(fx+e)+\sqrt{2}\,\mathrm{t}}{3f\sqrt{-c(\sec(fx+e)-1)}}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/3*a^2/f*2^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)*(12*arctan(1/2*2^(1/2)/(-cos(f
*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*tan(f*x+e)-
7*2^(1/2)*tan(f*x+e)-2^(1/2)*tan(f*x+e)*sec(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.93

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= \left[\frac{2\left(3\sqrt{2}a^2c\sqrt{-\frac{1}{c}}\cos\left(fx+e\right)\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin\right.}{3\,cf\cos\left(fx+e\right)\sin\left(fx+e\right)}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [2/3*(3*sqrt(2)*a^2*c*sqrt(-1/c)*cos(f*x + e)*log(-(2*sqrt(2)*(cos(f*x + e)
^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*
cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x
+ e) - (7*a^2*cos(f*x + e)^2 + 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)*sin(f*x + e)), 2/3*(6*sqrt(2)*a^2*
sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)*sin(f*x + e) - (7*a^2*cos(f*x + e)^2
+ 8*a^2*cos(f*x + e) + a^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*c
os(f*x + e)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = a^2\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx + \int \frac{2\sec^2\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx + \int \frac{\sec^3\left(e+fx\right)}{\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(2*sec(
e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**3/sqrt(-
c*sec(e + f*x) + c), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^2\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 1.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{4\,a^2\left(\frac{3\,\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\left(3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-4\,c\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}}\right)}{3\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 4/3*a^2*(3*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(
c) + sqrt(2)*(3*c*tan(1/2*f*x + 1/2*e)^2 - 4*c)/(c*tan(1/2*f*x + 1/2*e)^2 -
 c)^(3/2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos(e+f\,x)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.76 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 113

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{2a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{2a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 3*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2)
/c^(3/2)/f-2*a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-2*a^2*tan(f*x+e)/c/f/(
c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4042, 4041, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{3\sqrt{2}a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{f(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (3*Sqrt[2]*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*
x]])])/(c^(3/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(f*(c - c*Sec[
e + f*x])^(3/2)) - (3*a^2*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(3a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{(3a^2)\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}$$

$$= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} + \frac{(6a^2)\,\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{cf}$$

$$= \frac{3\sqrt{2}a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{3a^2\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^2\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^2\tan(e+fx)}{10cf\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] -1/10*(a^2*Hypergeometric2F1[2, 5/2, 7/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
+ f*x])^2*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

method	result
default	$\frac{a^2\sqrt{2}\left(3\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\tan(fx+e)+2\sqrt{2}\cot(fx+e)+\sqrt{2}\csc(fx+e)-\sqrt{2}\sec(fx+e)\csc(fx+e)\right)}{cf\sqrt{-c(\sec(fx+e)-1)}}$
parts	$-\frac{a^2\sqrt{2}(1-\cos(fx+e))\left(\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}-\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,(1-\cos(fx+e))^2\csc(fx+e)^2-\text{arc}\right.}{4f\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{3}{2}}\left((1-\cos(fx+e))^2\csc(fx\right.}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] a^2/c/f*2^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)*(3*arctan(1/2*2^(1/2)/(-cos(f*x+e
)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*tan(f*x+e)+2*2^
(1/2)*cot(f*x+e)+2^(1/2)*csc(f*x+e)-2^(1/2)*sec(f*x+e)*csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.29

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = \left[\frac{3\sqrt{2}(a^2c\cos(fx+e)-a^2c)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\ldots}{(\cos(fx+e)\ldots}\right.\ldots}{\ldots}\right.$$

$$-\frac{\frac{3\sqrt{2}(a^2c\cos(fx+e)-a^2c)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}}-2\left(2a^2\cos(fx+e)^2+a^2\cos(fx+e)-a^2\right)\sqrt{\frac{c\ldots}{\ldots}}}{(c^2f\cos(fx+e)-c^2f)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/2*(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*sqrt(-1/c)*log((2*sqrt(2)*(cos
(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1
/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))
*sin(f*x + e) + 4*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
, -(3*sqrt(2)*(a^2*c*cos(f*x + e) - a^2*c)*arctan(sqrt(2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)/sq
rt(c) - 2*(2*a^2*cos(f*x + e)^2 + a^2*cos(f*x + e) - a^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = a^2\left(\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+\ldots}}\ldots\right.$$
$$+\int \frac{2\sec^2(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx$$
$$\left.+\int \frac{\sec^3(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*
sqrt(-c*sec(e + f*x) + c)), x) + Integral(2*sec(e + f*x)**2/(-c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integral(s
ec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e
 + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^2\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Giac [A] (verification not implemented)

none

Time = 1.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^2\left(\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{2}\left(3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}+\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c\right)c}\right)}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -a^2*(3*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(3/2)
+ sqrt(2)*(3*c*tan(1/2*f*x + 1/2*e)^2 - c)/(((c*tan(1/2*f*x + 1/2*e)^2 - c)
^(3/2) + sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c)*c))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.77 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 117

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{3a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} - \frac{a^2\tan(e+fx)}{f(c-c\sec(e+fx))^{5/2}} + \frac{5a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -3/8*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(5
/2)/f*2^(1/2)-a^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+5/4*a^2*tan(f*x+e)/c/
f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {4042, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{3a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} - \frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-3*a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/
(4*Sqrt[2]*c^(5/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(2*f*(c - c
```

```
*Sec[e + f*x])^(5/2)) + (3*a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(3
/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} - \frac{(3a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c} \\
&= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \frac{(3a^2)\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{8c^2} \\
&= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} \\
&\quad - \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{4c^2f} \\
&= -\frac{3a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}c^{5/2}f} \\
&\quad - \frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{3a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx =$$
$$-\frac{a^{3/2}\left(\sqrt{a}(-1+4\sec(e+fx)+5\sec^2(e+fx))+6\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right)\sec^2(e+fx)\sqrt{a(1+\sec(e}{4c^2f(-1+\sec(e+fx))^2(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(5/2),
x]
```

```
[Out] -1/4*(a^(3/2)*(Sqrt[a]*(-1 + 4*Sec[e + f*x] + 5*Sec[e + f*x]^2) + 6*Sqrt[2]
*ArcTanh[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqrt[a])]*Sec[e + f*x]^2*Sqrt[
a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^4)*Tan[e + f*x])/(c^2*f*(-1 + Sec[e
+ f*x])^2*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(100) = 200.

Time = 4.58 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.95

method	result
default	$\frac{a^2\sqrt{2}\left(3\arctan\left(\frac{1}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)(1-\cos(fx+e))^4\csc(fx+e)-3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,(1-\cos(fx+e))^2}{8c^2f\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))^3\sqrt{(1-\cos(fx+e))^2\csc(fx}}$
parts	Expression too large to display

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/8*a^2/c^2/f*2^(1/2)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
*csc(f*x+e)^2)^(1/2)/(1-cos(f*x+e))^3/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*(3*arctan(1/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(1-cos(f*x+e))^4*cs
c(f*x+e)-3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(1-cos(f*x+e))^2*sin(f*x
+e)-2*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*sin(f*x+e)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.67

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{3\sqrt{2}\left(a^2\cos(fx+e)^2-2a^2\cos(fx+e)+a^2\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}(\cos\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(-c)*
log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 4*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x +
e)^2 - 5*a^2*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f
*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1/8*(3*sqrt(
2)*(a^2*cos(f*x + e)^2 - 2*a^2*cos(f*x + e) + a^2)*sqrt(c)*arctan(sqrt(2)*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))
*sin(f*x + e) - 2*(a^2*cos(f*x + e)^3 - 4*a^2*cos(f*x + e)^2 - 5*a^2*cos(f*
x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^3*f*cos(f*x + e)^2 - 2
*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = a^2\left(\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)}}\right.$$
$$+\int \frac{2\sec^2(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}}$$
$$+\int \frac{\sec^3(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+c}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2
 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x
) + c)), x) + Integral(2*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-
c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**3/(c**2*sqrt(-c*sec(e + f
*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) +
c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^2\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm ="maxima")

[Out] integrate((a*sec(f*x + e) + a)^2*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2), x)

Giac [A] (verification not implemented)

none

Time = 1.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(3\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c+5\sqrt{c}}{c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}\right.}{8c^3f}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm ="giac")

[Out] 1/8*sqrt(2)*(3*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)) + (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 5*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))*a^2/(c^3*f)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)

[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)

3.78 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 164

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}-\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}-\frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -1/32*a^2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(
7/2)/f*2^(1/2)-1/3*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)
+1/4*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)-1/16*a^2*tan(f*x+e)/c^2/f/(c
-c*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4042, 3881, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f}-\frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}-\frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{3f(c-c\sec(e+fx))^{7/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/16*(a^2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])
])/(Sqrt[2]*c^(7/2)*f) - ((a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(3*f*(c -
c*Sec[e + f*x])^(7/2)) + (a^2*Tan[e + f*x])/(4*c*f*(c - c*Sec[e + f*x])^(5/
2)) - (a^2*Tan[e + f*x])/(16*c^2*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}}-\frac{a\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{(c-c\sec(e+fx))^{5/2}}\,dx}{2c}\\ &= -\frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}}+\frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}}+\frac{a^2\int\frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{8c^2}\end{aligned}$$

$$= -\frac{(a^2 + a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}} + \frac{a^2\int\frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{32c^3}$$

$$= -\frac{(a^2 + a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}} - \frac{a^2\text{Subst}\left(\int\frac{1}{2c+x^2}dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{16c^3f}$$

$$= -\frac{a^2\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}c^{7/2}f} - \frac{(a^2 + a^2\sec(e+fx))\tan(e+fx)}{3f(c-c\sec(e+fx))^{7/2}} + \frac{a^2\tan(e+fx)}{4cf(c-c\sec(e+fx))^{5/2}} - \frac{a^2\tan(e+fx)}{16c^2f(c-c\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}dx = -\frac{a^2\,\text{Hypergeometric2F1}\left(\frac{5}{2},4,\frac{7}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^2\tan(e+fx)}{40c^3f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c - c*Sec[e + f*x])^(7/2),
x]
```

```
[Out] -1/40*(a^2*Hypergeometric2F1[5/2, 4, 7/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
+ f*x])^2*Tan[e + f*x])/(c^3*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(141) = 282.

Time = 5.49 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.95

method	result
default	$-\frac{a^2\sqrt{2}\left(3\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}(1-\cos(fx+e))^4\sin(fx+e)-3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,(1-\cos(fx+e))^6\csc(\ldots\right.}{96c^3f\sqrt{\frac{c}{(1\ldots}}}$
parts	Expression too large to display

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/96*a^2/c^3/f*2^(1/2)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-
1)*csc(f*x+e)^2)^(1/2)/(1-cos(f*x+e))^5/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(
1/2)*(3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(1-cos(f*x+e))^4*sin(f*x+e)
-3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(1-cos(f*x+e))^6*csc(f*x+e)-3*ar
ctan(1/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(1-cos(f*x+e))^6*csc(f*x+e)
+6*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*(1-cos(f*x+e))^2*sin(f*x+e)^3+8*
((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)*sin(f*x+e)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.15

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \left[-\frac{3\sqrt{2}\left(a^2\cos\left(fx+e\right)^3-3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)-\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f
*x + e) - a^2)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt
(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f
*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*(7*a^2*cos(f*x
 + e)^4 + 29*a^2*cos(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*
f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e)), 1/96*(3*sqr
t(2)*(a^2*cos(f*x + e)^3 - 3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) - a^2)
*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e
)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*(7*a^2*cos(f*x + e)^4 + 29*a^2*c
os(f*x + e)^3 + 25*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e)))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 +
 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = a^2\left(\int \frac{}{-c^3\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3c^3\sqrt{-c\sec(e+f}}\right.$$
$$+\int \frac{2\sec^2(e+fx)}{-c^3\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3c^3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-3c^3\sqrt{-c\sec(e+fx)}}$$
$$+\int \frac{\sec^3(e+fx)}{-c^3\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3c^3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-3c^3\sqrt{-c\sec(e+fx)}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**
3 + 3*c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e
 + f*x) + c)*sec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x) + Integral(
2*sec(e + f*x)**2/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*c**3
*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e + f*x) +
c)*sec(e + f*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*
x)**3/(-c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*c**3*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x)**2 - 3*c**3*sqrt(-c*sec(e + f*x) + c)*sec(e + f
*x) + c**3*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.85

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{2}\left(3a^2\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}a^2c+8}{}\right.}{96c^4f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] 1/96*sqrt(2)*(3*a^2*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(
c)) + (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*a^2*c + 8*(c*tan(1/2*f*x + 1/
2*e)^2 - c)^(3/2)*a^2*c^2 - 3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^2*c^3)/(
c^3*tan(1/2*f*x + 1/2*e)^6))/(c^4*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)), x)
```

3.79 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 171

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = -\frac{256c^4(a+a\sec(e+fx))^3\tan(e+fx)}{3003f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{429f} - \frac{24c^2(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{143f} - \frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{13f}$$

```
[Out] -24/143*c^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/13*c*(
a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f-256/3003*c^4*(a+a*sec
(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-64/429*c^3*(a+a*sec(f*x+e))^
3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = -\frac{256c^4\tan(e+fx)(a\sec(e+fx)+a)^3}{3003f\sqrt{c-c\sec(e+fx)}} - \frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{429f} - \frac{24c^2\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{143f} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}{13f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-256*c^4*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(3003*f*Sqrt[c - c*Sec[e + f
*x]]) - (64*c^3*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x
])/(429*f) - (24*c^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[
e + f*x])/(143*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)*
Tan[e + f*x])/(13*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{13f} \\
&\quad+\frac{1}{13}(12c)\int\sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx \\
&= -\frac{24c^2(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{143f} \\
&\quad-\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{13f} \\
&\quad+\frac{1}{143}\left(96c^2\right)\int\sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx \\
&= -\frac{64c^3(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{429f} \\
&\quad-\frac{24c^2(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{143f} \\
&\quad-\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{13f} \\
&\quad+\frac{1}{429}\left(128c^3\right)\int\sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{256c^4(a+a\sec(e+fx))^3\tan(e+fx)}{3003f\sqrt{c-c\sec(e+fx)}} \\
&\quad-\frac{64c^3(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{429f} \\
&\quad-\frac{24c^2(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{143f} \\
&\quad-\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{13f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.77 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

$$
\int\sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \frac{4a^3c^3\cos^6\left(\frac{1}{2}(e+fx)\right)(-3766+6285\cos(e+fx)-2842\cos(2(e+fx))+835\cos(3(e+fx))}{3003f}
$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(7/2),x]
[Out] (4*a^3*c^3*Cos[(e + f*x)/2]^6*(-3766 + 6285*Cos[e + f*x] - 2842*Cos[2*(e +
f*x)] + 835*Cos[3*(e + f*x)])*Cot[(e + f*x)/2]*Sec[e + f*x]^6*Sqrt[c - c*Se
c[e + f*x]])/(3003*f)
```

Maple [A] (verified)

Time = 41.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.46

method	result
default	$\frac{2c^{3}a^{3}\left(835\cos(fx+e)^{3}-1421\cos(fx+e)^{2}+945\cos(fx+e)-231\right)\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)^{4}\sec(fx+e)^{6}\csc(fx+e)}{3003f}$
parts	$-\frac{2a^{3}(\sec(fx+e)-1)^{3}\left(177\cos(fx+e)^{3}-71\cos(fx+e)^{2}+27\cos(fx+e)-5\right)c^{3}\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)\csc(fx+e)}{35f(\cos(fx+e)-1)^{3}}+\frac{2a}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVE
RBOSE)
[Out] 2/3003*c^3*a^3/f*(835*cos(f*x+e)^3-1421*cos(f*x+e)^2+945*cos(f*x+e)-231)*(-
c*(sec(f*x+e)-1))^(1/2)*(cos(f*x+e)+1)^4*sec(f*x+e)^6*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3}(c-c\sec(e+fx))^{7/2}\,dx=\frac{2\left(835\,a^{3}c^{3}\cos\left(fx+e\right)^{7}+1919\,a^{3}c^{3}\cos\left(fx+e\right)^{6}+271\,a^{3}c^{3}\cos\left(fx+e\right)^{5}-1637\,a^{3}c^{3}\cos\left(f\right.\right.}{3003\,f\cos}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="fricas")
[Out] 2/3003*(835*a^3*c^3*cos(f*x + e)^7 + 1919*a^3*c^3*cos(f*x + e)^6 + 271*a^3*
c^3*cos(f*x + e)^5 - 1637*a^3*c^3*cos(f*x + e)^4 - 103*a^3*c^3*cos(f*x + e)
^3 + 973*a^3*c^3*cos(f*x + e)^2 + 21*a^3*c^3*cos(f*x + e) - 231*a^3*c^3)*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^6*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3}(c-c\sec(e+fx))^{7/2}\,dx=\text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(7/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.64

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \frac{128\sqrt{2}\left(429\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^3c^4+1001\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^5+819\left(c\tan\left(\frac{1}{2}\right.\right.\right.}{3003\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{13}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(7/2),x, algorithm
="giac")
```

```
[Out] 128/3003*sqrt(2)*(429*(c*tan(1/2*f*x + 1/2*e)^2 - c)^3*c^4 + 1001*(c*tan(1/
2*f*x + 1/2*e)^2 - c)^2*c^5 + 819*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^6 + 231*
c^7)*a^3*c^3/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(13/2)*f)
```

Mupad [B] (verification not implemented)

Time = 27.14 (sec) , antiderivative size = 710, normalized size of antiderivative = 4.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{7/2}\,dx = \frac{\left(\frac{a^3\,c^3\,2\mathrm{i}}{f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1670\mathrm{i}}{3003\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1} + \frac{\left(\frac{a^3\,c^3\,128\mathrm{i}}{13\,f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,128\mathrm{i}}{13\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^6} - \frac{\left(\frac{a^3\,c^3\,384\mathrm{i}}{11\,f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,3456\mathrm{i}}{143\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^5} - \frac{\left(\frac{a^3\,c^3\,8\mathrm{i}}{f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,2168\mathrm{i}}{3003\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} + \frac{\left(\frac{a^3\,c^3\,24\mathrm{i}}{f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,5464\mathrm{i}}{1001\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} + \frac{\left(\frac{a^3\,c^3\,160\mathrm{i}}{3\,f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,11360\mathrm{i}}{429\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4} - \frac{\left(\frac{a^3\,c^3\,320\mathrm{i}}{7\,f}+\frac{a^3\,c^3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,46400\mathrm{i}}{3003\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] (((a^3*c^3*2i)/f + (a^3*c^3*exp(e*1i + f*x*1i)*1670i)/(3003*f))*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1
) + (((a^3*c^3*128i)/(13*f) + (a^3*c^3*exp(e*1i + f*x*1i)*128i)/(13*f))*(c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*
x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^6) - (((a^3*c^3*384i)/(11*f) + (a^3*c^3
*exp(e*1i + f*x*1i)*3456i)/(143*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^
5) - (((a^3*c^3*8i)/f + (a^3*c^3*exp(e*1i + f*x*1i)*2168i)/(3003*f))*(c - c
/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1
i) - 1)*(exp(e*2i + f*x*2i) + 1)) + (((a^3*c^3*24i)/f + (a^3*c^3*exp(e*1i +
 f*x*1i)*5464i)/(1001*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^
3*c^3*160i)/(3*f) + (a^3*c^3*exp(e*1i + f*x*1i)*11360i)/(429*f))*(c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) -
 1)*(exp(e*2i + f*x*2i) + 1)^4) - (((a^3*c^3*320i)/(7*f) + (a^3*c^3*exp(e*1
```

```
i + f*x*1i)*46400i)/(3003*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

3.80 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 128

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3(a+a\sec(e+fx))^3\tan(e+fx)}{693f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{99f} - \frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{11f}$$

```
[Out] -2/11*c*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-64/693*c^3*(
a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-16/99*c^2*(a+a*sec(f*
x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used

$= \{4040, 4038\}$

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^3}{693f\sqrt{c-c\sec(e+fx)}} - \frac{16c^2\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{99f} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}{11f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(693*f*Sqrt[c - c*Sec[e + f*x
]]) - (16*c^2*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(99*f) - (2*c*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*
x])/(11*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{2c(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{11f} + \frac{1}{11}(8c)\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx$$

$$= -\frac{16c^2(a + a\sec(e + fx))^3\sqrt{c - c\sec(e + fx)}\tan(e + fx)}{99f} - \frac{2c(a + a\sec(e + fx))^3(c - c\sec(e + fx))^{3/2}\tan(e + fx)}{11f} + \frac{1}{99}(32c^2)\int\sec(e + fx)(a + a\sec(e + fx))^3\sqrt{c - c\sec(e + fx)}\,dx$$

$$= -\frac{64c^3(a + a\sec(e + fx))^3\tan(e + fx)}{693f\sqrt{c - c\sec(e + fx)}} - \frac{16c^2(a + a\sec(e + fx))^3\sqrt{c - c\sec(e + fx)}\tan(e + fx)}{99f} - \frac{2c(a + a\sec(e + fx))^3(c - c\sec(e + fx))^{3/2}\tan(e + fx)}{11f}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int\sec(e + fx)(a + a\sec(e + fx))^3(c - c\sec(e + fx))^{5/2}\,dx = \frac{8a^3c^2\cos^6\left(\frac{1}{2}(e + fx)\right)(277 - 364\cos(e + fx) + 151\cos(2(e + fx)))\cot\left(\frac{1}{2}(e + fx)\right)\sec^5(e +}{693f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (8*a^3*c^2*Cos[(e + f*x)/2]^6*(277 - 364*Cos[e + f*x] + 151*Cos[2*(e + f*x)
])*Cot[(e + f*x)/2]*Sec[e + f*x]^5*Sqrt[c - c*Sec[e + f*x]])/(693*f)
```

Maple [A] (verified)

Time = 38.80 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.54

method	result
default	$\frac{2c^2a^3\left(151\cos(fx+e)^2-182\cos(fx+e)+63\right)\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)^4\sec(fx+e)^5\csc(fx+e)}{693f}$
parts	$\frac{2a^3(\sec(fx+e)-1)^2\left(43\cos(fx+e)^2-14\cos(fx+e)+3\right)c^2\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)\csc(fx+e)}{15f(\cos(fx+e)-1)^2} - \frac{2a^3\left(1136\cos(fx+e)^5\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/693*c^2*a^3/f*(151*cos(f*x+e)^2-182*cos(f*x+e)+63)*(-c*(sec(f*x+e)-1))^(1
/2)*(cos(f*x+e)+1)^4*sec(f*x+e)^5*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \frac{2\left(151\,a^3c^2\cos\left(fx+e\right)^6+422\,a^3c^2\cos\left(fx+e\right)^5+241\,a^3c^2\cos\left(fx+e\right)^4-236\,a^3c^2\cos\left(fx+\right.\right.}{693\,f\cos\left(fx+e\right)^5\sin\left(f\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2/693*(151*a^3*c^2*cos(f*x + e)^6 + 422*a^3*c^2*cos(f*x + e)^5 + 241*a^3*c^
2*cos(f*x + e)^4 - 236*a^3*c^2*cos(f*x + e)^3 - 199*a^3*c^2*cos(f*x + e)^2
+ 70*a^3*c^2*cos(f*x + e) + 63*a^3*c^2)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = a^3\left(\int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx + \int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)dx + \int\left(-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)\right)dx + \int\left(-2c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^4\left(e+fx\right)\right)dx + \int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^5\left(e+fx\right)dx + \int c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^6\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**3*(Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(c
**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-2*c**2*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x)**3, x) + Integral(-2*c**2*sqrt(-c*sec(e +
f*x) + c)*sec(e + f*x)**4, x) + Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec
(e + f*x)**5, x) + Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**6,
 x))
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \frac{64\sqrt{2}\left(99\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^4+154\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+63\,c^6\right)a^3c^2}{693\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{11}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 64/693*sqrt(2)*(99*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^4 + 154*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c^5 + 63*c^6)*a^3*c^2/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(11
/2)*f)
```

Mupad [B] (verification not implemented)

Time = 25.00 (sec) , antiderivative size = 607, normalized size of antiderivative = 4.74

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}\,dx = \frac{\left(\frac{a^3\,c^2\,2\mathrm{i}}{f}+\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,302\mathrm{i}}{693\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1} - \frac{\left(\frac{a^3\,c^2\,64\mathrm{i}}{11\,f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,64\mathrm{i}}{11\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^5} + \frac{\left(\frac{a^3\,c^2\,16\mathrm{i}}{f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,944\mathrm{i}}{231\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2} + \frac{\left(\frac{a^3\,c^2\,160\mathrm{i}}{9\,f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1120\mathrm{i}}{99\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^4} - \frac{\left(\frac{a^3\,c^2\,20\mathrm{i}}{3\,f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,844\mathrm{i}}{693\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} - \frac{\left(\frac{a^3\,c^2\,160\mathrm{i}}{7\,f}-\frac{a^3\,c^2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,6880\mathrm{i}}{693\,f}\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}}{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^3}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] (((a^3*c^2*2i)/f + (a^3*c^2*exp(e*1i + f*x*1i)*302i)/(693*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/(exp(e*1i + f*x*1i) - 1)
- (((a^3*c^2*64i)/(11*f) - (a^3*c^2*exp(e*1i + f*x*1i)*64i)/(11*f))*(c - c/
(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i
) - 1)*(exp(e*2i + f*x*2i) + 1)^5) + (((a^3*c^2*16i)/f - (a^3*c^2*exp(e*1i
+ f*x*1i)*944i)/(231*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i
)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^2) + (((a^3
*c^2*160i)/(9*f) - (a^3*c^2*exp(e*1i + f*x*1i)*1120i)/(99*f))*(c - c/(exp(-
 e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)
*(exp(e*2i + f*x*2i) + 1)^4) - (((a^3*c^2*20i)/(3*f) - (a^3*c^2*exp(e*1i +
f*x*1i)*844i)/(693*f))*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/
2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) - (((a^3*c^2
*160i)/(7*f) - (a^3*c^2*exp(e*1i + f*x*1i)*6880i)/(693*f))*(c - c/(exp(- e*
1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2))/((exp(e*1i + f*x*1i) - 1)*(e
xp(e*2i + f*x*2i) + 1)^3)
```

3.81 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 85

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2(a+a\sec(e+fx))^3\tan(e+fx)}{63f\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{9f}$$

```
[Out] -8/63*c^2*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-2/9*c*(a+a
*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^3}{63f\sqrt{c-c\sec(e+fx)}} - \frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}{9f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(63*f*Sqrt[c - c*Sec[e + f*x]]
) - (2*c*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(9*f
)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{9f} \\
&\quad+\frac{1}{9}(4c)\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{8c^2(a+a\sec(e+fx))^3\tan(e+fx)}{63f\sqrt{c-c\sec(e+fx)}} \\
&\quad-\frac{2c(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{9f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \frac{16a^3c\cos^6\left(\frac{1}{2}(e+fx)\right)(-7+11\cos(e+fx))\cot\left(\frac{1}{2}(e+fx)\right)\sec^4(e+fx)\sqrt{c-c\sec(e+fx)}}{63f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (16*a^3*c*Cos[(e + f*x)/2]^6*(-7 + 11*Cos[e + f*x])*Cot[(e + f*x)/2]*Sec[e
+ f*x]^4*Sqrt[c - c*Sec[e + f*x]])/(63*f)
```

Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

method	result
default	$\frac{2a^3c(11\cos(fx+e)-7)\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)^4\sec(fx+e)^4\csc(fx+e)}{63f}$
parts	$-\frac{2a^3(\sec(fx+e)-1)(5\cos(fx+e)-1)c\sqrt{-c(\sec(fx+e)-1)}\,(\cos(fx+e)+1)\csc(fx+e)}{3f(\cos(fx+e)-1)}+\frac{2a^3(\sec(fx+e)-1)\left(272\cos(fx+e)^4-13\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/63*a^3*c/f*(11*cos(f*x+e)-7)*(-c*(sec(f*x+e)-1))^(1/2)*(cos(f*x+e)+1)^4*s
ec(f*x+e)^4*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\left(11\,a^3c\cos\left(fx+e\right)^5+37\,a^3c\cos\left(fx+e\right)^4+38\,a^3c\cos\left(fx+e\right)^3+2\,a^3c\cos\left(fx+e\right)^2-17\right.}{63\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] 2/63*(11*a^3*c*cos(f*x + e)^5 + 37*a^3*c*cos(f*x + e)^4 + 38*a^3*c*cos(f*x
+ e)^3 + 2*a^3*c*cos(f*x + e)^2 - 17*a^3*c*cos(f*x + e) - 7*a^3*c)*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c&-c\sec(e+fx))^{3/2}\,dx = a^3\left(\int c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx\right.\\&+\int 2c\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)dx\\&+\int\left(-2c\sqrt{-c\sec\left(e+fx\right)+c}\sec^4\left(e+fx\right)\right)dx\\&\left.+\int\left(-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^5\left(e+fx\right)\right)dx\right)\end{aligned}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(2*c*
sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(-2*c*sqrt(-c*sec(e
 + f*x) + c)*sec(e + f*x)**4, x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**5, x))
```

Maxima [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.86 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx = \frac{32\sqrt{2}\left(9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^5+7c^6\right)a^3}{63\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{9}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 32/63*sqrt(2)*(9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^5 + 7*c^6)*a^3/((c*tan(1/
2*f*x + 1/2*e)^2 - c)^(9/2)*f)
```

Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 471, normalized size of antiderivative = 5.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}\,dx=\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^3\,c\,2i}{f}+\frac{a^3\,c\,e^{e\,1i+f\,x\,1i}\,22i}{63\,f}\right)}{e^{e\,1i+f\,x\,1i}-1}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^3\,c\,32i}{9\,f}+\frac{a^3\,c\,e^{e\,1i+f\,x\,1i}\,32i}{9\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^4}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^3\,c\,8i}{3\,f}-\frac{a^3\,c\,e^{e\,1i+f\,x\,1i}\,200i}{63\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)}+\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^3\,c\,32i}{7\,f}+\frac{a^3\,c\,e^{e\,1i+f\,x\,1i}\,608i}{63\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^3}-\frac{a^3\,c\,e^{e\,1i+f\,x\,1i}\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,160i}{21\,f\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^2}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*2i)/
f + (a^3*c*exp(e*1i + f*x*1i)*22i)/(63*f)))/(exp(e*1i + f*x*1i) - 1) - ((c
- c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*32i)/(9*
f) + (a^3*c*exp(e*1i + f*x*1i)*32i)/(9*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(
e*2i + f*x*2i) + 1)^4) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*((a^3*c*8i)/(3*f) - (a^3*c*exp(e*1i + f*x*1i)*200i)/(63*f)))/(
(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i -
f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*c*32i)/(7*f) + (a^3*c*exp(e*
1i + f*x*1i)*608i)/(63*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) +
 1)^3) - (a^3*c*exp(e*1i + f*x*1i)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1
i + f*x*1i)/2))^(1/2)*160i)/(21*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*
2i) + 1)^2)
```

3.82 $\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2/7*c*(a+a*sec(f*x+e))^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^3}{7f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(7*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{2c(a+a\sec(e+fx))^3\tan(e+fx)}{7f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = \frac{16a^3\cos^6\left(\frac{1}{2}(e+fx)\right)\cot\left(\frac{1}{2}(e+fx)\right)\sec^3(e+fx)\sqrt{c-c\sec(e+fx)}}{7f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]],x]
[Out] (16*a^3*Cos[(e + f*x)/2]^6*Cot[(e + f*x)/2]*Sec[e + f*x]^3*Sqrt[c - c*Sec[e
 + f*x]])/(7*f)
```

Maple [A] (verified)

Time = 5.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

method	result
default	$\frac{2a^3(\cos(fx+e)+1)^4\sqrt{-c(\sec(fx+e)-1)}\,\sec(fx+e)^3\csc(fx+e)}{7f}$
parts	$-\frac{2a^3\sqrt{-c(\sec(fx+e)-1)}\,\sin(fx+e)}{f(\cos(fx+e)-1)}-\frac{2a^3\sqrt{-c(\sec(fx+e)-1)}\left(-5+16\cos(fx+e)^4+8\cos(fx+e)^3-2\cos(fx+e)^2+\cos(fx+e)\right)}{35f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
[Out] 2/7*a^3/f*(cos(f*x+e)+1)^4*(-c*(sec(f*x+e)-1))^(1/2)*sec(f*x+e)^3*csc(f*x+e
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.37

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\left(a^3\cos\left(fx+e\right)^4+4\,a^3\cos\left(fx+e\right)^3+6\,a^3\cos\left(fx+e\right)^2+4\,a^3\cos\left(fx+e\right)+a^3\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{7\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2/7*(a^3*cos(f*x + e)^4 + 4*a^3*cos(f*x + e)^3 + 6*a^3*cos(f*x + e)^2 + 4*a
^3*cos(f*x + e) + a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x +
 e)^3*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx = a^3\left(\int\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)dx+\int 3\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)dx+\int 3\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)dx+\int\sqrt{-c\sec\left(e+fx\right)+c}\sec^4\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x), x) + Integral(3*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2, x) + Integral(3*sqrt(-c*sec(e + f*x)
 + c)*sec(e + f*x)**3, x) + Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)
**4, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int (a\sec(fx+e)+a)^3\sqrt{-c\sec(fx+e)+c}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] 2/7*(7*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
3/4)*(5*(a^3*f*cos(2*f*x + 2*e)^2 + a^3*f*sin(2*f*x + 2*e)^2 + 2*a^3*f*cos(
2*f*x + 2*e) + a^3*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*
cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4
*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e
)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*
e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e
)^2)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*
e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x +
 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x +
 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x
 + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(7/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*
f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2
*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(
8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos
(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2
*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(
2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4
*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) +
4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(7/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4
*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 +
 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x +
 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*
cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*
f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) +
6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10
*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + s
in(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x
 + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*s
```

```
in(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^
2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8
*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*c
os(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*
f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 1
2*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x +
 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos
(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x
+ 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6
*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6
*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*
cos(2*f*x + 2*e) + 1)^(1/4)), x) + 5*(a^3*f*cos(2*f*x + 2*e)^2 + a^3*f*sin(
2*f*x + 2*e)^2 + 2*a^3*f*cos(2*f*x + 2*e) + a^3*f)*integrate((((cos(10*f*x
+ 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*
x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x
 + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*
f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin
(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*si
n(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*s
in(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*
sin(2*f*x + 2*e))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*
sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*
e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10
*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x +
6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(5/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*
e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2
*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x
 + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f
*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*
x + 2*e)^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e) +
 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 1
0*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) +
cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*
x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*
cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)
^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(
2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x +
```

```
6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f
*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f
*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x
 + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(
6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e
) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(
6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x
+ 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*si
n(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e)
+ sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2
*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4
*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4
*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x) - 9*(a^3*f*co
s(2*f*x + 2*e)^2 + a^3*f*sin(2*f*x + 2*e)^2 + 2*a^3*f*cos(2*f*x + 2*e) + a^
3*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*
cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)
*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e
) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*
e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(2*f*x + 2*e)*sin(10*f*x + 1
0*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x +
 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x
+ 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x
 + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*sin(3/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*
f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4
*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(
2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(
10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*c
os(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + co
s(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)
*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e
)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)
))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos
(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x +
 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*
f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e)
+ 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*
```

```
f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e
) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x
+ 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f
*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2
 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (2*(4*cos(8*f*x + 8*e) + 6*c
os(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e)
 + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(
2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x +
4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(
4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 +
 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e)
 + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x +
 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*
sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x +
 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/4)), x) - (a^3*f*cos(2*f*x + 2*e)^2 + a^3*f*sin(2*f*x + 2*e)^2 + 2*
a^3*f*cos(2*f*x + 2*e) + a^3*f)*integrate((((cos(10*f*x + 10*e)*cos(2*f*x +
 2*e) + 4*cos(8*f*x + 8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x
+ 2*e) + 4*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*
f*x + 10*e)*sin(2*f*x + 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(
6*f*x + 6*e)*sin(2*f*x + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2
*f*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (cos(
2*f*x + 2*e)*sin(10*f*x + 10*e) + 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*c
os(2*f*x + 2*e)*sin(6*f*x + 6*e) + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - co
s(10*f*x + 10*e)*sin(2*f*x + 2*e) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6
*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)) - ((cos(2*f*x + 2*e)*sin(10*f*x + 10*e)
+ 4*cos(2*f*x + 2*e)*sin(8*f*x + 8*e) + 6*cos(2*f*x + 2*e)*sin(6*f*x + 6*e)
 + 4*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - cos(10*f*x + 10*e)*sin(2*f*x + 2*e
) - 4*cos(8*f*x + 8*e)*sin(2*f*x + 2*e) - 6*cos(6*f*x + 6*e)*sin(2*f*x + 2*
e) - 4*cos(4*f*x + 4*e)*sin(2*f*x + 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))) - (cos(10*f*x + 10*e)*cos(2*f*x + 2*e) + 4*cos(8*f*x +
8*e)*cos(2*f*x + 2*e) + 6*cos(6*f*x + 6*e)*cos(2*f*x + 2*e) + 4*cos(4*f*x +
 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + sin(10*f*x + 10*e)*sin(2*f*x
+ 2*e) + 4*sin(8*f*x + 8*e)*sin(2*f*x + 2*e) + 6*sin(6*f*x + 6*e)*sin(2*f*x
 + 2*e) + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)))/(((2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e)
 + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x +
 10*e)^2 + 8*(6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*c
```

```
os(8*f*x + 8*e) + 16*cos(8*f*x + 8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*
x + 2*e))*cos(6*f*x + 6*e) + 36*cos(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2
+ 8*cos(4*f*x + 4*e)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x
 + 8*e) + 6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(1
0*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x
+ 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4
*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e
)^2 + 16*sin(4*f*x + 4*e)^2 + 8*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f
*x + 2*e)^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (
2*(4*cos(8*f*x + 8*e) + 6*cos(6*f*x + 6*e) + 4*cos(4*f*x + 4*e) + cos(2*f*x
 + 2*e))*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 8*(6*cos(6*f*x + 6*e)
+ 4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(8*f*x + 8*e) + 16*cos(8*f*x +
8*e)^2 + 12*(4*cos(4*f*x + 4*e) + cos(2*f*x + 2*e))*cos(6*f*x + 6*e) + 36*c
os(6*f*x + 6*e)^2 + 16*cos(4*f*x + 4*e)^2 + 8*cos(4*f*x + 4*e)*cos(2*f*x +
2*e) + cos(2*f*x + 2*e)^2 + 2*(4*sin(8*f*x + 8*e) + 6*sin(6*f*x + 6*e) + 4*
sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e
)^2 + 8*(6*sin(6*f*x + 6*e) + 4*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*
f*x + 8*e) + 16*sin(8*f*x + 8*e)^2 + 12*(4*sin(4*f*x + 4*e) + sin(2*f*x + 2
*e))*sin(6*f*x + 6*e) + 36*sin(6*f*x + 6*e)^2 + 16*sin(4*f*x + 4*e)^2 + 8*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + sin(2*f*x + 2*e)^2)*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2
*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)), x))*sqrt(c) - (7*(a^3*sin(6*f*x + 6
*e) + 5*a^3*sin(4*f*x + 4*e) + 3*a^3*sin(2*f*x + 2*e))*cos(7/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - (7*a^3*cos(6*f*x + 6*e) + 35*a^3*cos
(4*f*x + 4*e) + 21*a^3*cos(2*f*x + 2*e) + a^3)*sin(7/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)))*sqrt(c))/((f*cos(2*f*x + 2*e)^2 + f*sin(2*f*x
 + 2*e)^2 + 2*f*cos(2*f*x + 2*e) + f)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e
)^2 + 2*cos(2*f*x + 2*e) + 1)^(3/4))
```

Giac [A] (verification not implemented)

none

Time = 0.82 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx=\frac{16\sqrt{2}a^3c^4}{7\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{7}{2}}f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 16/7*sqrt(2)*a^3*c^4/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(7/2)*f)
```

Mupad [B] (verification not implemented)

Time = 17.16 (sec) , antiderivative size = 375, normalized size of antiderivative = 9.15

$$\int \sec(e+fx)(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^3\,2i}{f}+\frac{a^3\,e^{e\,1i+f\,x\,1i}\,2i}{7\,f}\right)}{e^{e\,1i+f\,x\,1i}-1}-\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^3\,8i}{f}+\frac{a^3\,e^{e\,1i+f\,x\,1i}\,8i}{7\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^2}+\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^3\,4i}{f}+\frac{a^3\,e^{e\,1i+f\,x\,1i}\,36i}{7\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)}+\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{a^3\,16i}{7\,f}-\frac{a^3\,e^{e\,1i+f\,x\,1i}\,16i}{7\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,2i+f\,x\,2i}+1\right)^3}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*2i)/f
+ (a^3*exp(e*1i + f*x*1i)*2i)/(7*f)))/(exp(e*1i + f*x*1i) - 1) - ((c - c/(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a^3*8i)/f + (a^3*exp
(e*1i + f*x*1i)*8i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) +
 1)^2) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((a
^3*4i)/f + (a^3*exp(e*1i + f*x*1i)*36i)/(7*f)))/((exp(e*1i + f*x*1i) - 1)*(
exp(e*2i + f*x*2i) + 1)) + ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*((a^3*16i)/(7*f) - (a^3*exp(e*1i + f*x*1i)*16i)/(7*f)))/((ex
p(e*1i + f*x*1i) - 1)*(exp(e*2i + f*x*2i) + 1)^3)
```

3.83 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}} dx$

Optimal result

Integrand size = 34, antiderivative size = 164

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}} dx = -\frac{8\sqrt{2}a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -8*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/f/c^(1/2)+8*a^3*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+2/5*a*(a+a*sec(f*x+e)
)^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+4/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)
/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used

$= \{4041, 3880, 209\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{8\sqrt{2}a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{4\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3f\sqrt{c-c\sec(e+fx)}} + \frac{2a\tan(e+fx)(a\sec(e+fx)+a)^2}{5f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-8*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(Sqrt[c]*f) + (8*a^3*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]) + (
2*a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*Sqrt[c - c*Sec[e + f*x]]) + (
4*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + (2a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx$$

$$= \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (4a^2)\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}dx$$

$$= \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} + (8a^3)\int\frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx$$

$$= \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{(16a^3)\,\text{Subst}\left(\int\frac{1}{2c+x^2}dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{f}$$

$$= -\frac{8\sqrt{2}a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f} + \frac{8a^3\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{2a(a+a\sec(e+fx))^2\tan(e+fx)}{5f\sqrt{c-c\sec(e+fx)}} + \frac{4(a^3+a^3\sec(e+fx))\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}dx$$
$$= \frac{2a^3\left(-60\sqrt{2}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a(1+\sec(e+fx))}}{\sqrt{2}\sqrt{a}}\right) + \sqrt{a(1+\sec(e+fx))}(73+16\sec(e+fx)+3\sec^2(e+fx))\right)}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/Sqrt[c - c*Sec[e + f*x]],x]
[Out] (2*a^3*(-60*Sqrt[2]*Sqrt[a]*ArcTanh[Sqrt[a*(1 + Sec[e + f*x])]/(Sqrt[2]*Sqr
t[a])] + Sqrt[a*(1 + Sec[e + f*x])]*(73 + 16*Sec[e + f*x] + 3*Sec[e + f*x]^
2))*Tan[e + f*x])/(15*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]
)
```

Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

method	result
default	$-\frac{a^{3}\sqrt{2}\left(120\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)^{2}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)-73\sqrt{2}\,\cos(fx+e)^{2}-16\sqrt{2}\,\cos(fx+e)-3\sqrt{2}\right)\tan(fx+e)\sec(}{15f\sqrt{-c(\sec(fx+e)-1)}}$
parts	$\frac{a^{3}\sqrt{2}\,\sin(fx+e)\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)}{f(\cos(fx+e)+1)\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}+\frac{a^{3}\sqrt{2}\left(-15\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\cos(fx+e)^{2}\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)+13}{15f\sqrt{-c(\sec(fx+}}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/15*a^3/f*2^(1/2)*(120*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*ar
ctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-73*2^(1/2)*cos(f*x+e)^
2-16*2^(1/2)*cos(f*x+e)-3*2^(1/2))/(-c*(sec(f*x+e)-1))^(1/2)*tan(f*x+e)*sec
(f*x+e)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.30

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx$$

$$=\left[\frac{2\left(30\sqrt{2}a^{3}c\sqrt{-\frac{1}{c}}\cos\left(fx+e\right)^{2}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^{2}+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin}{15\,cf\cos\left(fx+e\right)^{2}s}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [2/15*(30*sqrt(2)*a^3*c*sqrt(-1/c)*cos(f*x + e)^2*log(-(2*sqrt(2)*(cos(f*x
+ e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) -
 (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(
f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos(f*x
+ e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x + e)^2*
sin(f*x + e)), 2/15*(60*sqrt(2)*a^3*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x
+ e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*cos(f*x + e)^2
*sin(f*x + e) - (73*a^3*cos(f*x + e)^3 + 89*a^3*cos(f*x + e)^2 + 19*a^3*cos
(f*x + e) + 3*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/(c*f*cos(f*x +
e)^2*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx = a^3\left(\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx + \int \frac{3\sec^2(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx + \int \frac{3\sec^3(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx + \int \frac{\sec^4(e+fx)}{\sqrt{-c\sec(e+fx)+c}}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(
e + f*x)**2/sqrt(-c*sec(e + f*x) + c), x) + Integral(3*sec(e + f*x)**3/sqrt
(-c*sec(e + f*x) + c), x) + Integral(sec(e + f*x)**4/sqrt(-c*sec(e + f*x) +
 c), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^3\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Giac [A] (verification not implemented)

none

Time = 1.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{8\,a^3\left(\frac{15\,\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{2}\left(15\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2-5\,\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c+3\,c^2\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{5}{2}}}\right)}{15\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 8/15*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqr
t(c) + sqrt(2)*(15*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 - 5*(c*tan(1/2*f*x + 1/
2*e)^2 - c)*c + 3*c^2)/(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^3}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)
```

3.84 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 168

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{10\sqrt{2}a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 10*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))*2^(1/2
)/c^(3/2)/f-a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-10*a^3
*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(1/2)-5/3*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/
c/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4042, 4041, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{10\sqrt{2}a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{3cf\sqrt{c-c\sec(e+fx)}} - \frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{f(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (10*Sqrt[2]*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f
*x]])])/(c^(3/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(f*(c - c*Sec
[e + f*x])^(3/2)) - (10*a^3*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]]) -
(5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(3*c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{(5a)\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{\sqrt{c-c\sec(e+fx)}}\,dx}{2c}\\
&= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}\\
&\quad -\frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf\sqrt{c-c\sec(e+fx)}}-\frac{(5a^2)\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}
\end{aligned}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf\sqrt{c-c\sec(e+fx)}} - \frac{(10a^3)\int\frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{c}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf\sqrt{c-c\sec(e+fx)}} + \frac{(20a^3)\,\text{Subst}\left(\int\frac{1}{2c+x^2}dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{cf}$$

$$= \frac{10\sqrt{2}a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{c^{3/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{10a^3\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{3cf\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.38

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}dx = -\frac{a^3\,\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^3\tan(e+fx)}{14cf\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(3/2), x]
[Out] -1/14*(a^3*Hypergeometric2F1[2, 7/2, 9/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e + f*x])^3*Tan[e + f*x])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 7.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

method	result
default	$\frac{a^3\sqrt{2}\left(30\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\tan(fx+e)+19\sqrt{2}\cot(fx+e)+7\sqrt{2}\csc(fx+e)-13\sqrt{2}\sec(fx+e)\csc(fx+e\right.}{3cf\sqrt{-c(\sec(fx+e)-1)}}$
parts	Expression too large to display

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3*a^3/c/f*2^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)*(30*arctan(1/2*2^(1/2)/(-cos(
f*x+e)/(cos(f*x+e)+1))^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*tan(f*x+e)
+19*2^(1/2)*cot(f*x+e)+7*2^(1/2)*csc(f*x+e)-13*2^(1/2)*sec(f*x+e)*csc(f*x+e
)-2^(1/2)*sec(f*x+e)^2*csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = \left[\frac{15\sqrt{2}\left(a^3c\cos(fx+e)^2 - a^3c\cos(fx+e)\right)\sqrt{-\frac{1}{c}}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e\right.}{}\right.}{}\right.$$

$$-\frac{2\left(\frac{15\sqrt{2}\left(a^3c\cos(fx+e)^2-a^3c\cos(fx+e)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{c}\sin(fx+e)}\right)\sin(fx+e)}{\sqrt{c}} - \left(19\,a^3\cos(fx+e)^3 + 7\,a^3\cos(fx\right.\right.}{3\left(c^2f\cos(fx+e)^2 - c^2f\cos(fx+e)\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/3*(15*sqrt(2)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*sqrt(-1/c)*log
((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*sqrt(-1/c) + (3*cos(f*x + e) + 1)*sin(f*x + e))/((cos(f*x + e) - 1
)*sin(f*x + e)))*sin(f*x + e) + 2*(19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x +
e)^2 - 13*a^3*cos(f*x + e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/
((c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e))*sin(f*x + e)), -2/3*(15*sqrt(2
)*(a^3*c*cos(f*x + e)^2 - a^3*c*cos(f*x + e))*arctan(sqrt(2)*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e)
/sqrt(c) - (19*a^3*cos(f*x + e)^3 + 7*a^3*cos(f*x + e)^2 - 13*a^3*cos(f*x +
 e) - a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c^2*f*cos(f*x + e)^2
- c^2*f*cos(f*x + e))*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = a^3\left(\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)}}\right.$$

$$+\int \frac{3\sec^2\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx$$

$$+\int \frac{3\sec^3\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx$$

$$\left.+\int \frac{\sec^4\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)+c\sqrt{-c\sec\left(e+fx\right)+c}}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*
sqrt(-c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**2/(-c*sqrt(-c*sec
(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x) + Integral(3
*sec(e + f*x)**3/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec
(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(-c*sqrt(-c*sec(e + f*x) + c
)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a\sec\left(fx+e\right)+a\right)^3\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^3*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Giac [A] (verification not implemented)

none

Time = 1.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{2\,a^3\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\left(6\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-7\,c\right)}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}c}+\frac{3\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}\right)}{3\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] -2/3*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(
3/2) + 2*sqrt(2)*(6*c*tan(1/2*f*x + 1/2*e)^2 - 7*c)/((c*tan(1/2*f*x + 1/2*e
)^2 - c)^(3/2)*c) + 3*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/(c^2*tan(1
/2*f*x + 1/2*e)^2))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^3}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.85 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 174

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{15a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -15/4*a^3*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/c^(
5/2)/f*2^(1/2)-1/2*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)
+5/4*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(3/2)+15/4*a^3*ta
n(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4042, 4041, 3880, 209}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{15a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}} + \frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{4cf(c-c\sec(e+fx))^{3/2}} - \frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{2f(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-15*a^3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])
/(2*Sqrt[2]*c^(5/2)*f) - (a*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(2*f*(c -
c*Sec[e + f*x])^(5/2)) + (5*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(4*c*f*(
c - c*Sec[e + f*x])^(3/2)) + (15*a^3*Tan[e + f*x])/(4*c^2*f*Sqrt[c - c*Sec[
e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.))/
Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*d*Cot[e +
f*x]*((c + d*Csc[e + f*x])^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])),
 x] + Dist[2*c*((2*n - 1)/(2*n - 1)), Int[Csc[e + f*x]*((c + d*Csc[e + f*x]
)^(n - 1)/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]
```

Rule 4042

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0] && LtQ[m, -2^(
-1)] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}-\frac{(5a)\int\frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c-c\sec(e+fx))^{3/2}}\,dx}{4c}\\
&=-\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}}+\frac{(15a^2)\int\frac{\sec(e+fx)(a+a\sec(e+fx))}{\sqrt{c-c\sec(e+fx)}}\,dx}{8c^2}
\end{aligned}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}} + \frac{(15a^3)\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{4c^2}$$

$$= -\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}} - \frac{(15a^3)\,\mathrm{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{2c^2f}$$

$$= -\frac{15a^3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} - \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{4cf(c-c\sec(e+fx))^{3/2}} + \frac{15a^3\tan(e+fx)}{4c^2f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.62 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.37

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^3\,\mathrm{Hypergeometric2F1}\left(3,\frac{7}{2},\frac{9}{2},\frac{1}{2}(1+\sec(e+fx))\right)(1+\sec(e+fx))^3\tan(e+fx)}{28c^2f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c - c*Sec[e + f*x])^(5/2),
x]
[Out] -1/28*(a^3*Hypergeometric2F1[3, 7/2, 9/2, (1 + Sec[e + f*x])/2]*(1 + Sec[e
+ f*x])^3*Tan[e + f*x])/(c^2*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 6.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.28

method	result
default	$\frac{a^3\sqrt{2}\left(15\arctan\left(\frac{1}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)(1-\cos(fx+e))^4\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\csc(fx+e)-15(1-\cos(fx+e)}{4c^2f\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))^3\left((1-\cos(fx+e))^2\csc(fx+\right.}$
parts	Expression too large to display

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/4*a^3/c^2/f*2^(1/2)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)
*csc(f*x+e)^2)^(1/2)/(1-cos(f*x+e))^3/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(15
*arctan(1/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(1-cos(f*x+e))^4*((1-cos
(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*csc(f*x+e)-15*(1-cos(f*x+e))^4*csc(f*x+e)+
5*(1-cos(f*x+e))^2*sin(f*x+e)+2*sin(f*x+e)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.53

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{15\sqrt{2}\left(a^3\cos(fx+e)^2 - 2a^3\cos(fx+e) + a^3\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}(c\cdots}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(-c)*
log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e
) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e)
- 1)*sin(f*x + e)))*sin(f*x + e) + 4*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x
+ e)^2 - 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
)))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e)), 1
/4*(15*sqrt(2)*(a^3*cos(f*x + e)^2 - 2*a^3*cos(f*x + e) + a^3)*sqrt(c)*arct
an(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*si
n(f*x + e)))*sin(f*x + e) - 2*(9*a^3*cos(f*x + e)^3 - 8*a^3*cos(f*x + e)^2
- 13*a^3*cos(f*x + e) + 4*a^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((c
^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = a^3\left(\int \frac{\sec(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx}}\right.$$

$$+\int \frac{3\sec^2(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+}}$$

$$+\int \frac{3\sec^3(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+}}$$

$$+\int \frac{\sec^4(e+fx)}{c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)-2c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+c^2\sqrt{-c\sec(e+fx)+}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2
 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x
) + c)), x) + Integral(3*sec(e + f*x)**2/(c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-
c*sec(e + f*x) + c)), x) + Integral(3*sec(e + f*x)**3/(c**2*sqrt(-c*sec(e +
 f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)
+ c**2*sqrt(-c*sec(e + f*x) + c)), x) + Integral(sec(e + f*x)**4/(c**2*sqrt
(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x))
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 1.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^3\left(\frac{15\sqrt{2}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{8\sqrt{2}}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}c^2}+\frac{7\sqrt{2}\left(c\tan\left(\frac{1}{2}\right.\right.}{}\right.}{4f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/4*a^3*(15*sqrt(2)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/c^(5
/2) + 8*sqrt(2)/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2) + (7*sqrt(2)*(c*ta
n(1/2*f*x + 1/2*e)^2 - c)^(3/2) + 9*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 -
 c)*c)/(c^4*tan(1/2*f*x + 1/2*e)^4))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^3}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.86 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 142

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}} + \frac{32c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5af} + \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] 12/5*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/a/f+2*c*(c-c*sec(f*x+e))^(5/2)*t
an(f*x+e)/f/(a+a*sec(f*x+e))+128/5*c^4*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2
)+32/5*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {4039, 3878, 3877}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}} + \frac{32c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5af} + \frac{12c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (128*c^4*Tan[e + f*x])/(5*a*f*Sqrt[c - c*Sec[e + f*x]]) + (32*c^3*Sqrt[c -
c*Sec[e + f*x]]*Tan[e + f*x])/(5*a*f) + (12*c^2*(c - c*Sec[e + f*x])^(3/2)*
Tan[e + f*x])/(5*a*f) + (2*c*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(a
 + a*Sec[e + f*x]))
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(6c)\int\sec(e+fx)(c-c\sec(e+fx))^{5/2}\,dx}{a}\\
&= \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af}+\frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))}\\
&\quad-\frac{(48c^2)\int\sec(e+fx)(c-c\sec(e+fx))^{3/2}\,dx}{5a}\\
&= \frac{32c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5af}+\frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af}\\
&\quad+\frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{(64c^3)\int\sec(e+fx)\sqrt{c-c\sec(e+fx)}\,dx}{5a}
\end{aligned}
$$

$$= \frac{128c^4\tan(e+fx)}{5af\sqrt{c-c\sec(e+fx)}} + \frac{32c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5af} + \frac{12c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{2c^4(91+43\sec(e+fx)-7\sec^2(e+fx)+\sec^3(e+fx))\tan(e+}{5af(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c^4*(91 + 43*Sec[e + f*x] - 7*Sec[e + f*x]^2 + Sec[e + f*x]^3)*Tan[e + f
*x])/(5*a*f*(1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2c^3(\sec(fx+e)-1)^3\sqrt{-c(\sec(fx+e)-1)}\left(91\cos(fx+e)^3+43\cos(fx+e)^2-7\cos(fx+e)+1\right)\cot(fx+e)}{5af(\cos(fx+e)-1)^3}$	81

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/5/a/f*c^3*(sec(f*x+e)-1)^3*(-c*(sec(f*x+e)-1))^(1/2)*(91*cos(f*x+e)^3+43*
cos(f*x+e)^2-7*cos(f*x+e)+1)/(cos(f*x+e)-1)^3*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = -\frac{2\left(91\,c^3\cos\left(fx+e\right)^3+43\,c^3\cos\left(fx+e\right)^2-7\,c^3\cos\left(fx+e\right)+c^3\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{5\,af\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2/5*(91*c^3*cos(f*x + e)^3 + 43*c^3*cos(f*x + e)^2 - 7*c^3*cos(f*x + e) +
c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)^2*sin(f*x +
e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = \frac{8\left(16\sqrt{2}c^{\frac{7}{2}} - \frac{56\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{70\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{35\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right.}{5\,af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{7}{2}}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 8/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2)
*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8)/(a*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)
*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.83 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = -\frac{8\sqrt{2}c^3\left(\frac{5\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{a} - \frac{15\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c+5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^2+c^3}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}a}\right)}{5f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -8/5*sqrt(2)*c^3*(5*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/a - (15*(c*tan(1/2*f
*x + 1/2*e)^2 - c)^2*c + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2 + c^3)/((c*ta
n(1/2*f*x + 1/2*e)^2 - c)^(5/2)*a))/f
```

Mupad [B] (verification not implemented)

Time = 17.98 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{a+a\sec(e+fx)}\,dx = -\frac{2\,c^3\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,86{}\mathrm{i}+{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,245{}\mathrm{i}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,180{}\mathrm{i}+{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,245{}\mathrm{i}+{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,86}{5\,a\,f\,\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}-1\right)\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^2}$$

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(2*c^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(
e*1i + f*x*1i)*86i + exp(e*2i + f*x*2i)*245i + exp(e*3i + f*x*3i)*180i + ex
p(e*4i + f*x*4i)*245i + exp(e*5i + f*x*5i)*86i + exp(e*6i + f*x*6i)*91i + 9
1i))/(5*a*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*2i + f*x*2i) + 1)^2)
```

3.87 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 108

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}} + \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3af} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] 2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))+32/3*c^3*tan(f*x+e
)/a/f/(c-c*sec(f*x+e))^(1/2)+8/3*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {4039, 3878, 3877}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}} + \frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3af} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (32*c^3*Tan[e + f*x])/(3*a*f*Sqrt[c - c*Sec[e + f*x]]) + (8*c^2*Sqrt[c - c*
Sec[e + f*x]]*Tan[e + f*x])/(3*a*f) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e
 + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(4c)\int \sec(e+fx)(c-c\sec(e+fx))^{3/2}\,dx}{a} \\
&= \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3af} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))} \\
&\quad - \frac{(16c^2)\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}\,dx}{3a} \\
&= \frac{32c^3\tan(e+fx)}{3af\sqrt{c-c\sec(e+fx)}} + \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3af} \\
&\quad + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = -\frac{2c^3(-23-10\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{3af(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x]),x]
[Out] (-2*c^3*(-23 - 10*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e + f*x])/(3*a*f*(1 +
Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 5.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{2c^2(\sec(fx+e)-1)^2\sqrt{-c(\sec(fx+e)-1)}\left(23\cos(fx+e)^2+10\cos(fx+e)-1\right)\cot(fx+e)}{3af(\cos(fx+e)-1)^2}$	71

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERB
OSE)
[Out] -2/3/a/f*c^2*(sec(f*x+e)-1)^2*(-c*(sec(f*x+e)-1))^(1/2)*(23*cos(f*x+e)^2+10
*cos(f*x+e)-1)/(cos(f*x+e)-1)^2*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = -\frac{2\left(23\,c^2\cos\left(fx+e\right)^2+10\,c^2\cos\left(fx+e\right)-c^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,af\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
[Out] -2/3*(23*c^2*cos(f*x + e)^2 + 10*c^2*cos(f*x + e) - c^2)*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/(a*f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c^2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \left(-\frac{2c^2\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)}{\sec(e+fx)+1}\right)}{a}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x
) + Integral(-2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f*x
) + 1), x) + Integral(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3/(sec(e
 + f*x) + 1), x))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = -\frac{4\left(8\sqrt{2}c^{\frac{5}{2}} - \frac{20\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{3\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)}{3\,af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] -4/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*sqrt(2)*
c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a*f*(sin(f*x + e)/(cos(f*x +
e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = -\frac{4\sqrt{2}c^2\left(\frac{3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{a} - \frac{6\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+c^2}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}a}\right)}{3f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -4/3*sqrt(2)*c^2*(3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/a - (6*(c*tan(1/2*f*
x + 1/2*e)^2 - c)*c + c^2)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a))/f
```

Mupad [B] (verification not implemented)

Time = 15.56 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx = \frac{2\,c^2\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,\left(2\,\sin\left(e+f\,x\right)-44\,\sin\left(2\,e+2\,f\,x\right)+25\,\sin\left(\right.\right.}{3\,a\,f\,\left(\cos\left(3\,e+3\,f\,x\right)-2\,\cos\left(e+f\,x\right)-2\,\cos\right.}$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (2*c^2*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(2*sin(e + f*x) - 44*sin
(2*e + 2*f*x) + 25*sin(3*e + 3*f*x) - 26*sin(4*e + 4*f*x) + 23*sin(5*e + 5*
f*x)))/(3*a*f*(cos(3*e + 3*f*x) - 2*cos(e + f*x) - 2*cos(4*e + 4*f*x) + cos
(5*e + 5*f*x) + 2))
```

3.88 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 72

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] 4*c^2*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(1/2)+2*c*(c-c*sec(f*x+e))^(1/2)*tan(
f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4039, 3877}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}} + \frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (4*c^2*Tan[e + f*x])/(a*f*Sqrt[c - c*Sec[e + f*x]]) + (2*c*Sqrt[c - c*Sec[e
 + f*x]]*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
```

```
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(2c)\int\sec(e+fx)\sqrt{c-c\sec(e+fx)}\,dx}{a}\\ &= \frac{4c^2\tan(e+fx)}{af\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = \frac{2c^2(3+\sec(e+fx))\tan(e+fx)}{af(1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c^2*(3 + Sec[e + f*x])*Tan[e + f*x])/(a*f*(1 + Sec[e + f*x])*Sqrt[c - c*
Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2c(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\,(3\cos(fx+e)+1)\cot(fx+e)}{af(\cos(fx+e)-1)}$	57

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2/a/f*c*(sec(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*(3*cos(f*x+e)+1)/(cos(f*x+
e)-1)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=-\frac{2\,(3\,c\cos\left(fx+e\right)+c)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{af\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] -2*(3*c*cos(f*x + e) + c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*f*sin(
f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\frac{\int\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec\left(e+fx\right)+1}\,dx+\int\left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)}{\sec\left(e+fx\right)+1}\right)\,dx}{a}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x) +
 Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/(sec(e + f*x) + 1),
x))/a
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.53

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx=\frac{2\left(2\sqrt{2}c^{\frac{3}{2}}-\frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}\right)}{af\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] 2*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2 + sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a*f*(sin(f*x + e)
/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx = -\frac{2\sqrt{2}\left(\frac{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}c}{a}-\frac{c^2}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}a}\right)}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] -2*sqrt(2)*(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c/a - c^2/(sqrt(c*tan(1/2*f*
x + 1/2*e)^2 - c)*a))/f
```

Mupad [B] (verification not implemented)

Time = 13.69 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx =$$
$$-\frac{c\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}\,\left(2\,\sin\left(e+f\,x\right)+6\,\sin\left(2\,e+2\,f\,x\right)+2\,\sin\left(3\,e+3\,f\,x\right)+3\,\sin\left(4\,e+4\,f\,x\right)\right)}{a\,f\,{\sin\left(2\,e+2\,f\,x\right)}^2}$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] -(c*(c - c/cos(e + f*x))^(1/2)*(2*sin(e + f*x) + 6*sin(2*e + 2*f*x) + 2*sin
(3*e + 3*f*x) + 3*sin(4*e + 4*f*x)))/(a*f*sin(2*e + 2*f*x)^2)
```

3.89 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 39

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4038}

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=\frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
[Out] (2*c*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral}=\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=-\frac{2\cot(e+fx)\sqrt{c-c\sec(e+fx)}}{af}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x]),x]
[Out] (-2*Cot[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f)
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{2\sqrt{-c(\sec(fx+e)-1)}\,\cot(fx+e)}{af}$	28

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x,method=_RETURNVERB
OSE)
[Out] -2/a/f*(-c*(sec(f*x+e)-1))^(1/2)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=-\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{af\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
fricas")
[Out] -2*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(a*f*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e)),x)
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(37) = 74.

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.15

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{\sqrt{2}\sqrt{c}-\frac{\sqrt{2}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}}{af\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
maxima")
[Out] -(sqrt(2)*sqrt(c) - sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/(a
*f*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e
) + 1) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx = -\frac{\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}(\cos(fx+e))}{af}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e)),x, algorithm="
giac")
[Out] -sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*sgn(tan(1/2*f*x + 1/2*e)^3 + ta
n(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a*f)
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx=-\frac{\sin\left(2\,e+2\,f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{a\,f\,{\sin\left(e+f\,x\right)}^2}$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
[Out] -(sin(2*e + 2*f*x)*(c - c/cos(e + f*x))^(1/2))/(a*f*sin(e + f*x)^2)
```

3.90 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 89

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/f*2^(1
/2)/c^(1/2)+tan(f*x+e)/f/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {4045, 3880, 209}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{f(a\sec(e+fx)+a)\sqrt{c-c\sec(e+fx)}} - \frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -(ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqrt[2
]*a*Sqrt[c]*f)) + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f
*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\text{integral} = \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{2a}$$

$$= \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{af}$$

$$= -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan\left(\frac{1}{2}(e+fx)\right)}{af\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Hypergeometric2F1[-1/2, 1, 1/2, (1 + Sec[e + f*x])/2]*Tan[(e + f*x)/2])/(a
*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\sqrt{2}\,\sin(fx+e)\left(\sqrt{2}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\right)}{2af(\cos(fx+e)+1)\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}$	112

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/a/f*2^(1/2)*sin(f*x+e)*(2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+arct
an(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)))/(cos(f*x+e)+1)/(-c*(sec
(f*x+e)-1))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.02

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \left[\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\log\left(-\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\sqrt{-\frac{1}{c}}-(3\cos(fx+e)+1)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(fx+e)-4\sqrt{\frac{c\cos(}{\cos}}}{4acf\sin(fx+e)}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*(sqrt(2)*c*sqrt(-1/c)*log(-(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*sqrt(-1/c) - (3*cos(f*x + e) + 1)*s
in(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt
(2)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x
+ e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(1/2),x)

[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec(e + f*x) + c)), x)/a

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)\sqrt{-c\sec(fx+e)+c}}\,dx$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)), x)

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\sqrt{2}\left(\frac{\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{c}\right)}{2af}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(2)*(arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/c)/(a*f)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(1/2)), x)
```

3.91 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}} dx$

Optimal result

Integrand size = 34, antiderivative size = 122

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} - \frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -3/8*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/c^(3/2
)/f*2^(1/2)-3/4*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(3/2)+tan(f*x+e)/f/(a+a*sec
(f*x+e))/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4045, 3881, 3880, 209}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} - \frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-3*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(4*S
qrt[2]*a*c^(3/2)*f) - (3*Tan[e + f*x])/(4*a*f*(c - c*Sec[e + f*x])^(3/2)) +
 Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}+\frac{3\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{2a} \\
&= -\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} \\
&\quad+\frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}+\frac{3\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{8ac} \\
&= -\frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}} \\
&\quad-\frac{3\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx,x,\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{4acf}
\end{aligned}
$$

$$= -\frac{3\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}ac^{3/2}f} - \frac{3\tan(e+fx)}{4af(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan\left(\frac{1}{2}(e$$

$$\overline{2acf\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (Hypergeometric2F1[-1/2, 2, 1/2, (1 + Sec[e + f*x])/2]*Tan[(e + f*x)/2])/(2
*a*c*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.57

method	result
default	$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+3\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)-3\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}-3\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right.\right.}{8af(\cos(fx+e)+1)c(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/8/a/f*2^(1/2)*(2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+3*a
rctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)-3*2^(1/2)*
(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)))/(cos(f*x+e)+1)/c/(sec(f*x+e)-1)/(-c*(sec(f*x+e)-1))^(1/2)
/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.70

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{3\sqrt{2}\sqrt{-c}(\cos(fx+e)-1)\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+\right.}{}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/16*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) - 1)*log((2*sqrt(2)*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*c
os(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x +
 e) + 4*(cos(f*x + e)^2 - 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e)))/((a*c^2*f*cos(f*x + e) - a*c^2*f)*sin(f*x + e)), 1/8*(3*sqrt(2)*sqr
t(c)*(cos(f*x + e) - 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x +
e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(cos(f*x + e)^2 -
 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^2*f*cos(f*x
 + e) - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+c\sqrt{-c\sec(e+fx)+c}}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqr
t(-c*sec(e + f*x) + c)), x)/a
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(3/2)),
x)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(3\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-2\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}\right.}{8\,ac^2f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(3/2),x, algorithm="
giac")
```

```
[Out] 1/8*sqrt(2)*(3*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)) -
 2*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c) - sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/
tan(1/2*f*x + 1/2*e)^2)/(a*c^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(3/2)), x)
```

3.92 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 156

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{15\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} - \frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}} - \frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -15/64*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a/c^(5
/2)/f*2^(1/2)-5/8*tan(f*x+e)/a/f/(c-c*sec(f*x+e))^(5/2)+tan(f*x+e)/f/(a+a*s
ec(f*x+e))/(c-c*sec(f*x+e))^(5/2)-15/32*tan(f*x+e)/a/c/f/(c-c*sec(f*x+e))^(
3/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4045, 3881, 3880, 209}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{15\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} - \frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}} - \frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a\sec(e+fx)+a)(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-15*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(32
*Sqrt[2]*a*c^(5/2)*f) - (5*Tan[e + f*x])/(8*a*f*(c - c*Sec[e + f*x])^(5/2))
 + Tan[e + f*x]/(f*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)) - (15*T
an[e + f*x])/(32*a*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}+\frac{5\int\frac{\sec(e+fx)}{(c-c\sec(e+fx))^{5/2}}\,dx}{2a}\\
&= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}+\frac{15\int\frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{16ac}
\end{aligned}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}} - \frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}} + \frac{15\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{64ac^2}$$

$$= -\frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}} - \frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}} - \frac{15\text{Subst}\left(\int \frac{1}{2c+x^2}dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{32ac^2f}$$

$$= -\frac{15\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{32\sqrt{2}ac^{5/2}f} - \frac{5\tan(e+fx)}{8af(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{f(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}} - \frac{15\tan(e+fx)}{32acf(c-c\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.37

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan\left(\frac{1}{2}\right.\cdots}{4ac^2f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (Hypergeometric2F1[-1/2, 3, 1/2, (1 + Sec[e + f*x])/2]*Tan[(e + f*x)/2])/(4
*a*c^2*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.72

method	result
default	$-\frac{\sqrt{2}\left(3\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)^2+20\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)-15\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)^2-15\sqrt{2}\cdots\right.}{64af(\cos(fx+e)+1)c^2(\sec(fx+e)-1)^2\sqrt{-c\cdots}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/64/a/f*2^(1/2)*(3*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^
2+20*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-15*arctan(1/2*2^
```

(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)^2-15*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+30*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)-15*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)))/(cos(f*x+e)+1)/c^2/(sec(f*x+e)-1)^2/(-c*(sec(f*x+e)-1))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*tan(f*x+e)*sec(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.57

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \Bigg[-\frac{15\sqrt{2}\left(\cos\left(fx+e\right)^2-2\cos\left(fx+e\right)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{}}{}\right.}{}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [-1/128*(15*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) - 4*(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e)), 1/64*(15*sqrt(2)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) + 2*(3*cos(f*x + e)^3 + 20*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin(f*x + e))]

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)-c^2\sqrt{-c\sec\left(e+fx\right)+c}\sec^2\left(e+fx\right)-c^2\sqrt{-c}}}{a}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))**(5/2),x)

[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 - c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x)/a

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)*(-c*sec(f*x + e) + c)^(5/2)),
x)
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(15\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-8\sqrt{c\tan\left(\frac{1}{2}fx\right.}\right.}{64\,a}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c-c*sec(f*x+e))^(5/2),x, algorithm="
giac")
```

```
[Out] 1/64*sqrt(2)*(15*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))
 - 8*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c) - (9*(c*tan(1/2*f*x + 1/2*e)^2 - c)
^(3/2)*c + 7*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2
*e)^4))/(a*c^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)\,{\left(c-\frac{c}{\cos(e+f\,x)}\right)}^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c - c/cos(e + f*x))^(5/2)), x)
```

3.93 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 155

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3a^2f} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\,(a^2+a^2\sec(e+fx))} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] -4*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+2/3*c*(c-c*
sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-64/3*c^4*tan(f*x+e)/a^2/f
/(c-c*sec(f*x+e))^(1/2)-16/3*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a^2/f
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {4039, 3878, 3877}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3a^2f} - \frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f\,(a^2\sec(e+fx)+a^2)} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-64*c^4*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (16*c^3*Sqrt[c
- c*Sec[e + f*x]]*Tan[e + f*x])/(3*a^2*f) - (4*c^2*(c - c*Sec[e + f*x])^(3/
2)*Tan[e + f*x])/(f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c*Sec[e + f*x])^(
5/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{(2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{a+a\sec(e+fx)}\,dx}{a}\\
&= -\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)}+\frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}\\
&\quad+\frac{\left(8c^2\right)\int \sec(e+fx)(c-c\sec(e+fx))^{3/2}\,dx}{a^2}\\
&= -\frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3a^2f}-\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)}\\
&\quad+\frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{\left(32c^3\right)\int \sec(e+fx)\sqrt{c-c\sec(e+fx)}\,dx}{3a^2}
\end{aligned}
$$

$$= -\frac{64c^4\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3a^2f} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f\left(a^2+a^2\sec(e+fx)\right)} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.46

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c^4(-45-69\sec(e+fx)-15\sec^2(e+fx)+\sec^3(e+fx))\tan(e}{3a^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^2,
x]
[Out] (2*c^4*(-45 - 69*Sec[e + f*x] - 15*Sec[e + f*x]^2 + Sec[e + f*x]^3)*Tan[e +
 f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 12.58 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^3c^3(3\cos(fx+e)+1)\left(15\cos(fx+e)^2+18\cos(fx+e)-1\right)\cot(fx+e)^2\csc(fx+e)}{3a^2f(\cos(fx+e)-1)^2}$	89

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNVE
RBOSE)
[Out] 2/3/a^2/f*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^3*c^3*(3*cos(f*x+e)+1)*(
15*cos(f*x+e)^2+18*cos(f*x+e)-1)/(cos(f*x+e)-1)^2*cot(f*x+e)^2*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\left(45\,c^3\cos\left(fx+e\right)^3+69\,c^3\cos\left(fx+e\right)^2+15\,c^3\cos\left(fx+e\right)-c^3\right.}{3\left(a^2f\cos\left(fx+e\right)^2+a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right.}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(45*c^3*cos(f*x + e)^3 + 69*c^3*cos(f*x + e)^2 + 15*c^3*cos(f*x + e) -
c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e)^2 + a^2*f
*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{4\left(16\sqrt{2}c^{\frac{7}{2}} - \frac{56\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{70\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{35\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{4\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8} + \frac{\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}\right)}{3a^2f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{7}{2}}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -4/3*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2
)*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*sqrt(2)*c^(7/2)*sin(f*x +
 e)^8/(cos(f*x + e) + 1)^8 + sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e)
+ 1)^10)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 0.93 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{4\sqrt{2}c^3\left(\frac{9\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c+c^2}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}a^2}-\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}a^4c^2+9\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}a^4c^3}{a^6c^3}\right)}{3f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -4/3*sqrt(2)*c^3*((9*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + c^2)/((c*tan(1/2*f*
x + 1/2*e)^2 - c)^(3/2)*a^2) - ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*a^4*c^
2 + 9*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^4*c^3)/(a^6*c^3))/f
```

Mupad [B] (verification not implemented)

Time = 17.74 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,c^3\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,138\mathrm{i}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,195\mathrm{i}+\mathrm{e}^{e\,3\mathrm{i}}\right.}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1}\right.}$$

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c^3*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e
*1i + f*x*1i)*138i + exp(e*2i + f*x*2i)*195i + exp(e*3i + f*x*3i)*268i + ex
p(e*4i + f*x*4i)*195i + exp(e*5i + f*x*5i)*138i + exp(e*6i + f*x*6i)*45i +
45i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1)^3*(exp(e*1i + f*x*1i) - exp(e*2i +
f*x*2i) + exp(e*3i + f*x*3i) - 1))
```

3.94 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2} dx$

Optimal result

Integrand size = 34, antiderivative size = 123

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2} dx = -\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f(a^2+a^2\sec(e+fx))} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] 2/3*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-16/3*c^3*tan(f
*x+e)/a^2/f/(c-c*sec(f*x+e))^(1/2)-8/3*c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e
)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4039, 3877}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2} dx = -\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a^2\sec(e+fx)+a^2)} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-16*c^3*Tan[e + f*x])/(3*a^2*f*Sqrt[c - c*Sec[e + f*x]]) - (8*c^2*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + (2*c*(c - c
*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{(4c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx}{3a}\\
&=-\frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}+\frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}\\
&\quad+\frac{(8c^2)\int\sec(e+fx)\sqrt{c-c\sec(e+fx)}\,dx}{3a^2}\\
&=-\frac{16c^3\tan(e+fx)}{3a^2f\sqrt{c-c\sec(e+fx)}}-\frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}\\
&\quad+\frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.52

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c^3(11+18\sec(e+fx)+3\sec^2(e+fx))\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] (-2*c^3*(11 + 18*Sec[e + f*x] + 3*Sec[e + f*x]^2)*Tan[e + f*x])/(3*a^2*f*(1
 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 12.52 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{2\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2 c^2 \left(11\cos(fx+e)^2+18\cos(fx+e)+3\right)\cot(fx+e)^2\csc(fx+e)}{3a^2 f(\cos(fx+e)-1)}$	79

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3/a^2/f*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(11*cos(f*x+e)^2+
18*cos(f*x+e)+3)/(cos(f*x+e)-1)*cot(f*x+e)^2*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\left(11\,c^2\cos\left(fx+e\right)^2+18\,c^2\cos\left(fx+e\right)+3\,c^2\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\left(a^2 f\cos\left(fx+e\right)+a^2 f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(11*c^2*cos(f*x + e)^2 + 18*c^2*cos(f*x + e) + 3*c^2)*sqrt((c*cos(f*x +
 e) - c)/cos(f*x + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.33

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\left(8\sqrt{2}c^{\frac{5}{2}} - \frac{20\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{2\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \right.}{3\,a^2f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 2/3*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*sqrt(2)*c
^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - sqrt(2)*c^(5/2)*sin(f*x + e)^8
/(cos(f*x + e) + 1)^8)/(a^2*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.81 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2\sqrt{2}c^2\left(\frac{3\,c}{\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,a^2} - \frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}a^4c^2+6\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,a^4c^3}{a^6c^3}\right)}{3\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] -2/3*sqrt(2)*c^2*(3*c/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^2) - ((c*tan(1/
2*f*x + 1/2*e)^2 - c)^(3/2)*a^4*c^2 + 6*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*
a^4*c^3)/(a^6*c^3))/f
```

Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,c^2\,\sqrt{c-\frac{c}{\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,36{}\mathrm{i}+{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,34{}\mathrm{i}+{\mathrm{e}}^{e\,3{}\mathrm{i}+\dots}\right.}{3\,a^2\,f\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}-1\right)\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+1\right)}$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c^2*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e
*1i + f*x*1i)*36i + exp(e*2i + f*x*2i)*34i + exp(e*3i + f*x*3i)*36i + exp(e
*4i + f*x*4i)*11i + 11i))/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x
*1i) + 1)^3)
```

3.95 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 89

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{4c^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] -4/3*c^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)+2/3*c*(c-
c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4039, 4038}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{3f(a\sec(e+fx)+a)^2} - \frac{4c^2\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-4*c^2*Tan[e + f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]
]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^
2)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{(2c)\int\frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx}{3a}$$

$$= -\frac{4c^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} + \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2c^2(1+3\sec(e+fx))\tan(e+fx)}{3a^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^2,
x]
[Out] (-2*c^2*(1 + 3*Sec[e + f*x])*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])^2*Sq
rt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

method	result	size
default	$\frac{2\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)c(\cos(fx+e)+3)\cot(fx+e)^{2}\csc(fx+e)}{3a^{2}f}$	53

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3/a^2/f*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)*c*(cos(f*x+e)+3)*cot(f*x
+e)^2*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\left(c\cos\left(fx+e\right)^{2}+3\,c\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\left(a^{2}f\cos\left(fx+e\right)+a^{2}f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] 2/3*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx + \int \left(-\frac{c\sqrt{-c\sec\left(e+fx\right)+c}\sec^{2}\left(e+fx\right)}{\sec^{2}\left(e+fx\right)+2\sec\left(e+fx\right)+1}\right)dx}{a^{2}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec
(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2/
(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2\sqrt{2}c^{\frac{3}{2}} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}{3\,a^2 f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -1/3*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + sqrt(2)*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^2*f*(sin(f*x
 + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(
3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^{\frac{3}{2}}}{a^2} + \frac{3\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}\,c}{a^2}}{3\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/3*(sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)/a^2 + 3*sqrt(2)*sqrt(c*ta
n(1/2*f*x + 1/2*e)^2 - c)*c/a^2)/f
```

Mupad [B] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,c\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1i-f\,x\,1i}}{2}+\frac{\mathrm{e}^{e\,1i+f\,x\,1i}}{2}}}\,\left(\mathrm{e}^{e\,1i+f\,x\,1i}\,6i+\mathrm{e}^{e\,2i+f\,x\,2i}\,2i+\mathrm{e}^{e\,3i+f\,x\,3}\right.}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1i+f\,x\,1i}-1\right)\,{\left(\mathrm{e}^{e\,1i+f\,x\,1i}+1\right)}^3}$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*c*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*(exp(e*1
i + f*x*1i)*6i + exp(e*2i + f*x*2i)*2i + exp(e*3i + f*x*3i)*6i + exp(e*4i +
 f*x*4i)*1i + 1i))/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) +
1)^3)
```

3.96 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2/3*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4038}

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = \frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
[Out] (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = -\frac{\cos^2(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{6a^2f}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^2,x]
[Out] -1/6*(Cos[e + f*x]^2*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e +
 f*x]])/(a^2*f)
```

Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{2\sqrt{-c(\sec(fx+e)-1)}\,\cos(fx+e)\cot(fx+e)}{3a^2f(\cos(fx+e)+1)}$	44

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x,method=_RETURNVE
RBOSE)
[Out] -2/3/a^2/f*(-c*(sec(f*x+e)-1))^(1/2)/(cos(f*x+e)+1)*cos(f*x+e)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = -\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos{(fx+e)}^2}{3\,(a^2f\cos{(fx+e)}+a^2f)\sin{(fx+e)}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
[Out] -2/3*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2/((a^2*f*cos(f*x
 + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x)/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(37) = 74$.

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.66

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = -\frac{\sqrt{2}\sqrt{c} - \frac{2\sqrt{2}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{\sqrt{2}\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{6a^2f\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] -1/6*(sqrt(2)*sqrt(c) - 2*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(a^2*f*sqrt(sin(f
*x + e)/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = \frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}(\cos(fx+e))}{6a^2cf}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] 1/6*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*sgn(tan(1/2*f*x + 1/2*e)^3
 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a^2*c*f)
```

Mupad [B] (verification not implemented)

Time = 17.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.29

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx = \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^2\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{3\,a^2\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3}$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] ((exp(e*2i + f*x*2i)*1i + 1i)^2*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2))^(1/2)*2i)/(3*a^2*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^3)
```

3.97 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 138

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{2f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/4*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/f*2^
(1/2)/c^(1/2)+1/3*tan(f*x+e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+1/
2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {4045, 3880, 209}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} + \frac{\tan(e+fx)}{2f\left(a^2\sec(e+fx)+a^2\right)\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqr
t[2]*a^2*Sqrt[c]*f) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*S
ec[e + f*x]]) + Tan[e + f*x]/(2*f*(a^2 + a^2*Sec[e + f*x])*Sqrt[c - c*Sec[e
 + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}+\frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx}{2a}\\
&= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{\tan(e+fx)}{2f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}+\frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{4a^2}\\
&= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{\tan(e+fx)}{2f\left(a^2+a^2\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}\\
&\quad-\frac{\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx,x,\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{2a^2f}
\end{aligned}
$$

$$= -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{2f(a^2+a^2\sec(e+fx))\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.43

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{3f(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Hypergeometric2F1[-3/2, 1, -1/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(3*f*
(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.33

method	result
default	$\frac{\sqrt{2}\left(-\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}+3\arctan\left(\frac{1}{\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\right)+3\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)(-\cot($
	$12a^2f\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/12/a^2/f*2^(1/2)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*cs
c(f*x+e)^2)^(1/2)*(-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)+3*arctan(1/((1-
cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))+3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1
/2))/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{3\sqrt{2}\sqrt{-c}(\cos(fx+e)+1)\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3\,c\cos(fx+e)+c)\sin(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right)\sin(f}{24\left(a^2cf\cos(fx+e)+a^2cf\right)\sin(fx+e)}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*sqrt(-c)*(cos(f*x + e) + 1)*log((2*sqrt(2)*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*c
os(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x +
 e) + 4*(5*cos(f*x + e)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e)))/((a^2*c*f*cos(f*x + e) + a^2*c*f)*sin(f*x + e)), 1/12*(3*sqrt(2)*
sqrt(c)*(cos(f*x + e) + 1)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e
)^2 + 3*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c*f*co
s(f*x + e) + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+2\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a^2}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + 2*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**2
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^2\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*sqrt(-c*sec(f*x + e) + c)),
x)
```

Giac [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\sqrt{2}\left(\frac{3\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^4-3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^5}{c^6}\right)}{12a^2f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/12*sqrt(2)*(3*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c)
+ ((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^4 - 3*sqrt(c*tan(1/2*f*x + 1/2*e)
^2 - c)*c^5)/c^6)/(a^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+fx)\left(a+\frac{a}{\cos(e+fx)}\right)^2\sqrt{c-\frac{c}{\cos(e+fx)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(1/2)), x)
```

3.98 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 169

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{5\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} - \frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{5\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -5/16*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/c^(
3/2)/f*2^(1/2)-5/8*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(3/2)+1/3*tan(f*x+e)/f
/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2)+5/6*tan(f*x+e)/f/(a^2+a^2*sec(f*
x+e))/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4045, 3881, 3880, 209}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{5\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} - \frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{5\tan(e+fx)}{6f\left(a^2\sec(e+fx)+a^2\right)(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-5*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(8*S
qrt[2]*a^2*c^(3/2)*f) - (5*Tan[e + f*x])/(8*a^2*f*(c - c*Sec[e + f*x])^(3/2
)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)) +
 (5*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}+\frac{5\int\frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx}{6a}\\
&= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{5\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}+\frac{5\int\frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{4a^2}
\end{aligned}$$

$$= -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{5\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}} + \frac{5\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}dx}{16a^2c}$$

$$= -\frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{5\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}} - \frac{5\text{Subst}\left(\int \frac{1}{2c+x^2}dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{8a^2cf}$$

$$= -\frac{5\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} - \frac{5\tan(e+fx)}{8a^2f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{5\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.49 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.38

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},2,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{6a^2cf(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/2)),
x]
[Out] (Hypergeometric2F1[-3/2, 2, -1/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(6*a^
2*c*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.18

method	result
default	$\frac{\sqrt{2}\left(-13\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)^2+15\sin(fx+e)^2\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)+10\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)+15\sqrt{2}\sqrt{}\right.}{48a^2f(\sec(fx+e)-1)\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{-c(\sec(fx+e)-1)}\,c(\cos(fx+e)+1)^2}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
```

[Out] 1/48/a^2/f*2^(1/2)*(-13*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2+15*sin(f*x+e)^2*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+10*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+15*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))/(sec(f*x+e)-1)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(cos(f*x+e)+1)^2*tan(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.18

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{15\sqrt{2}\left(\cos(fx+e)^2-1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(f\right.}{}\right.}{}\right.$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] [-1/96*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e)), 1/48*(15*sqrt(2)*(cos(f*x + e)^2 - 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(13*cos(f*x + e)^3 - 10*cos(f*x + e)^2 - 15*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec(e+fx)}{-c\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)-c\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+c\sqrt{-c}\cdots}}{a^2}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(3/2),x)

[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**2

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(3/2))
, x)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.76

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{2}\left(15\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-\frac{3\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right.}{48a^2c}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/48*sqrt(2)*(15*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))
 - 3*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/tan(1/2*f*x + 1/2*e)^2 + 2*((c*tan(
1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^2 - 6*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^
3)/c^3)/(a^2*c^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx=\int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^2\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(3/2)), x)
```

3.99 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 203

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{35\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} - \frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{7\tan(e+fx)}{6f\,(a^2+a^2\sec(e+fx))\,(c-c\sec(e+fx))^{5/2}} - \frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -35/128*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^2/c
^(5/2)/f*2^(1/2)-35/48*tan(f*x+e)/a^2/f/(c-c*sec(f*x+e))^(5/2)+1/3*tan(f*x+
e)/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2)+7/6*tan(f*x+e)/f/(a^2+a^2*se
c(f*x+e))/(c-c*sec(f*x+e))^(5/2)-35/64*tan(f*x+e)/a^2/c/f/(c-c*sec(f*x+e))^
(3/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4045, 3881, 3880, 209}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{35\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f} - \frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}} - \frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}} + \frac{7\tan(e+fx)}{6f\left(a^2\sec(e+fx)+a^2\right)(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{3f(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-35*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(64
*Sqrt[2]*a^2*c^(5/2)*f) - (35*Tan[e + f*x])/(48*a^2*f*(c - c*Sec[e + f*x])^
(5/2)) + Tan[e + f*x]/(3*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2
)) + (7*Tan[e + f*x])/(6*f*(a^2 + a^2*Sec[e + f*x])*(c - c*Sec[e + f*x])^(5
/2)) - (35*Tan[e + f*x])/(64*a^2*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
```

```
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}+\frac{7\int\frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx}{6a}\\
&=\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{7\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}+\frac{35\int\frac{\sec(e+fx)}{(c-c\sec(e+fx))^{5/2}}\,dx}{12a^2}\\
&=-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{7\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}+\frac{35\int\frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{32a^2c}\\
&=-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{7\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}+\frac{35\int\frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{128a^2c^2}\\
&=-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{7\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}-\frac{35\text{Subst}\left(\int\frac{1}{2c+x^2}\,dx,x,\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{64a^2c^2f}\\
&=-\frac{35\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{64\sqrt{2}a^2c^{5/2}f}-\frac{35\tan(e+fx)}{48a^2f(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{\tan(e+fx)}{3f(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{7\tan(e+fx)}{6f\left(a^2+a^2\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}-\frac{35\tan(e+fx)}{64a^2cf(c-c\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.80 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.32

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{12a^2c^2f(1+\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(5/2)),
x]
```

```
[Out] (Hypergeometric2F1[-3/2, 3, -1/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(12*a
^2*c^2*f*(1 + Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.38

method	result
default	$-\frac{\sqrt{2}\left(-43\cos(fx+e)^3\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+105\sin(fx+e)^2\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx+e)+161\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(\ldots\right.}{384a^2f\sqrt{-c(\sec(fx+e)-1)}(\sec(f\ldots}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/384/a^2/f*2^(1/2)*(-43*cos(f*x+e)^3*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)+105*sin(f*x+e)^2*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/
2))*cos(f*x+e)+161*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-
105*sin(f*x+e)^2*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))+35*
2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-105*2^(1/2)*(-cos(f*x
+e)/(cos(f*x+e)+1))^(1/2))/(-c*(sec(f*x+e)-1))^(1/2)/(sec(f*x+e)-1)^2/(-cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)/c^2/(cos(f*x+e)+1)^2*tan(f*x+e)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{105\sqrt{2}\left(\cos{(fx+e)}^3-\cos{(fx+e)}^2-\cos{(fx+e)}+\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/768*(105*sqrt(2)*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*s
qrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f
*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(43*cos(f*x + e)^4 - 161*cos(f
*x + e)^3 - 35*cos(f*x + e)^2 + 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)
/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2
*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)), 1/384*(105*sqrt(2)*(cos(f*x
 + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c
*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f
*x + e) - 2*(43*cos(f*x + e)^4 - 161*cos(f*x + e)^3 - 35*cos(f*x + e)^2 + 1
05*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f
*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*
sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\int \frac{\sec{(e+fx)}}{c^2\sqrt{-c\sec{(e+fx)}+c}\sec^4{(e+fx)}-2c^2\sqrt{-c\sec{(e+fx)}+c}\sec^2{(e+fx)}+c^2\sqrt{}}}{a^2}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c
**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c**2*sqrt(-c*sec(e + f*x) +
 c)), x)/a**2
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^2(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^2*(-c*sec(f*x + e) + c)^(5/2))
, x)
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.79

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(105\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)+\frac{8\left(\left(c\tan\left(\frac{1}{2}fx+\right.\right.\right.}{}\right.}{}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/384*sqrt(2)*(105*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c
)) + 8*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^2 - 9*sqrt(c*tan(1/2*f*x + 1
/2*e)^2 - c)*c^3)/c^3 - 3*(13*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 11*s
qrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4))/(a^2*c
^3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+fx)\left(a+\frac{a}{\cos(e+fx)}\right)^2\left(c-\frac{c}{\cos(e+fx)}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))^(5/2)), x)
```

3.100 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3} dx$

Optimal result

Integrand size = 34, antiderivative size = 169

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3} dx = \frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}} + \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a^3+a^3\sec(e+fx))} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

```
[Out] -4/5*c^2*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+2/5*c*(c-
c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+32/5*c^4*tan(f*x+e)/a^3
/f/(c-c*sec(f*x+e))^(1/2)+16/5*c^3*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a^3
+a^3*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4039, 3877}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3} dx = \frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}} + \frac{16c^3\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a^3\sec(e+fx)+a^3)} - \frac{4c^2\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5af(a\sec(e+fx)+a)^2} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (32*c^4*Tan[e + f*x])/(5*a^3*f*Sqrt[c - c*Sec[e + f*x]]) + (16*c^3*Sqrt[c -
 c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a^3 + a^3*Sec[e + f*x])) - (4*c^2*(c -
 c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*a*f*(a + a*Sec[e + f*x])^2) + (2*c*
(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(6c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^2}\,dx}{5a}$$

$$= -\frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(8c^2)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{a+a\sec(e+fx)}\,dx}{5a^2}$$

$$= \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(16c^3)\int\sec(e+fx)\sqrt{c-c\sec(e+fx)}\,dx}{5a^3}$$

$$= \frac{32c^4\tan(e+fx)}{5a^3f\sqrt{c-c\sec(e+fx)}} + \frac{16c^3\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f\left(a^3+a^3\sec(e+fx)\right)} - \frac{4c^2(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.44

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c^4(23+55\sec(e+fx)+45\sec^2(e+fx)+5\sec^3(e+fx))\tan(e+fx)}{5a^3f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(7/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] (2*c^4*(23 + 55*Sec[e + f*x] + 45*Sec[e + f*x]^2 + 5*Sec[e + f*x]^3)*Tan[e
+ f*x])/(5*a^3*f*(1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 36.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{2\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^3 c^3\left(23\cos(fx+e)^3+55\cos(fx+e)^2+45\cos(fx+e)+5\right)\cot(fx+e)^3\csc(fx+e)^2}{5a^3 f(\cos(fx+e)-1)}$	91

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/5/a^3/f*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^3*c^3*(23*cos(f*x+e)^3+5
5*cos(f*x+e)^2+45*cos(f*x+e)+5)/(cos(f*x+e)-1)*cot(f*x+e)^3*csc(f*x+e)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.64

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\left(23c^3\cos(fx+e)^3+55c^3\cos(fx+e)^2+45c^3\cos(fx+e)+5c^3\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{5\left(a^3f\cos(fx+e)^2+2a^3f\cos(fx+e)+a^3f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/5*(23*c^3*cos(f*x + e)^3 + 55*c^3*cos(f*x + e)^2 + 45*c^3*cos(f*x + e) +
 5*c^3)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*
a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(7/2)/(a+a*sec(f*x+e))**3,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.27

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2\left(16\sqrt{2}c^{\frac{7}{2}} - \frac{56\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{70\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{35\sqrt{2}c^{\frac{7}{2}}\sin(fx+e)}{(\cos(fx+e)+1)^6}\right.}{5a^3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{7}{2}}\left(\frac{}{c}\right.}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
[Out] 2/5*(16*sqrt(2)*c^(7/2) - 56*sqrt(2)*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + 70*sqrt(2)*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 35*sqrt(2)
*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(7/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8 - sqrt(2)*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) +
 1)^10 + sqrt(2)*c^(7/2)*sin(f*x + e)^12/(cos(f*x + e) + 1)^12)/(a^3*f*(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)^(7/2)*(sin(f*x + e)/(cos(f*x + e) + 1) -
1)^(7/2))
```

Giac [A] (verification not implemented)

none

Time = 1.02 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.75

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2\sqrt{2}c^3\left(\frac{5c}{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}a^3} - \frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}a^{12}c^8+5\left(c\tan\left(\frac{1}{2}fx+\right.\right.}{a}\right.}{5f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
[Out] 2/5*sqrt(2)*c^3*(5*c/(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^3) - ((c*tan(1/2
*f*x + 1/2*e)^2 - c)^(5/2)*a^12*c^8 + 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2
)*a^12*c^9 + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*a^12*c^10)/(a^15*c^10))/
f
```

Mupad [B] (verification not implemented)

Time = 22.30 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.91

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{7/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\left(\frac{c^3\,46i}{5\,a^3\,f}+\frac{c^3\,e^{e\,1i+f\,x\,1i}\,4i}{a^3\,f}+\frac{c^3\,e^{e\,2i+f\,x\,2i}\,46i}{5\,a^3\,f}\right)}{\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,1i+f\,x\,1i}+1\right)} - \frac{c^3\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,16i}{5\,a^3\,f\,\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,1i+f\,x\,1i}+1\right)^2} - \frac{c^3\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,48i}{5\,a^3\,f\,\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,1i+f\,x\,1i}+1\right)^3} + \frac{c^3\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,128i}{5\,a^3\,f\,\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,1i+f\,x\,1i}+1\right)^4} - \frac{c^3\left(e^{e\,2i+f\,x\,2i}+1\right)\sqrt{c-\frac{c}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}}\,64i}{5\,a^3\,f\,\left(e^{e\,1i+f\,x\,1i}-1\right)\left(e^{e\,1i+f\,x\,1i}+1\right)^5}$$

```
[In] int((c - c/cos(e + f*x))^(7/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2)*128i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^4) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*
1i + f*x*1i) + 1)^2) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f
*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*48i)/(5*a^3*f*(exp(e*1i + f*x*1i) -
 1)*(exp(e*1i + f*x*1i) + 1)^3) - ((c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1
i + f*x*1i)/2))^(1/2)*((c^3*46i)/(5*a^3*f) + (c^3*exp(e*1i + f*x*1i)*4i)/(a
^3*f) + (c^3*exp(e*2i + f*x*2i)*46i)/(5*a^3*f)))/((exp(e*1i + f*x*1i) - 1)*
(exp(e*1i + f*x*1i) + 1)) - (c^3*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1
i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(e*1i + f*x*
1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

3.101 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 135

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3} \, dx = \frac{16c^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

```
[Out] 2/5*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+16/15*c^3*tan(
f*x+e)/f/(a^3+a^3*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)-8/15*c^2*(c-c*sec(f*x+
e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4039, 4038}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3} \, dx = \frac{16c^3\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}} - \frac{8c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{15af(a\sec(e+fx)+a)^2} + \frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (16*c^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x
]]) - (8*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(15*a*f*(a + a*Sec[e +
f*x])^2) + (2*c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(5*f*(a + a*Sec[e
+ f*x])^3)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{(4c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^2}\,dx}{5a} \\
&= -\frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{15af(a+a\sec(e+fx))^2} \\
&\quad + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(8c^2)\int\frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{a+a\sec(e+fx)}\,dx}{15a^2} \\
&= \frac{16c^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{8c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{2c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c^3(7+10\sec(e+fx)+15\sec^2(e+fx))\tan(e+fx)}{15a^3f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] (2*c^3*(7 + 10*Sec[e + f*x] + 15*Sec[e + f*x]^2)*Tan[e + f*x])/(15*a^3*f*(1
 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 41.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{2\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^{2} c^{2} \left(7\cos(fx+e)^{2}+10\cos(fx+e)+15\right)\cot(fx+e)^{3}\csc(fx+e)^{2}}{15a^{3}f}$	71

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15/a^3/f*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(7*cos(f*x+e)^2+
10*cos(f*x+e)+15)*cot(f*x+e)^3*csc(f*x+e)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.77

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\left(7\,c^{2}\cos\left(fx+e\right)^{3}+10\,c^{2}\cos\left(fx+e\right)^{2}+15\,c^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{15\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/15*(7*c^2*cos(f*x + e)^3 + 10*c^2*cos(f*x + e)^2 + 15*c^2*cos(f*x + e))*
sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*co
s(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.40

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{8\sqrt{2}c^{\frac{5}{2}} - \frac{20\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{15\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{5\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8} - \frac{3\sqrt{2}c^{\frac{5}{2}}\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}}{15\,a^3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/15*(8*sqrt(2)*c^(5/2) - 20*sqrt(2)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 15*sqrt(2)*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5*sqrt(2)
*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 5*sqrt(2)*c^(5/2)*sin(f*x +
e)^8/(cos(f*x + e) + 1)^8 - 3*sqrt(2)*c^(5/2)*sin(f*x + e)^10/(cos(f*x + e)
 + 1)^10)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(5/2)*(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.86 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\frac{15\sqrt{2}\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}c^2}{a^3} + \frac{3\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}+10\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c}{a^3}}{15\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/15*(15*sqrt(2)*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2/a^3 + (3*sqrt(2)*(
c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 10*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2
- c)^(3/2)*c)/a^3)/f
```

Mupad [B] (verification not implemented)

Time = 19.27 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.38

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{c^2\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,14\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)} + \frac{c^2\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^2} - \frac{c^2\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,112\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3} + \frac{c^2\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,64\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^4} - \frac{c^2\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,32\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^5}$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f
*x*1i)/2))^(1/2)*16i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)^2) - (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(1/2)*14i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e
*1i + f*x*1i) + 1)) - (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*
x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*112i)/(15*a^3*f*(exp(e*1i + f*x*1i)
- 1)*(exp(e*1i + f*x*1i) + 1)^3) + (c^2*(exp(e*2i + f*x*2i) + 1)*(c - c/(ex
p(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*64i)/(5*a^3*f*(exp(e*1i
 + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^4) - (c^2*(exp(e*2i + f*x*2i) + 1)
*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*32i)/(5*a^3*
f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

3.102 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 88

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{4c^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}$$

```
[Out] -4/15*c^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+2/5*c*(c
-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4039, 4038}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{5f(a\sec(e+fx)+a)^3} - \frac{4c^2\tan(e+fx)}{15af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-4*c^2*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e + f*x
]]) + (2*c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])
^3)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{(2c)\int\frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^2}\,dx}{5a}\\ &=-\frac{4c^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}+\frac{2c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx=-\frac{2c^2(-1+5\sec(e+fx))\tan(e+fx)}{15a^3f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^3,
x]
```

```
[Out] (-2*c^2*(-1 + 5*Sec[e + f*x])*Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^3*
Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{2(\sec(fx+e)-1)(\cos(fx+e)-5)\sqrt{-c(\sec(fx+e)-1)}\,c\cos(fx+e)^{2}\cot(fx+e)}{15a^{3}f(\cos(fx+e)+1)^{2}(\cos(fx+e)-1)}$	73

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/15/a^3/f*(sec(f*x+e)-1)*(cos(f*x+e)-5)*(-c*(sec(f*x+e)-1))^(1/2)*c/(cos(f
*x+e)+1)^2/(cos(f*x+e)-1)*cos(f*x+e)^2*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\left(c\cos\left(fx+e\right)^{3}-5\,c\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{15\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -2/15*(c*cos(f*x + e)^3 - 5*c*cos(f*x + e)^2)*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x +
 e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \frac{c\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \left(-\frac{c\sqrt{-c\sec(e+fx)+c}\,\mathrm{se}}{\sec^3(e+fx)+3\sec^2(e+fx)}\right.}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec
(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(-c*sqrt(-c*sec(e + f*x) +
 c)*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) +
 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(80) = 160.

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.85

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\sqrt{2}c^{\frac{3}{2}} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{7\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{3\sqrt{2}c^{\frac{3}{2}}\sin(fx+e)^8}{(\cos(fx+e)+1)^8}}{30\,a^3 f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/30*(2*sqrt(2)*c^(3/2) - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7*sqrt(2)*c
^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 3*sqrt(2)*c^(3/2)*sin(f*x + e)
^8/(cos(f*x + e) + 1)^8)/(a^3*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)
*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\sqrt{2}\left(3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}+5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c\right)}{30\,a^3cf}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/30*sqrt(2)*(3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2) + 5*(c*tan(1/2*f*x +
1/2*e)^2 - c)^(3/2)*c)/(a^3*c*f)
```

Mupad [B] (verification not implemented)

Time = 18.94 (sec) , antiderivative size = 446, normalized size of antiderivative = 5.07

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^3}\,dx = -\frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)} + \frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,28\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^2} - \frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,76\mathrm{i}}{15\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^3} + \frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,32\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^4} - \frac{c\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)^5}$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x
*1i)/2))^(1/2)*28i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i)
+ 1)^2) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(
e*1i + f*x*1i)/2))^(1/2)*2i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i +
 f*x*1i) + 1)) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2
 + exp(e*1i + f*x*1i)/2))^(1/2)*76i)/(15*a^3*f*(exp(e*1i + f*x*1i) - 1)*(ex
p(e*1i + f*x*1i) + 1)^3) + (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i -
 f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*32i)/(5*a^3*f*(exp(e*1i + f*x*1i)
 - 1)*(exp(e*1i + f*x*1i) + 1)^4) - (c*(exp(e*2i + f*x*2i) + 1)*(c - c/(exp
(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i
+ f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)^5)
```

3.103 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2/5*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4038}

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = \frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\cos^3(e+fx)\csc\left(\frac{1}{2}(e+fx)\right)\sec^5\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{20a^3f}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^3,x]
[Out] -1/20*(Cos[e + f*x]^3*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^5*Sqrt[c - c*Sec[e
+ f*x]])/(a^3*f)
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{2\sqrt{-c(\sec(fx+e)-1)}\,\cos(fx+e)^2\cot(fx+e)}{5a^3f(\cos(fx+e)+1)^2}$	46

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x,method=_RETURNVE
RBOSE)
[Out] -2/5/a^3/f*(-c*(sec(f*x+e)-1))^(1/2)/(cos(f*x+e)+1)^2*cos(f*x+e)^2*cot(f*x+
e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{2\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^3}{5\left(a^3f\cos(fx+e)^2+2a^3f\cos(fx+e)+a^3f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="fricas")
[Out] -2/5*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^3/((a^3*f*cos(f*x
 + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \frac{\sqrt{-c\sec(e+fx)+c}\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**3,x)
```

```
[Out] Integral(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x)/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(37) = 74$.

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.32

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\sqrt{2}\sqrt{c} - \frac{3\sqrt{2}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3\sqrt{2}\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{\sqrt{2}\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}{20\,a^3 f\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="maxima")
```

```
[Out] -1/20*(sqrt(2)*sqrt(c) - 3*sqrt(2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 3*sqrt(2)*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - sqrt(2)*sqrt(
c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/(a^3*f*sqrt(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\sqrt{2}\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)\operatorname{sgn}(\cos(fx+e))}{20\,a^3c^2f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] -1/20*sqrt(2)*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*sgn(tan(1/2*f*x + 1/2*e)
^3 + tan(1/2*f*x + 1/2*e))*sgn(cos(f*x + e))/(a^3*c^2*f)
```

Mupad [B] (verification not implemented)

Time = 19.91 (sec) , antiderivative size = 441, normalized size of antiderivative = 10.76

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^3}\,dx = -\frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,2\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)} + \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,8\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^2} - \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^3} + \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,16\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^4} - \frac{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{c-\frac{c}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,8\mathrm{i}}{5\,a^3\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)}^5}$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(1/2)*8i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i) + 1)
^2) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*1i +
 f*x*1i)/2))^(1/2)*2i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f*x*1i
) + 1)) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e*1i + f
*x*1i) + 1)^3) + ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*1i)/2 +
 exp(e*1i + f*x*1i)/2))^(1/2)*16i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*(exp(e
*1i + f*x*1i) + 1)^4) - ((exp(e*2i + f*x*2i) + 1)*(c - c/(exp(- e*1i - f*x*
1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*8i)/(5*a^3*f*(exp(e*1i + f*x*1i) - 1)*
(exp(e*1i + f*x*1i) + 1)^5)
```

3.104 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 181

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx &= -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\\&\quad+\frac{\tan(e+fx)}{6af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\\&\quad+\frac{\tan(e+fx)}{4f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] -1/8*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/f*2^
(1/2)/c^(1/2)+1/5*tan(f*x+e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2)+1/
6*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2/(c-c*sec(f*x+e))^(1/2)+1/4*tan(f*x+e)/f
/(a^3+a^3*sec(f*x+e))/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used

$= \{4045, 3880, 209\}$

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} + \frac{\tan(e+fx)}{4f\left(a^3\sec(e+fx)+a^3\right)\sqrt{c-c\sec(e+fx)}}$$
$$+ \frac{\tan(e+fx)}{6af(a\sec(e+fx)+a)^2\sqrt{c-c\sec(e+fx)}}$$
$$+ \frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])]/(Sqr
t[2]*a^3*Sqrt[c]*f) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*Sqrt[c - c*S
ec[e + f*x]]) + Tan[e + f*x]/(6*a*f*(a + a*Sec[e + f*x])^2*Sqrt[c - c*Sec[e
 + f*x]]) + Tan[e + f*x]/(4*f*(a^3 + a^3*Sec[e + f*x])*Sqrt[c - c*Sec[e + f
*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\text{integral} = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}}\,dx}{2a}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{6af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))\sqrt{c-c\sec(e+fx)}}\,dx}{4a^2}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{6af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{8a^3}$$

$$= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{6af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{4a^3f}$$

$$= -\frac{\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{4\sqrt{2}a^3\sqrt{c}f} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{6af(a+a\sec(e+fx))^2\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4f\left(a^3+a^3\sec(e+fx)\right)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.75 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.33

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan(e+fx)}{5f(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]]),x]
[Out] (Hypergeometric2F1[-5/2, 1, -3/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(5*f*
(a + a*Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17

method	result
default	$\frac{\sqrt{2}\left(3\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{5}{2}}-5\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^{\frac{3}{2}}+15\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}+15\arctan\left(\right.\right.}{120a^3f\sqrt{\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}$

[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/120/a^3/f*2^(1/2)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)^2)^(1/2)*(3*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(5/2)-5*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(3/2)+15*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)+15*arctan(1/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)))/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-cot(f*x+e)+csc(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.22

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\left[-\frac{15\sqrt{2}\left(\cos(fx+e)^2+2\cos(fx+e)+1\right)\sqrt{-c}\log\left(\frac{2\sqrt{2}\left(\cos(fx+e)^2+\cos(fx+e)\right)\sqrt{-c}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+(3c\cos(fx+e)}{(\cos(fx+e)-1)\sin(fx+e)}\right.}{240\left(a^3cf\cos(fx+e)^2+2a^3cf\cos\right.}\right.$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/240*(15*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e)), 1/120*(15*sqrt(2)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(37*cos(f*x + e)^3 + 40*cos(f*x + e)^2 + 15*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin(f*x + e))]

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\frac{\int \frac{\sec(e+fx)}{\sqrt{-c\sec(e+fx)+c}\sec^3(e+fx)+3\sqrt{-c\sec(e+fx)+c}\sec^2(e+fx)+3\sqrt{-c\sec(e+fx)+c}\sec(e+fx)+\sqrt{-c\sec(e+fx)+c}}\,dx}{a^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 3*sqrt(-
c*sec(e + f*x) + c)*sec(e + f*x)**2 + 3*sqrt(-c*sec(e + f*x) + c)*sec(e + f
*x) + sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{(a\sec(fx+e)+a)^3\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*sqrt(-c*sec(f*x + e) + c)),
x)
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx$$
$$=\frac{\sqrt{2}\left(\frac{15\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{3\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{5}{2}}c^{12}-5\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^{\frac{3}{2}}c^{13}+15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}\,c^{14}}{c^{15}}\right)}{120\,a^3f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/120*sqrt(2)*(15*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c))/sqrt(c
) - (3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^12 - 5*(c*tan(1/2*f*x + 1/2*e
)^2 - c)^(3/2)*c^13 + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^14)/c^15)/(a^
3*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^3\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(1/2)), x)
```

3.105 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 212

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{7\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} \\ &\quad - \frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} \\ &\quad + \frac{7\tan(e+fx)}{30af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} \\ &\quad + \frac{7\tan(e+fx)}{12f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}\end{aligned}$$

```
[Out] -7/32*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/c^(
3/2)/f*2^(1/2)-7/16*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(3/2)+1/5*tan(f*x+e)/
f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2)+7/30*tan(f*x+e)/a/f/(a+a*sec(f*
x+e))^2/(c-c*sec(f*x+e))^(3/2)+7/12*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))/(c-c*
sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used

$= \{4045, 3881, 3880, 209\}$

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{7\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} - \frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{7\tan(e+fx)}{12f\,(a^3\sec(e+fx)+a^3)\,(c-c\sec(e+fx))^{3/2}} + \frac{7\tan(e+fx)}{30af(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (-7*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(16*
Sqrt[2]*a^3*c^(3/2)*f) - (7*Tan[e + f*x])/(16*a^3*f*(c - c*Sec[e + f*x])^(3
/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2))
 + (7*Tan[e + f*x])/(30*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^(3/
2)) + (7*Tan[e + f*x])/(12*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*x])^
(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
```

```
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\text{integral} = \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}+\frac{7\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}\,dx}{10a}$$

$$=\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{30af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}+\frac{7\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{3/2}}\,dx}{12a^2}$$

$$=\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{30af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{12f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}+\frac{7\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{8a^3}$$

$$=-\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{30af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{12f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}+\frac{7\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{32a^3c}$$

$$=-\frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{30af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}}+\frac{7\tan(e+fx)}{12f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{3/2}}-\frac{7\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx,x,\frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{16a^3cf}$$

$$= -\frac{7\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{16\sqrt{2}a^3c^{3/2}f} - \frac{7\tan(e+fx)}{16a^3f(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}} + \frac{7\tan(e+fx)}{30af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{3/2}} + \frac{7\tan(e+fx)}{12f\,(a^3+a^3\sec(e+fx))\,(c-c\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.84 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.30

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},2,-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan(e}{10a^3cf(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(3/2)),
x]
[Out] (Hypergeometric2F1[-5/2, 2, -3/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(10*a
^3*c*f*(1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.30

method	result
default	$-\frac{\sqrt{2}\left(139\cos(fx+e)^3\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+21\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)^2-105\sin(fx+e)^2\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)\cos(fx\right.}{480a^3f\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{-c(\sec(fx+e)-}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x,method=_RETURNVE
RBOSE)
[Out] -1/480/a^3/f*2^(1/2)*(139*cos(f*x+e)^3*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)+21*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-105*sin(f
*x+e)^2*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))*cos(f*x+e)-1
75*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-105*sin(f*x+e)^2*a
rctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-105*2^(1/2)*(-cos(f*x
+e)/(cos(f*x+e)+1))^(1/2))/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(-c*(sec(f*x+
e)-1))^(1/2)/(sec(f*x+e)-1)/c/(cos(f*x+e)+1)^3*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.28

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{105\sqrt{2}\left(\cos\left(fx+e\right)^3+\cos\left(fx+e\right)^2-\cos\left(fx+e\right)\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/960*(105*sqrt(2)*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*s
qrt(-c)*log((2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f
*x + e) - 1)*sin(f*x + e)))*sin(f*x + e) + 4*(139*cos(f*x + e)^4 + 21*cos(f
*x + e)^3 - 175*cos(f*x + e)^2 - 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c
)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^
3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)), 1/480*(105*sqrt(2)*(cos(f*
x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(c)*arctan(sqrt(2)*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(
f*x + e) - 2*(139*cos(f*x + e)^4 + 21*cos(f*x + e)^3 - 175*cos(f*x + e)^2 -
 105*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos
(f*x + e)^3 + a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f
)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\int \frac{\sec\left(e+fx\right)}{-c\sqrt{-c\sec\left(e+fx\right)+c}\sec^4\left(e+fx\right)-2c\sqrt{-c\sec\left(e+fx\right)+c}\sec^3\left(e+fx\right)+2c}}{a^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/(-c*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c*s
qrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 + 2*c*sqrt(-c*sec(e + f*x) + c)*se
c(e + f*x) + c*sqrt(-c*sec(e + f*x) + c)), x)/a**3
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec\left(fx+e\right)}{\left(a\sec\left(fx+e\right)+a\right)^3\left(-c\sec\left(fx+e\right)+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^(3/2))
, x)
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(105\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right) - \frac{15\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}\cdots\right.}}{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\cdots} \cdots\right.}{\cdots}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/480*sqrt(2)*(105*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c
)) - 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/tan(1/2*f*x + 1/2*e)^2 - 2*(3*(c
*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^8 - 10*(c*tan(1/2*f*x + 1/2*e)^2 - c)^
(3/2)*c^9 + 45*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^10)/c^10)/(a^3*c^2*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^3\,{\left(c-\frac{c}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(3/2)), x)
```

3.106 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 246

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{63\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} - \frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{3\tan(e+fx)}{10af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{21\tan(e+fx)}{20f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}} - \frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -63/256*arctan(1/2*c^(1/2)*tan(f*x+e)*2^(1/2)/(c-c*sec(f*x+e))^(1/2))/a^3/c
^(5/2)/f*2^(1/2)-21/32*tan(f*x+e)/a^3/f/(c-c*sec(f*x+e))^(5/2)+1/5*tan(f*x+
e)/f/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2)+3/10*tan(f*x+e)/a/f/(a+a*sec
(f*x+e))^2/(c-c*sec(f*x+e))^(5/2)+21/20*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))/(
c-c*sec(f*x+e))^(5/2)-63/128*tan(f*x+e)/a^3/c/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

$= \{4045, 3881, 3880, 209\}$

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{63\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} - \frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}} - \frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{21\tan(e+fx)}{20f\,(a^3\sec(e+fx)+a^3)\,(c-c\sec(e+fx))^{5/2}} + \frac{3\tan(e+fx)}{10af(a\sec(e+fx)+a)^2(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a\sec(e+fx)+a)^3(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (-63*ArcTan[(Sqrt[c]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sec[e + f*x]])])/(12
8*Sqrt[2]*a^3*c^(5/2)*f) - (21*Tan[e + f*x])/(32*a^3*f*(c - c*Sec[e + f*x])
^(5/2)) + Tan[e + f*x]/(5*f*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/
2)) + (3*Tan[e + f*x])/(10*a*f*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x])^
(5/2)) + (21*Tan[e + f*x])/(20*f*(a^3 + a^3*Sec[e + f*x])*(c - c*Sec[e + f*
x])^(5/2)) - (63*Tan[e + f*x])/(128*a^3*c*f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
```

```
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}+\frac{9\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\,dx}{10a}\\
&= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{3\tan(e+fx)}{10af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{21\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c-c\sec(e+fx))^{5/2}}\,dx}{20a^2}\\
&= \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{3\tan(e+fx)}{10af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{21\tan(e+fx)}{20f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}+\frac{21\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{5/2}}\,dx}{8a^3}\\
&= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{3\tan(e+fx)}{10af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{21\tan(e+fx)}{20f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}+\frac{63\int \frac{\sec(e+fx)}{(c-c\sec(e+fx))^{3/2}}\,dx}{64a^3c}\\
&= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}}+\frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{3\tan(e+fx)}{10af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{21\tan(e+fx)}{20f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}}\\
&\quad-\frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}+\frac{63\int \frac{\sec(e+fx)}{\sqrt{c-c\sec(e+fx)}}\,dx}{256a^3c^2}
\end{aligned}$$

$$= -\frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{3\tan(e+fx)}{10af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{21\tan(e+fx)}{20f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}} - \frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}} - \frac{63\text{Subst}\left(\int \frac{1}{2c+x^2}\,dx, x, \frac{c\tan(e+fx)}{\sqrt{c-c\sec(e+fx)}}\right)}{128a^3c^2f}$$

$$= -\frac{63\arctan\left(\frac{\sqrt{c}\tan(e+fx)}{\sqrt{2}\sqrt{c-c\sec(e+fx)}}\right)}{128\sqrt{2}a^3c^{5/2}f} - \frac{21\tan(e+fx)}{32a^3f(c-c\sec(e+fx))^{5/2}} + \frac{\tan(e+fx)}{5f(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}} + \frac{3\tan(e+fx)}{10af(a+a\sec(e+fx))^2(c-c\sec(e+fx))^{5/2}} + \frac{21\tan(e+fx)}{20f\left(a^3+a^3\sec(e+fx)\right)(c-c\sec(e+fx))^{5/2}} - \frac{63\tan(e+fx)}{128a^3cf(c-c\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},3,-\frac{3}{2},\frac{1}{2}(1+\sec(e+fx))\right)\tan(e}{20a^3c^2f(1+\sec(e+fx))^3\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x])^(5/2)),
x]
```

```
[Out] (Hypergeometric2F1[-5/2, 3, -3/2, (1 + Sec[e + f*x])/2]*Tan[e + f*x])/(20*a
^3*c^2*f*(1 + Sec[e + f*x])^3*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.11

method	result
default	$\frac{\sqrt{2}\left(257\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\cos(fx+e)^4-354\cos(fx+e)^3\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}+315\sin(fx+e)^4\arctan\left(\frac{\sqrt{2}}{2\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}}\right)-588\sqrt{2}\right.}{1280a^3f\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{-c(\sec(fx+e)-1)}\,(\mathrm{se}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/1280/a^3/f*2^(1/2)*(257*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*
x+e)^4-354*cos(f*x+e)^3*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+315*sin(
f*x+e)^4*arctan(1/2*2^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2))-588*2^(1/2)
*(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2+210*2^(1/2)*(-cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)*cos(f*x+e)+315*2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(1
/2))/(-cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/(sec(f*x+
e)-1)^2/c^2/(cos(f*x+e)+1)^3*tan(f*x+e)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{315\sqrt{2}\left(\cos(fx+e)^4-2\cos(fx+e)^2+1\right)\sqrt{-c}\log}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/2560*(315*sqrt(2)*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-c)*log(
(2*sqrt(2)*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(-c)*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)) + (3*c*cos(f*x + e) + c)*sin(f*x + e))/((cos(f*x + e) - 1)
*sin(f*x + e)))*sin(f*x + e) + 4*(257*cos(f*x + e)^5 - 354*cos(f*x + e)^4 -
 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 315*cos(f*x + e))*sqrt((c*cos(f*
x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x
 + e)^2 + a^3*c^3*f)*sin(f*x + e)), 1/1280*(315*sqrt(2)*(cos(f*x + e)^4 - 2
*cos(f*x + e)^2 + 1)*sqrt(c)*arctan(sqrt(2)*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/(sqrt(c)*sin(f*x + e)))*sin(f*x + e) - 2*(257*cos(f*x
 + e)^5 - 354*cos(f*x + e)^4 - 588*cos(f*x + e)^3 + 210*cos(f*x + e)^2 + 31
5*cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*
x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\int \frac{}{c^2\sqrt{-c\sec{(e+fx)}+c}\sec^5{(e+fx)}+c^2\sqrt{-c\sec{(e+fx)}+c}\sec^4{(e+fx)}-2c^2\sqrt{}}}{}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c-c*sec(f*x+e))**(5/2),x)

[Out] Integral(sec(e + f*x)/(c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**5 + c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**4 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**3 - 2*c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x)**2 + c**2*sqrt(-c*sec(e + f*x) + c)*sec(e + f*x) + c**2*sqrt(-c*sec(e + f*x) + c)), x)/a**3

Maxima [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sec{(fx+e)}}{(a\sec{(fx+e)}+a)^3(-c\sec{(fx+e)}+c)^{\frac{5}{2}}}\,dx$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(f*x + e)/((a*sec(f*x + e) + a)^3*(-c*sec(f*x + e) + c)^(5/2)), x)

Giac [A] (verification not implemented)

none

Time = 0.71 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.75

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\left(315\sqrt{c}\arctan\left(\frac{\sqrt{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}}{\sqrt{c}}\right)-\frac{5\left(17\left(c\tan\left(\frac{1}{2}fx+\right.\right.\right.}{}\right.}{}$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c-c*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] 1/1280*sqrt(2)*(315*sqrt(c)*arctan(sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)/sqrt(c)) - 5*(17*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c + 15*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4) - 8*((c*tan(1/2*f*x + 1/2*e)^2 - c)^(5/2)*c^8 - 5*(c*tan(1/2*f*x + 1/2*e)^2 - c)^(3/2)*c^9 + 30*sqrt(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^10)/c^10)/(a^3*c^3*f)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^3\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2)),x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c - c/cos(e + f*x))^(5/2)), x)
```

3.107 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\frac{a(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 1/3*a*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4038}

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\frac{a\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{3f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(3*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
```

```
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{a(c - c\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{ac^3\sec(e+fx)\left(3-3\sec(e+fx)+\sec^2(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2),
x]
[Out] -1/3*(a*c^3*Sec[e + f*x]*(3 - 3*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e + f*x]
)/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(37) = 74.

Time = 3.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{(\sec(fx+e)-1)^2\left(7\cos(fx+e)^2-4\cos(fx+e)+1\right)\sqrt{-c(\sec(fx+e)-1)}\,c^2\sqrt{a(\sec(fx+e)+1)}\,(\cos(fx+e)+1)\csc(fx+e)}{3f(\cos(fx+e)-1)^2}$	88
risch	$\frac{2ic^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(3\,e^{5i(fx+e)}-6\,e^{4i(fx+e)}+10\,e^{3i(fx+e)}-6\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\left(1+e^{2i(fx+e)}\right)^2 f}$	165

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
[Out] 1/3/f*(sec(f*x+e)-1)^2*(7*cos(f*x+e)^2-4*cos(f*x+e)+1)*(-c*(sec(f*x+e)-1))^
(1/2)*c^2*(a*(sec(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)/(cos(f*x+e)-1)^2*csc(f*x+
e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \frac{\left(3\,c^2\cos\left(fx+e\right)^2-3\,c^2\cos\left(fx+e\right)+c^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*c^2*cos(f*x + e)^2 - 3*c^2*cos(f*x + e) + c^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^
2*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(37) = 74$.

Time = 0.39 (sec) , antiderivative size = 638, normalized size of antiderivative = 14.84

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx = \frac{2\left(30\,c^2\cos\left(3\,fx+3\,e\right)\sin\left(2\,fx+2\,e\right)-9\,c^2\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)-3\,c^2\sin\left(fx+e\right)-\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(30*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*c^2*cos(2*f*x + 2*e)*sin(
f*x + e) - 3*c^2*sin(f*x + e) - (3*c^2*sin(5*f*x + 5*e) - 6*c^2*sin(4*f*x +
 4*e) + 10*c^2*sin(3*f*x + 3*e) - 6*c^2*sin(2*f*x + 2*e) + 3*c^2*sin(f*x +
```

```
e))*cos(6*f*x + 6*e) + 9*(c^2*sin(4*f*x + 4*e) + c^2*sin(2*f*x + 2*e))*cos(
5*f*x + 5*e) - 3*(10*c^2*sin(3*f*x + 3*e) + 3*c^2*sin(f*x + e))*cos(4*f*x +
 4*e) + (3*c^2*cos(5*f*x + 5*e) - 6*c^2*cos(4*f*x + 4*e) + 10*c^2*cos(3*f*x
 + 3*e) - 6*c^2*cos(2*f*x + 2*e) + 3*c^2*cos(f*x + e))*sin(6*f*x + 6*e) - 3
*(3*c^2*cos(4*f*x + 4*e) + 3*c^2*cos(2*f*x + 2*e) + c^2)*sin(5*f*x + 5*e) +
 3*(10*c^2*cos(3*f*x + 3*e) + 3*c^2*cos(f*x + e) + 2*c^2)*sin(4*f*x + 4*e)
- 10*(3*c^2*cos(2*f*x + 2*e) + c^2)*sin(3*f*x + 3*e) + 3*(3*c^2*cos(f*x + e
) + 2*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(3*cos(4*f*x + 4*e) + 3*co
s(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x + 6*e)^2 + 6*(3*cos(2*f*x
+ 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2 + 9*cos(2*f*x + 2*e)^2
+ 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + sin(6*f*x + 6*
e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*sin(
2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\int \sqrt{a\sec\left(fx+e\right)+a}(-c\sec\left(fx+e\right)+c)^{\frac{5}{2}}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.16

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx=\frac{2\,c^2\,\sqrt{\frac{a\,\left(\cos(e+f\,x)+1\right)}{\cos(e+f\,x)}}\,\sqrt{\frac{c\,\left(\cos(e+f\,x)-1\right)}{\cos(e+f\,x)}}\,\left(10\,\sin\left(e+f\,x\right)-12\,\sin\left(2\,e+2\,f\,x\right)+13\,\sin\left(3\,e+3\,f\right.}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)-2\,\cos\left(4\,e+4\,f\,x\right)-\cos\left(6\,e+6\,f\right.}$$

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] (2*c^2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/
cos(e + f*x))^(1/2)*(10*sin(e + f*x) - 12*sin(2*e + 2*f*x) + 13*sin(3*e + 3
*f*x) - 6*sin(4*e + 4*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*e + 2*f*x) -
2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

3.108 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] 1/2*a*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4038}

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*Sqrt[a + a*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
```

```
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = \frac{a(c - c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c - c\sec(e+fx))^{3/2}\,dx = \frac{ac^2(-2+\sec(e+fx))\sec(e+fx)\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2),
x]
```

```
[Out] (a*c^2*(-2 + Sec[e + f*x])*Sec[e + f*x]*Tan[e + f*x])/(2*f*Sqrt[a*(1 + Sec[
e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

method	result	size
default	$-\frac{(\sec(fx+e)-1)(3\cos(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\,c\sqrt{a(\sec(fx+e)+1)}\,(\cos(fx+e)+1)\csc(fx+e)}{2f(\cos(fx+e)-1)}$	74
risch	$\frac{2ic\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{3i(fx+e)}-e^{2i(fx+e)}+e^{i(fx+e)}\right)}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\left(1+e^{2i(fx+e)}\right)f}$	137

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f*(sec(f*x+e)-1)*(3*cos(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*c*(a*(sec(
f*x+e)+1))^(1/2)*(cos(f*x+e)+1)/(cos(f*x+e)-1)*csc(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.81

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = \frac{(2\,c\cos(fx+e)-c)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\,f\cos(fx+e)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*c*cos(f*x + e) - c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = \int \sqrt{a(\sec(e+fx)+1)}(-c(\sec(e+fx)-1))^{\frac{3}{2}}\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**(3/2)*sec(e +
f*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(37) = 74.

Time = 0.37 (sec) , antiderivative size = 298, normalized size of antiderivative = 6.93

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\,(2\,c\cos(3\,fx+3\,e)\sin(2\,fx+2\,e)-2\,c\cos(2\,fx+2\,e)\sin(fx+e)-(c\sin(3\,fx+3\,e)-c}{(2\,(2\,\cos(2\,fx+2\,e)+1)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*c*cos(2*f*x + 2*e)*sin(f*x + e
) - (c*sin(3*f*x + 3*e) - c*sin(2*f*x + 2*e) + c*sin(f*x + e))*cos(4*f*x +
```

```
4*e) + (c*cos(3*f*x + 3*e) - c*cos(2*f*x + 2*e) + c*cos(f*x + e))*sin(4*f*x
 + 4*e) - (2*c*cos(2*f*x + 2*e) + c)*sin(3*f*x + 3*e) + (2*c*cos(f*x + e) +
 c)*sin(2*f*x + 2*e) - c*sin(f*x + e))*sqrt(a)*sqrt(c)/((2*(2*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin
(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^
2 + 4*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\int \sqrt{a\sec(fx+e)+a}(-c\sec(fx+e)+c)^{\frac{3}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.81

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}\,dx=\frac{c\sqrt{c-\frac{c}{\cos(e+fx)}}\sqrt{\frac{a(\cos(e+fx)+1)}{\cos(e+fx)}}\left(\sin(e+fx)-\sin(2e+2fx)+\sin(3e+3fx)\right)}{f\sin(2e+2fx)^2}$$

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] (c*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*(
sin(e + f*x) - sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(f*sin(2*e + 2*f*x)^2)
```

3.109 $\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 41

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4038}

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((c*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{c\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\sec(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]],x]
[Out] -((c*Sec[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result	size
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,\sin(fx+e)}{f(\cos(fx+e)-1)}$	47
risch	$\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,e^{i(fx+e)}}{\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$	102

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/f*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)*sin(f*x+e)/(cos(f*x+e)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{f\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))/(f*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \int \sqrt{a(\sec(e+fx)+1)}\sqrt{-c(\sec(e+fx)-1)}\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x
), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\sqrt{-a}\sqrt{c}}{f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-a)*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1))
```

Giac [F]

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \sqrt{a\sec\left(fx+e\right)+a}\sqrt{-c\sec\left(fx+e\right)+c}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \sec(e+fx)\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}\,dx=\frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}}{f\,\sin\left(e+f\,x\right)}$$

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2))/(f
*sin(e + f*x))
```

3.110 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 51

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] a*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/
2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4037}

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{a\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (a*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{a\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\log(1-\sec(e+fx))\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] (Log[1 - Sec[e + f*x]]*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt
[c - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(47) = 94.

Time = 3.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.92

method	result
default	$\frac{\sqrt{a(\sec(fx+e)+1)}\,(\ln(-\cot(fx+e)+\csc(fx+e)-1)+\ln(-\cot(fx+e)+\csc(fx+e)+1)-2\ln(-\cot(fx+e)+\csc(fx+e)))(\cot(fx+e)-}{f\sqrt{-c(\sec(fx+e)-1)}}$
risch	$-\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)\ln\left(1+e^{2i(fx+e)}\right)}{\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*(a*(sec(f*x+e)+1))^(1/2)*(ln(-cot(f*x+e)+csc(f*x+e)-1)+ln(-cot(f*x+e)+c
sc(f*x+e)+1)-2*ln(-cot(f*x+e)+csc(f*x+e)))/(-c*(sec(f*x+e)-1))^(1/2)*(cot(f
*x+e)-csc(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\sec(fx+e)+a}\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/(
c*sec(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sqrt{a(\sec(e+fx)+1)}\sec(e+fx)}{\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1)
), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{c}} + \frac{\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{c}} - \frac{2\sqrt{-a}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{\sqrt{c}}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(sqrt(-a)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(c) + sqrt(-a)*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - 2*sqrt(-a)*log(sin(f*x + e)/
(cos(f*x + e) + 1))/sqrt(c))/f
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

3.111 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}$$

```
[Out] -1/2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4035}

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{2f(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2),x]
[Out] -1/2*(Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{2f(c-c\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\cot(e+fx)\sqrt{a(1+\sec(e+fx))}}{cf\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(3/2
),x]
[Out] (Cot[e + f*x]*Sqrt[a*(1 + Sec[e + f*x])])/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\sqrt{a(\sec(fx+e)+1)}\,\tan(fx+e)}{2fc(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}}$	50
risch	$\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,e^{i(fx+e)}}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$	105

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
[Out] 1/2/f*(a*(sec(f*x+e)+1))^(1/2)/c/(sec(f*x+e)-1)/(-c*(sec(f*x+e)-1))^(1/2)*t
an(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx=\frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{(c^2f\cos(fx+e)-c^2f)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*cos(f*x + e)/((c^2*f*cos(f*x + e) - c^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a(\sec(e+fx)+1)}\sec(e+fx)}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(3/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(
3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(36) = 72.

Time = 0.38 (sec) , antiderivative size = 514, normalized size of antiderivative = 12.24

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx =$$

$$-\frac{}{(c^2\cos(4fx+4e)^2+4c^2\cos(3fx+3e)^2+4c^2\cos(2fx+2e)^2+4c^2\cos(fx+e)^2+c^2\sin(4fx+4e)^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
[Out] -2*((sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) - (cos(3*f*x + 3*e)
+ cos(f*x + e))*sin(4*f*x + 4*e) + (2*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e
) - 2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 2*cos(f*x + e)*sin(2*f*x + 2*e) +
 2*cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*x + e))*sqrt(a)*sqrt(c)/((c^2*cos(
4*f*x + 4*e)^2 + 4*c^2*cos(3*f*x + 3*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^
2*cos(f*x + e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(3*f*x + 3*e)^2 + 4*c^
2*sin(2*f*x + 2*e)^2 - 8*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*c^2*sin(f*x
+ e)^2 - 4*c^2*cos(f*x + e) + c^2 - 2*(2*c^2*cos(3*f*x + 3*e) - 2*c^2*cos(2
*f*x + 2*e) + 2*c^2*cos(f*x + e) - c^2)*cos(4*f*x + 4*e) - 4*(2*c^2*cos(2*f
*x + 2*e) - 2*c^2*cos(f*x + e) + c^2)*cos(3*f*x + 3*e) - 4*(2*c^2*cos(f*x +
 e) - c^2)*cos(2*f*x + 2*e) - 4*(c^2*sin(3*f*x + 3*e) - c^2*sin(2*f*x + 2*e
) + c^2*sin(f*x + e))*sin(4*f*x + 4*e) - 8*(c^2*sin(2*f*x + 2*e) - c^2*sin(
f*x + e))*sin(3*f*x + 3*e))*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{a\sec(fx+e)+a}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.81

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{2\sqrt{\frac{a(\cos(e+fx)+1)}{\cos(e+fx)}}\sqrt{\frac{c(\cos(e+fx)-1)}{\cos(e+fx)}}(\sin(e+fx)-2\sin(2e+2fx)+\sin(3e+3fx))}{c^2f(4\cos(e+fx)+4\cos(2e+2fx)-4\cos(3e+3fx)+\cos(4e+4fx)-5)}$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] -(2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/cos
(e + f*x))^(1/2)*(sin(e + f*x) - 2*sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(c
^2*f*(4*cos(e + f*x) + 4*cos(2*e + 2*f*x) - 4*cos(3*e + 3*f*x) + cos(4*e +
4*f*x) - 5))
```

3.112 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/2*a*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4038}

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(a*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2
))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{a\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{2c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] (a*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(2*c^3*f*(-1 + Sec[e + f*x])^3*Sq
rt[a*(1 + Sec[e + f*x])])
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

method	result	size
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\,(3\tan(fx+e)-\sec(fx+e)\tan(fx+e))}{8f(\sec(fx+e)-1)^2\sqrt{-c(\sec(fx+e)-1)}\,c^2}$	67
risch	$\frac{2i\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{3i(fx+e)}-e^{2i(fx+e)}+e^{i(fx+e)}\right)}{c^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$	126

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/f*(a*(sec(f*x+e)+1))^(1/2)/(sec(f*x+e)-1)^2/(-c*(sec(f*x+e)-1))^(1/2)/
c^2*(3*tan(f*x+e)-sec(f*x+e)*tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(37) = 74$.

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\left(2\cos\left(fx+e\right)^2-\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\left(c^3f\cos\left(fx+e\right)^2-2\,c^3f\cos\left(fx+e\right)+c^3f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*cos(f*x + e)^2 - cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^3*f*cos(f*x + e)^2 - 2*c^3*f
*cos(f*x + e) + c^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\left(-c\left(\sec\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(
5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(37) = 74$.

Time = 0.46 (sec) , antiderivative size = 758, normalized size of antiderivative = 17.63

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*((sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + (sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - cos(2*f*x + 2*e)*sin(4*f*x + 4*e
) + cos(4*f*x + 4*e)*sin(2*f*x + 2*e) - (cos(4*f*x + 4*e) + 2*cos(2*f*x + 2
*e) + 1)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (cos(4*f*x
+ 4*e) + 2*cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
```

```
x + 2*e))) + sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 3
6*c^3*cos(2*f*x + 2*e)^2 + 16*c^3*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e)))^2 + 16*c^3*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
^2 + c^3*sin(4*f*x + 4*e)^2 + 12*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 36
*c^3*sin(2*f*x + 2*e)^2 + 16*c^3*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 16*c^3*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 + 12*c^3*cos(2*f*x + 2*e) + c^3 + 2*(6*c^3*cos(2*f*x + 2*e) + c^3)*cos(4*
f*x + 4*e) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^3)*cos(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c^3*cos(4*f*x + 4*e) + 6*c^3*cos(2*f*
x + 2*e) + c^3)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*(c
^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e) - 4*c^3*sin(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 8*(c^3*sin(4*f*x + 4*e) + 6*c^3*sin(2*f*x + 2*e))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\left(-c\sec\left(fx+e\right)+c\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.72

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,4\mathrm{i}}{c^3\,f}+\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos(\cdots}}}{c^3\,f}\cdots\right.}{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sin\left(e+f\,x\right)\,10\mathrm{i}-\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,8\mathrm{i}\cdots}$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((exp(e*3i + f*x*3i)*(a + a/cos(e + f*x))^(1/2)
*4i)/(c^3*f) + (exp(e*3i + f*x*3i)*cos(2*e + 2*f*x)*(a + a/cos(e + f*x))^(1
/2)*4i)/(c^3*f) - (cos(e + f*x)*exp(e*3i + f*x*3i)*(a + a/cos(e + f*x))^(1/
2)*4i)/(c^3*f)))/(exp(e*3i + f*x*3i)*sin(e + f*x)*10i - exp(e*3i + f*x*3i)*
sin(2*e + 2*f*x)*8i + exp(e*3i + f*x*3i)*sin(3*e + 3*f*x)*2i)
```

3.113 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a^2(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{10f\sqrt{a+a\sec(e+fx)}} + \frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{5f}$$

```
[Out] 1/10*a^2*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/5*a*(
c-c*sec(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{10f\sqrt{a\sec(e+fx)+a}} + \frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{5f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^2*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(10*f*Sqrt[a + a*Sec[e + f*x]
]) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(
5*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{5f} \\ &\quad+\frac{1}{5}(2a)\int\sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx \\ &= \frac{a^2(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{10f\sqrt{a+a\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{5f}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int\sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx=\frac{a^2c^4\sec(e+fx)\left(-10+10\sec(e+fx)-5\sec^3(e+fx)+2\sec^4(e+fx)\right)\tan(e+f}{10f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2
),x]
```

```
[Out] (a^2*c^4*Sec[e + f*x]*(-10 + 10*Sec[e + f*x] - 5*Sec[e + f*x]^3 + 2*Sec[e +
 f*x]^4)*Tan[e + f*x])/(10*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e +
f*x]])
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

method	result
default	$-\frac{a\left(13\cos(fx+e)^3-16\cos(fx+e)^2+9\cos(fx+e)-2\right)(\sec(fx+e)-1)^3\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,c^3(\cos(fx+e)+1)^2\sec(}{10f(\cos(fx+e)-1)^3}$
risch	$\frac{2ia\,c^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(5\,e^{9i(fx+e)}-10\,e^{8i(fx+e)}+20\,e^{7i(fx+e)}-10\,e^{6i(fx+e)}+14\,e^{5i(fx+e)}-10\,e^{4i(fx+e)}+20\,e^{3i(}}{5\left(1+e^{2i(fx+e)}\right)^4\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/10/f*a*(13*cos(f*x+e)^3-16*cos(f*x+e)^2+9*cos(f*x+e)-2)*(sec(f*x+e)-1)^3
*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)*c^3*(cos(f*x+e)+1)^2/(c
os(f*x+e)-1)^3*sec(f*x+e)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{\left(10\,ac^3\cos\left(fx+e\right)^4-10\,ac^3\cos\left(fx+e\right)^3+5\,ac^3\cos\left(fx+e\right)-2\,ac^3\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}}{10\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/10*(10*a*c^3*cos(f*x + e)^4 - 10*a*c^3*cos(f*x + e)^3 + 5*a*c^3*cos(f*x +
 e) - 2*a*c^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. 2(77) = 154.

Time = 0.41 (sec) , antiderivative size = 1680, normalized size of antiderivative = 18.88

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/5*(100*a*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 25*a*c^3*cos(2*f*x + 2*e
)*sin(f*x + e) - 5*a*c^3*sin(f*x + e) - (5*a*c^3*sin(9*f*x + 9*e) - 10*a*c^
3*sin(8*f*x + 8*e) + 20*a*c^3*sin(7*f*x + 7*e) - 10*a*c^3*sin(6*f*x + 6*e)
+ 14*a*c^3*sin(5*f*x + 5*e) - 10*a*c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*
x + 3*e) - 10*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e))*cos(10*f*x + 1
0*e) + 25*(a*c^3*sin(8*f*x + 8*e) + 2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(
4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 5*(20*a*c^3*sin(7
*f*x + 7*e) + 10*a*c^3*sin(6*f*x + 6*e) + 14*a*c^3*sin(5*f*x + 5*e) + 10*a*
c^3*sin(4*f*x + 4*e) + 20*a*c^3*sin(3*f*x + 3*e) + 5*a*c^3*sin(f*x + e))*co
s(8*f*x + 8*e) + 100*(2*a*c^3*sin(6*f*x + 6*e) + 2*a*c^3*sin(4*f*x + 4*e) +
 a*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(14*a*c^3*sin(5*f*x + 5*e) +
 20*a*c^3*sin(3*f*x + 3*e) - 5*a*c^3*sin(2*f*x + 2*e) + 5*a*c^3*sin(f*x + e
))*cos(6*f*x + 6*e) + 70*(2*a*c^3*sin(4*f*x + 4*e) + a*c^3*sin(2*f*x + 2*e)
)*cos(5*f*x + 5*e) - 50*(4*a*c^3*sin(3*f*x + 3*e) - a*c^3*sin(2*f*x + 2*e)
+ a*c^3*sin(f*x + e))*cos(4*f*x + 4*e) + (5*a*c^3*cos(9*f*x + 9*e) - 10*a*c
^3*cos(8*f*x + 8*e) + 20*a*c^3*cos(7*f*x + 7*e) - 10*a*c^3*cos(6*f*x + 6*e)
 + 14*a*c^3*cos(5*f*x + 5*e) - 10*a*c^3*cos(4*f*x + 4*e) + 20*a*c^3*cos(3*f
*x + 3*e) - 10*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x + e))*sin(10*f*x +
10*e) - 5*(5*a*c^3*cos(8*f*x + 8*e) + 10*a*c^3*cos(6*f*x + 6*e) + 10*a*c^3*
cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(9*f*x + 9*e) + 5*(
20*a*c^3*cos(7*f*x + 7*e) + 10*a*c^3*cos(6*f*x + 6*e) + 14*a*c^3*cos(5*f*x
+ 5*e) + 10*a*c^3*cos(4*f*x + 4*e) + 20*a*c^3*cos(3*f*x + 3*e) + 5*a*c^3*co
s(f*x + e) + 2*a*c^3)*sin(8*f*x + 8*e) - 20*(10*a*c^3*cos(6*f*x + 6*e) + 10
*a*c^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(7*f*x + 7*e
) + 10*(14*a*c^3*cos(5*f*x + 5*e) + 20*a*c^3*cos(3*f*x + 3*e) - 5*a*c^3*cos
(2*f*x + 2*e) + 5*a*c^3*cos(f*x + e) + a*c^3)*sin(6*f*x + 6*e) - 14*(10*a*c
^3*cos(4*f*x + 4*e) + 5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(5*f*x + 5*e) +
10*(20*a*c^3*cos(3*f*x + 3*e) - 5*a*c^3*cos(2*f*x + 2*e) + 5*a*c^3*cos(f*x
+ e) + a*c^3)*sin(4*f*x + 4*e) - 20*(5*a*c^3*cos(2*f*x + 2*e) + a*c^3)*sin(
3*f*x + 3*e) + 5*(5*a*c^3*cos(f*x + e) + 2*a*c^3)*sin(2*f*x + 2*e))*sqrt(a)
*sqrt(c)/((2*(5*cos(8*f*x + 8*e) + 10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e
) + 5*cos(2*f*x + 2*e) + 1)*cos(10*f*x + 10*e) + cos(10*f*x + 10*e)^2 + 10*
(10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e) + 25*cos(8*f*x + 8*e)^2 + 20*(10*cos(4*f*x + 4*e) + 5*cos(2*f*x
```

```
+ 2*e) + 1)*cos(6*f*x + 6*e) + 100*cos(6*f*x + 6*e)^2 + 20*(5*cos(2*f*x + 2
*e) + 1)*cos(4*f*x + 4*e) + 100*cos(4*f*x + 4*e)^2 + 25*cos(2*f*x + 2*e)^2
+ 10*(sin(8*f*x + 8*e) + 2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) + sin(2*f*
x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 50*(2*sin(6*f*x + 6*e
) + 2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 25*sin(8*f*x
+ 8*e)^2 + 100*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 1
00*sin(6*f*x + 6*e)^2 + 100*sin(4*f*x + 4*e)^2 + 100*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 25*sin(2*f*x + 2*e)^2 + 10*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^{\frac{7}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 17.66 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.30

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx=\frac{\sqrt{c-\frac{c}{\cos(e+fx)}}\left(\frac{a\,c^3\,e^{e\,5i+f\,x\,5i}\sqrt{a+\frac{a}{\cos(e+fx)}}\,28i}{5\,f}-\frac{a\,c^3\,\cos(e+fx)\,e^{e\,5i+f\,x\,5i}\sqrt{a+\frac{a}{\cos(e+fx)}}\,8i}{f}+\frac{a\,c^3\,e^{\cdots}}{\cdots}\right.}{e^{e\,5i+f\,x\,5i}\,\sin(e+fx)\,4i+e^{e\,5i\cdots}}$$

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*c^3*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))
^(1/2)*28i)/(5*f) - (a*c^3*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f
*x))^(1/2)*8i)/f + (a*c^3*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a/cos(e
+ f*x))^(1/2)*16i)/f - (a*c^3*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x)*(a + a/co
s(e + f*x))^(1/2)*8i)/f + (a*c^3*exp(e*5i + f*x*5i)*cos(4*e + 4*f*x)*(a + a
/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*5i + f*x*5i)*sin(e + f*x)*4i + exp(e*5i
 + f*x*5i)*sin(3*e + 3*f*x)*6i + exp(e*5i + f*x*5i)*sin(5*e + 5*f*x)*2i)
```

3.114 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{a+a\sec(e+fx)}} + \frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{4f}$$

```
[Out] 1/6*a^2*(c-c*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+1/4*a*(c
-c*sec(f*x+e))^(5/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{a^2\tan(e+fx)(c-c\sec(e+fx))^{5/2}}{6f\sqrt{a\sec(e+fx)+a}} + \frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}{4f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (a^2*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(6*f*Sqrt[a + a*Sec[e + f*x]]
) + (a*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(4
*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{4f} \\ &\quad +\frac{1}{2}a\int\sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\,dx \\ &= \frac{a^2(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{a+a\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}\tan(e+fx)}{4f}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int\sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{ac^3(5\cos(e+fx)-3\cos(2(e+fx))+3\cos(3(e+fx)))\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{12f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] -1/12*(a*c^3*(5*Cos[e + f*x] - 3*Cos[2*(e + f*x)] + 3*Cos[3*(e + f*x)])*Sec
[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Sec[
e + f*x]])
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

method	result
default	$\frac{a(\sec(fx+e)-1)^2\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,c^2(\cos(fx+e)+1)^2(11\cot(fx+e)-10\csc(fx+e)+3\sec(fx+e)\csc(fx+e))}{12f(\cos(fx+e)-1)^2}$
risch	$\frac{2iac^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,\left(3\,e^{7i(fx+e)}-3\,e^{6i(fx+e)}+5\,e^{5i(fx+e)}+5\,e^{3i(fx+e)}-3\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3\left(1+e^{2i(fx+e)}\right)^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/12/f*a*(sec(f*x+e)-1)^2*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2
)*c^2*(cos(f*x+e)+1)^2/(cos(f*x+e)-1)^2*(11*cot(f*x+e)-10*csc(f*x+e)+3*sec(
f*x+e)*csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{\left(12\,ac^2\cos\left(fx+e\right)^3-6\,ac^2\cos\left(fx+e\right)^2-4\,ac^2\cos\left(fx+e\right)+3\,ac^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\,\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{12\,f\cos\left(fx+e\right)^3\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(12*a*c^2*cos(f*x + e)^3 - 6*a*c^2*cos(f*x + e)^2 - 4*a*c^2*cos(f*x +
e) + 3*a*c^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. 2(77) = 154.

Time = 0.39 (sec) , antiderivative size = 1105, normalized size of antiderivative = 12.42

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(20*a*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a*c^2*cos(2*f*x + 2*e)
*sin(f*x + e) - 3*a*c^2*sin(f*x + e) - (3*a*c^2*sin(7*f*x + 7*e) - 3*a*c^2*
sin(6*f*x + 6*e) + 5*a*c^2*sin(5*f*x + 5*e) + 5*a*c^2*sin(3*f*x + 3*e) - 3*
a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*a*c^
2*sin(6*f*x + 6*e) + 3*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*c
os(7*f*x + 7*e) - 2*(10*a*c^2*sin(5*f*x + 5*e) + 9*a*c^2*sin(4*f*x + 4*e) +
 10*a*c^2*sin(3*f*x + 3*e) + 6*a*c^2*sin(f*x + e))*cos(6*f*x + 6*e) + 10*(3
*a*c^2*sin(4*f*x + 4*e) + 2*a*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 6*(5
*a*c^2*sin(3*f*x + 3*e) - 3*a*c^2*sin(2*f*x + 2*e) + 3*a*c^2*sin(f*x + e))*
cos(4*f*x + 4*e) + (3*a*c^2*cos(7*f*x + 7*e) - 3*a*c^2*cos(6*f*x + 6*e) + 5
*a*c^2*cos(5*f*x + 5*e) + 5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*
e) + 3*a*c^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a*c^2*cos(6*f*x + 6*e) +
 6*a*c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(7*f*x + 7
*e) + (20*a*c^2*cos(5*f*x + 5*e) + 18*a*c^2*cos(4*f*x + 4*e) + 20*a*c^2*cos
(3*f*x + 3*e) + 12*a*c^2*cos(f*x + e) + 3*a*c^2)*sin(6*f*x + 6*e) - 5*(6*a*
c^2*cos(4*f*x + 4*e) + 4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(5*f*x + 5*e) +
 6*(5*a*c^2*cos(3*f*x + 3*e) - 3*a*c^2*cos(2*f*x + 2*e) + 3*a*c^2*cos(f*x +
 e))*sin(4*f*x + 4*e) - 5*(4*a*c^2*cos(2*f*x + 2*e) + a*c^2)*sin(3*f*x + 3*
e) + 3*(4*a*c^2*cos(f*x + e) + a*c^2)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2
*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f
*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)
 + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*s
in(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 17.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}\,dx=\frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a\,c^2\,\cos(e+f\,x)\,e^{e\,4i+f\,x\,4i}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,20i}{3\,f}-\frac{a\,c^2\,e^{e\,4i+f\,x\,4i}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\right.}{e^{e\,4i+f\,x\,4i}\,\sin(2\,e+2\,f\,x)\,4i+e^{e\,4i+f\,x\,4i}\,\sin(4\,e+\right.}$$

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*c^2*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/
cos(e + f*x))^(1/2)*20i)/(3*f) - (a*c^2*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a*c^2*exp(e*4i + f*x*4i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)
*4i + exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

3.115 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}} - \frac{c(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f}$$

```
[Out] -1/3*c^2*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/3*c*(
a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{3f\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}}{3f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(c^2*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*
x]]) - (c*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(3*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{c(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f}\\&\quad+\frac{1}{3}(2c)\int\sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx\\&=-\frac{c^2(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}\\&\quad-\frac{c(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{3f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int\sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{a^2c^2\sec(e+fx)\left(-3+\sec^2(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2
),x]
[Out] (a^2*c^2*Sec[e + f*x]*(-3 + Sec[e + f*x]^2)*Tan[e + f*x])/(3*f*Sqrt[a*(1 +
Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a(\sec(fx+e)-1)(2\cos(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\, c\sqrt{a(\sec(fx+e)+1)}\,(\cos(fx+e)+1)^2\sec(fx+e)\csc(fx+e)}{3f(\cos(fx+e)-1)}$	83
risch	$\frac{2iac\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(3\,e^{5i(fx+e)}+2\,e^{3i(fx+e)}+3\,e^{i(fx+e)}\right)}{3\left(1+e^{2i(fx+e)}\right)^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$	142

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/f*a*(sec(f*x+e)-1)*(2*cos(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*c*(a*(se
c(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)^2/(cos(f*x+e)-1)*sec(f*x+e)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{\left(3\,ac\cos\left(fx+e\right)^2-ac\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a*c*cos(f*x + e)^2 - a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^2*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(77) = 154$.

Time = 0.42 (sec) , antiderivative size = 550, normalized size of antiderivative = 6.18

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\,(6\,ac\cos{(3\,fx+3\,e)}\sin{(2\,fx+2\,e)}+9\,ac\cos{(fx+e)}\sin{(2\,fx+2\,e)}-9\,ac\cos{(}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(6*a*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 9*a*c*cos(f*x + e)*sin(2*f*x
 + 2*e) - 9*a*c*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a*c*sin(f*x + e) - (3*a*c
*sin(5*f*x + 5*e) + 2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(6*f*x
+ 6*e) + 9*(a*c*sin(4*f*x + 4*e) + a*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) -
 3*(2*a*c*sin(3*f*x + 3*e) + 3*a*c*sin(f*x + e))*cos(4*f*x + 4*e) + (3*a*c*
cos(5*f*x + 5*e) + 2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(6*f*x +
 6*e) - 3*(3*a*c*cos(4*f*x + 4*e) + 3*a*c*cos(2*f*x + 2*e) + a*c)*sin(5*f*x
 + 5*e) + 3*(2*a*c*cos(3*f*x + 3*e) + 3*a*c*cos(f*x + e))*sin(4*f*x + 4*e)
- 2*(3*a*c*cos(2*f*x + 2*e) + a*c)*sin(3*f*x + 3*e))*sqrt(a)*sqrt(c)/((2*(3
*cos(4*f*x + 4*e) + 3*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + cos(6*f*x +
6*e)^2 + 6*(3*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 9*cos(4*f*x + 4*e)^2
 + 9*cos(2*f*x + 2*e)^2 + 6*(sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(6*f*x
 + 6*e) + sin(6*f*x + 6*e)^2 + 9*sin(4*f*x + 4*e)^2 + 18*sin(4*f*x + 4*e)*s
in(2*f*x + 2*e) + 9*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx = \int (a\sec{(fx+e)}+a)^{\frac{3}{2}}(-c\sec{(fx+e)}+c)^{\frac{3}{2}}\sec{(fx+e)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.72 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{2\,a\,c\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,(2\,\sin(e+f\,x)+5\,\sin(3\,e+3\,f\,x)+3\,\sin(5\,e+}{3\,f\,\left(\cos(2\,e+2\,f\,x)-2\,\cos(4\,e+4\,f\,x)-\cos(6\,e+6\,f\,x)+2\right)}$$

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] (2*a*c*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/
2)*(2*sin(e + f*x) + 5*sin(3*e + 3*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*
e + 2*f*x) - 2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

3.116 $\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*c*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -1/2*(c*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]
])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{c(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{ac(1+2\cos(e+fx))\sec^2(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx)\right)}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]],
x]
[Out] -1/2*(a*c*(1 + 2*Cos[e + f*x])*Sec[e + f*x]^2*Sqrt[a*(1 + Sec[e + f*x])]*Ta
n[(e + f*x)/2])/(f*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{a(\cos(fx+e)+1)^2\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,\sec(fx+e)\csc(fx+e)}{2f}$	54
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{3i(fx+e)}+e^{2i(fx+e)}+e^{i(fx+e)}\right)}{\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$	135

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
[Out] 1/2/f*a*(cos(f*x+e)+1)^2*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)
*sec(f*x+e)*csc(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{(2\,a\cos(fx+e)+a)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\,f\cos(fx+e)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/2*(2*a*cos(f*x + e) + a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = \int (a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x) - 1))*sec(e +
f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2\sqrt{-a}a\sqrt{c}}{f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*sqrt(-a)*a*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x +
 e)/(cos(f*x + e) + 1) - 1)^2)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\int (a\sec(fx+e)+a)^{\frac{3}{2}}\sqrt{-c\sec(fx+e)+c}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}\,dx=\frac{a\sqrt{c-\frac{c}{\cos(e+f\,x)}}\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,(\sin(e+f\,x)+\sin(2\,e+2\,f\,x)+\sin(3\,e}{f\sin(2\,e+2\,f\,x)^2}$$

```
[In] int(((a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (a*(c - c/cos(e + f*x))^(1/2)*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*(
sin(e + f*x) + sin(2*e + 2*f*x) + sin(3*e + 3*f*x)))/(f*sin(2*e + 2*f*x)^2)
```

3.117 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 95

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 2*a^2*ln(1-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4040, 4037}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{2a^2\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (2*a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[c
 - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+(2a)\int\frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx\\ &= \frac{2a^2\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\frac{a^2(2\log(1-\sec(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/Sqrt[c - c*Sec[e + f*x]
],x]
```

```
[Out] (a^2*(2*Log[1 - Sec[e + f*x]] + Sec[e + f*x])*Tan[e + f*x])/(f*Sqrt[a*(1 +
Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.83 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.35

method	result
default	$\frac{a\sqrt{a(\sec(fx+e)+1)}\,(4\ln(-\cot(fx+e)+\csc(fx+e))\sin(fx+e)-2\ln(-\cot(fx+e)+\csc(fx+e)-1)\sin(fx+e)-2\ln(-\cot(fx+e)+\mathrm{cs}}{f(\cos(fx+e)+1)\sqrt{-c(\sec(fx+e)-1)}}$
risch	$-\frac{2ia\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,(2\ln(\mathrm{e}^{i(fx+e)}-1)\mathrm{e}^{3i(fx+e)}-\ln(1+\mathrm{e}^{2i(fx+e)})\mathrm{e}^{3i(fx+e)}+2\,\mathrm{e}^{i(fx+e)}\ln(\mathrm{e}^{i(fx+e)}-1)-\mathrm{e}^{i(fx+e)}\ln(1+\mathrm{e}^{2i(fx+}}{\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right.}{1+\mathrm{e}^{2i(fx+e}}}}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU RNVERBOSE)

[Out] 1/f*a*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)/(-c*(sec(f*x+e)-1))^(1/2)*(4* ln(-cot(f*x+e)+csc(f*x+e))*sin(f*x+e)-2*ln(-cot(f*x+e)+csc(f*x+e)-1)*sin(f* x+e)-2*ln(-cot(f*x+e)+csc(f*x+e)+1)*sin(f*x+e)+sin(f*x+e)+tan(f*x+e))

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor ithm="fricas")

[Out] integral(-(a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt (-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sec(e+fx)}{\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)

[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(87) = 174.

Time = 0.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx =$$
$$-\frac{2\left(a\cos\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)\sin\left(2\,fx+2\,e\right)+\left(a\cos\left(2\,fx+2\,e\right)^2+a\sin\left(2\,fx+2\right.\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e)
 + (a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*(a*cos(2*f*x + 2*e)^2
+ a*sin(2*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*arctan2(sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - 1) - (a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(2*f*x + 2*e)^2 + c*sin(2
*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

3.118 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{a^2\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -a*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-a^2*ln(1-sec(
f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4039, 4037}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^2\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)))
 - (a^2*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x]]*S
qrt[c - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{a\int\frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c}\\ &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{a^2\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^2\left(\log(1-\sec(e+fx))-\frac{2}{-1+\sec(e+fx)}\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(3
/2),x]
[Out] -((a^2*(Log[1 - Sec[e + f*x]] - 2/(-1 + Sec[e + f*x]))*Tan[e + f*x])/(c*f*S
qrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(91) = 182$.

Time = 3.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.01

method	result
default	$-\frac{a(\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-1)+\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)+1)-2\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx-}{f\sqrt{-c(\sec(fx+}}$
risch	$\frac{ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(2\,e^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-e^{2i(fx+e)}\ln\left(1+e^{2i(fx+e)}\right)-4\,e^{i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)+2\,e^{i(fx+e)}\ln\left(1+e^{2i(fx+e}\right.\right.}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*a*(cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-1)+cos(f*x+e)*ln(-cot(f*x+e)+c
sc(f*x+e)+1)-2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e))-ln(-cot(f*x+e)+csc(f*x
+e)-1)-ln(-cot(f*x+e)+csc(f*x+e)+1)+2*ln(-cot(f*x+e)+csc(f*x+e))-cos(f*x+e)
-1)*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/(sec(f*x+e)-1)/c/(co
s(f*x+e)+1)*tan(f*x+e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((a*sec(f*x + e)^2 + a*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt(
-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sec(e+fx)}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/(-c*(sec(e + f*x) - 1))
**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.23

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\frac{\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{\frac{3}{2}}} + \frac{\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}} - \frac{2\sqrt{-a}a\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{3}{2}}} \cdots}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] (sqrt(-a)*a*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(3/2) + sqrt(-a)*a*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 2*sqrt(-a)*a*log(sin(f*x
+ e)/(cos(f*x + e) + 1))/c^(3/2) + sqrt(-a)*a*(cos(f*x + e) + 1)^2/(c^(3/2)
*sin(f*x + e)^2))/f
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x
)
```

3.119 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/4*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{4f(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5/2),x]
[Out] -1/4*((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5/2
))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(a(1+\sec(e+fx)))^{3/2}\tan(e+fx)}{4f(c-c\sec(e+fx))^{5/2}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(5
/2),x]
```

```
[Out] -1/4*((a*(1 + Sec[e + f*x]))^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5
/2))
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.55

method	result	size
default	$-\frac{a\sqrt{a(\sec(fx+e)+1)}\,(\tan(fx+e)+\sec(fx+e)\tan(fx+e))}{4f(\sec(fx+e)-1)^2\sqrt{-c(\sec(fx+e)-1)}\,c^2}$	65
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{c^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$	116

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/f*a*(a*(sec(f*x+e)+1))^(1/2)/(sec(f*x+e)-1)^2/(-c*(sec(f*x+e)-1))^(1/2
)/c^2*(tan(f*x+e)+sec(f*x+e)*tan(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2}{\left(c^3f\cos(fx+e)^2-2\,c^3f\cos(fx+e)+c^3f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] a*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2/((c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) + c^3*f
)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a(\sec{(e+fx)}+1))^{\frac{3}{2}}\sec{(e+fx)}}{(-c\,(\sec{(e+fx)}-1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**(3/2)*sec(e + f*x)/(-c*(sec(e + f*x) - 1))
**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(36) = 72.

Time = 0.42 (sec) , antiderivative size = 533, normalized size of antiderivative = 12.69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{}{\left(c^3\cos\left(4\,fx+4\,e\right)^2+16\,c^3\cos\left(3\,fx+3\,e\right)^2+36\,c^3\cos\left(2\,fx+2\right.\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2*(6*a*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 6*a*cos(f*x + e)*sin(2*f*x + 2*e
) - 6*a*cos(2*f*x + 2*e)*sin(f*x + e) - (a*sin(3*f*x + 3*e) + a*sin(f*x + e
))*cos(4*f*x + 4*e) + (a*cos(3*f*x + 3*e) + a*cos(f*x + e))*sin(4*f*x + 4*e
) - (6*a*cos(2*f*x + 2*e) + a)*sin(3*f*x + 3*e) - a*sin(f*x + e))*sqrt(a)*s
qrt(c)/((c^3*cos(4*f*x + 4*e)^2 + 16*c^3*cos(3*f*x + 3*e)^2 + 36*c^3*cos(2*
f*x + 2*e)^2 + 16*c^3*cos(f*x + e)^2 + c^3*sin(4*f*x + 4*e)^2 + 16*c^3*sin(
3*f*x + 3*e)^2 + 36*c^3*sin(2*f*x + 2*e)^2 - 48*c^3*sin(2*f*x + 2*e)*sin(f*
x + e) + 16*c^3*sin(f*x + e)^2 - 8*c^3*cos(f*x + e) + c^3 - 2*(4*c^3*cos(3*
f*x + 3*e) - 6*c^3*cos(2*f*x + 2*e) + 4*c^3*cos(f*x + e) - c^3)*cos(4*f*x +
 4*e) - 8*(6*c^3*cos(2*f*x + 2*e) - 4*c^3*cos(f*x + e) + c^3)*cos(3*f*x + 3
*e) - 12*(4*c^3*cos(f*x + e) - c^3)*cos(2*f*x + 2*e) - 4*(2*c^3*sin(3*f*x +
 3*e) - 3*c^3*sin(2*f*x + 2*e) + 2*c^3*sin(f*x + e))*sin(4*f*x + 4*e) - 16*
(3*c^3*sin(2*f*x + 2*e) - 2*c^3*sin(f*x + e))*sin(3*f*x + 3*e))*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.93

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{2\,a\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,(6\,\sin(e+f\,x)-8\,\sin(2\,e+2\,f\,x)+7\,\sin(3\,e+3\,f\,x)-4\,\sin(4\,e+4}{c^3\,f\,(48\,\cos(e+f\,x)+15\,\cos(2\,e+2\,f\,x)-40\,\cos(3\,e+3\,f\,x)+26\,\cos(4\,e+4\,f\,x)-8\,\cos(5\,e+5\,f\,x)}$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*a*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/c
os(e + f*x))^(1/2)*(6*sin(e + f*x) - 8*sin(2*e + 2*f*x) + 7*sin(3*e + 3*f*x
) - 4*sin(4*e + 4*f*x) + sin(5*e + 5*f*x)))/(c^3*f*(48*cos(e + f*x) + 15*co
s(2*e + 2*f*x) - 40*cos(3*e + 3*f*x) + 26*cos(4*e + 4*f*x) - 8*cos(5*e + 5*
f*x) + cos(6*e + 6*f*x) - 42))
```

3.120 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 88

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}} - \frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{24cf(c-c\sec(e+fx))^{5/2}}$$

```
[Out] -1/6*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)-1/24*(a+a*s
ec(f*x+e))^(3/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{24cf(c-c\sec(e+fx))^{5/2}} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] -1/6*((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(7/2
)) - ((a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(24*c*f*(c - c*Sec[e + f*x])
^(5/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}+\frac{\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{5/2}}\,dx}{6c}\\ &= -\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}-\frac{(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{24cf(c-c\sec(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\frac{a^2(1+3\sec(e+fx))\tan(e+fx)}{6c^3f(-1+\sec(e+fx))^3\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(7
/2),x]
```

```
[Out] (a^2*(1 + 3*Sec[e + f*x])*Tan[e + f*x])/(6*c^3*f*(-1 + Sec[e + f*x])^3*Sqrt
[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{a(5\cos(fx+e)-1)\sqrt{a(\sec(fx+e)+1)}\,(\cos(fx+e)+1)\tan(fx+e)\sec(fx+e)^2}{24f(\sec(fx+e)-1)^3\sqrt{-c(\sec(fx+e)-1)}\,c^3}$	77
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(3\,e^{5i(fx+e)}-3\,e^{4i(fx+e)}+8\,e^{3i(fx+e)}-3\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3c^3\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^5\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$	153

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/24/f*a*(5*cos(f*x+e)-1)*(a*(sec(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)/(sec(f*x+e)-1)^3/(-c*(sec(f*x+e)-1))^(1/2)/c^3*tan(f*x+e)*sec(f*x+e)^2

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\left(6\,a\cos\left(fx+e\right)^3-3\,a\cos\left(fx+e\right)^2+a\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+\cdots}{\cos(fx+\cdots}}}{6\left(c^4f\cos\left(fx+e\right)^3-3\,c^4f\cos\left(fx+e\right)^2+3\,c^4f\cos\left(fx+e\right)-\cdots\right.}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algorithm="fricas")

[Out] 1/6*(6*a*cos(f*x + e)^3 - 3*a*cos(f*x + e)^2 + a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f*cos(f*x + e)^3 - 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(7/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(76) = 152$.

Time = 0.80 (sec) , antiderivative size = 1559, normalized size of antiderivative = 17.72

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(3*(a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*cos(6*f*x + 6*e) + 3*(a*si
n(6*f*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) - 4*a*sin(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 4*(2*a*sin(6*f*x + 6*e) + 15*a*sin(4*f*x + 4*e) +
 15*a*sin(2*f*x + 2*e) + 3*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(a*sin(6*f
*x + 6*e) + 9*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e))*cos(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a*cos(4*f*x + 4*e) + a*cos(2*f*x +
2*e))*sin(6*f*x + 6*e) + 3*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x + 2*e) - 3*(a
*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) - 4*a*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 4*(2*a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x +
 4*e) + 15*a*cos(2*f*x + 2*e) + 3*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + 2*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 3*(a*cos(6*f*x + 6*e) + 9*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) + a)*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^4*
cos(6*f*x + 6*e)^2 + 225*c^4*cos(4*f*x + 4*e)^2 + 225*c^4*cos(2*f*x + 2*e)^
2 + 36*c^4*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 400*c^4
*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 36*c^4*cos(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^4*sin(6*f*x + 6*e)^2 + 225
*c^4*sin(4*f*x + 4*e)^2 + 450*c^4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 225*c
^4*sin(2*f*x + 2*e)^2 + 36*c^4*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e)))^2 + 400*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 36*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 30*c^4*c
os(2*f*x + 2*e) + c^4 + 2*(15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e
) + c^4)*cos(6*f*x + 6*e) + 30*(15*c^4*cos(2*f*x + 2*e) + c^4)*cos(4*f*x +
4*e) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*
x + 2*e) - 20*c^4*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*
c^4*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*cos(6*f*x + 6*e) + 15*c
^4*cos(4*f*x + 4*e) + 15*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + c^4)*cos(3/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e))) - 12*(c^4*cos(6*f*x + 6*e) + 15*c^4*cos(4*f*x + 4*e) + 15*
c^4*cos(2*f*x + 2*e) + c^4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
```

```
*e))) + 30*(c^4*sin(4*f*x + 4*e) + c^4*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) -
 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*
e) - 20*c^4*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*c^4*si
n(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) - 40*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x
 + 4*e) + 15*c^4*sin(2*f*x + 2*e) - 6*c^4*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 12*(c^4*sin(6*f*x + 6*e) + 15*c^4*sin(4*f*x + 4*e) + 15*c^4*sin(2*f*x + 2*
e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.10

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+fx)}}\left(\frac{a\,e^{e\,4i+f\,x\,4i}\,\sqrt{a+\frac{a}{\cos(e+fx)}}\,4i}{c^4\,f} - \frac{a\,\cos(e+fx)\,e^{e\,4i+f\,x\,4i}\,\sqrt{a+\cdots}}{3\,c^4\,f}\cdots\right.}{e^{e\,4i+f\,x\,4i}\,\sin(e+fx)\,28i - e^{e\,4i+f\,x\,4i}\,\sin(2\,e+2\,f\cdots}$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*4i + f*x*4i)*(a + a/cos(e + f*x))^(1/
2)*4i)/(c^4*f) - (a*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/cos(e + f*x))^(1
/2)*44i)/(3*c^4*f) + (a*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)*(a + a/cos(e +
f*x))^(1/2)*4i)/(c^4*f) - (a*exp(e*4i + f*x*4i)*cos(3*e + 3*f*x)*(a + a/cos
(e + f*x))^(1/2)*4i)/(c^4*f)))/(exp(e*4i + f*x*4i)*sin(e + f*x)*28i - exp(e
*4i + f*x*4i)*sin(2*e + 2*f*x)*28i + exp(e*4i + f*x*4i)*sin(3*e + 3*f*x)*12
i - exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

3.121 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}} + \frac{a^2\tan(e+fx)}{12cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}$$

```
[Out] 1/12*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(7/2)/(a+a*sec(f*x+e))^(1/2)-1/4*a
*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4039, 4038}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{a^2\tan(e+fx)}{12cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}} - \frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{4f(c-c\sec(e+fx))^{9/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -1/4*(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(9/2
)) + (a^2*Tan[e + f*x])/(12*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x
])^(7/2))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}} - \frac{a\int\frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{7/2}}\,dx}{4c}$$

$$= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{4f(c-c\sec(e+fx))^{9/2}} + \frac{a^2\tan(e+fx)}{12cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 4.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^2(1+2\sec(e+fx))\tan(e+fx)}{6c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(9
/2),x]
```

```
[Out] -1/6*(a^2*(1 + 2*Sec[e + f*x])*Tan[e + f*x])/(c^4*f*(-1 + Sec[e + f*x])^4*S
qrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a\left(17\cos(fx+e)^{2}-6\cos(fx+e)+1\right)\sqrt{a(\sec(fx+e)+1)}\,(\cos(fx+e)+1)\tan(fx+e)\sec(fx+e)^{3}}{96f(\sec(fx+e)-1)^{4}\sqrt{-c(\sec(fx+e)-1)}\,c^{4}}$	87
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\left(3\,e^{7i(fx+e)}-6\,e^{6i(fx+e)}+17\,e^{5i(fx+e)}-16\,e^{4i(fx+e)}+17\,e^{3i(fx+e)}-6\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3c^{4}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{7}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,f}$	175

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/96/f*a*(17*cos(f*x+e)^2-6*cos(f*x+e)+1)*(a*(sec(f*x+e)+1))^(1/2)*(cos(f*
x+e)+1)/(sec(f*x+e)-1)^4/(-c*(sec(f*x+e)-1))^(1/2)/c^4*tan(f*x+e)*sec(f*x+e
)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.72

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\left(6\,a\cos\left(fx+e\right)^{4}-6\,a\cos\left(fx+e\right)^{3}+4\,a\cos\left(fx+e\right)^{2}-a\cos\left(f\right.\right.}{6\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}+6\,c^{5}f\cos\left(fx+e\right)^{2}-4\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a*cos(f*x + e)^4 - 6*a*cos(f*x + e)^3 + 4*a*cos(f*x + e)^2 - a*cos(f
*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/
cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c^5*f*cos
(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2608 vs. $2(80) = 160$.

Time = 3.32 (sec) , antiderivative size = 2608, normalized size of antiderivative = 28.35

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(28*a*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 28*a*cos(4*f*x + 4*e)*sin(2*f
*x + 2*e) + 2*(3*a*sin(6*f*x + 6*e) + 8*a*sin(4*f*x + 4*e) + 3*a*sin(2*f*x
+ 2*e))*cos(8*f*x + 8*e) + (3*a*sin(8*f*x + 8*e) + 36*a*sin(6*f*x + 6*e) +
82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e) - 32*a*sin(5/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17
*a*sin(8*f*x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140
*a*sin(2*f*x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a*sin(8*f*
x + 8*e) + 140*a*sin(6*f*x + 6*e) + 294*a*sin(4*f*x + 4*e) + 140*a*sin(2*f*
x + 2*e) + 32*a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a*sin(8*f*x + 8*e) + 3
6*a*sin(6*f*x + 6*e) + 82*a*sin(4*f*x + 4*e) + 36*a*sin(2*f*x + 2*e))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*(3*a*cos(6*f*x + 6*e) +
 8*a*cos(4*f*x + 4*e) + 3*a*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - 2*(14*a*co
s(4*f*x + 4*e) - 3*a)*sin(6*f*x + 6*e) + 4*(7*a*cos(2*f*x + 2*e) + 4*a)*sin
(4*f*x + 4*e) + 6*a*sin(2*f*x + 2*e) - (3*a*cos(8*f*x + 8*e) + 36*a*cos(6*f
*x + 6*e) + 82*a*cos(4*f*x + 4*e) + 36*a*cos(2*f*x + 2*e) - 32*a*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a*cos(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 3*a)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) - (17*a*cos(8*f*x + 8*e) + 140*a*cos(6*f*x + 6*e) + 294*a*cos(
4*f*x + 4*e) + 140*a*cos(2*f*x + 2*e) + 32*a*cos(1/2*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 17*a)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - (17*a*cos(8*f*x + 8*e) + 140*a*cos(6*f*x + 6*e) + 294*a*cos(4*f*
x + 4*e) + 140*a*cos(2*f*x + 2*e) + 32*a*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 17*a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - (3*a*cos(8*f*x + 8*e) + 36*a*cos(6*f*x + 6*e) + 82*a*cos(4*f*x + 4*e
) + 36*a*cos(2*f*x + 2*e) + 3*a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))))*sqrt(a)*sqrt(c)/((c^5*cos(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x +
6*e)^2 + 4900*c^5*cos(4*f*x + 4*e)^2 + 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*
cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 + c^5*sin(8*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*
```

e)^2 + 4900*c^5*sin(4*f*x + 4*e)^2 + 3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*c^5*sin(2*f*x + 2*e)^2 + 64*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c^5*cos(2*f*x + 2*e) + c^5 + 2*(28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*cos(2*f*x + 2*e) + c^5)*cos(4*f*x + 4*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 28*(2*c^5*sin(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 19.89 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{9/2}}\,dx=\frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(\frac{a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,68\mathrm{i}}{3\,c^5\,f}-\frac{a\,\cos(e+f\,x)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+}}{3\,c^5\,f}\right.}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,84\mathrm{i}-\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(2\,e+\right.}$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(9/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))^(1/
2)*68i)/(3*c^5*f) - (a*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))
^(1/2)*88i)/(3*c^5*f) + (a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a/cos(e
 + f*x))^(1/2)*80i)/(3*c^5*f) - (a*exp(e*5i + f*x*5i)*cos(3*e + 3*f*x)*(a +
 a/cos(e + f*x))^(1/2)*8i)/(c^5*f) + (a*exp(e*5i + f*x*5i)*cos(4*e + 4*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f)))/(exp(e*5i + f*x*5i)*sin(e + f*x)*
84i - exp(e*5i + f*x*5i)*sin(2*e + 2*f*x)*96i + exp(e*5i + f*x*5i)*sin(3*e
+ 3*f*x)*54i - exp(e*5i + f*x*5i)*sin(4*e + 4*f*x)*16i + exp(e*5i + f*x*5i)
*sin(5*e + 5*f*x)*2i)
```

3.122 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}} + \frac{a^2\tan(e+fx)}{20cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}$$

```
[Out] 1/20*a^2*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(9/2)/(a+a*sec(f*x+e))^(1/2)-1/5*a
*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4039, 4038}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{a^2\tan(e+fx)}{20cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{9/2}} - \frac{a\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{5f(c-c\sec(e+fx))^{11/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(11/2),x
]
```

```
[Out] -1/5*(a*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(11/
2)) + (a^2*Tan[e + f*x])/(20*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*
x])^(9/2))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}}-\frac{a\int\frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{(c-c\sec(e+fx))^{9/2}}\,dx}{5c}\\ &= -\frac{a\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{5f(c-c\sec(e+fx))^{11/2}}+\frac{a^2\tan(e+fx)}{20cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{9/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 5.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.74

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\frac{a^2(3+5\sec(e+fx))\tan(e+fx)}{20c^5f(-1+\sec(e+fx))^5\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(3/2))/(c - c*Sec[e + f*x])^(1
1/2),x]
```

```
[Out] (a^2*(3 + 5*Sec[e + f*x])*Tan[e + f*x])/(20*c^5*f*(-1 + Sec[e + f*x])^5*Sqr
t[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

method	result
default	$\frac{a\left(49\cos(fx+e)^3-23\cos(fx+e)^2+7\cos(fx+e)-1\right)\sqrt{a(\sec(fx+e)+1)}\,(\cos(fx+e)+1)\tan(fx+e)\sec(fx+e)^4}{320f(\sec(fx+e)-1)^5\sqrt{-c(\sec(fx+e)-1)}\,c^5}$
risch	$\frac{2ia\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(5\,e^{9i(fx+e)}-15\,e^{8i(fx+e)}+50\,e^{7i(fx+e)}-75\,e^{6i(fx+e)}+102\,e^{5i(fx+e)}-75\,e^{4i(fx+e)}+50\,e^{3i(fx+e)}-15\,e^{2i(fx+e)}\right.}{5c^5\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^9\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/320/f*a*(49*cos(f*x+e)^3-23*cos(f*x+e)^2+7*cos(f*x+e)-1)*(a*(sec(f*x+e)+1
))^(1/2)*(cos(f*x+e)+1)/(sec(f*x+e)-1)^5/(-c*(sec(f*x+e)-1))^(1/2)/c^5*tan(
f*x+e)*sec(f*x+e)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(80) = 160.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\left(20\,a\cos\left(fx+e\right)^5-30\,a\cos\left(fx+e\right)^4+30\,a\cos\left(fx+e\right)^3-15\,c\right.}{20\left(c^6f\cos\left(fx+e\right)^5-5\,c^6f\cos\left(fx+e\right)^4+10\,c^6f\cos\left(fx+e\right)^3-\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/20*(20*a*cos(f*x + e)^5 - 30*a*cos(f*x + e)^4 + 30*a*cos(f*x + e)^3 - 15*
a*cos(f*x + e)^2 + 3*a*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5 - 5*c^6*f*
cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^2 + 5*c^6*
f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(11/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3906 vs. $2(80) = 160$.

Time = 16.64 (sec) , antiderivative size = 3906, normalized size of antiderivative = 42.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="maxima")
[Out] -2/5*(225*a*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 225*a*cos(4*f*x + 4*e)*sin(
2*f*x + 2*e) - 15*(a*sin(8*f*x + 8*e) + 5*a*sin(6*f*x + 6*e) + 5*a*sin(4*f*
x + 4*e) + a*sin(2*f*x + 2*e))*cos(10*f*x + 10*e) - 225*(a*sin(6*f*x + 6*e)
 + a*sin(4*f*x + 4*e))*cos(8*f*x + 8*e) - 5*(a*sin(10*f*x + 10*e) + 15*a*si
n(8*f*x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*sin(2
*f*x + 2*e) - 20*a*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4
8*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*a*sin(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 10*(5*a*sin(10*f*x + 10*e) + 45*a*sin(8*f*x + 8*e) +
 150*a*sin(6*f*x + 6*e) + 150*a*sin(4*f*x + 4*e) + 45*a*sin(2*f*x + 2*e) -
36*a*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*a*sin(1/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 6*(17*a*sin(10*f*x + 10*e) + 135*a*sin(8*f*x + 8*e)
 + 420*a*sin(6*f*x + 6*e) + 420*a*sin(4*f*x + 4*e) + 135*a*sin(2*f*x + 2*e)
 + 60*a*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 40*a*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) - 50*(a*sin(10*f*x + 10*e) + 9*a*sin(8*f*x + 8*e)
+ 30*a*sin(6*f*x + 6*e) + 30*a*sin(4*f*x + 4*e) + 9*a*sin(2*f*x + 2*e) + 2*
a*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 5*(a*sin(10*f*x + 10*e) + 15*a*sin(8*f*
x + 8*e) + 60*a*sin(6*f*x + 6*e) + 60*a*sin(4*f*x + 4*e) + 15*a*sin(2*f*x +
 2*e))*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 15*(a*cos(8*f
*x + 8*e) + 5*a*cos(6*f*x + 6*e) + 5*a*cos(4*f*x + 4*e) + a*cos(2*f*x + 2*e
))*sin(10*f*x + 10*e) + 15*(15*a*cos(6*f*x + 6*e) + 15*a*cos(4*f*x + 4*e) -
```

a)*sin(8*f*x + 8*e) - 75*(3*a*cos(2*f*x + 2*e) + a)*sin(6*f*x + 6*e) - 75*(3*a*cos(2*f*x + 2*e) + a)*sin(4*f*x + 4*e) - 15*a*sin(2*f*x + 2*e) + 5*(a*cos(10*f*x + 10*e) + 15*a*cos(8*f*x + 8*e) + 60*a*cos(6*f*x + 6*e) + 60*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) - 20*a*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 48*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*(5*a*cos(10*f*x + 10*e) + 45*a*cos(8*f*x + 8*e) + 150*a*cos(6*f*x + 6*e) + 150*a*cos(4*f*x + 4*e) + 45*a*cos(2*f*x + 2*e) - 36*a*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 5*a)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(17*a*cos(10*f*x + 10*e) + 135*a*cos(8*f*x + 8*e) + 420*a*cos(6*f*x + 6*e) + 420*a*cos(4*f*x + 4*e) + 135*a*cos(2*f*x + 2*e) + 60*a*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 40*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 50*(a*cos(10*f*x + 10*e) + 9*a*cos(8*f*x + 8*e) + 30*a*cos(6*f*x + 6*e) + 30*a*cos(4*f*x + 4*e) + 9*a*cos(2*f*x + 2*e) + 2*a*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + a)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 5*(a*cos(10*f*x + 10*e) + 15*a*cos(8*f*x + 8*e) + 60*a*cos(6*f*x + 6*e) + 60*a*cos(4*f*x + 4*e) + 15*a*cos(2*f*x + 2*e) + a)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 10*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 + 44100*c^6*cos(6*f*x + 6*e)^2 + 44100*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 + 2025*c^6*sin(8*f*x + 8*e)^2 + 44100*c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*sin(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 63504*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) + c^6 + 2*(45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(10*f*x + 10*e) + 90*(210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(8*f*x + 8*e) + 420*(210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(6*f*x + 6*e) + 420*(45*c^6*cos(2*f*x + 2*e) + c^6)*cos(4*f*x + 4*e) - 20*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(9/2*a rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 252*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 504*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(3*c^6*sin(8*f*x + 8*e) + 14*c^6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + 1350*(14*c^6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6300*(14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 20*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 120*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 504*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{3}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 19.24 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,60\mathrm{i}}{c^6\,f}-\frac{a\,\cos(e+f\,x)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+\frac{a}{\cos\ldots}}}{5\,c^6\,f}\ldots\right.}{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(e+f\,x\right)\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(2\,e\ldots\right.}$$

```
[In] int((a + a/cos(e + f*x))^(3/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(11/2)),x
)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(1/
2)*60i)/(c^6*f) - (a*cos(e + f*x)*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(
1/2)*608i)/(5*c^6*f) + (a*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a/cos(e
+ f*x))^(1/2)*72i)/(c^6*f) - (a*exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*(a + a/
cos(e + f*x))^(1/2)*44i)/(c^6*f) + (a*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(
a + a/cos(e + f*x))^(1/2)*12i)/(c^6*f) - (a*exp(e*6i + f*x*6i)*cos(5*e + 5*
f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^6*f)))/(exp(e*6i + f*x*6i)*sin(e + f
*x)*264i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*6i)*si
n(3*e + 3*f*x)*220i - exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*88i + exp(e*6i +
f*x*6i)*sin(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

3.123 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 134

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a^3(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f} + \frac{a(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{6f}$$

```
[Out] 1/6*a*(a+a*sec(f*x+e))^(3/2)*(c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f+1/15*a^3*(
c-c*sec(f*x+e))^(7/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)+2/15*a^2*(c-c*sec
(f*x+e))^(7/2)*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used

$= \{4040, 4038\}$

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{a^3\tan(e+fx)(c-c\sec(e+fx))^{7/2}}{15f\sqrt{a\sec(e+fx)+a}} + \frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{7/2}}{15f} + \frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{7/2}}{6f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x])/(15*f*Sqrt[a + a*Sec[e + f*x]
]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(7/2)*Tan[e + f*x
])/(15*f) + (a*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(7/2)*Tan[e
+ f*x])/(6*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\text{integral} = \frac{a(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{6f} + \frac{1}{3}(2a)\int \sec(e+fx)(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\,dx$$

$$= \frac{2a^2\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f} + \frac{a(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{6f} + \frac{1}{15}(4a^2)\int\sec(e+fx)\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\,dx$$

$$= \frac{a^3(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f\sqrt{a+a\sec(e+fx)}} + \frac{2a^2\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{15f} + \frac{a(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{7/2}\tan(e+fx)}{6f}$$

Mathematica [A] (verified)

Time = 4.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int\sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{4a^2c^4(21+28\cos(e+fx)+11\cos(2(e+fx)))\sec^6(e+fx)\sqrt{a(1+\sec(e+fx))}\sin$$
$$15f\sqrt{c-c\sec(e+fx)}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(7/2
),x]
```

```
[Out] (4*a^2*c^4*(21 + 28*Cos[e + f*x] + 11*Cos[2*(e + f*x)])*Sec[e + f*x]^6*Sqrt
[a*(1 + Sec[e + f*x])]*Sin[(e + f*x)/2]^8*Tan[(e + f*x)/2])/(15*f*Sqrt[c -
c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$-\frac{a^2\left(21\cos(fx+e)^3-33\cos(fx+e)^2+21\cos(fx+e)-5\right)(\sec(fx+e)-1)^3\sqrt{a(\sec(fx+e)+1)}\sqrt{}}{30f(\cos(fx+e)-1)^3}$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x)
```

```
[Out] -1/30/f*a^2*(21*cos(f*x+e)^3-33*cos(f*x+e)^2+21*cos(f*x+e)-5)*(sec(f*x+e)-1
)^3*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)*c^3*(cos(f*x+e)+1)^3
/(cos(f*x+e)-1)^3*sec(f*x+e)^2*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \frac{\left(30\,a^2c^3\cos\left(fx+e\right)^5-15\,a^2c^3\cos\left(fx+e\right)^4-20\,a^2c^3\cos\left(fx+e\right)^3+15\,a^2c^3\cos\left(fx\right.\right.}{30\,f\cos\left(fx+e\right)^5\sin\left(fx\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/30*(30*a^2*c^3*cos(f*x + e)^5 - 15*a^2*c^3*cos(f*x + e)^4 - 20*a^2*c^3*co
s(f*x + e)^3 + 15*a^2*c^3*cos(f*x + e)^2 + 6*a^2*c^3*cos(f*x + e) - 5*a^2*c
^3)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))/(f*cos(f*x + e)^5*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2454 vs. 2(116) = 232.

Time = 0.43 (sec) , antiderivative size = 2454, normalized size of antiderivative = 18.31

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(210*a^2*c^3*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 90*a^2*c^3*cos(2*f*x
+ 2*e)*sin(f*x + e) - 15*a^2*c^3*sin(f*x + e) - (15*a^2*c^3*sin(11*f*x + 11
*e) - 15*a^2*c^3*sin(10*f*x + 10*e) + 35*a^2*c^3*sin(9*f*x + 9*e) + 78*a^2*
c^3*sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f*x +
```

5*e) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15*a^2*
c^3*sin(f*x + e))*cos(12*f*x + 12*e) + 15*(6*a^2*c^3*sin(10*f*x + 10*e) + 1
5*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^3*sin(4
*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(11*f*x + 11*e) - 3*(70*a^2*c^
3*sin(9*f*x + 9*e) + 75*a^2*c^3*sin(8*f*x + 8*e) + 156*a^2*c^3*sin(7*f*x +
7*e) + 156*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e) + 70*a^2*
c^3*sin(3*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(10*f*x + 10*e) + 35*(15
*a^2*c^3*sin(8*f*x + 8*e) + 20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c^3*sin(4*
f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e) - 15*(78*a^2*c^3*
sin(7*f*x + 7*e) - 50*a^2*c^3*sin(6*f*x + 6*e) + 78*a^2*c^3*sin(5*f*x + 5*e
) + 35*a^2*c^3*sin(3*f*x + 3*e) - 15*a^2*c^3*sin(2*f*x + 2*e) + 15*a^2*c^3*
sin(f*x + e))*cos(8*f*x + 8*e) + 78*(20*a^2*c^3*sin(6*f*x + 6*e) + 15*a^2*c
^3*sin(4*f*x + 4*e) + 6*a^2*c^3*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(15
6*a^2*c^3*sin(5*f*x + 5*e) + 75*a^2*c^3*sin(4*f*x + 4*e) + 70*a^2*c^3*sin(3
*f*x + 3*e) + 30*a^2*c^3*sin(f*x + e))*cos(6*f*x + 6*e) + 234*(5*a^2*c^3*si
n(4*f*x + 4*e) + 2*a^2*c^3*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 75*(7*a^2*c
^3*sin(3*f*x + 3*e) - 3*a^2*c^3*sin(2*f*x + 2*e) + 3*a^2*c^3*sin(f*x + e))*
cos(4*f*x + 4*e) + (15*a^2*c^3*cos(11*f*x + 11*e) - 15*a^2*c^3*cos(10*f*x +
10*e) + 35*a^2*c^3*cos(9*f*x + 9*e) + 78*a^2*c^3*cos(7*f*x + 7*e) - 50*a^2
*c^3*cos(6*f*x + 6*e) + 78*a^2*c^3*cos(5*f*x + 5*e) + 35*a^2*c^3*cos(3*f*x
+ 3*e) - 15*a^2*c^3*cos(2*f*x + 2*e) + 15*a^2*c^3*cos(f*x + e))*sin(12*f*x
+ 12*e) - 15*(6*a^2*c^3*cos(10*f*x + 10*e) + 15*a^2*c^3*cos(8*f*x + 8*e) +
20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2
*f*x + 2*e) + a^2*c^3)*sin(11*f*x + 11*e) + 3*(70*a^2*c^3*cos(9*f*x + 9*e)
+ 75*a^2*c^3*cos(8*f*x + 8*e) + 156*a^2*c^3*cos(7*f*x + 7*e) + 156*a^2*c^3*
cos(5*f*x + 5*e) + 75*a^2*c^3*cos(4*f*x + 4*e) + 70*a^2*c^3*cos(3*f*x + 3*e
) + 30*a^2*c^3*cos(f*x + e) + 5*a^2*c^3)*sin(10*f*x + 10*e) - 35*(15*a^2*c^
3*cos(8*f*x + 8*e) + 20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^2*c^3*cos(4*f*x + 4
*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(9*f*x + 9*e) + 15*(78*a^2*c
^3*cos(7*f*x + 7*e) - 50*a^2*c^3*cos(6*f*x + 6*e) + 78*a^2*c^3*cos(5*f*x +
5*e) + 35*a^2*c^3*cos(3*f*x + 3*e) - 15*a^2*c^3*cos(2*f*x + 2*e) + 15*a^2*c
^3*cos(f*x + e))*sin(8*f*x + 8*e) - 78*(20*a^2*c^3*cos(6*f*x + 6*e) + 15*a^
2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*c^3)*sin(7*f*x +
7*e) + 10*(156*a^2*c^3*cos(5*f*x + 5*e) + 75*a^2*c^3*cos(4*f*x + 4*e) + 70*
a^2*c^3*cos(3*f*x + 3*e) + 30*a^2*c^3*cos(f*x + e) + 5*a^2*c^3)*sin(6*f*x +
6*e) - 78*(15*a^2*c^3*cos(4*f*x + 4*e) + 6*a^2*c^3*cos(2*f*x + 2*e) + a^2*
c^3)*sin(5*f*x + 5*e) + 75*(7*a^2*c^3*cos(3*f*x + 3*e) - 3*a^2*c^3*cos(2*f*
x + 2*e) + 3*a^2*c^3*cos(f*x + e))*sin(4*f*x + 4*e) - 35*(6*a^2*c^3*cos(2*f
*x + 2*e) + a^2*c^3)*sin(3*f*x + 3*e) + 15*(6*a^2*c^3*cos(f*x + e) + a^2*c^
3)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2*(6*cos(10*f*x + 10*e) + 15*cos(8*f
*x + 8*e) + 20*cos(6*f*x + 6*e) + 15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e)
+ 1)*cos(12*f*x + 12*e) + cos(12*f*x + 12*e)^2 + 12*(15*cos(8*f*x + 8*e) +
20*cos(6*f*x + 6*e) + 15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(10*
f*x + 10*e) + 36*cos(10*f*x + 10*e)^2 + 30*(20*cos(6*f*x + 6*e) + 15*cos(4*
f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + 225*cos(8*f*x + 8*e

```
)^2 + 40*(15*cos(4*f*x + 4*e) + 6*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) +
400*cos(6*f*x + 6*e)^2 + 30*(6*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 225
*cos(4*f*x + 4*e)^2 + 36*cos(2*f*x + 2*e)^2 + 2*(6*sin(10*f*x + 10*e) + 15*
sin(8*f*x + 8*e) + 20*sin(6*f*x + 6*e) + 15*sin(4*f*x + 4*e) + 6*sin(2*f*x
+ 2*e))*sin(12*f*x + 12*e) + sin(12*f*x + 12*e)^2 + 12*(15*sin(8*f*x + 8*e)
 + 20*sin(6*f*x + 6*e) + 15*sin(4*f*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(10*f
*x + 10*e) + 36*sin(10*f*x + 10*e)^2 + 30*(20*sin(6*f*x + 6*e) + 15*sin(4*f
*x + 4*e) + 6*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 225*sin(8*f*x + 8*e)^2 +
 120*(5*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 400*sin(6
*f*x + 6*e)^2 + 225*sin(4*f*x + 4*e)^2 + 180*sin(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 36*sin(2*f*x + 2*e)^2 + 12*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^{\frac{7}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.29

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{7/2}\,dx=\frac{\sqrt{c-\frac{c}{\cos(e+fx)}}\left(-\frac{a^2\,c^3\,e^{e\,6i+f\,x\,6i}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,20i}{3\,f}+\frac{a^2\,c^3\,\cos(e+f\,x)\,e^{e\,6i+f\,x\,6i}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,104i}{5\,f}+\cdots\right)}{e^{e\,6i+f\,x\,6i}\,\sin(2\,e+2\,f\,x)\,10i\cdots}$$

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(7/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c^3*cos(e + f*x)*exp(e*6i + f*x*6i)*(a +
a/cos(e + f*x))^(1/2)*104i)/(5*f) - (a^2*c^3*exp(e*6i + f*x*6i)*(a + a/cos(
e + f*x))^(1/2)*20i)/(3*f) + (a^2*c^3*exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*(
a + a/cos(e + f*x))^(1/2)*28i)/(3*f) - (a^2*c^3*exp(e*6i + f*x*6i)*cos(4*e
+ 4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a^2*c^3*exp(e*6i + f*x*6i)*cos
(5*e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*6i + f*x*6i)*sin(2*
e + 2*f*x)*10i + exp(e*6i + f*x*6i)*sin(4*e + 4*f*x)*8i + exp(e*6i + f*x*6i
)*sin(6*e + 6*f*x)*2i)
```

3.124 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 134

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = & -\frac{2c^3(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} \\ & -\frac{c^2(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f} \\ & -\frac{c(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f}\end{aligned}$$

```
[Out] -1/5*c*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f-2/15*c^3*
(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/5*c^2*(a+a*sec
(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used

$= \{4040, 4038\}$

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2c^3\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{15f\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{5f} - \frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}{5f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*c^3*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(15*f*Sqrt[c - c*Sec[e + f
*x]]) - (c^2*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*
x])/(5*f) - (c*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)*Tan[e
+ f*x])/(5*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{c(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f} + \frac{1}{5}(4c)\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$$

$$
\begin{aligned}
&= -\frac{c^2(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f} \\
&\quad - \frac{c(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f} \\
&\quad + \frac{1}{5}(2c^2)\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{2c^3(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{15f\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{c^2(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{5f} \\
&\quad - \frac{c(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{5f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.57

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \\
&-\frac{a^3c^3\sec(e+fx)\left(15-10\sec^2(e+fx)+3\sec^4(e+fx)\right)\tan(e+fx)}{15f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2
),x]
```

```
[Out] -1/15*(a^3*c^3*Sec[e + f*x]*(15 - 10*Sec[e + f*x]^2 + 3*Sec[e + f*x]^4)*Tan
[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.75

$$
\frac{a^2(\sec(fx+e)-1)^2\left(8\cos(fx+e)^2-9\cos(fx+e)+3\right)\sqrt{-c(\sec(fx+e)-1)}\,c^2\sqrt{a(\sec(fx+e)+1)}}{15f(\cos(fx+e)-1)^2}
$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/15/f*a^2*(sec(f*x+e)-1)^2*(8*cos(f*x+e)^2-9*cos(f*x+e)+3)*(-c*(sec(f*x+e)
-1))^(1/2)*c^2*(a*(sec(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)^3/(cos(f*x+e)-1)^2*s
ec(f*x+e)^2*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{\left(15\,a^2c^2\cos\left(fx+e\right)^4-10\,a^2c^2\cos\left(fx+e\right)^2+3\,a^2c^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{15\,f\cos\left(fx+e\right)^4\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/15*(15*a^2*c^2*cos(f*x + e)^4 - 10*a^2*c^2*cos(f*x + e)^2 + 3*a^2*c^2)*sq
rt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e
))/(f*cos(f*x + e)^4*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1526 vs. 2(116) = 232.

Time = 0.40 (sec) , antiderivative size = 1526, normalized size of antiderivative = 11.39

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 2/15*(100*a^2*c^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 75*a^2*c^2*cos(f*x +
e)*sin(2*f*x + 2*e) - 75*a^2*c^2*cos(2*f*x + 2*e)*sin(f*x + e) - 15*a^2*c^2
*sin(f*x + e) - (15*a^2*c^2*sin(9*f*x + 9*e) + 20*a^2*c^2*sin(7*f*x + 7*e)
+ 58*a^2*c^2*sin(5*f*x + 5*e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*si
n(f*x + e))*cos(10*f*x + 10*e) + 75*(a^2*c^2*sin(8*f*x + 8*e) + 2*a^2*c^2*s
```

```
in(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*co
s(9*f*x + 9*e) - 5*(20*a^2*c^2*sin(7*f*x + 7*e) + 58*a^2*c^2*sin(5*f*x + 5*
e) + 20*a^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(8*f*x + 8*e
) + 100*(2*a^2*c^2*sin(6*f*x + 6*e) + 2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*
sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 10*(58*a^2*c^2*sin(5*f*x + 5*e) + 20*a
^2*c^2*sin(3*f*x + 3*e) + 15*a^2*c^2*sin(f*x + e))*cos(6*f*x + 6*e) + 290*(
2*a^2*c^2*sin(4*f*x + 4*e) + a^2*c^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 5
0*(4*a^2*c^2*sin(3*f*x + 3*e) + 3*a^2*c^2*sin(f*x + e))*cos(4*f*x + 4*e) +
(15*a^2*c^2*cos(9*f*x + 9*e) + 20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*c^2*cos
(5*f*x + 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(
10*f*x + 10*e) - 15*(5*a^2*c^2*cos(8*f*x + 8*e) + 10*a^2*c^2*cos(6*f*x + 6*
e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*si
n(9*f*x + 9*e) + 5*(20*a^2*c^2*cos(7*f*x + 7*e) + 58*a^2*c^2*cos(5*f*x + 5*
e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(8*f*x + 8*e
) - 20*(10*a^2*c^2*cos(6*f*x + 6*e) + 10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c
^2*cos(2*f*x + 2*e) + a^2*c^2)*sin(7*f*x + 7*e) + 10*(58*a^2*c^2*cos(5*f*x
+ 5*e) + 20*a^2*c^2*cos(3*f*x + 3*e) + 15*a^2*c^2*cos(f*x + e))*sin(6*f*x +
 6*e) - 58*(10*a^2*c^2*cos(4*f*x + 4*e) + 5*a^2*c^2*cos(2*f*x + 2*e) + a^2*
c^2)*sin(5*f*x + 5*e) + 50*(4*a^2*c^2*cos(3*f*x + 3*e) + 3*a^2*c^2*cos(f*x
+ e))*sin(4*f*x + 4*e) - 20*(5*a^2*c^2*cos(2*f*x + 2*e) + a^2*c^2)*sin(3*f*
x + 3*e))*sqrt(a)*sqrt(c)/((2*(5*cos(8*f*x + 8*e) + 10*cos(6*f*x + 6*e) + 1
0*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) + 1)*cos(10*f*x + 10*e) + cos(10*f*
x + 10*e)^2 + 10*(10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e) + 5*cos(2*f*x +
 2*e) + 1)*cos(8*f*x + 8*e) + 25*cos(8*f*x + 8*e)^2 + 20*(10*cos(4*f*x + 4*
e) + 5*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 100*cos(6*f*x + 6*e)^2 + 20
*(5*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 100*cos(4*f*x + 4*e)^2 + 25*co
s(2*f*x + 2*e)^2 + 10*(sin(8*f*x + 8*e) + 2*sin(6*f*x + 6*e) + 2*sin(4*f*x
+ 4*e) + sin(2*f*x + 2*e))*sin(10*f*x + 10*e) + sin(10*f*x + 10*e)^2 + 50*(
2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin(8*f*x + 8*e
) + 25*sin(8*f*x + 8*e)^2 + 100*(2*sin(4*f*x + 4*e) + sin(2*f*x + 2*e))*sin
(6*f*x + 6*e) + 100*sin(6*f*x + 6*e)^2 + 100*sin(4*f*x + 4*e)^2 + 100*sin(4
*f*x + 4*e)*sin(2*f*x + 2*e) + 25*sin(2*f*x + 2*e)^2 + 10*cos(2*f*x + 2*e)
+ 1)*f)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 17.36 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.60

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,116\mathrm{i}}{15\,f}+\frac{a^2\,c^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,16\mathrm{i}}{3\,f}+\cdots\right.}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(e+f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,6\mathrm{i}+\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\cdots}}$$

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c^2*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x
))^(1/2)*116i)/(15*f) + (a^2*c^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a
/cos(e + f*x))^(1/2)*16i)/(3*f) + (a^2*c^2*exp(e*5i + f*x*5i)*cos(4*e + 4*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*5i + f*x*5i)*sin(e + f*x)*4i
+ exp(e*5i + f*x*5i)*sin(3*e + 3*f*x)*6i + exp(e*5i + f*x*5i)*sin(5*e + 5*f
*x)*2i)
```

3.125 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{c-c\sec(e+fx)}} - \frac{c(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{4f}$$

```
[Out] -1/6*c^2*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)-1/4*c*(
a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = -\frac{c^2\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}{4f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(c^2*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*
x]]) - (c*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])
/(4*f)
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{c(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{4f} \\
&\quad+\frac{1}{2}c\int\sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx \\
&= -\frac{c^2(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f\sqrt{c-c\sec(e+fx)}} \\
&\quad-\frac{c(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{4f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int\sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx =$$

$$-\frac{a^2c^2(5\cos(e+fx)+3(\cos(2(e+fx))+\cos(3(e+fx))))\sec^4(e+fx)\sqrt{a(1+\sec(e+fx))}\tan\left(\frac{1}{2}(e+fx\right.}{12f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2
),x]
```

```
[Out] -1/12*(a^2*c^2*(5*Cos[e + f*x] + 3*(Cos[2*(e + f*x)] + Cos[3*(e + f*x)]))*S
ec[e + f*x]^4*Sqrt[a*(1 + Sec[e + f*x])]*Tan[(e + f*x)/2])/(f*Sqrt[c - c*Se
c[e + f*x]])
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{a^{2}\left(\sec\left(fx+e\right)-1\right)\left(5\cos\left(fx+e\right)-3\right)\sqrt{-c\left(\sec\left(fx+e\right)-1\right)}\,c\sqrt{a\left(\sec\left(fx+e\right)+1\right)}\,\left(\cos\left(fx+e\right)+1\right)^{3}\sec\left(fx+e\right)^{2}\csc\left(fx+e\right)}{12f\left(\cos\left(fx+e\right)-1\right)}$	87
risch	$\frac{2ia^{2}c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\left(3\,e^{7i(fx+e)}+3\,e^{6i(fx+e)}+5\,e^{5i(fx+e)}+5\,e^{3i(fx+e)}+3\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3\left(1+e^{2i(fx+e)}\right)^{3}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$	177

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/12/f*a^2*(sec(f*x+e)-1)*(5*cos(f*x+e)-3)*(-c*(sec(f*x+e)-1))^(1/2)*c*(a*
(sec(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)^3/(cos(f*x+e)-1)*sec(f*x+e)^2*csc(f*x+
e)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{\left(12\,a^{2}c\cos\left(fx+e\right)^{3}+6\,a^{2}c\cos\left(fx+e\right)^{2}-4\,a^{2}c\cos\left(fx+e\right)-3\,a^{2}c\right)\sqrt{\frac{a\cos(fx+e)+}{\cos(fx+e)}}}{12\,f\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*(12*a^2*c*cos(f*x + e)^3 + 6*a^2*c*cos(f*x + e)^2 - 4*a^2*c*cos(f*x +
e) - 3*a^2*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
- c)/cos(f*x + e))/(f*cos(f*x + e)^3*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1106 vs. 2(77) = 154.

Time = 0.39 (sec) , antiderivative size = 1106, normalized size of antiderivative = 12.43

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
[Out] 2/3*(20*a^2*c*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 12*a^2*c*cos(2*f*x + 2*e)
*sin(f*x + e) - 3*a^2*c*sin(f*x + e) - (3*a^2*c*sin(7*f*x + 7*e) + 3*a^2*c*
sin(6*f*x + 6*e) + 5*a^2*c*sin(5*f*x + 5*e) + 5*a^2*c*sin(3*f*x + 3*e) + 3*
a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(2*a^2*
c*sin(6*f*x + 6*e) + 3*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*c
os(7*f*x + 7*e) - 2*(10*a^2*c*sin(5*f*x + 5*e) - 9*a^2*c*sin(4*f*x + 4*e) +
 10*a^2*c*sin(3*f*x + 3*e) + 6*a^2*c*sin(f*x + e))*cos(6*f*x + 6*e) + 10*(3
*a^2*c*sin(4*f*x + 4*e) + 2*a^2*c*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 6*(5
*a^2*c*sin(3*f*x + 3*e) + 3*a^2*c*sin(2*f*x + 2*e) + 3*a^2*c*sin(f*x + e))*
cos(4*f*x + 4*e) + (3*a^2*c*cos(7*f*x + 7*e) + 3*a^2*c*cos(6*f*x + 6*e) + 5
*a^2*c*cos(5*f*x + 5*e) + 5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*
e) + 3*a^2*c*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(4*a^2*c*cos(6*f*x + 6*e) +
 6*a^2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(7*f*x + 7
*e) + (20*a^2*c*cos(5*f*x + 5*e) - 18*a^2*c*cos(4*f*x + 4*e) + 20*a^2*c*cos
(3*f*x + 3*e) + 12*a^2*c*cos(f*x + e) - 3*a^2*c)*sin(6*f*x + 6*e) - 5*(6*a^
2*c*cos(4*f*x + 4*e) + 4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(5*f*x + 5*e) +
 6*(5*a^2*c*cos(3*f*x + 3*e) + 3*a^2*c*cos(2*f*x + 2*e) + 3*a^2*c*cos(f*x +
 e))*sin(4*f*x + 4*e) - 5*(4*a^2*c*cos(2*f*x + 2*e) + a^2*c)*sin(3*f*x + 3*
e) + 3*(4*a^2*c*cos(f*x + e) - a^2*c)*sin(2*f*x + 2*e))*sqrt(a)*sqrt(c)/((2
*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f
*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e)
 + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1
)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*s
```

```
in(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e)
 + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*
f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*f)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\int (a\sec(fx+e)+a)^{\frac{5}{2}}(-c\sec(fx+e)+c)^{\frac{3}{2}}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 16.85 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}\,dx=\frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a^2\,c\,\cos(e+f\,x)\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,20\mathrm{i}}{3\,f}+\frac{a^2\,c\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{f}\right.}{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,4\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(4\,e+\right.}$$

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*c*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/
cos(e + f*x))^(1/2)*20i)/(3*f) + (a^2*c*exp(e*4i + f*x*4i)*cos(2*e + 2*f*x)
*(a + a/cos(e + f*x))^(1/2)*4i)/f + (a^2*c*exp(e*4i + f*x*4i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/f))/(exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)
*4i + exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

3.126 $\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,d$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/3*c*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{c\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{3f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -1/3*(c*(a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*Sqrt[c - c*Sec[e + f*x]
])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{c(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{3f\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = -\frac{a^3c\sec(e+fx)\left(3+3\sec(e+fx)+\sec^2(e+fx)\right)\tan(e+fx)}{3f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]],
x]
```

```
[Out] -1/3*(a^3*c*Sec[e + f*x]*(3 + 3*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e + f*x]
)/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{a^2(\cos(fx+e)+1)^3\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,\sec(fx+e)^2\csc(fx+e)}{3f}$	58
risch	$\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(3\,e^{5i(fx+e)}+6\,e^{4i(fx+e)}+10\,e^{3i(fx+e)}+6\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3\left(1+e^{2i(fx+e)}\right)^2\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)f}$	165

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3/f*a^2*(cos(f*x+e)+1)^3*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/
2)*sec(f*x+e)^2*csc(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{\left(3\,a^2\cos\left(fx+e\right)^2+3\,a^2\cos\left(fx+e\right)+a^2\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{3\,f\cos\left(fx+e\right)^2\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^2 + 3*a^2*cos(f*x + e) + a^2)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(f*cos(f*x + e)^
2*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{8\sqrt{-a}a^2\sqrt{c}}{3\,f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^3\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 8/3*sqrt(-a)*a^2*sqrt(c)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^3*(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)^3)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \int (a\sec(fx+e)+a)^{\frac{5}{2}}\sqrt{-c\sec(fx+e)+c}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)*(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 14.98 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.16

$$\int \sec(e+fx)(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\,a^2\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,(10\,\sin(e+f\,x)+12\,\sin(2\,e+2\,f\,x)}{3\,f\,(\cos(2\,e+2\,f\,x)-2\,\cos(4\,e+4}$$

```
[In] int(((a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
```

```
[Out] (2*a^2*((a*(cos(e + f*x) + 1))/cos(e + f*x))^(1/2)*((c*(cos(e + f*x) - 1))/
cos(e + f*x))^(1/2)*(10*sin(e + f*x) + 12*sin(2*e + 2*f*x) + 13*sin(3*e + 3
*f*x) + 6*sin(4*e + 4*f*x) + 3*sin(5*e + 5*f*x)))/(3*f*(cos(2*e + 2*f*x) -
2*cos(4*e + 4*f*x) - cos(6*e + 6*f*x) + 2))
```

3.127 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 141

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}} + \frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)+4*a^3*ln(1
-sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+2*a
^2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4040, 4037}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{f\sqrt{c-c\sec(e+fx)}} + \frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(f*Sq
rt[c - c*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f*
Sqrt[c - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}+(2a)\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx\\
&= \frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}\\
&\quad+\left(4a^2\right)\int\frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx\\
&= \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{f\sqrt{c-c\sec(e+fx)}}+\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\frac{a^3(1+8\log(1-\sec(e+fx))+6\sec(e+fx)+\sec^2(e+fx))\tan}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/Sqrt[c - c*Sec[e + f*x]
],x]
```

```
[Out] (a^3*(1 + 8*Log[1 - Sec[e + f*x]] + 6*Sec[e + f*x] + Sec[e + f*x]^2)*Tan[e
+ f*x])/(2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.05

method	result
default	$\frac{a^{2}\sqrt{a(\sec(fx+e)+1)}\,(16\ln(-\cot(fx+e)+\csc(fx+e))\sin(fx+e)-8\ln(-\cot(fx+e)+\csc(fx+e)-1)\sin(fx+e)-8\ln(-\cot(fx+e)+\mathrm{cs}}{2f(\cos(fx+e)+1)\sqrt{-c(\sec(fx+e)-1)}}$
risch	$-\frac{2ia^{2}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\left(3\,\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}+3\right)\left(\mathrm{e}^{2i(fx+e)}-\mathrm{e}^{i(fx+e)}\right)}{\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,f\left(1+\mathrm{e}^{2i(fx+e)}\right)^{2}}-\frac{8ia^{2}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{i(fx+e)}-1\right)}{\left(\mathrm{e}^{i(fx+e)}+1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,f}+$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)/(-c*(sec(f*x+e)-1))^(1/2)
*(16*ln(-cot(f*x+e)+csc(f*x+e))*sin(f*x+e)-8*ln(-cot(f*x+e)+csc(f*x+e)-1)*s
in(f*x+e)-8*ln(-cot(f*x+e)+csc(f*x+e)+1)*sin(f*x+e)+5*sin(f*x+e)+6*tan(f*x+
e)+sec(f*x+e)*tan(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{\frac{5}{2}}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c*sec(f*x + e) - c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(127) = 254$.

Time = 0.41 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.23

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx =$$
$$-\frac{2\left(a^2\cos\left(2\,fx+2\,e\right)\sin\left(4\,fx+4\,e\right)-a^2\cos\left(4\,fx+4\,e\right)\sin\left(2\,fx+2\,e\right)-a^2\sin\left(2\,fx+2\,e\right)+2\left(a^2\cos\left(\right.\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(a^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - a^2*cos(4*f*x + 4*e)*sin(2*f*x
+ 2*e) - a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x
 + 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(
2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + a^2*sin
(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x
 + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2*f*x + 2*e) + a^2)
*cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + 3*(a^2*si
n(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 3*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 3*(a^2*cos(4*f*x + 4*e)
+ 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - 3*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c*cos(4*f*
x + 4*e)^2 + 4*c*cos(2*f*x + 2*e)^2 + c*sin(4*f*x + 4*e)^2 + 4*c*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 4*c*sin(2*f*x + 2*e)^2 + 2*(2*c*cos(2*f*x + 2*e)
+ c)*cos(4*f*x + 4*e) + 4*c*cos(2*f*x + 2*e) + c)*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec\left(fx+e\right)+a)^{\frac{5}{2}}\sec\left(fx+e\right)}{\sqrt{-c\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{\sqrt{c-c\sec(e+fx)}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x
)
```

3.128 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=-\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}}-\frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)-4*a^3*ln(1-se
c(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-2*a^
2*(a+a*sec(f*x+e))^(1/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4039, 4040, 4037}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx=-\frac{4a^3\tan(e+fx)\log(1-\sec(e+fx))}{cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{2a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf\sqrt{c-c\sec(e+fx)}}-\frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{f(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(3/2)
)) - (4*a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c*f*Sqrt[a + a*Sec[e + f*x
]]*Sqrt[c - c*Sec[e + f*x]]) - (2*a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x]
)/(c*f*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{(2a)\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c} \\
&= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} \\
&\quad -\frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}} - \frac{(4a^2)\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c} \\
&= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{f(c-c\sec(e+fx))^{3/2}} - \frac{4a^3\log(1-\sec(e+fx))\tan(e+fx)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad -\frac{2a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{a^3\left(4\log(1-\sec(e+fx))-\frac{4}{-1+\sec(e+fx)}+\sec(e+fx)\right)\tan(e+fx)}{cf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((a^3*(4*Log[1 - Sec[e + f*x]] - 4/(-1 + Sec[e + f*x]) + Sec[e + f*x])*Tan
[e + f*x])/(c*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.62

method	result
default	$-\frac{a^2\sqrt{a(\sec(fx+e)+1)}\,(4\ln(-\cot(fx+e)+\csc(fx+e)-1)\sin(fx+e)+4\ln(-\cot(fx+e)+\csc(fx+e)+1)\sin(fx+e)-8\ln(-\cot(fx+$
risch	$\frac{2ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(5\,e^{3i(fx+e)}-2\,e^{2i(fx+e)}+5\,e^{i(fx+e)}\right)}{c\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f\left(1+e^{2i(fx+e)}\right)}+\frac{8ia^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{c\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{4ia^2\sqrt{}}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f*a^2*(a*(sec(f*x+e)+1))^(1/2)/(sec(f*x+e)-1)/(-c*(sec(f*x+e)-1))^(1/2)/
c/(cos(f*x+e)+1)*(4*ln(-cot(f*x+e)+csc(f*x+e)-1)*sin(f*x+e)+4*ln(-cot(f*x+e
)+csc(f*x+e)+1)*sin(f*x+e)-8*ln(-cot(f*x+e)+csc(f*x+e))*sin(f*x+e)-4*ln(-co
t(f*x+e)+csc(f*x+e)-1)*tan(f*x+e)-4*ln(-cot(f*x+e)+csc(f*x+e)+1)*tan(f*x+e)
+8*ln(-cot(f*x+e)+csc(f*x+e))*tan(f*x+e)-3*sin(f*x+e)-2*tan(f*x+e)+sec(f*x+
e)*tan(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^2*sec(f*x + e)^2 - 2*c^2
*sec(f*x + e) + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(133) = 266$.

Time = 0.51 (sec) , antiderivative size = 2035, normalized size of antiderivative = 14.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 2*(8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*a^2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*a
^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*a^2*cos(4*f*x + 4*e)*sin(2*f*x + 2
*e) + 2*a^2*sin(2*f*x + 2*e) + 2*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x
+ 2*e)^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2
```

```
*f*x + 2*e) + a^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1) - 4*(a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*co
s(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*s
in(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(3/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin(1/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a
^2*cos(2*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) - 4*(a^2*cos(4*f*x + 4*e) + 2*a
^2*cos(2*f*x + 2*e) - 2*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*
cos(4*f*x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*
e) - 2*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(a^2*sin(4*f*x + 4*e) + 2*a^
2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*a
rctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 1) + (16*a^2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) + 5*a^2*sin(4*f*x + 4*e) + 6*a^2*sin(2*f*x + 2*e) - 8*(a^2*cos(4*f*x +
 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (5*a^2*s
in(4*f*x + 4*e) + 6*a^2*sin(2*f*x + 2*e) - 8*(a^2*cos(4*f*x + 4*e) + 2*a^2*
cos(2*f*x + 2*e) + a^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (16*a^2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 5*a^2*cos(4*f*x + 4*e) - 6*a^2*cos(2*f*x + 2*e) - 5*a^2 - 8*(
a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) -
 (5*a^2*cos(4*f*x + 4*e) + 6*a^2*cos(2*f*x + 2*e) + 5*a^2 + 8*(a^2*sin(4*f*
x + 4*e) + 2*a^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt
(c)/((c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*cos(3/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*sin(3/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*sin(1/2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*
f*x + 2*e) + c^2)*cos(4*f*x + 4*e) - 4*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*
f*x + 2*e) - 2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c
^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*cos(4*f*x
 + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e) - 2*c^
2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f
*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x
)
```

3.129 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}} + \frac{a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf(c-c\sec(e+fx))^{3/2}} + \frac{a^3\log(1-\sec(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*a*(a+a*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)+a^2*(a+a*
sec(f*x+e))^(1/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(3/2)+a^3*ln(1-sec(f*x+e)
)*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4039, 4037}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \frac{a^3\tan(e+fx)\log(1-\sec(e+fx))}{c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{a^2\tan(e+fx)\sqrt{a\sec(e+fx)+a}}{cf(c-c\sec(e+fx))^{3/2}} - \frac{a\tan(e+fx)(a\sec(e+fx)+a)^{3/2}}{2f(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/2*(a*(a + a*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(5
/2)) + (a^2*Sqrt[a + a*Sec[e + f*x]]*Tan[e + f*x])/(c*f*(c - c*Sec[e + f*x]
)^(3/2)) + (a^3*Log[1 - Sec[e + f*x]]*Tan[e + f*x])/(c^2*f*Sqrt[a + a*Sec[e
 + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}-\frac{a\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{3/2}}{(c-c\sec(e+fx))^{3/2}}\,dx}{c}\\
&= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}\\
&\quad+\frac{a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf(c-c\sec(e+fx))^{3/2}}+\frac{a^2\int\frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\,dx}{c^2}\\
&= -\frac{a(a+a\sec(e+fx))^{3/2}\tan(e+fx)}{2f(c-c\sec(e+fx))^{5/2}}+\frac{a^2\sqrt{a+a\sec(e+fx)}\tan(e+fx)}{cf(c-c\sec(e+fx))^{3/2}}\\
&\quad+\frac{a^3\log(1-\sec(e+fx))\tan(e+fx)}{c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.56

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{a^3\left(-\log(1-\sec(e+fx))+\frac{-2+4\sec(e+fx)}{(-1+\sec(e+fx))^2}\right)\tan(e+fx)}{c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(5
/2),x]
```

```
[Out] -((a^3*(-Log[1 - Sec[e + f*x]] + (-2 + 4*Sec[e + f*x])/(-1 + Sec[e + f*x])^
2)*Tan[e + f*x])/(c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]
))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(131) = 262$.

Time = 3.60 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.88

method	result
default	$-\frac{\sqrt{2}\,a^{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^{2}\csc(fx+e)^{2}-1}}\,(1-\cos(fx+e))\left(2\ln(-\cot(fx+e)+\csc(fx+e)+1)(1-\cos(fx+e))^{4}\csc(fx+e)^{4}+2\ln(-\cot \cdots\right.}{4f\left((1-\cos(fx+e))^{2} \cdots\right.}$
risch	$\frac{8ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,e^{2i(fx+e)}}{c^{2}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{3}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,f}-\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{c^{2}\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,f}+\frac{ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1 \cdots\right.}{1+e^{2i(fx+e)}}}}{c^{2}\left(e^{i(fx+e} \cdots\right.}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/f*2^(1/2)*a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)
^2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(2*ln(-cot(f*x+e)+csc(f*x+e)+1)*(1
-cos(f*x+e))^4*csc(f*x+e)^4+2*ln(-cot(f*x+e)+csc(f*x+e)-1)*(1-cos(f*x+e))^4
*csc(f*x+e)^4-4*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e))^4*csc(f*x+e)^4+2*
(1-cos(f*x+e))^2*csc(f*x+e)^2+1)*csc(f*x+e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(a^2*sec(f*x + e)^3 + 2*a^2*sec(f*x + e)^2 + a^2*sec(f*x + e))*sq
rt(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(c^3*sec(f*x + e)^3 - 3*c^
3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{c^{\frac{5}{2}}} + \frac{2\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}} - \frac{4\sqrt{-a}a^2\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^{\frac{5}{2}}} + \frac{\left(\sqrt{-a}a^2\sqrt{c}+\frac{2\sqrt{-a}a^2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(f}{c^3\sin(fx+e)^4}}{2f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
[Out] -1/2*(2*sqrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/c^(5/2) + 2*s
qrt(-a)*a^2*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 4*sqrt(-a)*a
^2*log(sin(f*x + e)/(cos(f*x + e) + 1))/c^(5/2) + (sqrt(-a)*a^2*sqrt(c) + 2
*sqrt(-a)*a^2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) +
1)^4/(c^3*sin(f*x + e)^4))/f
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{5/2}}\,dx=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
[Out] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x
)
```

3.130 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/6*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7/2),x]
[Out] -1/6*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(7/2
))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = -\frac{(a(1+\sec(e+fx)))^{5/2}\tan(e+fx)}{6f(c-c\sec(e+fx))^{7/2}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(7
/2),x]
```

```
[Out] -1/6*((a*(1 + Sec[e + f*x]))^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(7
/2))
```

Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

method	result	size
default	$\frac{a^{2}(\cos(fx+e)+1)^{2}\sqrt{a(\sec(fx+e)+1)}\,\tan(fx+e)\sec(fx+e)^{2}}{6f(\sec(fx+e)-1)^{3}\sqrt{-c(\sec(fx+e)-1)}\,c^{3}}$	71
risch	$\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,\left(3\,e^{5i(fx+e)}+10\,e^{3i(fx+e)}+3\,e^{i(fx+e)}\right)}{3c^{3}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{5}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,f}$	133

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/6/f*a^2*(cos(f*x+e)+1)^2*(a*(sec(f*x+e)+1))^(1/2)/(sec(f*x+e)-1)^3/(-c*(s
ec(f*x+e)-1))^(1/2)/c^3*tan(f*x+e)*sec(f*x+e)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \frac{\left(3\,a^{2}\cos\left(fx+e\right)^{3}+a^{2}\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos \ldots}{\ldots}}}{3\left(c^{4}f\cos\left(fx+e\right)^{3}-3\,c^{4}f\cos\left(fx+e\right)^{2}+3\,c^{4}f\cos\left(fx+e\right)-\ldots\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*a^2*cos(f*x + e)^3 + a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^4*f*cos(f*x + e)^3 -
 3*c^4*f*cos(f*x + e)^2 + 3*c^4*f*cos(f*x + e) - c^4*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1815 vs. $2(36) = 72$.

Time = 0.41 (sec) , antiderivative size = 1815, normalized size of antiderivative = 43.21

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(208*a^2*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 48*a^2*cos(f*x + e)*sin(2*
f*x + 2*e) - 48*a^2*cos(2*f*x + 2*e)*sin(f*x + e) - 3*a^2*sin(f*x + e) - (3
*a^2*sin(7*f*x + 7*e) + 13*a^2*sin(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) +
 3*a^2*sin(f*x + e))*cos(8*f*x + 8*e) + 6*(8*a^2*sin(6*f*x + 6*e) + 15*a^2*
sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(13*a^2*si
n(5*f*x + 5*e) + 13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(6*f*x +
6*e) + 26*(15*a^2*sin(4*f*x + 4*e) + 8*a^2*sin(2*f*x + 2*e))*cos(5*f*x + 5*
e) - 30*(13*a^2*sin(3*f*x + 3*e) + 3*a^2*sin(f*x + e))*cos(4*f*x + 4*e) + (
3*a^2*cos(7*f*x + 7*e) + 13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e)
+ 3*a^2*cos(f*x + e))*sin(8*f*x + 8*e) - 3*(16*a^2*cos(6*f*x + 6*e) + 30*a^
2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2)*sin(7*f*x + 7*e) + 16*(
13*a^2*cos(5*f*x + 5*e) + 13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f*x + e))*sin
(6*f*x + 6*e) - 13*(30*a^2*cos(4*f*x + 4*e) + 16*a^2*cos(2*f*x + 2*e) + a^2
)*sin(5*f*x + 5*e) + 30*(13*a^2*cos(3*f*x + 3*e) + 3*a^2*cos(f*x + e))*sin(
4*f*x + 4*e) - 13*(16*a^2*cos(2*f*x + 2*e) + a^2)*sin(3*f*x + 3*e))*sqrt(a)
*sqrt(c)/((c^4*cos(8*f*x + 8*e)^2 + 36*c^4*cos(7*f*x + 7*e)^2 + 256*c^4*cos
(6*f*x + 6*e)^2 + 676*c^4*cos(5*f*x + 5*e)^2 + 900*c^4*cos(4*f*x + 4*e)^2 +
 676*c^4*cos(3*f*x + 3*e)^2 + 256*c^4*cos(2*f*x + 2*e)^2 + 36*c^4*cos(f*x +
 e)^2 + c^4*sin(8*f*x + 8*e)^2 + 36*c^4*sin(7*f*x + 7*e)^2 + 256*c^4*sin(6*
f*x + 6*e)^2 + 676*c^4*sin(5*f*x + 5*e)^2 + 900*c^4*sin(4*f*x + 4*e)^2 + 67
6*c^4*sin(3*f*x + 3*e)^2 + 256*c^4*sin(2*f*x + 2*e)^2 - 192*c^4*sin(2*f*x +
 2*e)*sin(f*x + e) + 36*c^4*sin(f*x + e)^2 - 12*c^4*cos(f*x + e) + c^4 - 2*
(6*c^4*cos(7*f*x + 7*e) - 16*c^4*cos(6*f*x + 6*e) + 26*c^4*cos(5*f*x + 5*e)
 - 30*c^4*cos(4*f*x + 4*e) + 26*c^4*cos(3*f*x + 3*e) - 16*c^4*cos(2*f*x + 2
```

```
*e) + 6*c^4*cos(f*x + e) - c^4)*cos(8*f*x + 8*e) - 12*(16*c^4*cos(6*f*x + 6
*e) - 26*c^4*cos(5*f*x + 5*e) + 30*c^4*cos(4*f*x + 4*e) - 26*c^4*cos(3*f*x
+ 3*e) + 16*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(7*f*x + 7*
e) - 32*(26*c^4*cos(5*f*x + 5*e) - 30*c^4*cos(4*f*x + 4*e) + 26*c^4*cos(3*f
*x + 3*e) - 16*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(6*f*x +
 6*e) - 52*(30*c^4*cos(4*f*x + 4*e) - 26*c^4*cos(3*f*x + 3*e) + 16*c^4*cos(
2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(5*f*x + 5*e) - 60*(26*c^4*cos(
3*f*x + 3*e) - 16*c^4*cos(2*f*x + 2*e) + 6*c^4*cos(f*x + e) - c^4)*cos(4*f*
x + 4*e) - 52*(16*c^4*cos(2*f*x + 2*e) - 6*c^4*cos(f*x + e) + c^4)*cos(3*f*
x + 3*e) - 32*(6*c^4*cos(f*x + e) - c^4)*cos(2*f*x + 2*e) - 4*(3*c^4*sin(7*
f*x + 7*e) - 8*c^4*sin(6*f*x + 6*e) + 13*c^4*sin(5*f*x + 5*e) - 15*c^4*sin(
4*f*x + 4*e) + 13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x + 2*e) + 3*c^4*sin
(f*x + e))*sin(8*f*x + 8*e) - 24*(8*c^4*sin(6*f*x + 6*e) - 13*c^4*sin(5*f*x
 + 5*e) + 15*c^4*sin(4*f*x + 4*e) - 13*c^4*sin(3*f*x + 3*e) + 8*c^4*sin(2*f
*x + 2*e) - 3*c^4*sin(f*x + e))*sin(7*f*x + 7*e) - 64*(13*c^4*sin(5*f*x + 5
*e) - 15*c^4*sin(4*f*x + 4*e) + 13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x +
 2*e) + 3*c^4*sin(f*x + e))*sin(6*f*x + 6*e) - 104*(15*c^4*sin(4*f*x + 4*e)
 - 13*c^4*sin(3*f*x + 3*e) + 8*c^4*sin(2*f*x + 2*e) - 3*c^4*sin(f*x + e))*s
in(5*f*x + 5*e) - 120*(13*c^4*sin(3*f*x + 3*e) - 8*c^4*sin(2*f*x + 2*e) + 3
*c^4*sin(f*x + e))*sin(4*f*x + 4*e) - 104*(8*c^4*sin(2*f*x + 2*e) - 3*c^4*s
in(f*x + e))*sin(3*f*x + 3*e))*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=\int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(7/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx=-\frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\,\left(\frac{a^2\,\cos(e+f\,x)\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,52\mathrm{i}}{3\,c^4\,f}+\frac{a^2\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\cos(3\,e+3\,f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,4\mathrm{i}}{c^4\,f}\right)}{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(e+f\,x\right)\,28\mathrm{i}-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,28\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,12\mathrm{i}-\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sin(}$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(7/2)),x)
```

```
[Out] -((c - c/cos(e + f*x))^(1/2)*((a^2*cos(e + f*x)*exp(e*4i + f*x*4i)*(a + a/c
os(e + f*x))^(1/2)*52i)/(3*c^4*f) + (a^2*exp(e*4i + f*x*4i)*cos(3*e + 3*f*x
)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^4*f)))/(exp(e*4i + f*x*4i)*sin(e + f*x)
*28i - exp(e*4i + f*x*4i)*sin(2*e + 2*f*x)*28i + exp(e*4i + f*x*4i)*sin(3*e
 + 3*f*x)*12i - exp(e*4i + f*x*4i)*sin(4*e + 4*f*x)*2i)
```

3.131 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 88

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{48cf(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/8*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(9/2)-1/48*(a+a*s
ec(f*x+e))^(5/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{48cf(c-c\sec(e+fx))^{7/2}} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{8f(c-c\sec(e+fx))^{9/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9/2),x]
```

```
[Out] -1/8*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(9/2
)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(48*c*f*(c - c*Sec[e + f*x])
^(7/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\text{integral} = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}}+\frac{\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx}{8c}$$

$$= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{8f(c-c\sec(e+fx))^{9/2}}-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{48cf(c-c\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int\frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = -\frac{a^3(1+2\sec(e+fx)+3\sec^2(e+fx))\tan(e+fx)}{6c^4f(-1+\sec(e+fx))^4\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(9
/2),x]
```

```
[Out] -1/6*(a^3*(1 + 2*Sec[e + f*x] + 3*Sec[e + f*x]^2)*Tan[e + f*x])/(c^4*f*(-1
+ Sec[e + f*x])^4*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^{2}(7\cos(fx+e)-1)\sqrt{a(\sec(fx+e)+1)}\,(\cos(fx+e)+1)^{2}\tan(fx+e)\sec(fx+e)^{3}}{48f(\sec(fx+e)-1)^{4}\sqrt{-c(\sec(fx+e)-1)}\,c^{4}}$	81
risch	$\frac{2ia^{2}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\left(3\,e^{7i(fx+e)}-3\,e^{6i(fx+e)}+17\,e^{5i(fx+e)}-10\,e^{4i(fx+e)}+17\,e^{3i(fx+e)}-3\,e^{2i(fx+e)}+3\,e^{i(fx+e)}\right)}{3c^{4}\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)^{7}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,f}$	177

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/48/f*a^2*(7*cos(f*x+e)-1)*(a*(sec(f*x+e)+1))^(1/2)*(cos(f*x+e)+1)^2/(sec
(f*x+e)-1)^4/(-c*(sec(f*x+e)-1))^(1/2)/c^4*tan(f*x+e)*sec(f*x+e)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(76) = 152.

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\left(6\,a^{2}\cos\left(fx+e\right)^{4}-3\,a^{2}\cos\left(fx+e\right)^{3}+4\,a^{2}\cos\left(fx+e\right)^{2}-a^{2}\,c\right.}{6\,\left(c^{5}f\cos\left(fx+e\right)^{4}-4\,c^{5}f\cos\left(fx+e\right)^{3}+6\,c^{5}f\cos\left(fx+e\right)^{2}+\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*(6*a^2*cos(f*x + e)^4 - 3*a^2*cos(f*x + e)^3 + 4*a^2*cos(f*x + e)^2 - a
^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x +
e) - c)/cos(f*x + e))/((c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 + 6*c
^5*f*cos(f*x + e)^2 - 4*c^5*f*cos(f*x + e) + c^5*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2719 vs. 2(76) = 152.

Time = 3.26 (sec) , antiderivative size = 2719, normalized size of antiderivative = 30.90

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="maxima")
```

```
[Out] 2/3*(70*a^2*cos(6*f*x + 6*e)*sin(4*f*x + 4*e) - 70*a^2*cos(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 3*a^2*sin(2*f*x + 2*e) + (3*a^2*sin(6*f*x + 6*e) + 10*a^2*s
in(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e))*cos(8*f*x + 8*e) + (3*a^2*sin(8*f
*x + 8*e) + 60*a^2*sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin
(2*f*x + 2*e) - 32*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 32*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e) + 308*
a^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x + 2*e)
+ 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(5/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (17*a^2*sin(8*f*x + 8*e) + 308*a
^2*sin(6*f*x + 6*e) + 630*a^2*sin(4*f*x + 4*e) + 308*a^2*sin(2*f*x + 2*e) +
 32*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (3*a^2*sin(8*f*x + 8*e) + 60*a^2*
sin(6*f*x + 6*e) + 130*a^2*sin(4*f*x + 4*e) + 60*a^2*sin(2*f*x + 2*e))*cos(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*cos(6*f*x + 6*e)
+ 10*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x + 2*e))*sin(8*f*x + 8*e) - (70*
a^2*cos(4*f*x + 4*e) - 3*a^2)*sin(6*f*x + 6*e) + 10*(7*a^2*cos(2*f*x + 2*e)
 + a^2)*sin(4*f*x + 4*e) - (3*a^2*cos(8*f*x + 8*e) + 60*a^2*cos(6*f*x + 6*e
) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) - 32*a^2*cos(5/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 32*a^2*cos(3/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e))) + 3*a^2)*sin(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) - (17*a^2*cos(8*f*x + 8*e) + 308*a^2*cos(6*f*x + 6*e) + 630
*a^2*cos(4*f*x + 4*e) + 308*a^2*cos(2*f*x + 2*e) + 32*a^2*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a^2)*sin(5/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) - (17*a^2*cos(8*f*x + 8*e) + 308*a^2*cos(6*f*x + 6*
e) + 630*a^2*cos(4*f*x + 4*e) + 308*a^2*cos(2*f*x + 2*e) + 32*a^2*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 17*a^2)*sin(3/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - (3*a^2*cos(8*f*x + 8*e) + 60*a^2*cos(6*f*
x + 6*e) + 130*a^2*cos(4*f*x + 4*e) + 60*a^2*cos(2*f*x + 2*e) + 3*a^2)*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((c^5*cos
(8*f*x + 8*e)^2 + 784*c^5*cos(6*f*x + 6*e)^2 + 4900*c^5*cos(4*f*x + 4*e)^2
+ 784*c^5*cos(2*f*x + 2*e)^2 + 64*c^5*cos(7/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + 3136*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 + 3136*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
```

```
+ 64*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^5*sin(8
*f*x + 8*e)^2 + 784*c^5*sin(6*f*x + 6*e)^2 + 4900*c^5*sin(4*f*x + 4*e)^2 +
3920*c^5*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 784*c^5*sin(2*f*x + 2*e)^2 + 6
4*c^5*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin
(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 3136*c^5*sin(3/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 64*c^5*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 56*c^5*cos(2*f*x + 2*e) + c^5 + 2*(28*c^5
*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) + c^5
)*cos(8*f*x + 8*e) + 56*(70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e)
+ c^5)*cos(6*f*x + 6*e) + 140*(28*c^5*cos(2*f*x + 2*e) + c^5)*cos(4*f*x + 4
*e) - 16*(c^5*cos(8*f*x + 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x
 + 4*e) + 28*c^5*cos(2*f*x + 2*e) - 56*c^5*cos(5/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)
*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x
+ 8*e) + 28*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f
*x + 2*e) - 56*c^5*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8
*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(5/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*cos(8*f*x + 8*e) + 28
*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e) -
 8*c^5*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^5)*cos(3/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*cos(8*f*x + 8*e) + 2
8*c^5*cos(6*f*x + 6*e) + 70*c^5*cos(4*f*x + 4*e) + 28*c^5*cos(2*f*x + 2*e)
+ c^5)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 28*(2*c^5*sin
(6*f*x + 6*e) + 5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(8*f*x
+ 8*e) + 784*(5*c^5*sin(4*f*x + 4*e) + 2*c^5*sin(2*f*x + 2*e))*sin(6*f*x +
6*e) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*
x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 56*c^5*sin(5/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) - 8*c^5*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e
) + 28*c^5*sin(6*f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x +
2*e) - 56*c^5*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 8*c^5*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(5/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) - 112*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*
f*x + 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e) - 8*c^5*sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) - 16*(c^5*sin(8*f*x + 8*e) + 28*c^5*sin(6*f*x +
 6*e) + 70*c^5*sin(4*f*x + 4*e) + 28*c^5*sin(2*f*x + 2*e))*sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(9/2),x, algor
ithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.98

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}\left(\frac{a^2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,68\mathrm{i}}{3\,c^5\,f}-\frac{a^2\,\cos(e+f\,x)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sqrt{a+}}{3\,c^5\,f}\right.}{\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin(e+f\,x)\,84\mathrm{i}-\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\sin(2\,e+}$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(9/2)),x)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*exp(e*5i + f*x*5i)*(a + a/cos(e + f*x))^(
1/2)*68i)/(3*c^5*f) - (a^2*cos(e + f*x)*exp(e*5i + f*x*5i)*(a + a/cos(e + f
*x))^(1/2)*52i)/(3*c^5*f) + (a^2*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a
/cos(e + f*x))^(1/2)*80i)/(3*c^5*f) - (a^2*exp(e*5i + f*x*5i)*cos(3*e + 3*f
*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f) + (a^2*exp(e*5i + f*x*5i)*cos(4*
e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^5*f)))/(exp(e*5i + f*x*5i)*sin
(e + f*x)*84i - exp(e*5i + f*x*5i)*sin(2*e + 2*f*x)*96i + exp(e*5i + f*x*5i
)*sin(3*e + 3*f*x)*54i - exp(e*5i + f*x*5i)*sin(4*e + 4*f*x)*16i + exp(e*5i
 + f*x*5i)*sin(5*e + 5*f*x)*2i)
```

3.132 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 133

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}} - \frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{240c^2f(c-c\sec(e+fx))^{7/2}}$$

```
[Out] -1/10*(a+a*sec(f*x+e))^(5/2)*tan(f*x+e)/f/(c-c*sec(f*x+e))^(11/2)-1/40*(a+a
*sec(f*x+e))^(5/2)*tan(f*x+e)/c/f/(c-c*sec(f*x+e))^(9/2)-1/240*(a+a*sec(f*x
+e))^(5/2)*tan(f*x+e)/c^2/f/(c-c*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4036, 4035}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{240c^2f(c-c\sec(e+fx))^{7/2}} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{40cf(c-c\sec(e+fx))^{9/2}} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^{5/2}}{10f(c-c\sec(e+fx))^{11/2}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(11/2),x
]
```

```
[Out] -1/10*((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(f*(c - c*Sec[e + f*x])^(11
/2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(40*c*f*(c - c*Sec[e + f*x
```

```
])^(9/2)) - ((a + a*Sec[e + f*x])^(5/2)*Tan[e + f*x])/(240*c^2*f*(c - c*Sec
[e + f*x])^(7/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{9/2}}\,dx}{5c}\\
&= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}}\\
&\quad -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}}+\frac{\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{7/2}}\,dx}{40c^2}\\
&= -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{10f(c-c\sec(e+fx))^{11/2}}-\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{40cf(c-c\sec(e+fx))^{9/2}}\\
&\quad -\frac{(a+a\sec(e+fx))^{5/2}\tan(e+fx)}{240c^2f(c-c\sec(e+fx))^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx=\frac{a^3(2+5\sec(e+fx)+5\sec^2(e+fx))\tan(e+fx)}{15c^5f(-1+\sec(e+fx))^5\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^(5/2))/(c - c*Sec[e + f*x])^(1
1/2),x]
```

```
[Out] (a^3*(2 + 5*Sec[e + f*x] + 5*Sec[e + f*x]^2)*Tan[e + f*x])/(15*c^5*f*(-1 +
Sec[e + f*x])^5*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

method	result
default	$\frac{a^{2}\left(31\cos\left(fx+e\right)^{2}-8\cos\left(fx+e\right)+1\right)\sqrt{a\left(\sec\left(fx+e\right)+1\right)}\,\left(\cos\left(fx+e\right)+1\right)^{2}\tan\left(fx+e\right)\sec\left(fx+e\right)^{4}}{240f\left(\sec\left(fx+e\right)-1\right)^{5}\sqrt{-c\left(\sec\left(fx+e\right)-1\right)}\,c^{5}}$
risch	$\frac{2ia^{2}\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,\left(15\,\mathrm{e}^{9i(fx+e)}-30\,\mathrm{e}^{8i(fx+e)}+140\,\mathrm{e}^{7i(fx+e)}-170\,\mathrm{e}^{6i(fx+e)}+282\,\mathrm{e}^{5i(fx+e)}-170\,\mathrm{e}^{4i(fx+e)}+140\,\mathrm{e}^{3i(fx+e)}-30\,\mathrm{e}\ldots\right.}{15c^{5}\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)^{9}\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^{2}}{1+\mathrm{e}^{2i(fx+e)}}}\,f}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] 1/240/f*a^2*(31*cos(f*x+e)^2-8*cos(f*x+e)+1)*(a*(sec(f*x+e)+1))^(1/2)*(cos(
f*x+e)+1)^2/(sec(f*x+e)-1)^5/(-c*(sec(f*x+e)-1))^(1/2)/c^5*tan(f*x+e)*sec(f
*x+e)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\left(15\,a^{2}\cos\left(fx+e\right)^{5}-15\,a^{2}\cos\left(fx+e\right)^{4}+20\,a^{2}\cos\left(fx+e\right)^{3}-\ldots\right.}{15\left(c^{6}f\cos\left(fx+e\right)^{5}-5\,c^{6}f\cos\left(fx+e\right)^{4}+10\,c^{6}f\cos\left(fx+e\ldots\right.\right.}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/15*(15*a^2*cos(f*x + e)^5 - 15*a^2*cos(f*x + e)^4 + 20*a^2*cos(f*x + e)^3
 - 10*a^2*cos(f*x + e)^2 + 2*a^2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((c^6*f*cos(f*x + e)^5
- 5*c^6*f*cos(f*x + e)^4 + 10*c^6*f*cos(f*x + e)^3 - 10*c^6*f*cos(f*x + e)^
2 + 5*c^6*f*cos(f*x + e) - c^6*f)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(11/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4108 vs. $2(115) = 230$.

Time = 17.08 (sec) , antiderivative size = 4108, normalized size of antiderivative = 30.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="maxima")
[Out] -2/15*(1350*a^2*cos(6*f*x + 6*e)*sin(2*f*x + 2*e) + 1350*a^2*cos(4*f*x + 4*
e)*sin(2*f*x + 2*e) - 30*a^2*sin(2*f*x + 2*e) - 10*(3*a^2*sin(8*f*x + 8*e)
+ 17*a^2*sin(6*f*x + 6*e) + 17*a^2*sin(4*f*x + 4*e) + 3*a^2*sin(2*f*x + 2*e
))*cos(10*f*x + 10*e) - 1350*(a^2*sin(6*f*x + 6*e) + a^2*sin(4*f*x + 4*e))*
cos(8*f*x + 8*e) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*e) +
290*a^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*
e) - 80*a^2*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 192*a^2*
sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*sin(3/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(9/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e
) + 450*a^2*sin(6*f*x + 6*e) + 450*a^2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x
 + 2*e) - 72*a^2*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*
a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(7/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 6*(47*a^2*sin(10*f*x + 10*e) + 855*a^
2*sin(8*f*x + 8*e) + 2730*a^2*sin(6*f*x + 6*e) + 2730*a^2*sin(4*f*x + 4*e)
+ 855*a^2*sin(2*f*x + 2*e) + 240*a^2*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 160*a^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)))*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(7*a^2*sin(10
*f*x + 10*e) + 135*a^2*sin(8*f*x + 8*e) + 450*a^2*sin(6*f*x + 6*e) + 450*a^
2*sin(4*f*x + 4*e) + 135*a^2*sin(2*f*x + 2*e) + 20*a^2*sin(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) - 5*(3*a^2*sin(10*f*x + 10*e) + 75*a^2*sin(8*f*x + 8*e) + 290*a
^2*sin(6*f*x + 6*e) + 290*a^2*sin(4*f*x + 4*e) + 75*a^2*sin(2*f*x + 2*e))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 10*(3*a^2*cos(8*f*x +
```

8*e) + 17*a^2*cos(6*f*x + 6*e) + 17*a^2*cos(4*f*x + 4*e) + 3*a^2*cos(2*f*x
+ 2*e))*sin(10*f*x + 10*e) + 30*(45*a^2*cos(6*f*x + 6*e) + 45*a^2*cos(4*f*
x + 4*e) - a^2)*sin(8*f*x + 8*e) - 10*(135*a^2*cos(2*f*x + 2*e) + 17*a^2)*s
in(6*f*x + 6*e) - 10*(135*a^2*cos(2*f*x + 2*e) + 17*a^2)*sin(4*f*x + 4*e) +
5*(3*a^2*cos(10*f*x + 10*e) + 75*a^2*cos(8*f*x + 8*e) + 290*a^2*cos(6*f*x
+ 6*e) + 290*a^2*cos(4*f*x + 4*e) + 75*a^2*cos(2*f*x + 2*e) - 80*a^2*cos(7/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 192*a^2*cos(5/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) - 80*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))) + 3*a^2)*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 20*(7*a^2*cos(10*f*x + 10*e) + 135*a^2*cos(8*f*x + 8*e) + 450*a^
2*cos(6*f*x + 6*e) + 450*a^2*cos(4*f*x + 4*e) + 135*a^2*cos(2*f*x + 2*e) -
72*a^2*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*a^2*cos(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7*a^2)*sin(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) + 6*(47*a^2*cos(10*f*x + 10*e) + 855*a^2*
cos(8*f*x + 8*e) + 2730*a^2*cos(6*f*x + 6*e) + 2730*a^2*cos(4*f*x + 4*e) +
855*a^2*cos(2*f*x + 2*e) + 240*a^2*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 160*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ 47*a^2)*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 20*(7*a^2
*cos(10*f*x + 10*e) + 135*a^2*cos(8*f*x + 8*e) + 450*a^2*cos(6*f*x + 6*e) +
450*a^2*cos(4*f*x + 4*e) + 135*a^2*cos(2*f*x + 2*e) + 20*a^2*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 7*a^2)*sin(3/2*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 5*(3*a^2*cos(10*f*x + 10*e) + 75*a^2*cos(8*f*x
+ 8*e) + 290*a^2*cos(6*f*x + 6*e) + 290*a^2*cos(4*f*x + 4*e) + 75*a^2*cos(
2*f*x + 2*e) + 3*a^2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))
*sqrt(a)*sqrt(c)/((c^6*cos(10*f*x + 10*e)^2 + 2025*c^6*cos(8*f*x + 8*e)^2 +
44100*c^6*cos(6*f*x + 6*e)^2 + 44100*c^6*cos(4*f*x + 4*e)^2 + 2025*c^6*cos
(2*f*x + 2*e)^2 + 100*c^6*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
)))^2 + 14400*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
63504*c^6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c^
6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*cos(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^6*sin(10*f*x + 10*e)^2 +
2025*c^6*sin(8*f*x + 8*e)^2 + 44100*c^6*sin(6*f*x + 6*e)^2 + 44100*c^6*sin
(4*f*x + 4*e)^2 + 18900*c^6*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 2025*c^6*si
n(2*f*x + 2*e)^2 + 100*c^6*sin(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + 14400*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
63504*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 14400*c
^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 100*c^6*sin(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 90*c^6*cos(2*f*x + 2*e) +
c^6 + 2*(45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(
4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(10*f*x + 10*e) + 90*(210*
c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) +
c^6)*cos(8*f*x + 8*e) + 420*(210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x +
2*e) + c^6)*cos(6*f*x + 6*e) + 420*(45*c^6*cos(2*f*x + 2*e) + c^6)*cos(4*f
*x + 4*e) - 20*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*
cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 120

```
*c^6*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 252*c^6*cos(5/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + c^6)*cos(9/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos
(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) - 252*c^
6*cos(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 120*c^6*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(7/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) - 504*(c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 21
0*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e)
 - 120*c^6*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*co
s(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + c^6)*cos(5/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*cos(10*f*x + 10*e) + 45*c^6*
cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e) + 210*c^6*cos(4*f*x + 4*e) + 45
*c^6*cos(2*f*x + 2*e) - 10*c^6*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + c^6)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 20*(
c^6*cos(10*f*x + 10*e) + 45*c^6*cos(8*f*x + 8*e) + 210*c^6*cos(6*f*x + 6*e)
 + 210*c^6*cos(4*f*x + 4*e) + 45*c^6*cos(2*f*x + 2*e) + c^6)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 30*(3*c^6*sin(8*f*x + 8*e) + 14*c^
6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) + 3*c^6*sin(2*f*x + 2*e))*sin(
10*f*x + 10*e) + 1350*(14*c^6*sin(6*f*x + 6*e) + 14*c^6*sin(4*f*x + 4*e) +
3*c^6*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 6300*(14*c^6*sin(4*f*x + 4*e) +
3*c^6*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 20*(c^6*sin(10*f*x + 10*e) + 45*
c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e)
+ 45*c^6*sin(2*f*x + 2*e) - 120*c^6*sin(7/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^6*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(9/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8
*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*
sin(2*f*x + 2*e) - 252*c^6*sin(5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e))) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 10*c^
6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(7/2*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 504*(c^6*sin(10*f*x + 10*e) + 45*c^6*si
n(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c
^6*sin(2*f*x + 2*e) - 120*c^6*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e))) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(
5/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 240*(c^6*sin(10*f*x + 10
*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x + 6*e) + 210*c^6*sin(4*f*
x + 4*e) + 45*c^6*sin(2*f*x + 2*e) - 10*c^6*sin(1/2*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e))))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
 - 20*(c^6*sin(10*f*x + 10*e) + 45*c^6*sin(8*f*x + 8*e) + 210*c^6*sin(6*f*x
 + 6*e) + 210*c^6*sin(4*f*x + 4*e) + 45*c^6*sin(2*f*x + 2*e))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^{\frac{5}{2}}\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{11}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(11/2),x, algo
rithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 18.62 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.15

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^{5/2}}{(c-c\sec(e+fx))^{11/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+fx)}}\left(\frac{a^2\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,136\mathrm{i}}{3\,c^6\,f}-\frac{a^2\,\cos(e+f\,x)\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sqrt{}}{15\,c^6\,f}\right.}{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin(e+f\,x)\,264\mathrm{i}-\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin}$$

```
[In] int((a + a/cos(e + f*x))^(5/2)/(cos(e + f*x)*(c - c/cos(e + f*x))^(11/2)),x
)
```

```
[Out] ((c - c/cos(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(a + a/cos(e + f*x))^(
1/2)*136i)/(3*c^6*f) - (a^2*cos(e + f*x)*exp(e*6i + f*x*6i)*(a + a/cos(e +
f*x))^(1/2)*1688i)/(15*c^6*f) + (a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a
 + a/cos(e + f*x))^(1/2)*160i)/(3*c^6*f) - (a^2*exp(e*6i + f*x*6i)*cos(3*e
+ 3*f*x)*(a + a/cos(e + f*x))^(1/2)*124i)/(3*c^6*f) + (a^2*exp(e*6i + f*x*6
i)*cos(4*e + 4*f*x)*(a + a/cos(e + f*x))^(1/2)*8i)/(c^6*f) - (a^2*exp(e*6i
+ f*x*6i)*cos(5*e + 5*f*x)*(a + a/cos(e + f*x))^(1/2)*4i)/(c^6*f)))/(exp(e*
6i + f*x*6i)*sin(e + f*x)*264i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i +
 exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*220i - exp(e*6i + f*x*6i)*sin(4*e + 4*
f*x)*88i + exp(e*6i + f*x*6i)*sin(5*e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin
(6*e + 6*f*x)*2i)
```

3.133 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 139

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}} - \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -1/2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)-4*c^3*ln(
1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-2*
c^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4037}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}} - \frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*S
qrt[a + a*Sec[e + f*x]]) - (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(2*f
*Sqrt[a + a*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}+(2c)\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= -\frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}} + \left(4c^2\right)\int\frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$$

$$= -\frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}-\frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c^3(1+8\log(1+\sec(e+fx))-6\sec(e+fx)+\sec^2(e+fx))\tan(e+fx)}{2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/Sqrt[a + a*Sec[e + f*x]
],x]
```

```
[Out] -1/2*(c^3*(1 + 8*Log[1 + Sec[e + f*x]] - 6*Sec[e + f*x] + Sec[e + f*x]^2)*T
an[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c^2\sqrt{a(\sec(fx+e)+1)}\,\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2\left(8\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)-1)+8\cos(fx+e)^2\ln(-\cot(\right.}{2fa(\cos(fx+e)-1)^2}$
risch	$-\frac{2ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(3\,e^{2i(fx+e)}-e^{i(fx+e)}+3\right)\left(e^{2i(fx+e)}+e^{i(fx+e)}\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f\left(1+e^{2i(fx+e)}\right)^2}+\frac{8ic^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\ln\left(e^{i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}-$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f/a*c^2*(a*(sec(f*x+e)+1))^(1/2)*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)
-1)^2*(8*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)-1)+8*cos(f*x+e)^2*ln(-cot(f
*x+e)+csc(f*x+e)+1)+7*cos(f*x+e)^2+6*cos(f*x+e)-1)/(cos(f*x+e)-1)^2*cot(f*x
+e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(-c*sec(f*x + e) + c)/sqrt(a*sec(f*x + e) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(125) = 250$.

Time = 0.40 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.30

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\left(c^2\cos\left(2\,fx+2\,e\right)\sin\left(4\,fx+4\,e\right)-c^2\cos\left(4\,fx+4\,e\right)\sin\left(2\,fx\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 2*(c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) - c^2*cos(4*f*x + 4*e)*sin(2*f*x +
 2*e) - c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x
+ 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2
*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + c^2*sin(
4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x
+ 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(2*f*x + 2*e) + c^2)*
cos(4*f*x + 4*e))*arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) - 3*(c^2*sin
(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(3/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) - 3*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*cos(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 3*(c^2*cos(4*f*x + 4*e) +
 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e))) + 3*(c^2*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*sin(1/2*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(4*f*x
 + 4*e)^2 + 4*a*cos(2*f*x + 2*e)^2 + a*sin(4*f*x + 4*e)^2 + 4*a*sin(4*f*x +
 4*e)*sin(2*f*x + 2*e) + 4*a*sin(2*f*x + 2*e)^2 + 2*(2*a*cos(2*f*x + 2*e) +
 a)*cos(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.55 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\,c^2\left(\frac{2\,\sqrt{-ac}c\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)}{a|c|}-\frac{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2\sqrt{-ac}c+4\left(c\tan\right.}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 2*c^2*(2*sqrt(-a*c)*c*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a*abs(c)) - (3*(c*
tan(1/2*f*x + 1/2*e)^2 - c)^2*sqrt(-a*c)*c + 4*(c*tan(1/2*f*x + 1/2*e)^2 -
c)*sqrt(-a*c)*c^2 + sqrt(-a*c)*c^3)/((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*a*abs
(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

3.134 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 94

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}$$

```
[Out] -2*c^2*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)
)^(1/2)-c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {4040, 4037}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{2c^2\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (-2*c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]]) - (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[
a + a*Sec[e + f*x]])
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}+(2c)\int\frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx\\
&= -\frac{2c^2\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}-\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.64

$$
\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{c^2(-2\log(1+\sec(e+fx))+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/Sqrt[a + a*Sec[e + f*x]
],x]
```

```
[Out] (c^2*(-2*Log[1 + Sec[e + f*x]] + Sec[e + f*x])*Tan[e + f*x])/(f*Sqrt[a*(1 +
 Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
default	$\frac{c(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,(2\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-1)+2\cos(fx+e)\ln(-\cot(fx+e)}{fa(\cos(fx+e)-1)}$
risch	$\frac{2ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,(2\ln(e^{i(fx+e)}+1)e^{3i(fx+e)}-\ln(1+e^{2i(fx+e)})e^{3i(fx+e)}+2\,e^{i(fx+e)}\ln(e^{i(fx+e)}+1)-e^{i(fx+e)}\ln(1+e^{2i(fx+e)})}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f/a*c*(sec(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*(
2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-1)+2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f
*x+e)+1)+cos(f*x+e)+1)/(cos(f*x+e)-1)*cot(f*x+e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(-c*sec(f*x + e) + c)/sqr
t(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/sqrt(a*(sec(e + f*x) +
 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(86) = 172$.

Time = 0.39 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.94

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx =$$
$$-\frac{2\left(c\cos\left(\frac{1}{2}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)\sin\left(2\,fx+2\,e\right)-\left(c\cos\left(2\,fx+2\,e\right)^{2}+c\sin\left(2\,fx+2\right.\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -2*(c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(2*f*x + 2*e)
 - (c*cos(2*f*x + 2*e)^2 + c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) + 2*(c*cos(2*f*x + 2*e)^2
+ c*sin(2*f*x + 2*e)^2 + 2*c*cos(2*f*x + 2*e) + c)*arctan2(sin(1/2*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + 1) - (c*cos(2*f*x + 2*e) + c)*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqrt(c)/((a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 + 2*a*cos(2*f*x + 2*e) + a)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{2\left(\frac{\sqrt{-ac}c^{2}\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)}{a|c|}-\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)\sqrt{-ac}c^{2}+\sqrt{-ac}c^{3}}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)a|c|}\right)\mathrm{sg}}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 2*(sqrt(-a*c)*c^2*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a*abs(c)) - ((c*tan(1/
2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)*c^2 + sqrt(-a*c)*c^3)/((c*tan(1/2*f*x + 1/
2*e)^2 - c)*a*abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
[Out] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

3.135 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 50

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=-\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -c*ln(1+sec(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4037}

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=-\frac{c\tan(e+fx)\log(\sec(e+fx)+1)}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] -((c*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c\log(1+\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],
x]
[Out] -((c*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt
[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

method	result	size
default	$-\frac{\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,(\ln(-\cot(fx+e)+\csc(fx+e)-1)+\ln(-\cot(fx+e)+\csc(fx+e)+1))\cot(fx+e)}{fa}$	75
risch	$\frac{2i(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}\,\ln(e^{i(fx+e)}+1)}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)f}-\frac{i(e^{i(fx+e)}+1)\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}\,\ln(1+e^{2i(fx+e)})}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)f}$	206

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
[Out] -1/f/a*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*(ln(-cot(f*x+e)+c
sc(f*x+e)-1)+ln(-cot(f*x+e)+csc(f*x+e)+1))*cot(f*x+e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{-c\sec(fx+e)+c}\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{-c(\sec(e+fx)-1)}\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)
), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{\frac{\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}} + \frac{\sqrt{c}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}}}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(-a) + sqrt(c)*log(s
in(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(-a))/f
```

Giac [A] (verification not implemented)

none

Time = 1.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = -\frac{c^2\log\left(\left|c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right|\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{-ac}f|c|}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] -c^2*log(abs(c*tan(1/2*f*x + 1/2*e)^2 - c))*sgn(tan(1/2*f*x + 1/2*e)^3 + ta
n(1/2*f*x + 1/2*e))/(sqrt(-a*c)*f*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
[Out] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

3.136 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 47

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -arctanh(cos(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4044, 3855}

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\tan(e+fx)\operatorname{arctanh}(\cos(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] -((ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
```

```
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\text{integral} = \frac{\tan(e+fx)\int \csc(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\operatorname{arctanh}(\sec(e+fx))\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),
x]
```

```
[Out] -((ArcTanh[Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

method	result	size
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\,\ln(-\cot(fx+e)+\csc(fx+e))(\cot(fx+e)-\csc(fx+e))}{fa\sqrt{-c(\sec(fx+e)-1)}}$	65
risch	$-\frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}+\frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}+1\right)}{\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$	228

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f/a*(a*(sec(f*x+e)+1))^(1/2)*ln(-cot(f*x+e)+csc(f*x+e))/(-c*(sec(f*x+e)-
1))^(1/2)*(cot(f*x+e)-csc(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.34

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{-ac}\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)^2+\left(ac\cos(fx+e)^2+ac\right)\sin(fx+e)\right)}{\left(\cos(fx+e)^2-1\right)\sin(fx+e)}\right)}{2\,acf},\ \frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{\frac{a\cdots}{}}}{}\right.}{a\cdots}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x +
e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))/(a*c*f), sqr
t(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))/(a*c*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x) - 1
))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\arctan(\sin(fx+e),\cos(fx+e)+1)-\arctan(\sin(fx+e),\cos(fx+e)-1)}{\sqrt{a}\sqrt{c}f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -(arctan2(sin(f*x + e), cos(f*x + e) + 1) - arctan2(sin(f*x + e), cos(f*x +
 e) - 1))/(sqrt(a)*sqrt(c)*f)
```

Giac [A] (verification not implemented)

none

Time = 1.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c^2\left(\frac{\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c}-\frac{\log(|c|)}{c}\right)}{2\sqrt{-ac}f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/2*c^2*(log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - log(abs(c))/c)/(sqrt(-a*c)
*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

3.137 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 95

$$\begin{aligned}\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} \\ &\quad -\frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\end{aligned}$$

```
[Out] -1/2*tan(f*x+e)/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)-1/2*arctanh
(cos(f*x+e))*tan(f*x+e)/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4045, 4044, 3855}

$$\begin{aligned}\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx &= -\frac{\tan(e+fx)\operatorname{arctanh}(\cos(e+fx))}{2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} \\ &\quad -\frac{\tan(e+fx)}{2f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}}\end{aligned}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] -1/2*Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) -
 (ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[
c - c*Sec[e + f*x]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx}{2c}\\
&= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}+\frac{\tan(e+fx)\int \csc(e+fx)\,dx}{2c\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{\tan(e+fx)}{2f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}-\frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(\frac{\operatorname{arctanh}(\sec(e+fx))}{2c^2}+\frac{1}{2c^2(1-\sec(e+fx))}\right)\tan(e+fx)}{f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)
),x]
[Out] -((c*(ArcTanh[Sec[e + f*x]]/(2*c^2) + 1/(2*c^2*(1 - Sec[e + f*x])))*Tan[e +
 f*x])/(f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.21

method	result
default	$-\frac{(2\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))-2\ln(-\cot(fx+e)+\csc(fx+e))-\cos(fx+e)-1)\sqrt{a(\sec(fx+e)+1)}\,\tan(fx+e)}{4fa\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)}$
risch	$\frac{i\left(\ln\left(e^{i(fx+e)}+1\right)e^{3i(fx+e)}-e^{3i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-e^{2i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)+e^{2i(fx+e)}\ln\left(e^{i(fx+e)}-1\right)-e^{i(fx+e)}\ln\left(e^{i(fx+e)}+1\right)\right.}{2c\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}-1\right)\sqrt{\frac{c\left(e^{i(f}\right.}{1+e^{2}}}}$

```
[In] int(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
[Out] -1/4/f/a*(2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e))-2*ln(-cot(f*x+e)+csc(f*x+
e))-cos(f*x+e)-1)*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec
(f*x+e)-1)/(cos(f*x+e)+1)*tan(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(83) = 166.

Time = 0.34 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.02

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{\sqrt{-ac}(\cos(fx+e)-1)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\,c}{}}\right.}{}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e) - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x
 + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e) - a*c^2*
f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) - 1)*arctan(sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(
a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a*c^2*f*cos(f*x + e)
 - a*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**
(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(83) = 166$.

Time = 0.38 (sec) , antiderivative size = 406, normalized size of antiderivative = 4.27

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{((2(2\cos(fx+e)-1)\cos(2fx+2e)-\cos(2fx+2e}{}$$

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(
f*x + e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f
*x + e)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - (
2*(2*cos(f*x + e) - 1)*cos(2*f*x + 2*e) - cos(2*f*x + 2*e)^2 - 4*cos(f*x +
e)^2 - sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e
)^2 + 4*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 2*cos(f
*x + e)*sin(2*f*x + 2*e) - 2*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin(f*x + e)
)*sqrt(a)*sqrt(c)/((a*c^2*cos(2*f*x + 2*e)^2 + 4*a*c^2*cos(f*x + e)^2 + a*c
^2*sin(2*f*x + 2*e)^2 - 4*a*c^2*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a*c^2*sin
(f*x + e)^2 - 4*a*c^2*cos(f*x + e) + a*c^2 - 2*(2*a*c^2*cos(f*x + e) - a*c^
2)*cos(2*f*x + 2*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.88 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}-\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)+\log\left(|c|\right)}{4\sqrt{-ac}f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)/(c*tan(1/2*f*x + 1/2*e)^2) - log(abs(c)
*tan(1/2*f*x + 1/2*e)^2) + log(abs(c)))/(sqrt(-a*c)*f*abs(c)*sgn(tan(1/2*f*
x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+fx)\,\sqrt{a+\frac{a}{\cos(e+fx)}}\,\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

3.138 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 140

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{4c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/4*tan(f*x+e)/f/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2)-1/4*tan(f*x
+e)/c/f/(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e
))*tan(f*x+e)/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used

$= \{4045, 4044, 3855\}$

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)\operatorname{arctanh}(\cos(e+fx))}{4c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4cf\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{3/2}} - \frac{\tan(e+fx)}{4f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] -1/4*Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)) -
 Tan[e + f*x]/(4*c*f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(3/2)) -
 (ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqr
t[c - c*Sec[e + f*x]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\text{integral} = -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}}\,dx}{2c}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx}{4c^2}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} + \frac{\tan(e+fx)\int \csc(e+fx)\,dx}{4c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{4f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}} - \frac{\tan(e+fx)}{4cf\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{3/2}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{4c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(2+\operatorname{arctanh}(\sec(e+fx))(-1+\sec(e+fx))^2-\sec(e+fx))\tan(e+fx)}{4c^2f(-1+\sec(e+fx))^2\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])^(5/2)
),x]
```

```
[Out] -1/4*((2 + ArcTanh[Sec[e + f*x]]*(-1 + Sec[e + f*x])^2 - Sec[e + f*x])*Tan[
e + f*x])/(c^2*f*(-1 + Sec[e + f*x])^2*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.12

method	result
default	$\frac{\left(4\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e))-8\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))-5\cos(fx+e)^2+4\ln(-\cot(fx+e)+\csc(fx+e)\right.}{16fa(\sec(fx+e)-1)^2\sqrt{-c(\sec(fx+e)-1)}\,c^2(\cos(fx+e)+1)}$
risch	$\frac{i\left(3\,e^{2i(fx+e)}-4\,e^{i(fx+e)}+3\right)\left(e^{2i(fx+e)}+e^{i(fx+e)}\right)}{2c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)f}+\frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}+1\right)}{4c^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)f}$

```
[In] int(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/f/a*(4*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e))-8*cos(f*x+e)*ln(-cot(f*
x+e)+csc(f*x+e))-5*cos(f*x+e)^2+4*ln(-cot(f*x+e)+csc(f*x+e))-2*cos(f*x+e)+3
)*(a*(sec(f*x+e)+1))^(1/2)/(sec(f*x+e)-1)^2/(-c*(sec(f*x+e)-1))^(1/2)/c^2/(
cos(f*x+e)+1)*tan(f*x+e)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.26

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx=\left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2-2\,\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\sqrt{\right.}{}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x
+ e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 - 2*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*sin
(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)*arctan(sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*cos(f*x + e)^2 - 2*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a*c^3*f*cos(f*x + e)^2 - 2*a*c^3*f*cos(f*x + e) + a*c^3*f)*
sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,(-c(\sec(e+fx)-1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(-c*(sec(e + f*x) - 1))**
(5/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(122) = 244$.

Time = 0.43 (sec) , antiderivative size = 1201, normalized size of antiderivative = 8.58

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] 1/4*((2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1)*cos(
4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x + e)
+ 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) - 1)*cos
(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x
+ 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(4*f*x
+ 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e) - 16*s
in(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x + e
) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos(f*x +
 e) + 1) - (2*(4*cos(3*f*x + 3*e) - 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) - 1
)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) - 4*cos(f*x
 + e) + 1)*cos(3*f*x + 3*e) - 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) -
1)*cos(2*f*x + 2*e) - 36*cos(2*f*x + 2*e)^2 - 16*cos(f*x + e)^2 + 4*(2*sin(
3*f*x + 3*e) - 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) - sin(
4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) - 2*sin(f*x + e))*sin(3*f*x + 3*e)
- 16*sin(3*f*x + 3*e)^2 - 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f
*x + e) - 16*sin(f*x + e)^2 + 8*cos(f*x + e) - 1)*arctan2(sin(f*x + e), cos
(f*x + e) - 1) - 2*(3*sin(3*f*x + 3*e) - 4*sin(2*f*x + 2*e) + 3*sin(f*x + e
))*cos(4*f*x + 4*e) + 2*(3*cos(3*f*x + 3*e) - 4*cos(2*f*x + 2*e) + 3*cos(f*
x + e))*sin(4*f*x + 4*e) - 2*(2*cos(2*f*x + 2*e) + 3)*sin(3*f*x + 3*e) + 4*
(cos(f*x + e) + 2)*sin(2*f*x + 2*e) + 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) -
 4*cos(2*f*x + 2*e)*sin(f*x + e) - 6*sin(f*x + e))*sqrt(a)*sqrt(c)/((a*c^3*
cos(4*f*x + 4*e)^2 + 16*a*c^3*cos(3*f*x + 3*e)^2 + 36*a*c^3*cos(2*f*x + 2*e
```

```
)^2 + 16*a*c^3*cos(f*x + e)^2 + a*c^3*sin(4*f*x + 4*e)^2 + 16*a*c^3*sin(3*f
*x + 3*e)^2 + 36*a*c^3*sin(2*f*x + 2*e)^2 - 48*a*c^3*sin(2*f*x + 2*e)*sin(f
*x + e) + 16*a*c^3*sin(f*x + e)^2 - 8*a*c^3*cos(f*x + e) + a*c^3 - 2*(4*a*c
^3*cos(3*f*x + 3*e) - 6*a*c^3*cos(2*f*x + 2*e) + 4*a*c^3*cos(f*x + e) - a*c
^3)*cos(4*f*x + 4*e) - 8*(6*a*c^3*cos(2*f*x + 2*e) - 4*a*c^3*cos(f*x + e) +
 a*c^3)*cos(3*f*x + 3*e) - 12*(4*a*c^3*cos(f*x + e) - a*c^3)*cos(2*f*x + 2*
e) - 4*(2*a*c^3*sin(3*f*x + 3*e) - 3*a*c^3*sin(2*f*x + 2*e) + 2*a*c^3*sin(f
*x + e))*sin(4*f*x + 4*e) - 16*(3*a*c^3*sin(2*f*x + 2*e) - 2*a*c^3*sin(f*x
+ e))*sin(3*f*x + 3*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.87

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\frac{3\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2+2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4} - 2\log\left(|c|\tan\left(\frac{1}{2}\right.\right.}{16\sqrt{-ac}cf|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\right.\right.}$$

```
[In] integrate(sec(f*x+e)/(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] 1/16*((3*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 2*(c*tan(1/2*f*x + 1/2*e)^2 - c
)*c)/(c^2*tan(1/2*f*x + 1/2*e)^4) - 2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) +
2*log(abs(c)))/(sqrt(-a*c)*c*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*
f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

3.139 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 142

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)+4*c^3*ln(1+sec
(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+2*c^2
*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4039, 4040, 4037}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{4c^3\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{2c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a\sec(e+fx)+a}} + \frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sq
rt[c - c*Sec[e + f*x]]) + (2*c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a*
f*Sqrt[a + a*Sec[e + f*x]]) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(
f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{(2c)\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a} \\
&= \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}} - \frac{(4c^2)\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a} \\
&= \frac{4c^3\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{2c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.59

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-4c^2\log(1+\sec(e+fx))+c^2\sec(e+fx)-\frac{4c^2}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(3
/2),x]
```

```
[Out] -((c*(-4*c^2*Log[1 + Sec[e + f*x]] + c^2*Sec[e + f*x] - (4*c^2)/(1 + Sec[e
+ f*x]))*Tan[e + f*x])/(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f
*x]]))
```

Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.35

method	result
default	$-\frac{\left(4\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)-1)+4\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e)+1)+\sin(fx+e)^2+4\cos(fx+e)\ln(-\mathrm{co}\right.}{}$
risch	$\frac{2ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(5e^{3i(fx+e)}+2e^{2i(fx+e)}+5e^{i(fx+e)}\right)}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f\left(1+e^{2i(fx+e)}\right)}-\frac{8ic^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\ln\left(e^{i(fx+e)}+1\right)}{a\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}+\frac{4ic^2\left(e^{i}\right.}{}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f/a^2*(4*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e)-1)+4*cos(f*x+e)^2*ln(-co
t(f*x+e)+csc(f*x+e)+1)+sin(f*x+e)^2+4*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-
1)+4*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)+1)+4*cos(f*x+e))*(-c*(sec(f*x+e)-
1))^(1/2)*(sec(f*x+e)-1)^2*c^2*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)-1)*cot(
f*x+e)^2*csc(f*x+e)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2
*sec(f*x + e) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(130) = 260$.

Time = 0.49 (sec) , antiderivative size = 2035, normalized size of antiderivative = 14.33

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -2*(8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*cos(3/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 +
 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(3/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 8*c^2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*
c^2*cos(2*f*x + 2*e)*sin(4*f*x + 4*e) + 2*c^2*cos(4*f*x + 4*e)*sin(2*f*x +
2*e) + 2*c^2*sin(2*f*x + 2*e) + 2*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x
 + 2*e)^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*c^2*cos(
```

```
2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1) - 4*(c^2*cos(4*f*x + 4*e)^2 + 4*c^2*cos(2*f*x + 2*e)^2 + 4*c^2*c
os(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c^2*sin(4*f*x + 4*e)^2 + 4*c^2*
sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*c^2*sin(2*f*x + 2*e)^2 + 4*c^2*sin(3/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*c^2*cos(2*f*x + 2*e) + c^2 + 2*(2*
c^2*cos(2*f*x + 2*e) + c^2)*cos(4*f*x + 4*e) + 4*(c^2*cos(4*f*x + 4*e) + 2*
c^2*cos(2*f*x + 2*e) + 2*c^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + c^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2
*cos(4*f*x + 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2
*e) + 2*c^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(c^2*sin(4*f*x + 4*e) + 2*c
^2*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*
arctan2(sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))), cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (16*c^2*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) - 5*c^2*sin(4*f*x + 4*e) - 6*c^2*sin(2*f*x + 2*e) + 8*(c^2*cos(4*f*x
+ 4*e) + 2*c^2*cos(2*f*x + 2*e) + c^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1))*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (5*c^2*
sin(4*f*x + 4*e) + 6*c^2*sin(2*f*x + 2*e) - 8*(c^2*cos(4*f*x + 4*e) + 2*c^2
*cos(2*f*x + 2*e) + c^2)*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*c
os(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + (16*c^2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e))) + 5*c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + 5*c^2 + 8*
(c^2*sin(4*f*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1))*sin(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))
+ (5*c^2*cos(4*f*x + 4*e) + 6*c^2*cos(2*f*x + 2*e) + 5*c^2 + 8*(c^2*sin(4*f
*x + 4*e) + 2*c^2*sin(2*f*x + 2*e))*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e) + 1))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(a)*sqr
t(c)/((a^2*cos(4*f*x + 4*e)^2 + 4*a^2*cos(2*f*x + 2*e)^2 + 4*a^2*cos(3/2*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e)))^2 + a^2*sin(4*f*x + 4*e)^2 + 4*a^2*sin(4*f*x
 + 4*e)*sin(2*f*x + 2*e) + 4*a^2*sin(2*f*x + 2*e)^2 + 4*a^2*sin(3/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*sin(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e)))^2 + 4*a^2*cos(2*f*x + 2*e) + a^2 + 2*(2*a^2*cos(2
*f*x + 2*e) + a^2)*cos(4*f*x + 4*e) + 4*(a^2*cos(4*f*x + 4*e) + 2*a^2*cos(2
*f*x + 2*e) + 2*a^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
a^2)*cos(3/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*cos(4*f*
x + 4*e) + 2*a^2*cos(2*f*x + 2*e) + a^2)*cos(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e))) + 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*f*x + 2*e) + 2*a
^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*sin(3/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*(a^2*sin(4*f*x + 4*e) + 2*a^2*sin(2*
f*x + 2*e))*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.53 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{2\,c^2\left(\frac{2\,\sqrt{-ac}c\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)}{a^2|c|}+\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}}{a^2|c|}-\frac{2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\sqrt{-ac}c+\sqrt{-ac}c^2}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)a^2|c|}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx\right.\right.}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] -2*c^2*(2*sqrt(-a*c)*c*log(c*tan(1/2*f*x + 1/2*e)^2 - c)/(a^2*abs(c)) + (c*
tan(1/2*f*x + 1/2*e)^2 - c)*sqrt(-a*c)/(a^2*abs(c)) - (2*(c*tan(1/2*f*x + 1
/2*e)^2 - c)*sqrt(-a*c)*c + sqrt(-a*c)*c^2)/((c*tan(1/2*f*x + 1/2*e)^2 - c)
*a^2*abs(c)))*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)), x
)
```

3.140 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 95

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^2\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] c^2*ln(1+sec(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))
^(1/2)+c*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4039, 4037}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{c^2\tan(e+fx)\log(\sec(e+fx)+1)}{af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{c\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{f(a\sec(e+fx)+a)^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]]) + (c*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a + a
*Sec[e + f*x])^(3/2))
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}-\frac{c\int\frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a}\\ &= \frac{c^2\log(1+\sec(e+fx))\tan(e+fx)}{af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}+\frac{c\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(a+a\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = -\frac{c\left(-c\log(1+\sec(e+fx))-\frac{2c}{1+\sec(e+fx)}\right)\tan(e+fx)}{af\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(3
/2),x]
[Out] -((c*(-(c*Log[1 + Sec[e + f*x]]) - (2*c)/(1 + Sec[e + f*x]))*Tan[e + f*x])/
(a*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.56

method	result
default	$\frac{\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)c\sqrt{a(\sec(fx+e)+1)}\,(\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e)-1)+\cos(fx+e)\ln(-\cot(fx+e)+c}{f\,a^2}$
risch	$-\frac{ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,(2\,e^{2i(fx+e)}\ln(e^{i(fx+e)}+1)-e^{2i(fx+e)}\ln(1+e^{2i(fx+e)})+4\,e^{i(fx+e)}\ln(e^{i(fx+e)}+1)-2\,e^{i(fx+e)}\ln(1+e^{2i(fx-}{a(e^{i(fx+e)}+1)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,(e^{i(fx+e)}-1)\,f}$

[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETU RNVERBOSE)

[Out] 1/f/a^2*(-c*(sec(f*x+e)-1))^(1/2)*(sec(f*x+e)-1)*c*(a*(sec(f*x+e)+1))^(1/2) *(cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e)-1)+cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x +e)+1)-cos(f*x+e)+ln(-cot(f*x+e)+csc(f*x+e)-1)+ln(-cot(f*x+e)+csc(f*x+e)+1) +1)*cot(f*x+e)^2*csc(f*x+e)

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-c\sec(fx+e)+c)^{\frac{3}{2}}\sec(fx+e)}{(a\sec(fx+e)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor ithm="fricas")

[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*sqrt(a*sec(f*x + e) + a)*sqrt (-c*sec(f*x + e) + c)/(a^2*sec(f*x + e)^2 + 2*a^2*sec(f*x + e) + a^2), x)

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}}\,dx$$

[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(3/2),x)

[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/(a*(sec(e + f*x) + 1)) **(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}a}+\frac{c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}a}+\frac{c^{\frac{3}{2}}\sin(fx+e)^2}{\sqrt{-a}a(\cos(fx+e)+1)^2}}{f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] (c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a) + c^(3/2)*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a) + c^(3/2)*sin(f*x + e)^
2/(sqrt(-a)*a*(cos(f*x + e) + 1)^2))/f
```

Giac [A] (verification not implemented)

none

Time = 1.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+c\log\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)-c\right)c^2\mathrm{sgn}(t}{\sqrt{-ac}af|c|}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] (c*tan(1/2*f*x + 1/2*e)^2 + c*log(c*tan(1/2*f*x + 1/2*e)^2 - c) - c)*c^2*sg
n(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a*f*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)
```

```
[Out] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)), x
)
```

3.141 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

```
[Out] 1/2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4035}

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{2f(a\sec(e+fx)+a)^{3/2}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(3/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)\sqrt{c-c\sec(e+fx)}}{af\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(3/2
),x]
[Out] (Csc[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{\sin(fx+e)\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,\cos(fx+e)}{2f\,a^2(\cos(fx+e)+1)^2}$	56
risch	$\frac{2i\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,e^{i(fx+e)}}{a\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)f}$	105

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
[Out] 1/2/f/a^2*sin(f*x+e)*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*cos
(f*x+e)/(cos(f*x+e)+1)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos(fx+e)}{(a^2f\cos(fx+e)+a^2f)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
[Out] sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*cos(f*x + e)/((a^2*f*cos(f*x + e) + a^2*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(3/2),x)
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**(
3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{2\sqrt{-a}af}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
[Out] 1/2*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(cos(f*x +
e) + 1) - 1)/(sqrt(-a)*a*f)
```

Giac [A] (verification not implemented)

none

Time = 1.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c}{2\sqrt{-ac}af|c|}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(3/2),x, algor
ithm="giac")
[Out] 1/2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c/(sqrt(-a*c)*a*f*abs(c))
```

Mupad [B] (verification not implemented)

Time = 14.63 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{a\,f\,\sin\left(e+f\,x\right)\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}}$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)),x)
```

```
[Out] (c - c/cos(e + f*x))^(1/2)/(a*f*sin(e + f*x)*((a*(cos(e + f*x) + 1))/cos(e
+ f*x))^(1/2))
```

3.142 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 95

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*tan(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)-1/2*arctanh(
cos(f*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4045, 4044, 3855}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{2f(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)\operatorname{arctanh}(\cos(e+fx))}{2af\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(2*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c -
 c*Sec[e + f*x]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx}{2a}\\
&= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\tan(e+fx)\int \csc(e+fx)\,dx}{2a\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{\tan(e+fx)}{2f(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}-\frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2af\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(-1+\operatorname{arctanh}(\sec(e+fx))(1+\sec(e+fx)))\tan(e+fx)}{2f(a(1+\sec(e+fx)))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] -1/2*((-1 + ArcTanh[Sec[e + f*x]]*(1 + Sec[e + f*x]))*Tan[e + f*x])/(f*(a*(
1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

method	result
default	$\frac{\sin(fx+e)(2\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))+2\ln(-\cot(fx+e)+\csc(fx+e))+\cos(fx+e)-1)\sqrt{a(\sec(fx+e)+1)}}{4f\,a^2(\cos(fx+e)+1)^2\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{i(\ln(e^{i(fx+e)}+1)e^{3i(fx+e)}-e^{3i(fx+e)}\ln(e^{i(fx+e)}-1)+e^{2i(fx+e)}\ln(e^{i(fx+e)}+1)-e^{2i(fx+e)}\ln(e^{i(fx+e)}-1)-e^{i(fx+e)}\ln(e^{i(fx+e)}+}{2a(e^{i(fx+e)}+1)\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}(1}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/f/a^2*sin(f*x+e)*(2*cos(f*x+e)*ln(-cot(f*x+e)+csc(f*x+e))+2*ln(-cot(f*x
+e)+csc(f*x+e))+cos(f*x+e)-1)*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)^2/(-c
*(sec(f*x+e)-1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(83) = 166.

Time = 0.32 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.00

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{-ac}(\cos(fx+e)+1)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\right.}{}\right.}{}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)
^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x
 + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*co
s(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e) + a^2*c*
f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e) + 1)*arctan(sqrt(a*c)*sqrt((
a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(
a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e))/((a^2*c*f*cos(f*x + e)
 + a^2*c*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(83) = 166.

Time = 0.38 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.18

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\left((2(2\cos(fx+e)+1)\cos(2fx+2e)+\cos(2fx+2e)^2+4\cos(fx+e)^2+\sin(2fx+2e)^2+4\sin(2\ldots\right.}{}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos
(f*x + e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(
f*x + e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) -
(2*(2*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + cos(2*f*x + 2*e)^2 + 4*cos(f*x +
 e)^2 + sin(2*f*x + 2*e)^2 + 4*sin(2*f*x + 2*e)*sin(f*x + e) + 4*sin(f*x +
e)^2 + 4*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x + e) - 1) - 2*cos(
f*x + e)*sin(2*f*x + 2*e) + 2*cos(2*f*x + 2*e)*sin(f*x + e) + 2*sin(f*x + e
))*sqrt(a)*sqrt(c)/((a^2*c*cos(2*f*x + 2*e)^2 + 4*a^2*c*cos(f*x + e)^2 + a^
2*c*sin(2*f*x + 2*e)^2 + 4*a^2*c*sin(2*f*x + 2*e)*sin(f*x + e) + 4*a^2*c*si
n(f*x + e)^2 + 4*a^2*c*cos(f*x + e) + a^2*c + 2*(2*a^2*c*cos(f*x + e) + a^2
*c)*cos(2*f*x + 2*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.90 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c^2\left(\frac{\log\left(|c|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\right)}{c}-\frac{\log(|c|)}{c}-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c}{c^2}\right)}{4\sqrt{-ac}af|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/4*c^2*(log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - log(abs(c))/c - (c*tan(1/2
*f*x + 1/2*e)^2 - c)/c^2)/(sqrt(-a*c)*a*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3
 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,{\left(a+\frac{a}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,\sqrt{c-\frac{c}{\cos\left(e+f\,x\right)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

3.143 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 104

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*csc(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/2*arct
anh(cos(f*x+e))*tan(f*x+e)/a/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1
/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4044, 2691, 3855}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)\operatorname{arctanh}(\cos(e+fx))}{2acf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] Csc[e + f*x]/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]) -
(ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(2*a*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt
[c - c*Sec[e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\text{integral} = -\frac{\tan(e+fx)\int \cot^2(e+fx)\csc(e+fx)\,dx}{ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)\int \csc(e+fx)\,dx}{2ac\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= \frac{\csc(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{2acf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\csc(e+fx)-\operatorname{arctanh}(\sec(e+fx))\tan(e+fx)}{2acf\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(3/
2)),x]
```

```
[Out] (Csc[e + f*x] - ArcTanh[Sec[e + f*x]]*Tan[e + f*x])/(2*a*c*f*Sqrt[a*(1 + Se
c[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\left(\cos(fx+e)^2 \ln(-\cot(fx+e)+\csc(fx+e))-\ln(-\cot(fx+e)+\csc(fx+e))-\cos(fx+e)\right)\sqrt{a(\sec(fx+e)+1)}\,\tan(fx+e)}{2f\,a^2\sqrt{-c(\sec(fx+e)-1)}\,c(\sec(fx+e)-1)(\cos(fx+e)+1)^2}$
risch	$-\frac{i(\mathrm{e}^{4i(fx+e)}\ln(\mathrm{e}^{i(fx+e)}-1)-\mathrm{e}^{4i(fx+e)}\ln(\mathrm{e}^{i(fx+e)}+1)-2\,\mathrm{e}^{2i(fx+e)}\ln(\mathrm{e}^{i(fx+e)}-1)+2\,\mathrm{e}^{2i(fx+e)}\ln(\mathrm{e}^{i(fx+e)}+1)-2\,\mathrm{e}^{3i(fx+e)}-2\,\mathrm{e}^{i(}}{2ac(\mathrm{e}^{i(fx+e)}+1)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,(\mathrm{e}^{i(fx+e)}-1)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,f\left(1+\mathrm{e}^{2i(fx+e)}\right)}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f/a^2*(cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e))-ln(-cot(f*x+e)+csc(f*x+
e))-cos(f*x+e))*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/c/(sec(f
*x+e)-1)/(cos(f*x+e)+1)^2*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{4\left(2\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\right.}{}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*c)*(cos(f*x + e)^2 - 1)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x
 + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x +
e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f
*x + e)))*sin(f*x + e) - 2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*
cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^2*f*cos(f*x + e)^2
- a^2*c^2*f)*sin(f*x + e)), 1/2*(sqrt(a*c)*(cos(f*x + e)^2 - 1)*arctan(sqrt
(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos
(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2)/((a^2*c^
2*f*cos(f*x + e)^2 - a^2*c^2*f)*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{3}{2}}(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(3/2)*(-c*(sec(e + f*x) - 1)
)**(3/2)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(92) = 184$.

Time = 0.41 (sec) , antiderivative size = 567, normalized size of antiderivative = 5.45

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{((2(2\cos(2fx+2e)-1)\cos(4fx+4e)-\cos(4fx}{}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x + 4*e)^2 - 4*
cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*
e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e), c
os(f*x + e) + 1) - (2*(2*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - cos(4*f*x
 + 4*e)^2 - 4*cos(2*f*x + 2*e)^2 - sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*
sin(2*f*x + 2*e) - 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) - 1)*arctan2(s
in(f*x + e), cos(f*x + e) - 1) - 2*(sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*
f*x + 4*e) + 2*(cos(3*f*x + 3*e) + cos(f*x + e))*sin(4*f*x + 4*e) + 2*(2*co
s(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e)
- 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*cos(2*f*x + 2*e)*sin(f*x + e) - 2*sin
(f*x + e))*sqrt(a)*sqrt(c)/((a^2*c^2*cos(4*f*x + 4*e)^2 + 4*a^2*c^2*cos(2*f
*x + 2*e)^2 + a^2*c^2*sin(4*f*x + 4*e)^2 - 4*a^2*c^2*sin(4*f*x + 4*e)*sin(2
*f*x + 2*e) + 4*a^2*c^2*sin(2*f*x + 2*e)^2 - 4*a^2*c^2*cos(2*f*x + 2*e) + a
^2*c^2 - 2*(2*a^2*c^2*cos(2*f*x + 2*e) - a^2*c^2)*cos(4*f*x + 4*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 2.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{c}+\frac{2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}-2\log\left(|c|\tan\left(\frac{1}{2}fx\right.\right.}{8\sqrt{-ac}af|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\right.\right.}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/8*((c*tan(1/2*f*x + 1/2*e)^2 - c)/c + (2*c*tan(1/2*f*x + 1/2*e)^2 - c)/(c
*tan(1/2*f*x + 1/2*e)^2) - 2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) + 2*log(abs
(c)) - 1)/(sqrt(-a*c)*a*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x +
 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+fx)\left(a+\frac{a}{\cos(e+fx)}\right)^{3/2}\left(c-\frac{c}{\cos(e+fx)}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

3.144 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 146

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} - \frac{3\text{arctanh}(\cos(e+fx))\tan(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 3/8*csc(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2)-3/8*arctanh(cos(f*
x+e))*tan(f*x+e)/a/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4045, 4044, 2691, 3855}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{3\tan(e+fx)\text{arctanh}(\cos(e+fx))}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{3\csc(e+fx)}{8ac^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{3/2}(c-c\sec(e+fx))^{5/2}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) - Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a*c^2*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} \\ &\quad+\frac{3\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx}{4c}\end{aligned}$$

$$= -\frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} - \frac{(3\tan(e+fx))\int\cot^2(e+fx)\csc(e+fx)\,dx}{4ac^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} + \frac{(3\tan(e+fx))\int\csc(e+fx)\,dx}{8ac^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{3\csc(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}} - \frac{3\text{arctanh}(\cos(e+fx))\tan(e+fx)}{8ac^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int\frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{(2+3\sec(e+fx)-3\sec^2(e+fx)+3\text{arctanh}(\sec(e+fx))(-1+\sec(e+fx))^2(1+\sec(e+fx)))\tan(e}{8c^2f(-1+\sec(e+fx))^2(a(1+\sec(e+fx)))^{3/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(3/2)*(c - c*Sec[e + f*x])^(5/
2)),x]
```

```
[Out] -1/8*((2 + 3*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 3*ArcTanh[Sec[e + f*x]]*(-1
+ Sec[e + f*x])^2*(1 + Sec[e + f*x]))*Tan[e + f*x])/(c^2*f*(-1 + Sec[e + f*
x])^2*(a*(1 + Sec[e + f*x]))^(3/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.48

method	result
default	$\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left(-2(1-\cos(fx+e))^6\csc(fx+e)^6+12\ln(-\cot(fx+e)+\csc(fx+e))(1-\cos(fx+e))\right.}{64 f a^2\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^2\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{5}{2}}}$
risch	$\frac{i\left(5\,e^{5i(fx+e)}-2\,e^{4i(fx+e)}+2\,e^{3i(fx+e)}-2\,e^{2i(fx+e)}+5\,e^{i(fx+e)}\right)}{4a\,c^2\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}+1\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}-\frac{3i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln(e^{i(}}{8a\,c^2\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c}{-}}}$

[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/64/f*2^(1/2)/a^2*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^2/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)^2)^(5/2)*(1-cos(f*x+e))*(-2*(1-cos(f*x+e))^6*csc(f*x+e)^6+12*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e))^4*csc(f*x+e)^4+6*(1-cos(f*x+e))^2*csc(f*x+e)^2-1)*csc(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.73

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\cos\left(fx+e\right)^3-\cos\left(fx+e\right)^2-\cos\left(fx+e\right)+1\right)\sqrt{}}{}\right.$$

[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [-1/16*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(-a*c)*log(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e)^3 - cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e)), 1/8*(3*(cos(f*x + e)^3 - cos(f*x + e)^2 - cos(f*x + e) + 1)*sqrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*x + e)^3 - cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq

```
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^2*c^3*f*cos(f*x + e)^3 - a^2*c^3
*f*cos(f*x + e)^2 - a^2*c^3*f*cos(f*x + e) + a^2*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [A] (verification not implemented)

none

Time = 1.92 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{\frac{2\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)}{c} + \frac{9\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-c\right)^{2}+12\left(c\tan\left(\frac{1}{2}\,fx+\right.\right.}{c^{2}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}}{32\,\sqrt{-ac}acf|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\right.\right.}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
[Out] 1/32*(2*(c*tan(1/2*f*x + 1/2*e)^2 - c)/c + (9*(c*tan(1/2*f*x + 1/2*e)^2 - c
)^2 + 12*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c + 4*c^2)/(c^2*tan(1/2*f*x + 1/2*e
)^4) - 6*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) + 6*log(abs(c)) - 4)/(sqrt(-a*c
)*a*c*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{3/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),
x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(3/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

3.145 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 145

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^3\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 1/2*c*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)-c^3*ln(1+s
ec(f*x+e))*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-c
^2*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4039, 4037}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^3\tan(e+fx)\log(\sec(e+fx)+1)}{a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}}{af(a\sec(e+fx)+a)^{3/2}} + \frac{c\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{2f(a\sec(e+fx)+a)^{5/2}}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5/2),x]
```

```
[Out] -((c^3*Log[1 + Sec[e + f*x]]*Tan[e + f*x])/(a^2*f*Sqrt[a + a*Sec[e + f*x]]*
Sqrt[c - c*Sec[e + f*x]])) - (c^2*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(a
*f*(a + a*Sec[e + f*x])^(3/2)) + (c*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x]
)/(2*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 4037

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[a*c*Log[1 + (b/
a)*Csc[e + f*x]]*(Cot[e + f*x]/(b*f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc
[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && Eq
Q[a^2 - b^2, 0]
```

Rule 4039

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[2*a*c*Cot[e +
f*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(b*f*(2*m + 1))),
 x] - Dist[d*((2*n - 1)/(b*(2*m + 1))), Int[Csc[e + f*x]*(a + b*Csc[e + f*x
])^(m + 1)*(c + d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && LtQ[m
, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} - \frac{c\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{3/2}}\,dx}{a} \\
&= -\frac{c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}} \\
&\quad + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}} + \frac{c^2\int\frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{a^2} \\
&= -\frac{c^3\log(1+\sec(e+fx))\tan(e+fx)}{a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad - \frac{c^2\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{af(a+a\sec(e+fx))^{3/2}} + \frac{c(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int\frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c\left(c^2\log(1+\sec(e+fx)) - \frac{2c^2}{(1+\sec(e+fx))^2} + \frac{4c^2}{1+\sec(e+fx)}\right)\tan(e+fx)}{a^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(5/2))/(a + a*Sec[e + f*x])^(5
/2),x]
```

```
[Out] -((c*(c^2*Log[1 + Sec[e + f*x]] - (2*c^2)/(1 + Sec[e + f*x])^2 + (4*c^2)/(1
 + Sec[e + f*x]))*Tan[e + f*x])/(a^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c -
c*Sec[e + f*x]]))
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.50

method	result
default	$\frac{\sqrt{2}\,\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)^3\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{5}{2}}\sin(fx+e)^5\left((1-\cos(fx+\right.}{4f\,a^3(1-\cos(fx+}$
risch	$-\frac{8ic^2\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,e^{2i(fx+e)}}{a^2\left(e^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}+\frac{2ic^2\left(e^{i(fx+e)}+1\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\ln\left(e^{i(fx+e)}+1\right)}{a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}-\frac{ic^2\left(e^{i(fx+e)}+1\right)}{a^2\sqrt{\frac{a\left(e\right.}{1-}}}$

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/f*2^(1/2)/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*
x+e))^2*csc(f*x+e)^2-1)^3*(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^
2-1)*csc(f*x+e)^2)^(5/2)/(1-cos(f*x+e))^5*sin(f*x+e)^5*((1-cos(f*x+e))^4*cs
c(f*x+e)^4+2*(1-cos(f*x+e))^2*csc(f*x+e)^2+2*ln(-cot(f*x+e)+csc(f*x+e)-1)+2
*ln(-cot(f*x+e)+csc(f*x+e)+1))
```

Fricas [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx=\int \frac{(-c\sec(fx+e)+c)^{\frac{5}{2}}\sec(fx+e)}{(a\sec(fx+e)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*sqr
t(a*sec(f*x + e) + a)*sqrt(-c*sec(f*x + e) + c)/(a^3*sec(f*x + e)^3 + 3*a^3
*sec(f*x + e)^2 + 3*a^3*sec(f*x + e) + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(5/2)/(a+a*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{\sqrt{-a}a^2}+\frac{2\,c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{\sqrt{-a}a^2}-\frac{\frac{2\,\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{-a}c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}{a^3}}{2\,f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
[Out] -1/2*(2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(sqrt(-a)*a^2) + 2
*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(sqrt(-a)*a^2) - (2*sqrt(
-a)*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(-a)*c^(5/2)*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 1.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c^4\left(\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^2+4\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^3}{c^4}+2\,\log\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\mathrm{t}\right.}{2\,\sqrt{-ac}a^2f|c|}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(5/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
[Out] -1/2*c^4*(((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 + 4*(c*tan(1/2*f*x + 1/2*e)
^2 - c)*c^3)/c^4 + 2*log(c*tan(1/2*f*x + 1/2*e)^2 - c))*sgn(tan(1/2*f*x + 1
/2*e)^3 + tan(1/2*f*x + 1/2*e))/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{5/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
[Out] int((c - c/cos(e + f*x))^(5/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)), x
)
```

3.146 $\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 42

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

```
[Out] 1/4*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4035}

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\tan(e+fx)(c-c\sec(e+fx))^{3/2}}{4f(a\sec(e+fx)+a)^{5/2}}$$

```
[In] Int[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5/2),x]
[Out] ((c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(4*f*(a + a*Sec[e + f*x])^(5/2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{c(-1+\sec(e+fx))\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{4f(a(1+\sec(e+fx)))^{5/2}}$$

```
[In] Integrate[(Sec[e + f*x]*(c - c*Sec[e + f*x])^(3/2))/(a + a*Sec[e + f*x])^(5
/2),x]
```

```
[Out] -1/4*(c*(-1 + Sec[e + f*x])*Sqrt[c - c*Sec[e + f*x]]*Tan[e + f*x])/(f*(a*(1
 + Sec[e + f*x]))^(5/2))
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

method	result	size
default	$-\frac{\sin(fx+e)(\sec(fx+e)-1)\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,c\cos(fx+e)^{2}}{4f\,a^{3}(\cos(fx+e)+1)^{3}}$	67
risch	$\frac{2ic\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^{2}}{1+e^{2i(fx+e)}}}\,\left(e^{3i(fx+e)}+e^{i(fx+e)}\right)}{a^{2}\left(e^{i(fx+e)}+1\right)^{3}\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^{2}}{1+e^{2i(fx+e)}}}\,\left(e^{i(fx+e)}-1\right)f}$	116

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/f/a^3*sin(f*x+e)*(sec(f*x+e)-1)*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+
e)+1))^(1/2)*c*cos(f*x+e)^2/(cos(f*x+e)+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{c\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}\cos\left(fx+e\right)^{2}}{\left(a^{3}f\cos\left(fx+e\right)^{2}+2\,a^{3}f\cos\left(fx+e\right)+a^{3}f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] c*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2/((a^3*f*cos(f*x + e)^2 + 2*a^3*f*cos(f*x + e) + a^3*f
)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{(-c(\sec(e+fx)-1))^{\frac{3}{2}}\sec(e+fx)}{(a(\sec(e+fx)+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(3/2)/(a+a*sec(f*x+e))**(5/2),x)
[Out] Integral((-c*(sec(e + f*x) - 1))**(3/2)*sec(e + f*x)/(a*(sec(e + f*x) + 1))
**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(36) = 72$.

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.33

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{-a}c^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\sin(fx+e)^4}{4\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)f(\cos(fx+e)+1)^4}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
[Out] -1/4*sqrt(-a)*c^(3/2)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)*sin(f*x + e)^4/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x +
 e) + 1)^2)*f*(cos(f*x + e) + 1)^4)
```

Giac [A] (verification not implemented)

none

Time = 1.54 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2+2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c\right)c}{4\sqrt{-ac}a^2f|c|}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(3/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
[Out] -1/4*((c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c
)*c/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [B] (verification not implemented)

Time = 15.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int \frac{\sec(e+fx)(c-c\sec(e+fx))^{3/2}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{2\,c\,\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,\left(\sin\left(e+f\,x\right)+2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{a^2\,f\,\sqrt{\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}}\,\left(4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(e+f\,x\right)+4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

```
[In] int((c - c/cos(e + f*x))^(3/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*c*((c*(cos(e + f*x) - 1))/cos(e + f*x))^(1/2)*(sin(e + f*x) + 2*sin(2*e
 + 2*f*x) + sin(3*e + 3*f*x)))/(a^2*f*((a*(cos(e + f*x) + 1))/cos(e + f*x))
^(1/2)*(4*cos(2*e + 2*f*x) - 4*cos(e + f*x) + 4*cos(3*e + 3*f*x) + cos(4*e
+ 4*f*x) - 5))
```

3.147 $\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 43

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/2*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {4038}

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx=\frac{c\tan(e+fx)}{2f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2),x]
[Out] (c*Tan[e + f*x])/(2*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral}=\frac{c\tan(e+fx)}{2f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{(1+2\cos(e+fx))\csc\left(\frac{1}{2}(e+fx)\right)\sec^3\left(\frac{1}{2}(e+fx)\right)\sqrt{c-c\sec(e+fx)}}{8a^2f\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c - c*Sec[e + f*x]])/(a + a*Sec[e + f*x])^(5/2
),x]
[Out] ((1 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]^3*Sqrt[c - c*Sec[e
+ f*x]])/(8*a^2*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{\sqrt{-c(\sec(fx+e)-1)}\,\sqrt{a(\sec(fx+e)+1)}\,\cos(fx+e)^2\cot(fx+e)}{2f\,a^3(\cos(fx+e)+1)^2}$	58
risch	$\frac{2i\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{3i(fx+e)}+e^{2i(fx+e)}+e^{i(fx+e)}\right)}{a^2\left(e^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(e^{i(fx+e)}-1\right)f}$	124

```
[In] int(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
[Out] -1/2/f/a^3*(-c*(sec(f*x+e)-1))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1
)^2*cos(f*x+e)^2*cot(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \frac{\left(2\cos\left(fx+e\right)^2+\cos\left(fx+e\right)\right)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{2\left(a^3f\cos\left(fx+e\right)^2+2\,a^3f\cos\left(fx+e\right)+a^3f\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
[Out] 1/2*(2*cos(f*x + e)^2 + cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((a^3*f*cos(f*x + e)^2 + 2*a^3*f
*cos(f*x + e) + a^3*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = \int \frac{\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(5/2),x)
[Out] Integral(sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)/(a*(sec(e + f*x) + 1))**(
5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\sqrt{c}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^2}{8\sqrt{-a}a^2f}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
[Out] -1/8*sqrt(c)*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^2*(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)^2/(sqrt(-a)*a^2*f)
```

Giac [A] (verification not implemented)

none

Time = 1.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2}{8\sqrt{-ac}a^2f|c|}$$

```
[In] integrate(sec(f*x+e)*(c-c*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(5/2),x, algor
ithm="giac")
[Out] -1/8*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2/(sqrt(-a*c)*a^2*f*abs(c))
```

Mupad [B] (verification not implemented)

Time = 14.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.79

$$\int \frac{\sec(e+fx)\sqrt{c-c\sec(e+fx)}}{(a+a\sec(e+fx))^{5/2}}\,dx = -\frac{2\,\left(3\,\sin\left(e+f\,x\right)+3\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)\,\sqrt{\frac{c\,\left(\cos(e+f\,x)-1\right)}{\cos(e+f\,x)}}}{a^2\,f\,\sqrt{\frac{a\,\left(\cos(e+f\,x)+1\right)}{\cos(e+f\,x)}}\,\left(4\,\cos\left(2\,e+2\,f\,x\right)-4\,\cos\left(e+f\,x\right)+4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)-5\right)}$$

```
[In] int((c - c/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)),x)
```

```
[Out] -(2*(3*sin(e + f*x) + 3*sin(2*e + 2*f*x) + sin(3*e + 3*f*x))*((c*(cos(e + f
*x) - 1))/cos(e + f*x))^(1/2))/(a^2*f*((a*(cos(e + f*x) + 1))/cos(e + f*x))
^(1/2)*(4*cos(2*e + 2*f*x) - 4*cos(e + f*x) + 4*cos(3*e + 3*f*x) + cos(4*e
+ 4*f*x) - 5))
```

3.148 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 140

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 1/4*tan(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2)+1/4*tan(f*x+
e)/a/f/(a+a*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))^(1/2)-1/4*arctanh(cos(f*x+e)
)*tan(f*x+e)/a^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4045, 4044, 3855}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\tan(e+fx)\operatorname{arctanh}(\cos(e+fx))}{4a^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4af(a\sec(e+fx)+a)^{3/2}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]) + Ta
n[e + f*x]/(4*a*f*(a + a*Sec[e + f*x])^(3/2)*Sqrt[c - c*Sec[e + f*x]]) - (A
rcTanh[Cos[e + f*x]]*Tan[e + f*x])/(4*a^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c
 - c*Sec[e + f*x]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}+\frac{\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\,dx}{2a}\\
&= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}+\frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx}{4a^2}\\
&= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}}\\
&\quad+\frac{\tan(e+fx)\int \csc(e+fx)\,dx}{4a^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4af(a+a\sec(e+fx))^{3/2}\sqrt{c-c\sec(e+fx)}} - \frac{\operatorname{arctanh}(\cos(e+fx))\tan(e+fx)}{4a^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{(-2-\sec(e+fx)+\operatorname{arctanh}(\sec(e+fx))(1+\sec(e+fx))^2)\tan(e+fx)}{4f(a(1+\sec(e+fx)))^{5/2}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] -1/4*((-2 - Sec[e + f*x] + ArcTanh[Sec[e + f*x]]*(1 + Sec[e + f*x])^2)*Tan[
e + f*x])/(f*(a*(1 + Sec[e + f*x]))^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

method	result
default	$\frac{\sin(fx+e)\left(4\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e))+8\cos(fx+e)\ln(-\cot(fx+e)+\csc(fx+e))+5\cos(fx+e)^2+4\ln(-\cot(fx+e)+\right.}{16f\,a^3(\cos(fx+e)+1)^3\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{i\left(3\,e^{2i(fx+e)}+4\,e^{i(fx+e)}+3\right)\left(e^{2i(fx+e)}-e^{i(fx+e)}\right)}{2a^2\left(1+e^{2i(fx+e)}\right)\left(e^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f} - \frac{i\left(e^{i(fx+e)}+1\right)\left(e^{i(fx+e)}-1\right)\ln\left(e^{i(fx+e)}-1\right)}{4a^2\sqrt{\frac{a\left(e^{i(fx+e)}+1\right)^2}{1+e^{2i(fx+e)}}}\left(1+e^{2i(fx+e)}\right)\sqrt{\frac{c\left(e^{i(fx+e)}-1\right)^2}{1+e^{2i(fx+e)}}}\,f}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/f/a^3*sin(f*x+e)*(4*cos(f*x+e)^2*ln(-cot(f*x+e)+csc(f*x+e))+8*cos(f*x+
e)*ln(-cot(f*x+e)+csc(f*x+e))+5*cos(f*x+e)^2+4*ln(-cot(f*x+e)+csc(f*x+e))-2
*cos(f*x+e)-3)*(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)^3/(-c*(sec(f*x+e)-1)
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.26

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{-ac}\left(\cos\left(fx+e\right)^2+2\,\cos\left(fx+e\right)+1\right)\log\left(-\frac{4\left(2\right.}{}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(-4*(2*sqrt(-a*c
)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x
 + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f*x
+ e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(3*cos(f*x + e)^2 + 2*cos(f*x +
 e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(
f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*sin
(f*x + e)), 1/4*(sqrt(a*c)*(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)*arctan(sqr
t(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (3*cos(f*x + e)^2 + 2*cos(f*
x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/c
os(f*x + e)))/((a^3*c*f*cos(f*x + e)^2 + 2*a^3*c*f*cos(f*x + e) + a^3*c*f)*
sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\left(a\left(\sec\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}\sqrt{-c\left(\sec\left(e+fx\right)-1\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/((a*(sec(e + f*x) + 1))**(5/2)*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(122) = 244$.

Time = 0.43 (sec) , antiderivative size = 1191, normalized size of antiderivative = 8.51

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) + 1)*cos
(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*x + e)
 + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) + 1)*co
s(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin(3*f*x
 + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin(4*f*x
 + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e) + 16*
sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(f*x +
e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), cos(f*x
+ e) + 1) - (2*(4*cos(3*f*x + 3*e) + 6*cos(2*f*x + 2*e) + 4*cos(f*x + e) +
1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 8*(6*cos(2*f*x + 2*e) + 4*cos(f*
x + e) + 1)*cos(3*f*x + 3*e) + 16*cos(3*f*x + 3*e)^2 + 12*(4*cos(f*x + e) +
 1)*cos(2*f*x + 2*e) + 36*cos(2*f*x + 2*e)^2 + 16*cos(f*x + e)^2 + 4*(2*sin
(3*f*x + 3*e) + 3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(4*f*x + 4*e) + sin
(4*f*x + 4*e)^2 + 16*(3*sin(2*f*x + 2*e) + 2*sin(f*x + e))*sin(3*f*x + 3*e)
 + 16*sin(3*f*x + 3*e)^2 + 36*sin(2*f*x + 2*e)^2 + 48*sin(2*f*x + 2*e)*sin(
f*x + e) + 16*sin(f*x + e)^2 + 8*cos(f*x + e) + 1)*arctan2(sin(f*x + e), co
s(f*x + e) - 1) + 2*(3*sin(3*f*x + 3*e) + 4*sin(2*f*x + 2*e) + 3*sin(f*x +
e))*cos(4*f*x + 4*e) - 2*(3*cos(3*f*x + 3*e) + 4*cos(2*f*x + 2*e) + 3*cos(f
*x + e))*sin(4*f*x + 4*e) + 2*(2*cos(2*f*x + 2*e) + 3)*sin(3*f*x + 3*e) - 4
*(cos(f*x + e) - 2)*sin(2*f*x + 2*e) - 4*cos(3*f*x + 3*e)*sin(2*f*x + 2*e)
+ 4*cos(2*f*x + 2*e)*sin(f*x + e) + 6*sin(f*x + e))*sqrt(a)*sqrt(c)/((a^3*c
*cos(4*f*x + 4*e)^2 + 16*a^3*c*cos(3*f*x + 3*e)^2 + 36*a^3*c*cos(2*f*x + 2*
e)^2 + 16*a^3*c*cos(f*x + e)^2 + a^3*c*sin(4*f*x + 4*e)^2 + 16*a^3*c*sin(3*
f*x + 3*e)^2 + 36*a^3*c*sin(2*f*x + 2*e)^2 + 48*a^3*c*sin(2*f*x + 2*e)*sin(
f*x + e) + 16*a^3*c*sin(f*x + e)^2 + 8*a^3*c*cos(f*x + e) + a^3*c + 2*(4*a^
3*c*cos(3*f*x + 3*e) + 6*a^3*c*cos(2*f*x + 2*e) + 4*a^3*c*cos(f*x + e) + a^
3*c)*cos(4*f*x + 4*e) + 8*(6*a^3*c*cos(2*f*x + 2*e) + 4*a^3*c*cos(f*x + e)
+ a^3*c)*cos(3*f*x + 3*e) + 12*(4*a^3*c*cos(f*x + e) + a^3*c)*cos(2*f*x + 2
*e) + 4*(2*a^3*c*sin(3*f*x + 3*e) + 3*a^3*c*sin(2*f*x + 2*e) + 2*a^3*c*sin(
f*x + e))*sin(4*f*x + 4*e) + 16*(3*a^3*c*sin(2*f*x + 2*e) + 2*a^3*c*sin(f*x
 + e))*sin(3*f*x + 3*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.76 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{c^2\left(\frac{2\log\left(|c|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{c}-\frac{2\log(|c|)}{c}+\frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^3-2\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)c^4}{c^6}\right)}{16\sqrt{-ac}a^2f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] -1/16*c^2*(2*log(abs(c)*tan(1/2*f*x + 1/2*e)^2)/c - 2*log(abs(c))/c + ((c*t
an(1/2*f*x + 1/2*e)^2 - c)^2*c^3 - 2*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^4)/c^
6)/(sqrt(-a*c)*a^2*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*
e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(1/2)),
 x)
```

3.149 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 146

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} - \frac{3\text{arctanh}(\cos(e+fx))\tan(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 3/8*csc(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)+1/4*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2)-3/8*arctanh(cos(f*
x+e))*tan(f*x+e)/a^2/c/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4045, 4044, 2691, 3855}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{3\tan(e+fx)\text{arctanh}(\cos(e+fx))}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{3\csc(e+fx)}{8a^2cf\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{\tan(e+fx)}{4f(a\sec(e+fx)+a)^{5/2}(c-c\sec(e+fx))^{3/2}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x
]]) + Tan[e + f*x]/(4*f*(a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*c*f*Sqrt[a + a*Sec[e +
f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rule 4045

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[b*Cot[e + f*x]*
(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[
(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(c
 + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c
+ a*d, 0] && EqQ[a^2 - b^2, 0] && ((ILtQ[m, 0] && ILtQ[n - 1/2, 0]) || (ILt
Q[m - 1/2, 0] && ILtQ[n - 1/2, 0] && LtQ[m, n]))
```

Rubi steps

integral

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} + \frac{3\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{3/2}(c-c\sec(e+fx))^{3/2}}\,dx}{4a}$$

$$= \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} - \frac{(3\tan(e+fx))\int \cot^2(e+fx)\csc(e+fx)\,dx}{4a^2c\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$
\begin{aligned}
&= \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} \\
&\quad + \frac{(3\tan(e+fx))\int\csc(e+fx)\,dx}{8a^2c\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= \frac{3\csc(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad + \frac{\tan(e+fx)}{4f(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}} \\
&\quad - \frac{3\text{arctanh}(\cos(e+fx))\tan(e+fx)}{8a^2cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$
\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{(2-3\sec(e+fx)-3\sec^2(e+fx)+3\text{arctanh}(\sec(e+fx))(-1+\sec(e+fx))(1+\sec(e+fx))^2)\tan(e-}{8cf(-1+\sec(e+fx))(a(1+\sec(e+fx)))^{5/2}\sqrt{c-c\sec(e+fx)}}
$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(3/
2)),x]
```

```
[Out] -1/8*((2 - 3*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 3*ArcTanh[Sec[e + f*x]]*(-1
+ Sec[e + f*x])*(1 + Sec[e + f*x])^2)*Tan[e + f*x])/(c*f*(-1 + Sec[e + f*x]
)*(a*(1 + Sec[e + f*x]))^(5/2)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.47

method	result
default	$\frac{\sqrt{2}\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\,(1-\cos(fx+e))\left((1-\cos(fx+e))^6\csc(fx+e)^6-6(1-\cos(fx+e))^4\csc(fx+e)^4+12\ln(-\cot(fx+e)}{64f\,a^3\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\left(\frac{c(1-\cos(fx+e))^2\csc(fx+e)^2}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)^{\frac{3}{2}}}$
risch	$\frac{i\left(5\,\mathrm{e}^{5i(fx+e)}+2\,\mathrm{e}^{4i(fx+e)}+2\,\mathrm{e}^{3i(fx+e)}+2\,\mathrm{e}^{2i(fx+e)}+5\,\mathrm{e}^{i(fx+e)}\right)}{4a^2c\left(1+\mathrm{e}^{2i(fx+e)}\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,\left(\mathrm{e}^{i(fx+e)}-1\right)\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,f}+\frac{3i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\ln\left(\mathrm{e}^{i(f}\right.}{8a^2c\left(1+\mathrm{e}^{2i(fx+e)}\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\sqrt{\frac{c(}{}}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/64/f*2^(1/2)/a^3*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)/((1-cos(f
*x+e))^2*csc(f*x+e)^2-1)/(c*(1-cos(f*x+e))^2/((1-cos(f*x+e))^2*csc(f*x+e)^2
-1)*csc(f*x+e)^2)^(3/2)*(1-cos(f*x+e))*((1-cos(f*x+e))^6*csc(f*x+e)^6-6*(1-
cos(f*x+e))^4*csc(f*x+e)^4+12*ln(-cot(f*x+e)+csc(f*x+e))*(1-cos(f*x+e))^2*c
sc(f*x+e)^2+2)*csc(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.67

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \left[-\frac{3\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(-a*c)*l
og(-4*(2*sqrt(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
 + e) - c)/cos(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*
x + e))/((cos(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x +
e)^3 + cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 +
a^3*c^2*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e)
), 1/8*(3*(cos(f*x + e)^3 + cos(f*x + e)^2 - cos(f*x + e) - 1)*sqrt(a*c)*ar
ctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
 - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*x + e)^3 +
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^2*f*cos(f*x + e)^3 + a^3*c^2
*f*cos(f*x + e)^2 - a^3*c^2*f*cos(f*x + e) - a^3*c^2*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [A] (verification not implemented)

none

Time = 1.90 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \frac{\frac{2\left(3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)}{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2} - \frac{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c\right)^2c^2-4\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}{c^4}}{32\sqrt{-ac}a^2f|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx\right.\right.}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] 1/32*(2*(3*c*tan(1/2*f*x + 1/2*e)^2 - c)/(c*tan(1/2*f*x + 1/2*e)^2) - ((c*t
an(1/2*f*x + 1/2*e)^2 - c)^2*c^2 - 4*(c*tan(1/2*f*x + 1/2*e)^2 - c)*c^3)/c^
4 - 6*log(abs(c)*tan(1/2*f*x + 1/2*e)^2) + 6*log(abs(c)) - 4)/(sqrt(-a*c)*a
^2*f*abs(c)*sgn(tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),
x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(3/2)),
 x)
```

3.150 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 160

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{3\text{arctanh}(\cos(e+fx))\tan(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] 3/8*csc(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)-1/4*
cot(f*x+e)^2*csc(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(
1/2)-3/8*arctanh(cos(f*x+e))*tan(f*x+e)/a^2/c^2/f/(a+a*sec(f*x+e))^(1/2)/(c
-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4044, 2691, 3855}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{3\tan(e+fx)\text{arctanh}(\cos(e+fx))}{8a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} + \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/2)),x]
```

```
[Out] (3*Csc[e + f*x])/(8*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f
*x]]) - (Cot[e + f*x]^2*Csc[e + f*x])/(4*a^2*c^2*f*Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c - c*Sec[e + f*x]]) - (3*ArcTanh[Cos[e + f*x]]*Tan[e + f*x])/(8*a^2*
c^2*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4044

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[((-a)*c)^(m + 1
/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Int
[Csc[e + f*x]*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)\int \cot^4(e+fx)\csc(e+fx)\,dx}{a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= -\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad -\frac{(3\tan(e+fx))\int \cot^2(e+fx)\csc(e+fx)\,dx}{4a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad -\frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&\quad +\frac{(3\tan(e+fx))\int \csc(e+fx)\,dx}{8a^2c^2\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

$$= \frac{3\csc(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\cot^2(e+fx)\csc(e+fx)}{4a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{3\text{arctanh}(\cos(e+fx))\tan(e+fx)}{8a^2c^2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.48

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \frac{(3-2\cot^2(e+fx))\csc(e+fx) - 3\text{arctanh}(\sec(e+fx)}{8a^2c^2f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+f}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^(5/2)*(c - c*Sec[e + f*x])^(5/
2)),x]
```

```
[Out] ((3 - 2*Cot[e + f*x]^2)*Csc[e + f*x] - 3*ArcTanh[Sec[e + f*x]]*Tan[e + f*x]
)/(8*a^2*c^2*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c - c*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

method	result
default	$\frac{\left(3\cos(fx+e)^4\ln(-\cot(fx+e)+\csc(fx+e))-6\cos(fx+e)^2\ln(-\cot(fx+e)+\csc(fx+e))-5\cos(fx+e)^3+3\ln(-\cot(fx+e)+\csc(fx+e\right.}{8fa^3\sqrt{-c(\sec(fx+e)-1)}\,(\sec(fx+e)-1)^2c^2(\cos(fx+e)+1)^3}$
risch	$\frac{i(5\,\mathrm{e}^{7i(fx+e)}+3\,\mathrm{e}^{5i(fx+e)}+3\,\mathrm{e}^{3i(fx+e)}+5\,\mathrm{e}^{i(fx+e)})}{4a^2c^2\left(1+\mathrm{e}^{2i(fx+e)}\right)\left(\mathrm{e}^{i(fx+e)}+1\right)^3\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\left(\mathrm{e}^{i(fx+e)}-1\right)^3\sqrt{\frac{c\left(\mathrm{e}^{i(fx+e)}-1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,f} + \frac{3i\left(\mathrm{e}^{i(fx+e)}+1\right)\left(\mathrm{e}^{i(fx+e)}-1\right)\ln(\mathrm{e}}{8a^2c^2\left(1+\mathrm{e}^{2i(fx+e)}\right)\sqrt{\frac{a\left(\mathrm{e}^{i(fx+e)}+1\right)^2}{1+\mathrm{e}^{2i(fx+e)}}}\,\sqrt{}}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/f/a^3*(3*cos(f*x+e)^4*ln(-cot(f*x+e)+csc(f*x+e))-6*cos(f*x+e)^2*ln(-cot
(f*x+e)+csc(f*x+e))-5*cos(f*x+e)^3+3*ln(-cot(f*x+e)+csc(f*x+e))+3*cos(f*x+e
))*(a*(sec(f*x+e)+1))^(1/2)/(-c*(sec(f*x+e)-1))^(1/2)/(sec(f*x+e)-1)^2/c^2/
(cos(f*x+e)+1)^3*tan(f*x+e)*sec(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 482, normalized size of antiderivative = 3.01

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \left[-\frac{3\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{-ac}\log\left(-\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-a*c)*log(-4*(2*sqrt
(-a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/co
s(f*x + e))*cos(f*x + e)^2 + (a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos
(f*x + e)^2 - 1)*sin(f*x + e)))*sin(f*x + e) - 2*(5*cos(f*x + e)^4 - 3*cos(
f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) -
c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f*cos(f*x + e)^2 +
 a^3*c^3*f)*sin(f*x + e)), 1/8*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*s
qrt(a*c)*arctan(sqrt(a*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*c
os(f*x + e) - c)/cos(f*x + e))/(a*c*sin(f*x + e)))*sin(f*x + e) + (5*cos(f*
x + e)^4 - 3*cos(f*x + e)^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((
c*cos(f*x + e) - c)/cos(f*x + e)))/((a^3*c^3*f*cos(f*x + e)^4 - 2*a^3*c^3*f
*cos(f*x + e)^2 + a^3*c^3*f)*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1659 vs. $2(142) = 284$.

Time = 0.55 (sec) , antiderivative size = 1659, normalized size of antiderivative = 10.37

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/8*(3*(2*(4*cos(6*f*x + 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1
)*cos(8*f*x + 8*e) - cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f
*x + 2*e) + 1)*cos(6*f*x + 6*e) - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x +
 2*e) - 1)*cos(4*f*x + 4*e) - 36*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2
 + 4*(2*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f
*x + 8*e) - sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e
))*sin(6*f*x + 6*e) - 16*sin(6*f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*si
n(4*f*x + 4*e)*sin(2*f*x + 2*e) - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e
) - 1)*arctan2(sin(f*x + e), cos(f*x + e) + 1) - 3*(2*(4*cos(6*f*x + 6*e) -
 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*cos(8*f*x + 8*e) - cos(8*f*x
+ 8*e)^2 + 8*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e)
 - 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) - 1)*cos(4*f*x + 4*e) - 3
6*cos(4*f*x + 4*e)^2 - 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) - 3*si
n(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - sin(8*f*x + 8*e)^2
+ 16*(3*sin(4*f*x + 4*e) - 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) - 16*sin(6*
f*x + 6*e)^2 - 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e)
 - 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) - 1)*arctan2(sin(f*x + e), co
s(f*x + e) - 1) - 2*(5*sin(7*f*x + 7*e) + 3*sin(5*f*x + 5*e) + 3*sin(3*f*x
+ 3*e) + 5*sin(f*x + e))*cos(8*f*x + 8*e) - 20*(2*sin(6*f*x + 6*e) - 3*sin(
4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 8*(3*sin(5*f*x + 5*e)
 + 3*sin(3*f*x + 3*e) + 5*sin(f*x + e))*cos(6*f*x + 6*e) + 12*(3*sin(4*f*x
+ 4*e) - 2*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) - 12*(3*sin(3*f*x + 3*e) + 5*
sin(f*x + e))*cos(4*f*x + 4*e) + 2*(5*cos(7*f*x + 7*e) + 3*cos(5*f*x + 5*e)
 + 3*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(8*f*x + 8*e) + 10*(4*cos(6*f*x
+ 6*e) - 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) - 1)*sin(7*f*x + 7*e) - 8*
(3*cos(5*f*x + 5*e) + 3*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(6*f*x + 6*e)
 - 6*(6*cos(4*f*x + 4*e) - 4*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e) + 12*(3
*cos(3*f*x + 3*e) + 5*cos(f*x + e))*sin(4*f*x + 4*e) + 6*(4*cos(2*f*x + 2*e
) - 1)*sin(3*f*x + 3*e) - 24*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 40*cos(f*x
 + e)*sin(2*f*x + 2*e) + 40*cos(2*f*x + 2*e)*sin(f*x + e) - 10*sin(f*x + e)
)*sqrt(a)*sqrt(c)/((a^3*c^3*cos(8*f*x + 8*e)^2 + 16*a^3*c^3*cos(6*f*x + 6*e
)^2 + 36*a^3*c^3*cos(4*f*x + 4*e)^2 + 16*a^3*c^3*cos(2*f*x + 2*e)^2 + a^3*c
^3*sin(8*f*x + 8*e)^2 + 16*a^3*c^3*sin(6*f*x + 6*e)^2 + 36*a^3*c^3*sin(4*f*
x + 4*e)^2 - 48*a^3*c^3*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*a^3*c^3*sin(
```

```
2*f*x + 2*e)^2 - 8*a^3*c^3*cos(2*f*x + 2*e) + a^3*c^3 - 2*(4*a^3*c^3*cos(6*
f*x + 6*e) - 6*a^3*c^3*cos(4*f*x + 4*e) + 4*a^3*c^3*cos(2*f*x + 2*e) - a^3*
c^3)*cos(8*f*x + 8*e) - 8*(6*a^3*c^3*cos(4*f*x + 4*e) - 4*a^3*c^3*cos(2*f*x
 + 2*e) + a^3*c^3)*cos(6*f*x + 6*e) - 12*(4*a^3*c^3*cos(2*f*x + 2*e) - a^3*
c^3)*cos(4*f*x + 4*e) - 4*(2*a^3*c^3*sin(6*f*x + 6*e) - 3*a^3*c^3*sin(4*f*x
 + 4*e) + 2*a^3*c^3*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 16*(3*a^3*c^3*sin(
4*f*x + 4*e) - 2*a^3*c^3*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

Giac [A] (verification not implemented)

none

Time = 1.86 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\frac{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2c^2-6\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c^3}{c^4}-\frac{18\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)^2+28\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c\right)c+11\,c^2}{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}+12\log\left(|c|\,\mathrm{t}\right.}{64\sqrt{-ac}a^2cf|c|\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] -1/64*(((c*tan(1/2*f*x + 1/2*e)^2 - c)^2*c^2 - 6*(c*tan(1/2*f*x + 1/2*e)^2
- c)*c^3)/c^4 - (18*(c*tan(1/2*f*x + 1/2*e)^2 - c)^2 + 28*(c*tan(1/2*f*x +
1/2*e)^2 - c)*c + 11*c^2)/(c^2*tan(1/2*f*x + 1/2*e)^4) + 12*log(abs(c)*tan(
1/2*f*x + 1/2*e)^2) - 12*log(abs(c)) + 11)/(sqrt(-a*c)*a^2*c*f*abs(c)*sgn(t
an(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^{5/2}(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^{5/2}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,d$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(5/2)*(c - c/cos(e + f*x))^(5/2)),
 x)
```

3.151 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$

Optimal result

Integrand size = 32, antiderivative size = 101

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx =$$
$$-\frac{2^{\frac{1}{2}+n}c\,\text{Hypergeometric2F1}\left(\frac{1}{2}+m,\frac{1}{2}-n,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{\frac{1}{2}-n}(a+a\sec(e+}{f(1+2m)}$$

```
[Out] -2^(1/2+n)*c*hypergeom([1/2-n, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f*
x+e))^(1/2-n)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1+n)*tan(f*x+e)/f/(1+2*
m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {4046, 72, 71}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx =$$
$$-\frac{c2^{n+\frac{1}{2}}\tan(e+fx)(1-\sec(e+fx))^{\frac{1}{2}-n}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{n-1}\,\text{Hypergeometric2F1}}{f(2m+1)}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
```

```
[Out] -((2^(1/2 + n)*c*Hypergeometric2F1[1/2 + m, 1/2 - n, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 - n)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 + n)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}+n}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+n}ac(c-c\sec(e+fx))^{-1+n}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}-n}\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}+n}(a+a

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1}{2}+n,\frac{3}{2}+n,\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+}{f+2fn}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^n,x]
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2 - m, 1/2 + n, 3/2 + n, (1 - Sec[e + f*x]
)/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e +
f*x])^n*Tan[e + f*x])/(f + 2*f*n)
```

Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^n\,dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="fr
icas")
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a(\sec(e+fx)+1))^m\,(-c(\sec(e+fx)-1))^n\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**n,x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**n*sec(e + f*x),
 x)
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="ma
xima")
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^n\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^n,x, algorithm="gi
ac")
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^n*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^n\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^n}{\cos\left(e+f\,x\right)}\,dx$$

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n)/cos(e + f*x),x)
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^n)/cos(e + f*x), x)
```

3.152 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 32, antiderivative size = 92

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx = \frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx))^{-1-}}{5f}$$

```
[Out] 1/5*2^(1/2+m)*a*hypergeom([5/2, 1/2-m],[7/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x
+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*(c-c*sec(f*x+e))^2*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4046, 72, 71}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx = \frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))^2(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,\text{Hypergeometric2F1}\,(}{5f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])^2*Tan[e + f*x])/(5*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{3/2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{x}{2}\right)^{-\frac{1}{2}+m}(c-\right.}{f\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+f}{5f}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int\sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$$
$$= \frac{2^{\frac{1}{2}+m}c^2\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}-m,\frac{7}{2},\frac{1}{2}(1-\sec(e+fx))\right)(-1+\sec(e+fx))^2(1+\sec(e+fx))^{-\frac{1}{2}-m}(}{5f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^2,x]
```

```
[Out] (2^(1/2 + m)*c^2*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sec[e + f*x])/2]
*(-1 + Sec[e + f*x])^2*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))
^m*Tan[e + f*x])/(5*f)
```

Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^2\,dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x)
```

Fricas [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx \\ = \int (c\sec(fx+e)-c)^2(a\sec(fx+e)+a)^m\sec(fx+e)\,dx\end{aligned}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral((c^2*sec(f*x + e)^3 - 2*c^2*sec(f*x + e)^2 + c^2*sec(f*x + e))*(a*
sec(f*x + e) + a)^m, x)
```

Sympy [F]

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx \\ &= c^2\left(\int (a\sec(e+fx)+a)^m\sec(e+fx)\,dx\right. \\ &\quad+\int \left(-2(a\sec(e+fx)+a)^m\sec^2(e+fx)\right)\,dx \\ &\quad\left.+\int (a\sec(e+fx)+a)^m\sec^3(e+fx)\,dx\right)\end{aligned}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**2,x)
```

```
[Out] c**2*(Integral((a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral(-2*(a*se
c(e + f*x) + a)**m*sec(e + f*x)**2, x) + Integral((a*sec(e + f*x) + a)**m*s
ec(e + f*x)**3, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$$
$$=\int (c\sec(fx+e)-c)^2(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$$
$$=\int (c\sec(fx+e)-c)^2(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((c*sec(f*x + e) - c)^2*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^2\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}{\cos(e+f\,x)}\,dx$$

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^2)/cos(e + f*x), x)
```

3.153 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 90

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = \frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx))^{-1-}}{3f}$$

```
[Out] 1/3*2^(1/2+m)*a*hypergeom([3/2, 1/2-m],[5/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x
+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*(c-c*sec(f*x+e))*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4046, 72, 71}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = \frac{a2^{m+\frac{1}{2}}\tan(e+fx)(c-c\sec(e+fx))(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,\text{Hypergeometric2F1}\,(}{3f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

```
[Out] (2^(1/2 + m)*a*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])/2]*(
1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*(c - c*Sec[e + f*
x])*Tan[e + f*x])/(3*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int (a+ax)^{-\frac{1}{2}+m}\sqrt{c-cx}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx)\right)\text{Subst}\left(\int \left(\frac{1}{2}+\frac{x}{2}\right)^{-\frac{1}{2}+m}\sqrt{c-\right.}{f\sqrt{c-c\sec(e+fx)}}$$

$$= \frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+f}{3f}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx =$$

$$-\frac{2^{\frac{1}{2}+m}c\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}-m,\frac{5}{2},\frac{1}{2}(1-\sec(e+fx))\right)(-1+\sec(e+fx))(1+\sec(e+fx))^{-\frac{1}{2}-m}(}{3f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(2^(1/2 + m)*c*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sec[e + f*x])
/2]*(-1 + Sec[e + f*x])*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x])
)^m*Tan[e + f*x])/f
```

Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))\,dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$$
$$=\int -(c\sec(fx+e)-c)(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral(-(c*sec(f*x + e)^2 - c*sec(f*x + e))*(a*sec(f*x + e) + a)^m, x)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx$$
$$=-c\left(\int\left(-(a\sec(e+fx)+a)^m\sec(e+fx)\right)dx + \int (a\sec(e+fx)+a)^m\sec^2(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e)),x)
```

```
[Out] -c*(Integral(-(a*sec(e + f*x) + a)**m*sec(e + f*x), x) + Integral((a*sec(e
+ f*x) + a)**m*sec(e + f*x)**2, x))
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = \int -(c\sec(fx+e)-c)(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")

[Out] -integrate((c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = \int -(c\sec(fx+e)-c)(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e)),x, algorithm="giac
")

[Out] integrate(-(c*sec(f*x + e) - c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)}{\cos(e+f\,x)}\,dx$$

[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x)))/cos(e + f*x),x)

[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x)))/cos(e + f*x), x)

3.154 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 90

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx = -\frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+fx}{f(c-c\sec(e+fx))}$$

```
[Out] -2^(1/2+m)*a*hypergeom([-1/2, 1/2-m],[1/2],1/2-1/2*sec(f*x+e))*(1+sec(f*x+e
))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*tan(f*x+e)/f/(c-c*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4046, 72, 71}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx = -\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}-m,\frac{1}{2},\frac{1}{2}(\right.}{f(c-c\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
```

```
[Out] -((2^(1/2 + m)*a*Hypergeometric2F1[-1/2, 1/2 - m, 1/2, (1 - Sec[e + f*x])/2
]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*x])/
(f*(c - c*Sec[e + f*x])))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{x}{2}\right)^{-\frac{1}{2}+m}}{(c-cx)^{3/2}}\,dx, x, \text{s}\right.}{f\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{2}-m, \frac{1}{2}, \frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e}{f(c-c\sec(e+fx))}$$

Mathematica [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]),x]
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x]), x]
```

Maple [F]

$$\int \frac{\sec(fx+e)\,(a+a\sec(fx+e))^m}{c-c\sec(fx+e)}dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=\int -\frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{c\sec(fx+e)-c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="fric
as")
[Out] integral(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=-\frac{\int \frac{(a\sec(e+fx)+a)^m\sec(e+fx)}{\sec(e+fx)-1}\,dx}{c}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e)),x)
[Out] -Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x) - 1), x)/c
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=\int -\frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{c\sec(fx+e)-c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=\int -\frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{c\sec(fx+e)-c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{c-c\sec(e+fx)}\,dx=-\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{c-c\,\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))),x)
```

```
[Out] -int((a + a/cos(e + f*x))^m/(c - c*cos(e + f*x)), x)
```

3.155 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = -\frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec(e+f}{3f(c-c\sec(e+fx))^2}$$

```
[Out] -1/3*2^(1/2+m)*a*hypergeom([-3/2, 1/2-m],[-1/2],1/2-1/2*sec(f*x+e))*(1+sec(
f*x+e))^(1/2-m)*(a+a*sec(f*x+e))^(-1+m)*tan(f*x+e)/f/(c-c*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4046, 72, 71}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = -\frac{a2^{m+\frac{1}{2}}\tan(e+fx)(\sec(e+fx)+1)^{\frac{1}{2}-m}(a\sec(e+fx)+a)^{m-1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}-m,-\frac{1}{2},\frac{1}{2}\right.}{3f(c-c\sec(e+fx))^2}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(2^(1/2 + m)*a*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sec[e + f*x
])/2]*(1 + Sec[e + f*x])^(1/2 - m)*(a + a*Sec[e + f*x])^(-1 + m)*Tan[e + f*
x])/(f*(c - c*Sec[e + f*x])^2)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}+m}ac(a+a\sec(e+fx))^{-1+m}\left(\frac{a+a\sec(e+fx)}{a}\right)^{\frac{1}{2}-m}\tan(e+fx)\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{x}{2}\right)^{-\frac{1}{2}+m}}{(c-cx)^{5/2}}\,dx, x, \text{s}\right.}{f\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m}a\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{2}-m, -\frac{1}{2}, \frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{\frac{1}{2}-m}(a+a\sec}{3f(c-c\sec(e+fx))^2}$$

Mathematica [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx=\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2,x]
[Out] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^2, x]
```

Maple [F]

$$\int \frac{\sec\left(fx+e\right)\left(a+a\sec\left(fx+e\right)\right)^{m}}{\left(c-c\sec\left(fx+e\right)\right)^{2}}dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx=\int \frac{\left(a\sec\left(fx+e\right)+a\right)^{m}\sec\left(fx+e\right)}{\left(c\sec\left(fx+e\right)-c\right)^{2}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="fr
icas")
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2*sec(f*x + e)^2 - 2*c^2*se
c(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx=\frac{\int \frac{(a\sec(e+fx)+a)^m\sec(e+fx)}{\sec^2(e+fx)-2\sec(e+fx)+1}\,dx}{c^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**2,x)
[Out] Integral((a*sec(e + f*x) + a)**m*sec(e + f*x)/(sec(e + f*x)**2 - 2*sec(e +
f*x) + 1), x)/c**2
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(c\sec(fx+e)-c)^2}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="ma
xima")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(c\sec(fx+e)-c)^2}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^2,x, algorithm="gi
ac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*sec(f*x + e) - c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^2}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^2}\,dx$$

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^2),x)
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^2), x)
```

3.156 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 160

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3(a+a\sec(e+fx))^m\tan(e+fx)}{f(5+2m)\,(3+8m+4m^2)\,\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f\,(15+16m+4m^2)} - \frac{2c(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(5+2m)}$$

```
[Out] -2*c*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2)*tan(f*x+e)/f/(5+2*m)-64*c^3*
(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(5+2*m)/(4*m^2+8*m+3)/(c-c*sec(f*x+e))^(1/2
)-16*c^2*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(4*m^2+16*m
+15)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used

$= \{4040, 4038\}$

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = -\frac{64c^3\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+5)\left(4m^2+8m+3\right)\sqrt{c-c\sec(e+fx)}} - \frac{16c^2\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f\left(4m^2+16m+15\right)} - \frac{2c\tan(e+fx)(c-c\sec(e+fx))^{3/2}(a\sec(e+fx)+a)^m}{f(2m+5)}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-64*c^3*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(5 + 2*m)*(3 + 8*m + 4*m^2
)*Sqrt[c - c*Sec[e + f*x]]) - (16*c^2*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec
[e + f*x]]*Tan[e + f*x])/(f*(15 + 16*m + 4*m^2)) - (2*c*(a + a*Sec[e + f*x]
)^m*(c - c*Sec[e + f*x])^(3/2)*Tan[e + f*x])/(f*(5 + 2*m))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] && !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\text{integral} = -\frac{2c(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(5+2m)} + \frac{(8c)\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx}{5+2m}$$

$$= -\frac{16c^2(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(15+16m+4m^2)} - \frac{2c(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(5+2m)} + \frac{(32c^2)\int\sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx}{15+16m+4m^2}$$

$$= -\frac{64c^3(a+a\sec(e+fx))^m\tan(e+fx)}{f(15+46m+36m^2+8m^3)\sqrt{c-c\sec(e+fx)}} - \frac{16c^2(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(15+16m+4m^2)} - \frac{2c(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\tan(e+fx)}{f(5+2m)}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.68

$$\int\sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx =$$

$$-\frac{2c^3(a(1+\sec(e+fx)))^m\left(43+24m+4m^2-2(7+16m+4m^2)\sec(e+fx)+(3+8m+4m^2)\sec^2(e+\right.}{f(1+2m)(3+2m)(5+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*c^3*(a*(1 + Sec[e + f*x]))^m*(43 + 24*m + 4*m^2 - 2*(7 + 16*m + 4*m^2)*
Sec[e + f*x] + (3 + 8*m + 4*m^2)*Sec[e + f*x]^2)*Tan[e + f*x])/(f*(1 + 2*m)
*(3 + 2*m)*(5 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [F]

$$\int\sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^{\frac{5}{2}}\,dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = \frac{2\left(4\,c^2m^2+\left(4\,c^2m^2+24\,c^2m+43\,c^2\right)\cos\left(fx+e\right)^3+8\,c^2m-\left(4\,c^2m^2+8\,c^2m-29\,c^2\right)\cos\left(fx\right.\right.}{\left(8\,fm^3+36\,fm^2+46\,fm+15\,f\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] 2*(4*c^2*m^2 + (4*c^2*m^2 + 24*c^2*m + 43*c^2)*cos(f*x + e)^3 + 8*c^2*m - (
4*c^2*m^2 + 8*c^2*m - 29*c^2)*cos(f*x + e)^2 + 3*c^2 - (4*c^2*m^2 + 24*c^2*
m + 11*c^2)*cos(f*x + e))*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos
(f*x + e) - c)/cos(f*x + e))/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x
+ e)^2*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.42

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = -\frac{2\left(\frac{\sqrt{2}\left(2^{m+5}m+5\cdot2^{m+4}\right)(-a)^m c^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{\sqrt{2}\left(2^{m+4}m^2+2^{m+6}m+15\cdot2^{m+2}\right)(-a)^m c^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-2^{m+\frac{11}{2}}(-a)^m c^{\frac{5}{2}}\right)e^{\left(-m\log\right.}}{\left(8\,m^3+36\,m^2+46\,m+15\right)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{5}{2}}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] -2*(sqrt(2)*(2^(m + 5)*m + 5*2^(m + 4))*(-a)^m*c^(5/2)*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 - sqrt(2)*(2^(m + 4)*m^2 + 2^(m + 6)*m + 15*2^(m + 2))*(-a)
^m*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2^(m + 11/2)*(-a)^m*c^(5/2
))*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos
(f*x + e) + 1) - 1))/((8*m^3 + 36*m^2 + 46*m + 15)*f*(sin(f*x + e)/(cos(f*x
 + e) + 1) + 1)^(5/2)*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)^(5/2))
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = \int (-c\sec(fx+e)+c)^{\frac{5}{2}}(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^(5/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{5/2}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}{\cos(e+f\,x)}\,dx$$

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x),x)
```

```
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(5/2))/cos(e + f*x), x)
```

3.157 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 100

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2(a+a\sec(e+fx))^m\tan(e+fx)}{f(3+8m+4m^2)\sqrt{c-c\sec(e+fx)}} - \frac{2c(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(3+2m)}$$

```
[Out] -8*c^2*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(4*m^2+8*m+3)/(c-c*sec(f*x+e))^(1/2)
-2*c*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2)*tan(f*x+e)/f/(3+2*m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4040, 4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = -\frac{8c^2\tan(e+fx)(a\sec(e+fx)+a)^m}{f(4m^2+8m+3)\sqrt{c-c\sec(e+fx)}} - \frac{2c\tan(e+fx)\sqrt{c-c\sec(e+fx)}(a\sec(e+fx)+a)^m}{f(2m+3)}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-8*c^2*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(3 + 8*m + 4*m^2)*Sqrt[c -
c*Sec[e + f*x]]) - (2*c*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]]*Tan
[e + f*x])/(f*(3 + 2*m))
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
 && NeQ[m, -2^(-1)]
```

Rule 4040

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Simp[(-d)*Cot[e + f
*x]*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^(n - 1)/(f*(m + n))), x] +
 Dist[c*((2*n - 1)/(m + n)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(c + d
*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*
c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0] &&  !LtQ[m, -2^(-1)] &
&  !(IGtQ[m - 1/2, 0] && LtQ[m, n])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2c(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(3+2m)} \\ &\quad+\frac{(4c)\int\sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx}{3+2m} \\ &= -\frac{8c^2(a+a\sec(e+fx))^m\tan(e+fx)}{f\left(3+8m+4m^2\right)\sqrt{c-c\sec(e+fx)}} \\ &\quad-\frac{2c(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\tan(e+fx)}{f(3+2m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int\sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx=\frac{2c^2(a(1+\sec(e+fx)))^m(-5-2m+(1+2m)\sec(e+fx))\tan(e+fx)}{f(1+2m)(3+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*c^2*(a*(1 + Sec[e + f*x]))^m*(-5 - 2*m + (1 + 2*m)*Sec[e + f*x])*Tan[e +
 f*x])/(f*(1 + 2*m)*(3 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [F]

$$\int \sec(fx+e)\left(a+a\sec(fx+e)\right)^{m}\left(c-c\sec(fx+e)\right)^{\frac{3}{2}}dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = \frac{2\left((2\,cm+5\,c)\cos\left(fx+e\right)^2-2\,cm+4\,c\cos\left(fx+e\right)-c\right)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}}{\left(4\,fm^2+8\,fm+3\,f\right)\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
[Out] 2*((2*c*m + 5*c)*cos(f*x + e)^2 - 2*c*m + 4*c*cos(f*x + e) - c)*((a*cos(f*x
 + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))/((4*f*m^
2 + 8*f*m + 3*f)*cos(f*x + e)*sin(f*x + e))
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.71

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2\left(\sqrt{2}2^{m+2}(-a)^m c^{\frac{3}{2}} - \frac{\sqrt{2}(2^{m+2}m+3\cdot 2^{m+1})(-a)^m c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)e^{\left(-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\right)}}{(4m^2+8m+3)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)^{\frac{3}{2}}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] -2*(sqrt(2)*2^(m + 2)*(-a)^m*c^(3/2) - sqrt(2)*(2^(m + 2)*m + 3*2^(m + 1))*
(-a)^m*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*e^(-m*log(sin(f*x + e)/
(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/((4*m
^2 + 8*m + 3)*f*(sin(f*x + e)/(cos(f*x + e) + 1) + 1)^(3/2)*(sin(f*x + e)/(
cos(f*x + e) + 1) - 1)^(3/2))
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c - c\sec(e+fx))^{3/2}\,dx = \int (-c\sec(fx+e)+c)^{\frac{3}{2}}(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((-c*sec(f*x + e) + c)^(3/2)*(a*sec(f*x + e) + a)^m*sec(f*x + e),
x)
```

Mupad [B] (verification not implemented)

Time = 15.51 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{3/2}\,dx = -\frac{2\,c\left(\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}\right)^m\sqrt{\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}}\,(5\,\sin(e+f\,x)-2\,\sin(2\,e+2\,f\,x)+5\,\sin(3\,e+3\,f\,x)+2\,m\,\sin(e}{f\,(4\,m^2+8\,m+3)\,(3\,\cos(e+f\,x)-2\,\cos(2\,e+2\,f\,x)+\cos(3\,e}$$

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(3/2))/cos(e + f*x),x)
```

```
[Out] -(2*c*((a*(cos(e + f*x) + 1))/cos(e + f*x))^m*((c*(cos(e + f*x) - 1))/cos(e
 + f*x))^(1/2)*(5*sin(e + f*x) - 2*sin(2*e + 2*f*x) + 5*sin(3*e + 3*f*x) +
2*m*sin(e + f*x) - 4*m*sin(2*e + 2*f*x) + 2*m*sin(3*e + 3*f*x)))/(f*(8*m +
4*m^2 + 3)*(3*cos(e + f*x) - 2*cos(2*e + 2*f*x) + cos(3*e + 3*f*x) - 2))
```

3.158 $\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 46

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -2*c*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4038}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx = -\frac{2c\tan(e+fx)(a\sec(e+fx)+a)^m}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] (-2*c*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e +
f*x]])
```

Rule 4038

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[2*a*c*Cot[e + f
*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1)*Sqrt[c + d*Csc[e + f*x]])), x] /
; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

```
&& NeQ[m, -2^(-1)]
```

Rubi steps

$$\text{integral} = -\frac{2c(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.54

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx = \frac{\sqrt{2}e^{-\frac{1}{2}i(e+fx)}\left(1+e^{i(e+fx)}\right)\sqrt{\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}}\left(\frac{\left(1+e^{i(e+fx)}\right)^2}{1+e^{2i(e+fx)}}\right)^m\csc\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^{-m}(a(1+\sec(e+fx$$
$$(f+2fm)\sqrt{\sec(e+fx)}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*Sqrt[c - c*Sec[e + f*x]],x]
[Out] (Sqrt[2]*(1 + E^(I*(e + f*x)))*Sqrt[E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x)
))]*((1 + E^(I*(e + f*x)))^2/(1 + E^((2*I)*(e + f*x))))^m*Csc[(e + f*x)/2]*
(a*(1 + Sec[e + f*x]))^m*Sqrt[c - c*Sec[e + f*x]])/(E^((I/2)*(e + f*x))*(f
+ 2*f*m)*Sqrt[Sec[e + f*x]]*(1 + Sec[e + f*x])^m)
```

Maple [F]

$$\int \sec(fx+e)(a+a\sec(fx+e))^m\sqrt{c-c\sec(fx+e)}dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx = \frac{2\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}(\cos(fx+e)+1)}{(2fm+f)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] 2*((a*cos(f*x + e) + a)/cos(f*x + e))^m*sqrt((c*cos(f*x + e) - c)/cos(f*x +
 e))*(cos(f*x + e) + 1)/((2*f*m + f)*sin(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int (a(\sec{(e+fx)}+1))^m\sqrt{-c\left(\sec{(e+fx)}-1\right)}\sec{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sqrt(-c*(sec(e + f*x) - 1))*sec(e + f*x)
, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(44) = 88$.

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\frac{2^{m+\frac{3}{2}}(-a)^m\sqrt{c}e^{\left(-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\right)}}{f(2m+1)\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}+1}\sqrt{\frac{\sin(fx+e)}{\cos(fx+e)+1}-1}}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] 2^(m + 3/2)*(-a)^m*sqrt(c)*e^(-m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
 m*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1))/(f*(2*m + 1)*sqrt(sin(f*x + e)
/(cos(f*x + e) + 1) + 1)*sqrt(sin(f*x + e)/(cos(f*x + e) + 1) - 1))
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \sqrt{-c\sec(fx+e)+c}(a\sec(fx+e)+a)^m\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1/2),x, algorithm
="giac")
[Out] integrate(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m\sqrt{c-c\sec(e+fx)}\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\sqrt{c-\frac{c}{\cos(e+f\,x)}}}{\cos(e+f\,x)}\,dx$$

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x),x)
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1/2))/cos(e + f*x), x)
```

3.159 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -hypergeom([1, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*tan(f*
x+e)/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {4046, 70}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(1,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
```

```
[Out] -((Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*Sec[
e + f*x])^m*Tan[e + f*x])/(f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{c-cx}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(1, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sec(e+fx))\right)(a(1+\sec(e+fx)))^m\tan(e+fx)}{(f+2fm)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/Sqrt[c - c*Sec[e + f*x]],x]
[Out] -((Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a*(1 + Sec
[e + f*x]))^m*Tan[e + f*x])/((f + 2*f*m)*Sqrt[c - c*Sec[e + f*x]]))
```

Maple [F]

$$\int \frac{\sec (fx+e) (a+a\sec (fx+e))^m}{\sqrt{c-c\sec (fx+e)}}dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec (fx+e)+a)^m \sec (fx+e)}{\sqrt{-c\sec (fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c*
sec(f*x + e) - c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a(\sec (e+fx)+1))^m \sec (e+fx)}{\sqrt{-c (\sec (e+fx)-1)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/sqrt(-c*(sec(e + f*x) - 1))
, x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec (fx+e)+a)^m \sec (fx+e)}{\sqrt{-c\sec (fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{\sqrt{-c\sec(fx+e)+c}}\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(1/2),x, algorithm ="giac")

[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/sqrt(-c*sec(f*x + e) + c), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos(e+f\,x)\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)),x)

[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(1/2)), x)

3.160 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{2cf(1+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/2*hypergeom([2, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*ta
n(f*x+e)/c/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4046, 70}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(2,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{2cf(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/2*(Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*S
ec[e + f*x])^m*Tan[e + f*x])/(c*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{2cf(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(2,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a(1+\sec(e+fx)))^m\tan(e+fx)}{4cf\left(\frac{1}{2}+m\right)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(3/2),
x]
[Out] -1/4*(Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a*(1 +
Sec[e + f*x]))^m*Tan[e + f*x])/(c*f*(1/2 + m)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [F]

$$\int \frac{\sec(fx+e)(a+a\sec(fx+e))^m}{(c-c\sec(fx+e))^{\frac{3}{2}}}dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^2
*sec(f*x + e)^2 - 2*c^2*sec(f*x + e) + c^2), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a(\sec(e+fx)+1))^m\sec(e+fx)}{(-c(\sec(e+fx)-1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*sec(e + f*x)/(-c*(sec(e + f*x) - 1))**(3
/2), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Giac [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(3/2),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{3/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)),x)
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(3/2)), x)
```

3.161 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(3,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{4c^2f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

```
[Out] -1/4*hypergeom([3, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(a+a*sec(f*x+e))^m*ta
n(f*x+e)/c^2/f/(1+2*m)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4046, 70}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(3,m+\frac{1}{2},m+\frac{3}{2},\frac{1}{2}(\sec(e+fx)+1)\right)}{4c^2f(2m+1)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),x]
```

```
[Out] -1/4*(Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a + a*S
ec[e + f*x])^m*Tan[e + f*x])/(c^2*f*(1 + 2*m)*Sqrt[c - c*Sec[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{(c-cx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$
$$= -\frac{\text{Hypergeometric2F1}\left(3, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sec(e+fx))\right)(a+a\sec(e+fx))^m\tan(e+fx)}{4c^2f(1+2m)\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(3, \frac{1}{2}+m, \frac{3}{2}+m, \frac{1}{2}(1+\sec(e+fx))\right)(a(1+\sec(e+fx)))^m\tan(e+fx)}{8c^2f\left(\frac{1}{2}+m\right)\sqrt{c-c\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^(5/2),
x]
[Out] -1/8*(Hypergeometric2F1[3, 1/2 + m, 3/2 + m, (1 + Sec[e + f*x])/2]*(a*(1 +
Sec[e + f*x]))^m*Tan[e + f*x])/(c^2*f*(1/2 + m)*Sqrt[c - c*Sec[e + f*x]])
```

Maple [F]

$$\int \frac{\sec(fx+e)(a+a\sec(fx+e))^m}{(c-c\sec(fx+e))^{\frac{5}{2}}}dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x)
```

Fricas [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(-c*sec(f*x + e) + c)*(a*sec(f*x + e) + a)^m*sec(f*x + e)/(c^
3*sec(f*x + e)^3 - 3*c^3*sec(f*x + e)^2 + 3*c^3*sec(f*x + e) - c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/(c-c*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^(5/2),
x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/(c-c*sec(f*x+e))^(5/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,4,1,0]%%%}+%%%{2,[0,2,1,1]%%%}+%%%{1,[0,0,1,2]%%%} / %%%
{1,[0,
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^m}{(c-c\sec(e+fx))^{5/2}}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)),x)
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(5/2)), x)
```

3.162 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 169

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx &= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)} \\ &\quad+\frac{2(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{af\,(3+8m+4m^2)} \\ &\quad-\frac{2(a+a\sec(e+fx))^{2+m}(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{a^2f(1+2m)\,(15+16m+4m^2)}\end{aligned}$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/f/(1+2*m)+2*(a+a*sec
(f*x+e))^(1+m)*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/a/f/(4*m^2+8*m+3)-2*(a+a*
sec(f*x+e))^(2+m)*(c-c*sec(f*x+e))^(-3-m)*tan(f*x+e)/a^2/f/(1+2*m)/(4*m^2+1
6*m+15)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used

$= \{4036, 4035\}$

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx$$
$$= -\frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+2}(c-c\sec(e+fx))^{-m-3}}{a^2f(2m+1)\left(4m^2+16m+15\right)} + \frac{2\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-3}}{af\left(4m^2+8m+3\right)} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-3}}{f(2m+1)}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + (2*(a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-3 - m)*T
an[e + f*x])/(a*f*(3 + 8*m + 4*m^2)) - (2*(a + a*Sec[e + f*x])^(2 + m)*(c -
 c*Sec[e + f*x])^(-3 - m)*Tan[e + f*x])/(a^2*f*(1 + 2*m)*(15 + 16*m + 4*m^2
))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 &&  !LtQ[n, 0] &&  !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\text{integral} = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)} - \frac{2\int\sec(e+fx)(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-3-m}\,dx}{a(1+2m)}$$

$$
\begin{aligned}
&= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)} \\
&\quad + \frac{2(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{af\left(3+8m+4m^2\right)} \\
&\quad + \frac{2\int \sec(e+fx)(a+a\sec(e+fx))^{2+m}(c-c\sec(e+fx))^{-3-m}\,dx}{a^2\left(3+8m+4m^2\right)} \\
&= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{f(1+2m)} \\
&\quad + \frac{2(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{af\left(3+8m+4m^2\right)} \\
&\quad - \frac{2(a+a\sec(e+fx))^{2+m}(c-c\sec(e+fx))^{-3-m}\tan(e+fx)}{a^2f(5+2m)\left(3+8m+4m^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.62

$$
\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx
$$

$$
= \frac{(a(1+\sec(e+fx)))^m(c-c\sec(e+fx))^{-m}\left(7+12m+4m^2-2(3+2m)\sec(e+fx)+2\sec^2(e+fx)\right)}{c^3f(1+2m)(3+2m)(5+2m)(-1+\sec(e+fx))^3}
$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-3 - m)
,x]
```

```
[Out] ((a*(1 + Sec[e + f*x]))^m*(7 + 12*m + 4*m^2 - 2*(3 + 2*m)*Sec[e + f*x] + 2*
Sec[e + f*x]^2)*Tan[e + f*x])/(c^3*f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(-1 + Se
c[e + f*x])^3*(c - c*Sec[e + f*x])^m)
```

Maple [F]

$$
\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^{-3-m}\,dx
$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = -\frac{\left((4\,m^2+12\,m+7)\cos\left(fx+e\right)^2-2\,(2\,m+3)\cos\left(fx+e\right)+2\right)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-3}\sin}{(8\,fm^3+36\,fm^2+46\,fm+15\,f)\cos\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="fricas")
```

```
[Out] -((4*m^2 + 12*m + 7)*cos(f*x + e)^2 - 2*(2*m + 3)*cos(f*x + e) + 2)*((a*cos
(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^(-m - 3)
*sin(f*x + e)/((8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = \int (a(\sec(e+fx)+1))^m\,(-c(\sec(e+fx)-1))^{-m-3}\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-3-m),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(-m - 3)*sec(e
+ f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = \frac{\left((4\,m^2+8\,m+3)(-a)^m-\frac{2\,(4\,m^2+12\,m+5)(-a)^m\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{(4\,m^2+16\,m+15)(-a)^m\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)c^{-m-3}(\cos\left(fx+e\right)}{4\,(8\,m^3+36\,m^2+46\,m+15)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{2\,m}\sin\left(fx+e\right)^5}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="maxima")
```

```
[Out] 1/4*((4*m^2 + 8*m + 3)*(-a)^m - 2*(4*m^2 + 12*m + 5)*(-a)^m*sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + (4*m^2 + 16*m + 15)*(-a)^m*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4)*c^(-m - 3)*(cos(f*x + e) + 1)^5/((8*m^3 + 36*m^2 + 46*m + 15)*f
*(sin(f*x + e)/(cos(f*x + e) + 1))^(2*m)*sin(f*x + e)^5)
```

Giac [F]

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx\\&=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m-3}\sec(fx+e)\,dx\end{aligned}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-3-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 3)*sec(f*x + e
), x)
```

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.72

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-3-m}\,dx = -\frac{(\cos(3\,e+3\,f\,x)-\sin(3\,e+3\,f\,x)\;1\mathrm{i})\left(\frac{\sin(e+f\,x)\left(a+\frac{a}{\cos(e+f\,x)}\right)^m(\cos(3\,e+3\,f\,x)+\sin(3\,e+3\,f\,x)\;1\mathrm{i})\,(4\,m^2+12\,m+15)\;2\mathrm{i}}{f\,(m^3\,8\mathrm{i}+m^2\,36\mathrm{i}+m\,46\mathrm{i}+15\mathrm{i})}\right.}{8\cos}$$

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 3)),x)
```

```
[Out] -((cos(3*e + 3*f*x) - sin(3*e + 3*f*x)*1i)*((sin(e + f*x)*(a + a/cos(e + f*
x))^m*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*(12*m + 4*m^2 + 15)*2i)/(f*(
m*46i + m^2*36i + m^3*8i + 15i)) - (sin(2*e + 2*f*x)*(8*m + 12)*(a + a/cos(
e + f*x))^m*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*2i)/(f*(m*46i + m^2*36
i + m^3*8i + 15i)) + (sin(3*e + 3*f*x)*(a + a/cos(e + f*x))^m*(cos(3*e + 3*
f*x) + sin(3*e + 3*f*x)*1i)*(12*m + 4*m^2 + 7)*2i)/(f*(m*46i + m^2*36i + m^
3*8i + 15i))))/(8*cos(e + f*x)^3*(c - c/cos(e + f*x))^(m + 3))
```

3.163 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 104

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{f(1+2m)} + \frac{(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{af\,(3+8m+4m^2)}$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m)*tan(f*x+e)/f/(1+2*m)+(a+a*sec(f
*x+e))^(1+m)*(c-c*sec(f*x+e))^(-2-m)*tan(f*x+e)/a/f/(4*m^2+8*m+3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {4036, 4035}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx = \frac{\tan(e+fx)(a\sec(e+fx)+a)^{m+1}(c-c\sec(e+fx))^{-m-2}}{af\,(4m^2+8m+3)} - \frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-2}}{f(2m+1)}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)*Tan[e + f*x])/(f*(1
 + 2*m))) + ((a + a*Sec[e + f*x])^(1 + m)*(c - c*Sec[e + f*x])^(-2 - m)*Tan
[e + f*x])/(a*f*(3 + 8*m + 4*m^2))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rule 4036

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist
[(m + n + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(
c + d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
 && !LtQ[n, 0] && !(IGtQ[n + 1/2, 0] && LtQ[n + 1/2, -(m + n)])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{f(1+2m)} \\ &\quad -\frac{\int \sec(e+fx)(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-2-m}\,dx}{a(1+2m)} \\ &= -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{f(1+2m)} \\ &\quad +\frac{(a+a\sec(e+fx))^{1+m}(c-c\sec(e+fx))^{-2-m}\tan(e+fx)}{af\,(3+8m+4m^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.73

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx \\ &= \frac{(a(1+\sec(e+fx)))^m(-2(1+m)+\sec(e+fx))(c-c\sec(e+fx))^{-m}\tan(e+fx)}{c^2f(1+2m)(3+2m)(-1+\sec(e+fx))^2}\end{aligned}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-2 - m)
,x]
```

```
[Out] ((a*(1 + Sec[e + f*x]))^m*(-2*(1 + m) + Sec[e + f*x])*Tan[e + f*x])/(c^2*f*
(1 + 2*m)*(3 + 2*m)*(-1 + Sec[e + f*x])^2*(c - c*Sec[e + f*x])^m)
```

Maple [A] (verified)

Time = 5.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{\left(-\frac{2a}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}\right)^m \cot\left(\frac{fx}{2}+\frac{e}{2}\right)\left(-\frac{1}{2}+\cos(fx+e)(m+1)\right)\csc\left(\frac{fx}{2}+\frac{e}{2}\right)^2\left(\frac{c(\cos(fx+e)-1)}{\cos(fx+e)}\right)^{-m}}{f(3+2m)(1+2m)c^2}$	97

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x,method=_RETURNV
ERBOSE)
```

```
[Out] -(-2/(tan(1/2*f*x+1/2*e)^2-1)*a)^m*cot(1/2*f*x+1/2*e)*(-1/2+cos(f*x+e)*(m+1
))*csc(1/2*f*x+1/2*e)^2*(c*(cos(f*x+e)-1)/cos(f*x+e))^(-m)/f/(3+2*m)/(1+2*m
)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$= -\frac{(2(m+1)\cos(fx+e)-1)\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-2}\sin(fx+e)}{(4fm^2+8fm+3f)\cos(fx+e)^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="fricas")
```

```
[Out] -(2*(m + 1)*cos(f*x + e) - 1)*((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos
(f*x + e) - c)/cos(f*x + e))^(-m - 2)*sin(f*x + e)/((4*f*m^2 + 8*f*m + 3*f)
*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$= \int (a(\sec(e+fx)+1))^m(-c(\sec(e+fx)-1))^{-m-2}\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-2-m),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(-m - 2)*sec(e
+ f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$=-\frac{\left((-a)^m\,(2\,m+1)-\frac{(-a)^m(2\,m+3)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)c^{-m-2}(\cos\left(fx+e\right)+1)^3}{2\left(4\,m^2+8\,m+3\right)f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{2\,m}\sin\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="maxima")
```

```
[Out] -1/2*((-a)^m*(2*m + 1) - (-a)^m*(2*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*c^(-m - 2)*(cos(f*x + e) + 1)^3/((4*m^2 + 8*m + 3)*f*(sin(f*x + e)/(cos
(f*x + e) + 1))^(2*m)*sin(f*x + e)^3)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$=\int (a\sec\left(fx+e\right)+a)^m(-c\sec\left(fx+e\right)+c)^{-m-2}\sec\left(fx+e\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-2-m),x, algorith
m="giac")
```

```
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 2)*sec(f*x + e
), x)
```

Mupad [B] (verification not implemented)

Time = 19.41 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.39

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-2-m}\,dx$$
$$=\frac{\sin\left(e+f\,x\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,1\mathrm{i}}{f\,\cos\left(e+f\,x\right)^2\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{m+2}\,\left(m^2\,4\mathrm{i}+m\,8\mathrm{i}+3\mathrm{i}\right)}-\frac{\sin\left(2\,e+2\,f\,x\right)\,\left(2\,m+2\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,1\mathrm{i}}{2\,f\,\cos\left(e+f\,x\right)^2\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^{m+2}\,\left(m^2\,4\mathrm{i}+m\,8\mathrm{i}+3\mathrm{i}\right)}$$

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 2)),x)
```

```
[Out] (sin(e + f*x)*(a + a/cos(e + f*x))^m*1i)/(f*cos(e + f*x)^2*(c - c/cos(e + f
*x))^(m + 2)*(m*8i + m^2*4i + 3i)) - (sin(2*e + 2*f*x)*(2*m + 2)*(a + a/cos
(e + f*x))^m*1i)/(2*f*cos(e + f*x)^2*(c - c/cos(e + f*x))^(m + 2)*(m*8i + m
^2*4i + 3i))
```

3.164 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 47

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx = -\frac{(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\tan(e+fx)}{f(1+2m)}$$

```
[Out] -(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m)*tan(f*x+e)/f/(1+2*m)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$, Rules used = {4035}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx = -\frac{\tan(e+fx)(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}}{f(2m+1)}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m),x]
```

```
[Out] -(((a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)*Tan[e + f*x])/(f*(1
 + 2*m)))
```

Rule 4035

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Simp[b*Cot[e + f*x]
*(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(a*f*(2*m + 1))), x] /; Fre
```

```
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& EqQ[m + n + 1, 0] && NeQ[2*m + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a + a\sec(e + fx))^m (c - c\sec(e + fx))^{-1-m} \tan(e + fx)}{f(1 + 2m)}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \sec(e + fx)(a + a\sec(e + fx))^m (c - c\sec(e + fx))^{-1-m} dx = -\frac{(a(1 + \sec(e + fx)))^m (c - c\sec(e + fx))^{-1-m} \tan(e + fx)}{2f\left(\frac{1}{2} + m\right)}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(-1 - m)
,x]
```

```
[Out] -1/2*((a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e + f*x])^(-1 - m)*Tan[e + f*x])/
(f*(1/2 + m))
```

Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

method	result	size
parallelrisch	$\frac{\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}\right)^{-m} \left(-\frac{a}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}\right)^m \cot\left(\frac{fx}{2}+\frac{e}{2}\right)}{f(1+2m)c}$	76

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/f/(1+2*m)/c*(1/(tan(1/2*f*x+1/2*e)^2-1)*tan(1/2*f*x+1/2*e)^2*c)^(-m)*(-1/
(tan(1/2*f*x+1/2*e)^2-1)*a)^m*cot(1/2*f*x+1/2*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$=-\frac{\left(\frac{a\cos(fx+e)+a}{\cos(fx+e)}\right)^m\left(\frac{c\cos(fx+e)-c}{\cos(fx+e)}\right)^{-m-1}\sin(fx+e)}{(2\,fm+f)\cos(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="fricas")
```

```
[Out] -((a*cos(f*x + e) + a)/cos(f*x + e))^m*((c*cos(f*x + e) - c)/cos(f*x + e))^
(-m - 1)*sin(f*x + e)/((2*f*m + f)*cos(f*x + e))
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$=\int (a(\sec(e+fx)+1))^m\,(-c(\sec(e+fx)-1))^{-m-1}\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(-1-m),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(-m - 1)*sec(e
+ f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$=\frac{(-a)^m\,c^{-m-1}(\cos(fx+e)+1)}{f(2\,m+1)\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)^{2\,m}\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorith
m="maxima")
```

```
[Out] (-a)^m*c^(-m - 1)*(cos(f*x + e) + 1)/(f*(2*m + 1)*(sin(f*x + e)/(cos(f*x +
e) + 1))^(2*m)*sin(f*x + e))
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m-1}\sec(fx+e)\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m),x, algorithm="giac")

[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m - 1)*sec(f*x + e), x)

Mupad [B] (verification not implemented)

Time = 14.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-1-m}\,dx =$$
$$-\frac{(\sin(e+f\,x)+\sin(3\,e+3\,f\,x))\,\left(\frac{a\,(\cos(e+f\,x)+1)}{\cos(e+f\,x)}\right)^m}{c\,f\,(2\,m+1)\,\left(\frac{c\,(\cos(e+f\,x)-1)}{\cos(e+f\,x)}\right)^m\,(3\,\cos(e+f\,x)-2\,\cos(2\,e+2\,f\,x)+\cos(3\,e+3\,f\,x)-2)}$$

[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^(m + 1)),x)

[Out] -((sin(e + f*x) + sin(3*e + 3*f*x))*((a*(cos(e + f*x) + 1))/cos(e + f*x))^m)/(c*f*(2*m + 1)*((c*(cos(e + f*x) - 1))/cos(e + f*x))^m*(3*cos(e + f*x) - 2*cos(2*e + 2*f*x) + cos(3*e + 3*f*x) - 2))

3.165 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 101

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = -\frac{2^{\frac{1}{2}-m}c\,\text{Hypergeometric2F1}\left(\frac{1}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{\frac{1}{2}+m}(a+a\sec$$
$$f(1+2m)$$

```
[Out] -2^(1/2-m)*c*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f*
x+e))^(1/2+m)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(-1-m)*tan(f*x+e)/f/(1+2*
m)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {4046, 72, 71}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = -\frac{c2^{\frac{1}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m+\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m-1}\,\text{Hypergeometric2}$$
$$f(2m+1)$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 - m)*c*Hypergeometric2F1[1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
f*x])/2]*(1 - Sec[e + f*x])^(1/2 + m)*(a + a*Sec[e + f*x])^m*(c - c*Sec[e +
 f*x])^(-1 - m)*Tan[e + f*x])/(f*(1 + 2*m)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{-\frac{1}{2}-m}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{-\frac{1}{2}-m}ac(c-c\sec(e+fx))^{-1-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{\frac{1}{2}+m}\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{1}{2}-\frac{x}{2}\right)^{-\frac{1}{2}-m}(a+\right.}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2^{\frac{1}{2}-m}c\,\text{Hypergeometric2F1}\left(\frac{1}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{\frac{1}{2}+m}(a-}{f(1+2m)}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = -\frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1}{2}-m,\frac{3}{2}-m,\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\text{se}}{f(-1+2m)}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^m)/(c - c*Sec[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*Hypergeometric2F1[1/2 - m, 1/2 - m, 3/2 - m, (1 - Sec[e + f*
x])/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*Tan[e + f*x])
/(f*(-1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Maple [F]

$$\int \sec(fx+e)(a+a\sec(fx+e))^m(-c(\sec(fx+e)-1))^{-m}\,dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^m}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m/((c-c*sec(f*x+e))**m),x)
[Out] Timed out
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^m}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = \int \frac{(a\sec(fx+e)+a)^m\sec(fx+e)}{(-c\sec(fx+e)+c)^m}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m/((c-c*sec(f*x+e))^m),x, algorithm="
giac")
[Out] integrate((a*sec(f*x + e) + a)^m*sec(f*x + e)/(-c*sec(f*x + e) + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{-m}\,dx = \int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m}{\cos\left(e+f\,x\right)\,\left(c-\frac{c}{\cos(e+f\,x)}\right)^m}\,dx$$

```
[In] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^m),x)
[Out] int((a + a/cos(e + f*x))^m/(cos(e + f*x)*(c - c/cos(e + f*x))^m), x)
```

3.166 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx =$$
$$-\frac{2^{\frac{3}{2}-m}c\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}+m}(a+a\,\mathrm{se}}{f(1+2m)}$$

```
[Out] -2^(3/2-m)*c*hypergeom([-1/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec(f
*x+e))^(-1/2+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/((c-c*sec(f*x+e))^m
)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4046, 72, 71}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx =$$
$$-\frac{c2^{\frac{3}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,\text{Hypergeometric2F1}}{f(2m+1)}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),x]
```

```
[Out] -((2^(3/2 - m)*c*Hypergeometric2F1[-1/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e +
 f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x]
)/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{\frac{1}{2}-m}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{\frac{1}{2}-m}ac(c-c\sec(e+fx))^{-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{-\frac{1}{2}+m}\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{1}{2}-\frac{x}{2}\right)^{\frac{1}{2}-m}(a+ax\ldots\right.}{f\sqrt{a+a\sec(e+fx)}\ldots}$$

$$= -\frac{2^{\frac{3}{2}-m}c\,\text{Hypergeometric2F1}\left(-\frac{1}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}+\ldots}}{f(1+2m)\ldots}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$=\frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{3}{2}-m,\frac{5}{2}-m,\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e+fx)))^m(c-c\sec(e+fx))^{1-m}\tan(e+fx)}{f(3-2m)}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(1 - m),
x]
```

```
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2 - m, 3/2 - m, 5/2 - m, (1 - Sec[e + f*x]
)/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e +
f*x])^(1 - m)*Tan[e + f*x])/(f*(3 - 2*m))
```

Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^{1-m}\,dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

```
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$=\int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+1}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="fricas")
```

```
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e)
, x)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$= \int (a(\sec(e+fx)+1))^m\,(-c(\sec(e+fx)-1))^{1-m}\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(1-m),x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(1 - m)*sec(e +
 f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$= \int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+1}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$= \int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+1}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(1-m),x, algorithm
="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 1)*sec(f*x + e
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{1-m}\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^{1-m}}{\cos\left(e+f\,x\right)}\,dx$$

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1 - m))/cos(e + f*x),x)
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(1 - m))/cos(e + f*x), x)
```

3.167 $\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 101

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx = -\frac{2^{\frac{5}{2}-m}c^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}+m}(a+a}{f(1+2m)}$$

```
[Out] -2^(5/2-m)*c^2*hypergeom([-3/2+m, 1/2+m],[3/2+m],1/2+1/2*sec(f*x+e))*(1-sec
(f*x+e))^(-1/2+m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/(1+2*m)/((c-c*sec(f*x+e))
^m)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4046, 72, 71}

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx = -\frac{c^2 2^{\frac{5}{2}-m}\tan(e+fx)(1-\sec(e+fx))^{m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c-c\sec(e+fx))^{-m}\,\text{Hypergeometric2I}}{f(2m+1)}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),x]
```

```
[Out] -((2^(5/2 - m)*c^2*Hypergeometric2F1[-3/2 + m, 1/2 + m, 3/2 + m, (1 + Sec[e
 + f*x])/2]*(1 - Sec[e + f*x])^(-1/2 + m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*(1 + 2*m)*(c - c*Sec[e + f*x])^m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 4046

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a*c*(Cot[e + f
*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])), Subst[Int[(a +
b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int(a+ax)^{-\frac{1}{2}+m}(c-cx)^{\frac{3}{2}-m}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\left(2^{\frac{3}{2}-m}ac^2(c-c\sec(e+fx))^{-m}\left(\frac{c-c\sec(e+fx)}{c}\right)^{-\frac{1}{2}+m}\tan(e+fx)\right)\text{Subst}\left(\int\left(\frac{1}{2}-\frac{x}{2}\right)^{\frac{3}{2}-m}(a+ax}{f\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{2^{\frac{5}{2}-m}c^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2}+m,\frac{1}{2}+m,\frac{3}{2}+m,\frac{1}{2}(1+\sec(e+fx))\right)(1-\sec(e+fx))^{-\frac{1}{2}+m}}{f(1+2m)}$$

Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$= \frac{2^{\frac{1}{2}+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{5}{2}-m,\frac{7}{2}-m,\frac{1}{2}(1-\sec(e+fx))\right)(1+\sec(e+fx))^{-\frac{1}{2}-m}(a(1+\sec(e}{f(5-2m)}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^m*(c - c*Sec[e + f*x])^(2 - m),
x]
[Out] (2^(1/2 + m)*Hypergeometric2F1[1/2 - m, 5/2 - m, 7/2 - m, (1 - Sec[e + f*x]
)/2]*(1 + Sec[e + f*x])^(-1/2 - m)*(a*(1 + Sec[e + f*x]))^m*(c - c*Sec[e +
f*x])^(2 - m)*Tan[e + f*x])/(f*(5 - 2*m))
```

Maple [F]

$$\int \sec(fx+e)\,(a+a\sec(fx+e))^m\,(c-c\sec(fx+e))^{2-m}\,dx$$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
[Out] int(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x)
```

Fricas [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$= \int (a\sec(fx+e)+a)^m(-c\sec(fx+e)+c)^{-m+2}\sec(fx+e)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="fricas")
[Out] integral((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e)
, x)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$=\int (a(\sec{(e+fx)}+1))^m\,(-c(\sec{(e+fx)}-1))^{2-m}\sec{(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**m*(c-c*sec(f*x+e))**(2-m),x)
[Out] Integral((a*(sec(e + f*x) + 1))**m*(-c*(sec(e + f*x) - 1))**(2 - m)*sec(e +
 f*x), x)
```

Maxima [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$=\int (a\sec{(fx+e)}+a)^m(-c\sec{(fx+e)}+c)^{-m+2}\sec{(fx+e)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="maxima")
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

Giac [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$=\int (a\sec{(fx+e)}+a)^m(-c\sec{(fx+e)}+c)^{-m+2}\sec{(fx+e)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^m*(c-c*sec(f*x+e))^(2-m),x, algorithm
="giac")
[Out] integrate((a*sec(f*x + e) + a)^m*(-c*sec(f*x + e) + c)^(-m + 2)*sec(f*x + e
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(e+fx)(a+a\sec(e+fx))^m(c-c\sec(e+fx))^{2-m}\,dx$$
$$=\int \frac{\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\left(c-\frac{c}{\cos(e+f\,x)}\right)^{2-m}}{\cos\left(e+f\,x\right)}\,dx$$

```
[In] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(2 - m))/cos(e + f*x),x)
[Out] int(((a + a/cos(e + f*x))^m*(c - c/cos(e + f*x))^(2 - m))/cos(e + f*x), x)
```

3.168 $\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 105

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = \frac{a^3c\,\text{arctanh}(\sin(e+fx))}{4f}+\frac{a^3c\sec(e+fx)\tan(e+fx)}{4f} - \frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f}-\frac{2a^3c\tan^3(e+fx)}{3f}-\frac{a^3c\tan^5(e+fx)}{5f}$$

```
[Out] 1/4*a^3*c*arctanh(sin(f*x+e))/f+1/4*a^3*c*sec(f*x+e)*tan(f*x+e)/f-1/2*a^3*c
*sec(f*x+e)^3*tan(f*x+e)/f-2/3*a^3*c*tan(f*x+e)^3/f-1/5*a^3*c*tan(f*x+e)^5/
f
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {4047, 2687, 30, 2691, 3853, 3855, 14}

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = \frac{a^3c\,\text{arctanh}(\sin(e+fx))}{4f}-\frac{a^3c\tan^5(e+fx)}{5f}-\frac{2a^3c\tan^3(e+fx)}{3f} - \frac{a^3c\tan(e+fx)\sec^3(e+fx)}{2f}+\frac{a^3c\tan(e+fx)\sec(e+fx)}{4f}$$

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^3*c*ArcTanh[Sin[e + f*x]])/(4*f) + (a^3*c*Sec[e + f*x]*Tan[e + f*x])/(4*
f) - (a^3*c*Sec[e + f*x]^3*Tan[e + f*x])/(2*f) - (2*a^3*c*Tan[e + f*x]^3)/(
3*f) - (a^3*c*Tan[e + f*x]^5)/(5*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
```

```
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left((ac)\int \left(a^2\sec^2(e+fx)\tan^2(e+fx)+2a^2\sec^3(e+fx)\tan^2(e+fx)\right.\right.\\
&\qquad\left.\left.+a^2\sec^4(e+fx)\tan^2(e+fx)\right)\,dx\right)\\
&= -\left(\left(a^3c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)\\
&\quad-\left(a^3c\right)\int \sec^4(e+fx)\tan^2(e+fx)\,dx-\left(2a^3c\right)\int \sec^3(e+fx)\tan^2(e+fx)\,dx\\
&= -\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\left(a^3c\right)\int \sec^3(e+fx)\,dx\\
&\quad-\frac{\left(a^3c\right)\operatorname{Subst}\left(\int x^2\,dx,x,\tan(e+fx)\right)}{f}\\
&\quad-\frac{\left(a^3c\right)\operatorname{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a^3c\sec(e+fx)\tan(e+fx)}{4f}-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f}-\frac{a^3c\tan^3(e+fx)}{3f}\\
&\quad+\frac{1}{4}\left(a^3c\right)\int \sec(e+fx)\,dx-\frac{\left(a^3c\right)\operatorname{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a^3c\operatorname{arctanh}(\sin(e+fx))}{4f}+\frac{a^3c\sec(e+fx)\tan(e+fx)}{4f}\\
&\quad-\frac{a^3c\sec^3(e+fx)\tan(e+fx)}{2f}-\frac{2a^3c\tan^3(e+fx)}{3f}-\frac{a^3c\tan^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{a^3c(15\operatorname{arctanh}(\sin(e+fx))-\tan(e+fx)\left(-15\sec(e+fx)+30\sec^3(e+fx)+40\tan^2(e+fx)+12\tan^4(e+fx)\right))}{60f}$$

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^3*(c - c*Sec[e + f*x]),x]
[Out] (a^3*c*(15*ArcTanh[Sin[e + f*x]] - Tan[e + f*x]*(-15*Sec[e + f*x] + 30*Sec[
e + f*x]^3 + 40*Tan[e + f*x]^2 + 12*Tan[e + f*x]^4)))/(60*f)
```

Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^3c\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}\right.}{f}$
default	$\frac{a^3c\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)-2a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan}{8}\right.}{f}$
parts	$\frac{a^3c\tan(fx+e)}{f}+\frac{a^3c\sec(fx+e)\tan(fx+e)}{f}+\frac{a^3c\ln(\sec(fx+e)+\tan(fx+e))}{f}-\frac{2a^3c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e}{8}\right.\right.}{}$
norman	$\frac{\frac{a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}+\frac{25a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{64a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}+\frac{7a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{3f}-\frac{a^3c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{2f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}-\frac{a^3c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{4f}$
risch	$-\frac{ia^3c(15\,e^{9i(fx+e)}-60\,e^{8i(fx+e)}-90\,e^{7i(fx+e)}-240\,e^{6i(fx+e)}-40\,e^{4i(fx+e)}+90\,e^{3i(fx+e)}-80\,e^{2i(fx+e)}-15\,e^{i(fx+e)}}{30f\left(1+e^{2i(fx+e)}\right)^5}$
parallelrisch	$-\frac{3\left(\left(\frac{5\cos(fx+e)}{6}+\frac{5\cos(3fx+3e)}{12}+\frac{\cos(5fx+5e)}{12}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(-\frac{5\cos(fx+e)}{6}-\frac{5\cos(3fx+3e)}{12}-\frac{\cos(5fx+5e)}{12}\right)\ln(}{f(\cos(5fx+5e)+5\cos(3fx+3e)+10}$

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(a^3*c*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)-2*a^3*c*(-
(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e))
)+2*a^3*c*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^3*c*t
an(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$=\frac{15\,a^3c\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\,a^3c\cos\left(fx+e\right)^5\log\left(-\sin\left(fx+e\right)+1\right)+2\left(28\,a^3c\cos\left(f\right.\right.}{120\,f\cos\left(fx\right.}$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*a^3*c*cos(f*x + e)^5*log(sin(f*x + e) + 1) - 15*a^3*c*cos(f*x + e
)^5*log(-sin(f*x + e) + 1) + 2*(28*a^3*c*cos(f*x + e)^4 + 15*a^3*c*cos(f*x
+ e)^3 - 16*a^3*c*cos(f*x + e)^2 - 30*a^3*c*cos(f*x + e) - 12*a^3*c)*sin(f*
x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -a^3c\left(\int \left(-\sec^2(e+fx)\right)\,dx + \int \left(-2\sec^3(e+fx)\right)\,dx + \int 2\sec^5(e+fx)\,dx + \int \sec^6(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**3*(c-c*sec(f*x+e)),x)
[Out] -a**3*c*(Integral(-sec(e + f*x)**2, x) + Integral(-2*sec(e + f*x)**3, x) +
Integral(2*sec(e + f*x)**5, x) + Integral(sec(e + f*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.64

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = -\frac{8\left(3\tan(fx+e)^5+10\tan(fx+e)^3+15\tan(fx+e)\right)a^3c-15a^3c\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}-3\log(\cdots$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="ma
xima")
[Out] -1/120*(8*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*c -
15*a^3*c*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x
 + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 60*a^3*
c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*
x + e) - 1)) - 120*a^3*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.38

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx = \frac{15a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15a^3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(15a^3c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^9-70a^3c\tan\left(\frac{1}{2}fx\cdots\right.\right.}{\cdots}}{60f}$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^3*(c-c*sec(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/60*(15*a^3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*a^3*c*log(abs(tan(1/
2*f*x + 1/2*e) - 1)) - 2*(15*a^3*c*tan(1/2*f*x + 1/2*e)^9 - 70*a^3*c*tan(1/
2*f*x + 1/2*e)^7 + 128*a^3*c*tan(1/2*f*x + 1/2*e)^5 - 250*a^3*c*tan(1/2*f*x
 + 1/2*e)^3 - 15*a^3*c*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5
)/f
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^3(c-c\sec(e+fx))\,dx$$
$$= \frac{-\frac{c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}{2}+\frac{7\,c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{3}-\frac{64\,c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{15}+\frac{25\,c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{3}+\frac{c\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{2}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8+10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-10\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)} + \frac{a^3\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{2\,f}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

```
[Out] ((a^3*c*tan(e/2 + (f*x)/2))/2 + (25*a^3*c*tan(e/2 + (f*x)/2)^3)/3 - (64*a^3
*c*tan(e/2 + (f*x)/2)^5)/15 + (7*a^3*c*tan(e/2 + (f*x)/2)^7)/3 - (a^3*c*tan
(e/2 + (f*x)/2)^9)/2)/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 + (f*x)/2)^4
+ 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10
- 1)) + (a^3*c*atanh(tan(e/2 + (f*x)/2)))/(2*f)
```

3.169 $\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\sin(e+fx))}{8f} + \frac{a^2c\sec(e+fx)\tan(e+fx)}{8f} - \frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f} - \frac{a^2c\tan^3(e+fx)}{3f}$$

```
[Out] 1/8*a^2*c*arctanh(sin(f*x+e))/f+1/8*a^2*c*sec(f*x+e)*tan(f*x+e)/f-1/4*a^2*c
*sec(f*x+e)^3*tan(f*x+e)/f-1/3*a^2*c*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4047, 2687, 30, 2691, 3853, 3855}

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = \frac{a^2c\,\text{arctanh}(\sin(e+fx))}{8f} - \frac{a^2c\tan^3(e+fx)}{3f} - \frac{a^2c\tan(e+fx)\sec^3(e+fx)}{4f} + \frac{a^2c\tan(e+fx)\sec(e+fx)}{8f}$$

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*c*ArcTanh[Sin[e + f*x]])/(8*f) + (a^2*c*Sec[e + f*x]*Tan[e + f*x])/(8*
f) - (a^2*c*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) - (a^2*c*Tan[e + f*x]^3)/(3*
f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left((ac)\int \left(a\sec^2(e+fx)\tan^2(e+fx)+a\sec^3(e+fx)\tan^2(e+fx)\right)\,dx\right)\\
&= -\left(\left(a^2c\right)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right)-\left(a^2c\right)\int \sec^3(e+fx)\tan^2(e+fx)\,dx\\
&= -\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f}+\frac{1}{4}\left(a^2c\right)\int \sec^3(e+fx)\,dx\\
&\quad -\frac{\left(a^2c\right)\operatorname{Subst}\left(\int x^2\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a^2c\sec(e+fx)\tan(e+fx)}{8f}-\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f}\\
&\quad -\frac{a^2c\tan^3(e+fx)}{3f}+\frac{1}{8}\left(a^2c\right)\int \sec(e+fx)\,dx\\
&= \frac{a^2c\operatorname{arctanh}(\sin(e+fx))}{8f}+\frac{a^2c\sec(e+fx)\tan(e+fx)}{8f}\\
&\quad -\frac{a^2c\sec^3(e+fx)\tan(e+fx)}{4f}-\frac{a^2c\tan^3(e+fx)}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$
\begin{aligned}
&\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx\\
&=\frac{a^2c(3\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(3\sec(e+fx)-6\sec^3(e+fx)-8\tan^2(e+fx)\right))}{24f}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
[Out] (a^2*c*(3*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*Sec[e + f*x] - 6*Sec[e +
f*x]^3 - 8*Tan[e + f*x]^2)))/(24*f)
```

Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{-a^2c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+a^2c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+a^2}{f}$
default	$\frac{-a^2c\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+a^2c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+a^2}{f}$
parts	$\frac{a^2c\tan(fx+e)}{f}+\frac{a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{a^2c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}-\frac{a^2c(}{}$
norman	$\frac{-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{53a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12f}+\frac{11a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}-\frac{a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{a^2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{8f}+\frac{a^2c\ln(}{}$
risch	$-\frac{ia^2c\left(3e^{7i(fx+e)}-24e^{6i(fx+e)}-21e^{5i(fx+e)}-24e^{4i(fx+e)}+21e^{3i(fx+e)}-8e^{2i(fx+e)}-3e^{i(fx+e)}-8\right)}{12f\left(1+e^{2i(fx+e)}\right)^4}+\frac{a^2c\ln(e^{i(f}}{8f}$
parallelrisch	$\frac{a^2c\left(\left(-2\cos(2fx+2e)-\frac{\cos(4fx+4e)}{2}-\frac{3}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(2\cos(2fx+2e)+\frac{\cos(4fx+4e)}{2}+\frac{3}{2}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4f(3+\cos(4fx+4e)+4\cos(2fx+2e))}$

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-a^2*c*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+
e)+tan(f*x+e)))+a^2*c*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+a^2*c*(1/2*sec(f*x
+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^2*c*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\frac{3\,a^2c\cos(fx+e)^4\log(\sin(fx+e)+1)-3\,a^2c\cos(fx+e)^4\log(-\sin(fx+e)+1)+2\left(8\,a^2c\cos(fx+\right.}{48\,f\cos(fx+e)^4}$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*a^2*c*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 3*a^2*c*cos(f*x + e)^4
*log(-sin(f*x + e) + 1) + 2*(8*a^2*c*cos(f*x + e)^3 + 3*a^2*c*cos(f*x + e)^
2 - 8*a^2*c*cos(f*x + e) - 6*a^2*c)*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= -a^2c\left(\int \left(-\sec^2\left(e+fx\right)\right)\,dx + \int \left(-\sec^3\left(e+fx\right)\right)\,dx + \int \sec^4\left(e+fx\right)dx + \int \sec^5\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
[Out] -a**2*c*(Integral(-sec(e + f*x)**2, x) + Integral(-sec(e + f*x)**3, x) + In
tegral(sec(e + f*x)**4, x) + Integral(sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(78) = 156.

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.86

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx =$$
$$-\frac{16\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c-3\,a^2c\left(\frac{2\left(3\,\sin(fx+e)^3-5\,\sin(fx+e)\right)}{\sin(fx+e)^4-2\,\sin(fx+e)^2+1}-3\,\log\left(\sin\left(fx+e\right)+1\right)+3\,\mathrm{l}\right.}{}$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="ma
xima")
[Out] -1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c - 3*a^2*c*(2*(3*sin(f*x +
 e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin
(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) + 12*a^2*c*(2*sin(f*x + e)/(sin(f
*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*
tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$= \frac{3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\,a^2c\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(3\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-11\,a^2c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.\right.}{\left(\tan\left(\frac{1}{2}\,fx\right.\right.}}{24\,f}$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*a^2*c*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*a^2*c*log(abs(tan(1/2*
f*x + 1/2*e) - 1)) - 2*(3*a^2*c*tan(1/2*f*x + 1/2*e)^7 - 11*a^2*c*tan(1/2*f
*x + 1/2*e)^5 + 53*a^2*c*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*c*tan(1/2*f*x + 1/2
*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 16.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.70

$$\int \sec^2(e+fx)(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\frac{a^2\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{4\,f}$$
$$-\frac{\frac{c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7}{4}-\frac{11\,c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{12}+\frac{53\,c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3}{12}+\frac{c\,a^2\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{4}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right)}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

```
[Out] (a^2*c*atanh(tan(e/2 + (f*x)/2)))/(4*f) - ((a^2*c*tan(e/2 + (f*x)/2))/4 + (
53*a^2*c*tan(e/2 + (f*x)/2)^3)/12 - (11*a^2*c*tan(e/2 + (f*x)/2)^5)/12 + (a
^2*c*tan(e/2 + (f*x)/2)^7)/4)/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x
)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.170 $\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 30, antiderivative size = 17

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\tan^3(e+fx)}{3f}$$

```
[Out] -1/3*a*c*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4047, 2687, 30}

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\tan^3(e+fx)}{3f}$$

```
[In] Int[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(a*c*Tan[e + f*x]^3)/f
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\left((ac)\int \sec^2(e+fx)\tan^2(e+fx)\,dx\right) \\ &= -\frac{(ac)\text{Subst}\left(\int x^2\,dx, x, \tan(e+fx)\right)}{f} \\ &= -\frac{ac\tan^3(e+fx)}{3f} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\tan^3(e+fx)}{3f}$$

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
[Out] -1/3*(a*c*Tan[e + f*x]^3)/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(15) = 30.

Time = 1.97 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

method	result	size
norman	$\frac{8ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{3}}$	34
risch	$\frac{2iac\left(3\,\mathrm{e}^{4i(fx+e)}+1\right)}{3f\left(1+\mathrm{e}^{2i(fx+e)}\right)^{3}}$	35
derivativedivides	$\frac{ac\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)+ac\tan(fx+e)}{f}$	36
default	$\frac{ac\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)+ac\tan(fx+e)}{f}$	36
parts	$\frac{ac\tan(fx+e)}{f}+\frac{ac\left(-\frac{2}{3}-\frac{\sec(fx+e)^{2}}{3}\right)\tan(fx+e)}{f}$	38
parallelrisch	$\frac{8ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}$	45

```
[In] int(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 8/3*a*c/f*tan(1/2*f*x+1/2*e)^3/(tan(1/2*f*x+1/2*e)^2-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \frac{\left(ac\cos\left(fx+e\right)^2-ac\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fric
as")
[Out] 1/3*(a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.

Time = 0.85 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.00

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$= \begin{cases} \frac{-ac\left(\frac{\tan^3(e+fx)}{3}+\tan(e+fx)\right)+ac\tan(e+fx)}{f} & \text{for } f \neq 0 \\ x(a\sec(e)+a)\left(-c\sec(e)+c\right)\sec^2(e) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(f*x+e)**2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)
```

```
[Out] Piecewise(((-a*c*(tan(e + f*x)**3/3 + tan(e + f*x)) + a*c*tan(e + f*x))/f,
Ne(f, 0)), (x*(a*sec(e) + a)*(-c*sec(e) + c)*sec(e)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$= -\frac{\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)ac-3\,ac\tan\left(fx+e\right)}{3\,f}$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*a*c - 3*a*c*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\tan\left(fx+e\right)^3}{3\,f}$$

```
[In] integrate(sec(f*x+e)^2*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/3*a*c*tan(f*x + e)^3/f
```

Mupad [B] (verification not implemented)

Time = 13.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sec^2(e+fx)(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{a\,c\tan(e+f\,x)^3}{3\,f}$$

```
[In] int(((a + a/cos(e + f*x))*(c - c/cos(e + f*x)))/cos(e + f*x)^2,x)
```

```
[Out] -(a*c*tan(e + f*x)^3)/(3*f)
```

3.171 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 56

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{2c\operatorname{arctanh}(\sin(e+fx))}{af} - \frac{c\tan(e+fx)}{af} - \frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] 2*c*arctanh(sin(f*x+e))/a/f-c*tan(f*x+e)/a/f-2*c*tan(f*x+e)/f/(a+a*sec(f*x+
e))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4093, 3872, 3855, 3852, 8}

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{2c\operatorname{arctanh}(\sin(e+fx))}{af} - \frac{c\tan(e+fx)}{af} - \frac{2c\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*c*ArcTanh[Sin[e + f*x]])/(a*f) - (c*Tan[e + f*x])/(a*f) - (2*c*Tan[e + f
*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{\int \sec(e+fx)(-2ac+ac\sec(e+fx))\,dx}{a^2}\\
&= -\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{c\int \sec^2(e+fx)\,dx}{a}+\frac{(2c)\int \sec(e+fx)\,dx}{a}\\
&= \frac{2c\text{arctanh}(\sin(e+fx))}{af}-\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{c\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{af}\\
&= \frac{2c\text{arctanh}(\sin(e+fx))}{af}-\frac{c\tan(e+fx)}{af}-\frac{2c\tan(e+fx)}{f(a+a\sec(e+fx))}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(56) = 112.

Time = 0.60 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.75

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}-\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f}+\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(e\right.}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}\right.}{a}$$

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*((2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/f - (2*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]])/f + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])) + Sin[(e + f*x)/2]/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) +
 (2*Tan[(e + f*x)/2])/f))/a)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39

method	result	s
derivativedivides	$\frac{2c\left(-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2}-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2}\right)}{fa}$	7
default	$\frac{2c\left(-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-2}-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\frac{1}{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+2}\right)}{fa}$	7
parallelrisch	$-\frac{c\left(2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)\cos(fx+e)-2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\cos(fx+e)+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cos(fx+e)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{af\cos(fx+e)}$	8
risch	$-\frac{2ic\left(2\,\mathrm{e}^{2i(fx+e)}+\mathrm{e}^{i(fx+e)}+3\right)}{fa\left(\mathrm{e}^{i(fx+e)}+1\right)\left(1+\mathrm{e}^{2i(fx+e)}\right)}+\frac{2c\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{af}-\frac{2c\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{af}$	1
norman	$\frac{-\frac{4c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{6c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}+\frac{2c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}$	1

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c/a*(-tan(1/2*f*x+1/2*e)+1/2/(tan(1/2*f*x+1/2*e)-1)-ln(tan(1/2*f*x+1/2*
e)-1)+ln(tan(1/2*f*x+1/2*e)+1)+1/2/(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.88

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{(c\cos(fx+e)^2+c\cos(fx+e))\log(\sin(fx+e)+1)-(c\cos(fx+e)^2+c\cos(fx+e))\log(-\sin(fx}{af\cos(fx+e)^2+af\cos(fx+e)}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fric
as")
[Out] ((c*cos(f*x + e)^2 + c*cos(f*x + e))*log(sin(f*x + e) + 1) - (c*cos(f*x + e
)^2 + c*cos(f*x + e))*log(-sin(f*x + e) + 1) - (3*c*cos(f*x + e) + c)*sin(f
*x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(\int \left(-\frac{\sec^2(e+fx)}{\sec(e+fx)+1}\right)dx+\int \frac{\sec^3(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x) + 1), x) + Integral(sec(e + f*x
)**3/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(56) = 112$.

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.46

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+c\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right.}{a}\right.}{f}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] (c*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*(log(sin
(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) -
 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{2\left(\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}+\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a}\right)}{f}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 2*(c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - c*log(abs(tan(1/2*f*x + 1/2*e)
- 1))/a - c*tan(1/2*f*x + 1/2*e)/a + c*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x +
 1/2*e)^2 - 1)*a))/f
```

Mupad [B] (verification not implemented)

Time = 13.56 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{4\,c\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a\,f} - \frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)} - \frac{2\,c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f}$$

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))),x)
```

```
[Out] (4*c*atanh(tan(e/2 + (f*x)/2)))/(a*f) - (2*c*tan(e/2 + (f*x)/2))/(f*(a - a*
tan(e/2 + (f*x)/2)^2)) - (2*c*tan(e/2 + (f*x)/2))/(a*f)
```

3.172 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 70

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\operatorname{arctanh}(\sin(e+fx))}{a^2f}+\frac{7c\tan(e+fx)}{3a^2f(1+\sec(e+fx))}-\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] -c*arctanh(sin(f*x+e))/a^2/f+7/3*c*tan(f*x+e)/a^2/f/(1+sec(f*x+e))-2/3*c*ta
n(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4093, 4083, 3855, 3879}

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\operatorname{arctanh}(\sin(e+fx))}{a^2f}+\frac{7c\tan(e+fx)}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -((c*ArcTanh[Sin[e + f*x]])/(a^2*f)) + (7*c*Tan[e + f*x])/(3*a^2*f*(1 + Sec
[e + f*x])) - (2*c*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{\int\frac{\sec(e+fx)(-4ac+3ac\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^2}\\
&= -\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2}-\frac{c\int\sec(e+fx)\,dx}{a^2}+\frac{(7c)\int\frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{3a}\\
&= -\frac{c\text{arctanh}(\sin(e+fx))}{a^2f}-\frac{2c\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{7c\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int\frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\frac{c\left(-3\text{arctanh}(\sin(e+fx))+\frac{(5+7\sec(e+fx))\tan(e+fx)}{(1+\sec(e+fx))^2}\right)}{3a^2f}$$

```
[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
[Out] (c*(-3*ArcTanh[Sin[e + f*x]] + ((5 + 7*Sec[e + f*x])*Tan[e + f*x])/(1 + Sec
[e + f*x])^2))/(3*a^2*f)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{f\,a^{2}}$
default	$\frac{c\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{f\,a^{2}}$
parallelrisch	$\frac{c\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)\right)}{3a^{2}f}$
risch	$\frac{2ic\left(3\,\mathrm{e}^{2i(fx+e)}+12\,\mathrm{e}^{i(fx+e)}+5\right)}{3f\,a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}+\frac{c\ln\left(\mathrm{e}^{i(fx+e)}-i\right)}{a^{2}f}-\frac{c\ln\left(\mathrm{e}^{i(fx+e)}+i\right)}{a^{2}f}$
norman	$\frac{\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{11c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}+\frac{4c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{3af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{3af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}a}+\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{a^{2}f}-\frac{c\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}{a^{2}f}$

```
[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f/a^2*c*(1/3*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*x+1/2*e)+ln(tan(1/2*f*x+1/2
*e)-1)-ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{3\left(c\cos\left(fx+e\right)^{2}+2\,c\cos\left(fx+e\right)+c\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(c\cos\left(fx+e\right)^{2}+2\,c\cos\left(fx+e\right)+\right.}{6\left(a^{2}f\cos\left(fx+e\right)^{2}+2\,a^{2}f\cos\left(fx+e\right)+\right.}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(sin(f*x + e) + 1) - 3
*(c*cos(f*x + e)^2 + 2*c*cos(f*x + e) + c)*log(-sin(f*x + e) + 1) - 2*(5*c*
cos(f*x + e) + 7*c)*sin(f*x + e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x +
 e) + a^2*f)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$= -\frac{c\left(\int\left(-\frac{\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx+\int\frac{\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) +
Integral(sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(66) = 132$.

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{c\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6f}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
[Out] 1/6*(c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a^2) + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) +
 sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$= -\frac{\frac{3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2}-\frac{3c\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2}-\frac{a^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+6a^4c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a^6}}{3f}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*c*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 3*c*log(abs(tan(1/2*f*x
+ 1/2*e) - 1))/a^2 - (a^4*c*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c*tan(1/2*f*x +
1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{c\left(6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)-6\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\right)}{3\,a^2\,f}$$

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (c*(6*tan(e/2 + (f*x)/2) - 6*atanh(tan(e/2 + (f*x)/2)) + tan(e/2 + (f*x)/2)
^3))/(3*a^2*f)
```

3.173 $\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{4c\tan(e+fx)}{15f(a^3+a^3\sec(e+fx))}$$

```
[Out] -2/5*c*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+11/15*c*tan(f*x+e)/a/f/(a+a*sec(f*x+
e))^2-4/15*c*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {4093, 4085, 3879}

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{4c\tan(e+fx)}{15f(a^3\sec(e+fx)+a^3)} + \frac{11c\tan(e+fx)}{15af(a\sec(e+fx)+a)^2} - \frac{2c\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-2*c*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (11*c*Tan[e + f*x])/(15*
a*f*(a + a*Sec[e + f*x])^2) - (4*c*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f
*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3}-\frac{\int\frac{\sec(e+fx)(-6ac+5ac\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx}{5a^2}\\ &= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2}-\frac{(4c)\int\frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{15a^2}\\ &= -\frac{2c\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{11c\tan(e+fx)}{15af(a+a\sec(e+fx))^2}-\frac{4c\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.55

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{c(1+3\sec(e+fx)-4\sec^2(e+fx))\tan(e+fx)}{15a^3f(1+\sec(e+fx))^3}$$

[In] Integrate[(Sec[e + f*x]^2*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]

[Out] (c*(1 + 3*Sec[e + f*x] - 4*Sec[e + f*x]^2)*Tan[e + f*x])/(15*a^3*f*(1 + Sec[e + f*x])^3)

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

method	result	size
parallelrisch	$-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}\left(3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+5\right)}{30a^{3}f}$	36
derivativedivides	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}\right)}{2f\,a^{3}}$	37
default	$\frac{c\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}\right)}{2f\,a^{3}}$	37
risch	$\frac{2ic\left(15\,{\rm e}^{3i(fx+e)}-5\,{\rm e}^{2i(fx+e)}+5\,{\rm e}^{i(fx+e)}+1\right)}{15f\,a^{3}\left({\rm e}^{i(fx+e)}+1\right)^{5}}$	59
norman	$\frac{-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{6af}+\frac{7c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{30af}+\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}{30af}-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}{10af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)^{2}a^{2}}$	101

[In] int(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] -1/30*c*tan(1/2*f*x+1/2*e)^3*(3*tan(1/2*f*x+1/2*e)^2+5)/a^3/f

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\left(c\cos\left(fx+e\right)^2+3\,c\cos\left(fx+e\right)-4\,c\right)\sin\left(fx+e\right)}{15\,\left(a^3f\cos\left(fx+e\right)^3+3\,a^3f\cos\left(fx+e\right)^2+3\,a^3f\cos\left(fx+e\right)+a^3f\right)}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(c*cos(f*x + e)^2 + 3*c*cos(f*x + e) - 4*c)*sin(f*x + e)/(a^3*f*cos(f*
x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{c\left(\int\left(-\frac{\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\right)dx+\int\frac{\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx\right)}{a^3}$$

```
[In] integrate(sec(f*x+e)**2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] -c*(Integral(-sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x) + Integral(sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f
*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.34

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{\frac{c\left(\frac{15\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\,\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}-\frac{3\,c\left(\frac{5\,\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 3*c*(5*sin(f*x +
e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.43

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+5\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3}{30\,a^3f}$$

```
[In] integrate(sec(f*x+e)^2*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/30*(3*c*tan(1/2*f*x + 1/2*e)^5 + 5*c*tan(1/2*f*x + 1/2*e)^3)/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 13.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.41

$$\int \frac{\sec^2(e+fx)(c-c\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = -\frac{c\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+5\right)}{30\,a^3\,f}$$

```
[In] int((c - c/cos(e + f*x))/(cos(e + f*x)^2*(a + a/cos(e + f*x))^3),x)
```

```
[Out] -(c*tan(e/2 + (f*x)/2)^3*(3*tan(e/2 + (f*x)/2)^2 + 5))/(30*a^3*f)
```

3.174 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 34, antiderivative size = 140

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f} - \frac{a^2c\cos^2(e+fx)^{\frac{4+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{4+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^{1+p}\tan^3(e+fx)}{3fg}$$

```
[Out] -1/3*a^2*c*(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([3/2, 3/2+1/2*p],[5/2],sin(
f*x+e)^2)*(g*sec(f*x+e))^p*tan(f*x+e)^3/f-1/3*a^2*c*(cos(f*x+e)^2)^(2+1/2*p
)*hypergeom([3/2, 2+1/2*p],[5/2],sin(f*x+e)^2)*(g*sec(f*x+e))^(p+1)*tan(f*x
+e)^3/f/g
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {4047, 2697, 16}

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx = -\frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{p+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f} - \frac{a^2c\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+4}{2}}(g\sec(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{p+4}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3fg}$$

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(a^2*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2,
5/2, Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/f - (a^2*c*(Cos[e +
 f*x]^2)^((4 + p)/2)*Hypergeometric2F1[3/2, (4 + p)/2, 5/2, Sin[e + f*x]^2]
*(g*Sec[e + f*x])^(1 + p)*Tan[e + f*x]^3)/(3*f*g)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\text{integral} = -\left((ac)\int\left(a(g\sec(e+fx))^p\tan^2(e+fx)+a\sec(e+fx)(g\sec(e+fx))^p\tan^2(e+fx)\right)dx\right)$$

$$= -\left((a^2c)\int(g\sec(e+fx))^p\tan^2(e+fx)\,dx\right) - (a^2c)\int\sec(e+fx)(g\sec(e+fx))^p\tan^2(e+fx)\,dx$$

$$= -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f} - \frac{(a^2c)\int(g\sec(e+fx))^{1+p}\tan^2(e+fx)\,dx}{g}$$

$$= -\frac{a^2c\cos^2(e+fx)^{\frac{3+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f} - \frac{a^2c\cos^2(e+fx)^{\frac{4+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{4+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^{1+p}\tan^3(e+fx)}{3fg}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 325 vs. 2(140) = 280.

Time = 3.02 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.32

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$

$$= \frac{a^2\csc^2\left(\frac{1}{2}(e+fx)\right)\sec^4\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^p(1+\sec(e+fx))^2(c-c\sec(e+fx))\left(-2\cos^3(e+\right.}{}$$

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])^2*(c - c*Sec[e + f*x]),x]
```

```
[Out] (a^2*Csc[(e + f*x)/2]^2*Sec[(e + f*x)/2]^4*(g*Sec[e + f*x])^p*(1 + Sec[e +
f*x])^2*(c - c*Sec[e + f*x])*(-2*Cos[e + f*x]^3*(Cos[e + f*x]^2)^(p/2)*(2*H
ypergeometric2F1[1/2, (2 + p)/2, 3/2, Sin[e + f*x]^2] - Hypergeometric2F1[1
/2, (4 + p)/2, 3/2, Sin[e + f*x]^2])*Sin[e + f*x] - ((Sec[e + f*x]^2)^(-1 -
 p/2)*((4 + p)*Hypergeometric2F1[1/2, 1 - p/2, 3/2, -Tan[e + f*x]^2] - 3*(S
ec[e + f*x]^2)^(p/2))*Sin[e + f*x])/(1 + p) - (Hypergeometric2F1[1/2, (2 +
p)/2, (4 + p)/2, Sec[e + f*x]^2]*Sin[e + f*x])/((2 + p)*Sqrt[-Tan[e + f*x]^
2]) + (2*Cot[e + f*x]*Hypergeometric2F1[1/2, (3 + p)/2, (5 + p)/2, Sec[e +
f*x]^2]*Sqrt[-Tan[e + f*x]^2])/(3 + p)))/(32*f)
```

Maple [F]

$$\int (g\sec(fx+e))^p\,(a+a\sec(fx+e))^2\,(c-c\sec(fx+e))\,dx$$

```
[In] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x)
```

Fricas [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\int -(a\sec(fx+e)+a)^2(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(a^2*c*sec(f*x + e)^3 + a^2*c*sec(f*x + e)^2 - a^2*c*sec(f*x + e)
 - a^2*c)*(g*sec(f*x + e))^p, x)
```

Sympy [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=-a^2c\left(\int \left(-(g\sec(e+fx))^p\right)dx+\int \left(-(g\sec(e+fx))^p\sec(e+fx)\right)dx+\int (g\sec(e+fx))^p\sec^2(e+fx)\,dx+\int (g\sec(e+fx))^p\sec^3(e+fx)\,dx\right)$$

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))**2*(c-c*sec(f*x+e)),x)
```

```
[Out] -a**2*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral(-(g*sec(e + f*x))**p*
sec(e + f*x), x) + Integral((g*sec(e + f*x))**p*sec(e + f*x)**2, x) + Integ
ral((g*sec(e + f*x))**p*sec(e + f*x)**3, x))
```

Maxima [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\int -(a\sec(fx+e)+a)^2(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="maxima")
```

```
[Out] -integrate((a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
x)
```

Giac [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\int -(a\sec(fx+e)+a)^2(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))^2*(c-c*sec(f*x+e)),x, algorithm
="giac")
[Out] integrate(-(a*sec(f*x + e) + a)^2*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p,
x)
```

Mupad [F(-1)]

Timed out.

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))^2(c-c\sec(e+fx))\,dx$$
$$=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)^2\left(c-\frac{c}{\cos(e+f\,x)}\right)\left(\frac{g}{\cos(e+f\,x)}\right)^p\,dx$$

```
[In] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p,x)
[Out] int((a + a/cos(e + f*x))^2*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p, x)
```

3.175 $\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$

Optimal result

Integrand size = 32, antiderivative size = 65

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\cos^2(e+fx)^{\frac{3+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f}$$

```
[Out] -1/3*a*c*(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([3/2, 3/2+1/2*p],[5/2],sin(f*
x+e)^2)*(g*sec(f*x+e))^p*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4047, 2697}

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac\tan^3(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\sec(e+fx))^p\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{p+3}{2},\frac{5}{2},\sin^2(e+fx)\right)}{3f}$$

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
```

```
[Out] -1/3*(a*c*(Cos[e + f*x]^2)^((3 + p)/2)*Hypergeometric2F1[3/2, (3 + p)/2, 5/
2, Sin[e + f*x]^2]*(g*Sec[e + f*x])^p*Tan[e + f*x]^3)/f
```

Rule 2697

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.), x_Symbol] :> Dist
[((-a)*c)^m, Int[ExpandTrig[(g*csc[e + f*x])^p*cot[e + f*x]^(2*m), (c + d*c
sc[e + f*x])^(n - m), x], x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] &
& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n] && GeQ[n - m, 0
] && GtQ[m*n, 0]
```

Rubi steps

$$\text{integral} = -\left((ac)\int (g\sec(e+fx))^p\tan^2(e+fx)\,dx\right)$$
$$= -\frac{ac\cos^2(e+fx)^{\frac{3+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3+p}{2},\frac{5}{2},\sin^2(e+fx)\right)(g\sec(e+fx))^p\tan^3(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -\frac{ac(g\sec(e+fx))^p\tan(e+fx)\left(p+\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p}{2},\frac{2+p}{2},\sec^2(e+fx)\right)}{\sqrt{-\tan^2(e+fx)}}\right)}{fp(1+p)}$$

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sec[e + f*x])*(c - c*Sec[e + f*x]),x]
[Out] -((a*c*(g*Sec[e + f*x])^p*Tan[e + f*x]*(p + Hypergeometric2F1[1/2, p/2, (2
+ p)/2, Sec[e + f*x]^2]/Sqrt[-Tan[e + f*x]^2]))/(f*p*(1 + p)))
```

Maple [F]

$$\int (g\sec(fx+e))^p (a+a\sec(fx+e))(c-c\sec(fx+e))\,dx$$

[In] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)

[Out] int((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)

Fricas [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \int -(a\sec(fx+e)+a)(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] integral(-(a*c*sec(f*x + e)^2 - a*c)*(g*sec(f*x + e))^p, x)

Sympy [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = -ac\left(\int(-(g\sec(e+fx))^p)\,dx+\int(g\sec(e+fx))^p\sec^2(e+fx)\,dx\right)$$

[In] integrate((g*sec(f*x+e))**p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x)

[Out] -a*c*(Integral(-(g*sec(e + f*x))**p, x) + Integral((g*sec(e + f*x))**p*sec(e + f*x)**2, x))

Maxima [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx = \int -(a\sec(fx+e)+a)(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="maxima")

[Out] -integrate((a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x)

Giac [F]

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\int -(a\sec(fx+e)+a)(c\sec(fx+e)-c)(g\sec(fx+e))^p\,dx$$

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sec(f*x+e))*(c-c*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(-(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g\sec(e+fx))^p(a+a\sec(e+fx))(c-c\sec(e+fx))\,dx$$
$$=\int \left(a+\frac{a}{\cos(e+f\,x)}\right)\left(c-\frac{c}{\cos(e+f\,x)}\right)\left(\frac{g}{\cos(e+f\,x)}\right)^p\,dx$$

```
[In] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p,x)
```

```
[Out] int((a + a/cos(e + f*x))*(c - c/cos(e + f*x))*(g/cos(e + f*x))^p, x)
```

3.176 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 180

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{cg(1-2p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^{-1+p}\sin(e+fx)}{af(1-p)\sqrt{\sin^2(e+fx)}} + \frac{2c\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}} - \frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] -c*g*(1-2*p)*hypergeom([1/2, 1/2-1/2*p],[3/2-1/2*p],cos(f*x+e)^2)*(g*sec(f*
x+e))^(-1+p)*sin(f*x+e)/a/f/(1-p)/(sin(f*x+e)^2)^(1/2)+2*c*hypergeom([1/2,
-1/2*p],[1-1/2*p],cos(f*x+e)^2)*(g*sec(f*x+e))^p*sin(f*x+e)/a/f/(sin(f*x+e)
^2)^(1/2)-2*c*(g*sec(f*x+e))^p*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

$= \{4105, 3872, 3857, 2722\}$

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$
$$= -\frac{cg(1-2p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)}{af(1-p)\sqrt{\sin^2(e+fx)}}$$
$$+\frac{2c\sin(e+fx)(g\sec(e+fx))^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)}{af\sqrt{\sin^2(e+fx)}}$$
$$-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{f(a\sec(e+fx)+a)}$$

```
[In] Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] -((c*g*(1 - 2*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*x]^
2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(a*f*(1 - p)*Sqrt[Sin[e + f*x]^2
])) + (2*c*Hypergeometric2F1[1/2, -1/2*p, (2 - p)/2, Cos[e + f*x]^2]*(g*Sec
[e + f*x])^p*Sin[e + f*x])/(a*f*Sqrt[Sin[e + f*x]^2]) - (2*c*(g*Sec[e + f*x
])^p*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
```

```
0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\int (g\sec(e+fx))^p(ac(1-2p)+2acp\sec(e+fx))\,dx}{a^2}\\
&= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(c(1-2p))\int (g\sec(e+fx))^p\,dx}{a} + \frac{(2cp)\int (g\sec(e+fx))^{1+p}\,dx}{ag}\\
&= -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{\left(c(1-2p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\right)\int \left(\frac{\cos(e+fx)}{g}\right)^{-p}dx}{a}\\
&\quad + \frac{\left(2cp\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\right)\int \left(\frac{\cos(e+fx)}{g}\right)^{-1-p}dx}{ag}\\
&= -\frac{c(1-2p)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+fx)}{af(1-p)\sqrt{\sin^2(e+fx)}}\\
&\quad + \frac{2c\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+fx)}{af\sqrt{\sin^2(e+fx)}}\\
&\quad - \frac{2c(g\sec(e+fx))^p\tan(e+fx)}{f(a+a\sec(e+fx))}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 14.79 (sec) , antiderivative size = 3396, normalized size of antiderivative = 18.87

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (-6*c*Sec[e + f*x]^p*(g*Sec[e + f*x])^p*Tan[(e + f*x)/2]^3*(-((AppellF1[1/2
, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^
2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
```

*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(a*f*(3*Sec[(e + f*x)/2]^2*Sec[e + f*x]^p*(-((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 6*p*Sec[e + f*x]^(1 + p)*Sin[e + f*x]*Tan[(e + f*x)/2]*(-((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2)/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 6*Sec[e + f*x]^p*Tan[(e + f*x)/2]*((AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]*Sin[(e + f*x)/2])/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (Cos[(e + f*x)/2]^2*(-1/3*((1 - p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + ((p*AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (p*AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)/(3*AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (AppellF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(2*p*(AppellF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)

```
/2]^2] + AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((p*AppellF1[3/2, p, 1 - p, 5
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*
x)/2])/3 + (p*AppellF1[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*p*Tan[(e + f*x)/2]^2*
((-3*(1 - p)*AppellF1[5/2, p, 2 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (6*p*AppellF1[5/2, 1 + p,
1 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan
[(e + f*x)/2])/5 + (3*(1 + p)*AppellF1[5/2, 2 + p, -p, 7/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/(3*Appe
llF1[1/2, p, -p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*p*(Appel
lF1[3/2, p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1
[3/2, 1 + p, -p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)^2 + (AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Cos[(e + f*x)/2]^2*(2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]
 + 3*(-1/3*((1 - p)*AppellF1[3/2, p, 2 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (p*AppellF1[3/2, 1 +
p, 1 - p, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-1 + p)*((-3*(2 - p)*AppellF1
[5/2, p, 3 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)
/2]^2*Tan[(e + f*x)/2])/5 + (3*p*AppellF1[5/2, 1 + p, 2 - p, 7/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + p
*((-3*(1 - p)*AppellF1[5/2, 1 + p, 2 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + p)*AppellF1[5
/2, 2 + p, 1 - p, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*
x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, p, 1 - p, 3/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + p)*AppellF1[3/2, p, 2 - p, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[3/2, 1 + p, 1 - p, 5/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2)))
```

Maple [F]

$$\int \frac{(g\sec(fx+e))^p (c-c\sec(fx+e))}{a+a\sec(fx+e)}dx$$

```
[In] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\int -\frac{(c\sec(fx+e)-c)(g\sec(fx+e))^p}{a\sec(fx+e)+a}\,dx$$

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
fricas")
[Out] integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = -\frac{c\left(\int\left(-\frac{(g\sec(e+fx))^p}{\sec(e+fx)+1}\right)dx+\int\frac{(g\sec(e+fx))^p\sec(e+fx)}{\sec(e+fx)+1}\,dx\right)}{a}$$

```
[In] integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x)
[Out] -c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x) + 1), x) + Integral((g*sec(
e + f*x))**p*sec(e + f*x)/(sec(e + f*x) + 1), x))/a
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\int -\frac{(c\sec(fx+e)-c)(g\sec(fx+e))^p}{a\sec(fx+e)+a}\,dx$$

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
maxima")
[Out] -integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="
giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,2,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{2,[0,0,0,1]%%%} Err
or: Ba
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)\,\left(\frac{g}{\cos(e+f\,x)}\right)^p}{a+\frac{a}{\cos(e+f\,x)}}\,dx$$

```
[In] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x)),x)
[Out] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x)), x)
```

3.177 $\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 226

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{cg(3-4p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^{-1+p}\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} + \frac{c(5-4p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

```
[Out] -1/3*c*g*(3-4*p)*hypergeom([1/2, 1/2-1/2*p],[3/2-1/2*p],cos(f*x+e)^2)*(g*se
c(f*x+e))^(-1+p)*sin(f*x+e)/a^2/f/(sin(f*x+e)^2)^(1/2)+1/3*c*(5-4*p)*hyperg
eom([1/2, -1/2*p],[1-1/2*p],cos(f*x+e)^2)*(g*sec(f*x+e))^p*sin(f*x+e)/a^2/f
/(sin(f*x+e)^2)^(1/2)-1/3*c*(5-4*p)*(g*sec(f*x+e))^p*tan(f*x+e)/a^2/f/(1+se
c(f*x+e))-2/3*c*(g*sec(f*x+e))^p*tan(f*x+e)/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used

$= \{4105, 3872, 3857, 2722\}$

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$$
$$= -\frac{cg(3-4p)\sin(e+fx)(g\sec(e+fx))^{p-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}$$
$$+\frac{c(5-4p)\sin(e+fx)(g\sec(e+fx))^p\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)}{3a^2f\sqrt{\sin^2(e+fx)}}$$
$$-\frac{c(5-4p)\tan(e+fx)(g\sec(e+fx))^p}{3a^2f(\sec(e+fx)+1)}-\frac{2c\tan(e+fx)(g\sec(e+fx))^p}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] -1/3*(c*g*(3 - 4*p)*Hypergeometric2F1[1/2, (1 - p)/2, (3 - p)/2, Cos[e + f*
x]^2]*(g*Sec[e + f*x])^(-1 + p)*Sin[e + f*x])/(a^2*f*Sqrt[Sin[e + f*x]^2])
+ (c*(5 - 4*p)*Hypergeometric2F1[1/2, -1/2*p, (2 - p)/2, Cos[e + f*x]^2]*(g
*Sec[e + f*x])^p*Sin[e + f*x])/(3*a^2*f*Sqrt[Sin[e + f*x]^2]) - (c*(5 - 4*p
)*(g*Sec[e + f*x])^p*Tan[e + f*x])/(3*a^2*f*(1 + Sec[e + f*x])) - (2*c*(g*S
ec[e + f*x])^p*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
```

```
+ f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\int \frac{(g\sec(e+fx))^p(ac(3-2p)-2ac(1-p)\sec(e+fx))}{a+a\sec(e+fx)}\,dx}{3a^2}$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\int (g\sec(e+fx))^p\left(a^2c(3-4p)(1-p)+a^2c(5-4p)p\sec(e+fx)\right)\,dx}{3a^4}$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c(3-4p)(1-p))\int (g\sec(e+fx))^p\,dx}{3a^2} + \frac{(c(5-4p)p)\int (g\sec(e+fx))^{1+p}\,dx}{3a^2g}$$

$$= -\frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\left(c(3-4p)(1-p)\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\right)\int \left(\frac{\cos(e+fx)}{g}\right)^{-p}dx}{3a^2} + \frac{\left(c(5-4p)p\left(\frac{\cos(e+fx)}{g}\right)^p(g\sec(e+fx))^p\right)\int \left(\frac{\cos(e+fx)}{g}\right)^{-1-p}dx}{3a^2g}$$

$$= -\frac{c(3-4p)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+f}{3a^2f\sqrt{\sin^2(e+fx)}} + \frac{c(5-4p)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{p}{2},\frac{2-p}{2},\cos^2(e+fx)\right)(g\sec(e+fx))^p\sin(e+fx)}{3a^2f\sqrt{\sin^2(e+fx)}} - \frac{c(5-4p)(g\sec(e+fx))^p\tan(e+fx)}{3a^2f(1+\sec(e+fx))} - \frac{2c(g\sec(e+fx))^p\tan(e+fx)}{3f(a+a\sec(e+fx))^2}$$

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.83

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx =$$

$$-\frac{c(g\sec(e+fx))^p\left(2p(1+p)\tan(e+fx)+(1+\sec(e+fx))\left(-p(1+p)(-5+4p)\tan(e+fx)-((-\right.\right.}{}$$

```
[In] Integrate[((g*Sec[e + f*x])^p*(c - c*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,
x]
```

```
[Out] -1/3*(c*(g*Sec[e + f*x])^p*(2*p*(1 + p)*Tan[e + f*x] + (1 + Sec[e + f*x])*(
-(p*(1 + p)*(-5 + 4*p)*Tan[e + f*x]) - ((-1 + p)*(1 + p)*(-3 + 4*p)*Cot[e +
 f*x]*Hypergeometric2F1[1/2, p/2, (2 + p)/2, Sec[e + f*x]^2] + (5 - 4*p)*p^
2*Csc[e + f*x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sec[e + f*x]^2]
)*(1 + Sec[e + f*x])*Sqrt[-Tan[e + f*x]^2])))/(a^2*f*p*(1 + p)*(1 + Sec[e +
 f*x])^2)
```

Maple [F]

$$\int \frac{(g\sec(fx+e))^p (c-c\sec(fx+e))}{(a+a\sec(fx+e))^2}dx$$

```
[In] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

```
[Out] int((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \int -\frac{(c\sec(fx+e)-c)(g\sec(fx+e))^p}{(a\sec(fx+e)+a)^2}\,dx$$

```
[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral(-(c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a^2*sec(f*x + e)^2 + 2*a
^2*sec(f*x + e) + a^2), x)
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\left(\int \left(-\frac{(g\sec(e+fx))^p}{\sec^2(e+fx)+2\sec(e+fx)+1}\right)dx + \int \frac{(g\sec(e+fx))^p\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx\right)}{a^2}$$

```
[In] integrate((g*sec(f*x+e))**p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] -c*(Integral(-(g*sec(e + f*x))**p/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x
) + Integral((g*sec(e + f*x))**p*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e +
f*x) + 1), x))/a**2
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \int -\frac{(c\sec(fx+e)-c)(g\sec(fx+e))^p}{(a\sec(fx+e)+a)^2}\,dx$$

[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm ="maxima")

[Out] -integrate((c*sec(f*x + e) - c)*(g*sec(f*x + e))^p/(a*sec(f*x + e) + a)^2, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((g*sec(f*x+e))^p*(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm ="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun ding error%%%{-1,[0,1,4,0]%%%}+%%%{1,[0,1,0,0]%%%} / %%%{4,[0,0,0,2]%%%} Er ror: B

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^p(c-c\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \int \frac{\left(c-\frac{c}{\cos(e+f\,x)}\right)\,\left(\frac{g}{\cos(e+f\,x)}\right)^p}{\left(a+\frac{a}{\cos(e+f\,x)}\right)^2}\,dx$$

[In] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x))^2,x)

[Out] int(((c - c/cos(e + f*x))*(g/cos(e + f*x))^p)/(a + a/cos(e + f*x))^2, x)

3.178 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 104

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{2\sqrt{a}g^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{cf} + \frac{2g\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{cf}$$

```
[Out] -2*g^(3/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec
(f*x+e))^(1/2))*a^(1/2)/c/f+2*g*cot(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+a*sec(f*
x+e))^(1/2)/c/f
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4049, 49, 65, 223, 209}

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \frac{2ag^{3/2}\tan(e+fx)\arctan\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{2ag\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}$$

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e + f*x]),
x]
```

```
[Out] (-2*a*g*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c -
 c*Sec[e + f*x])) + (2*a*g^(3/2)*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqr
t[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e +
 f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(acg\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{gx}}{(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(ag^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{gx}\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(2ag\tan(e+fx))\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{cx^2}{g}}}\,dx, x, \sqrt{g\sec(e+fx)}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(2ag\tan(e+fx))\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{g}}\,dx, x, \frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{2ag\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{2ag^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 4.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.56

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \frac{2\cot\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^{3/2}\sqrt{a(1+\sec(e+fx))}\left(\sqrt{\text{se}\right.}{}$$

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c - c*Sec[e +
f*x]),x]
```

```
[Out] (2*Cot[(e + f*x)/2]*(g*Sec[e + f*x])^(3/2)*Sqrt[a*(1 + Sec[e + f*x])]*(Sqrt
[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (Log[1 + Sec[e + f*x]] - Log[Sqrt[S
ec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x
]^2]])*Sqrt[Tan[e + f*x]^2]))/(c*f*Sec[e + f*x]^(3/2)*(1 + Sec[e + f*x])^(3
/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(88) = 176$.

Time = 5.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.82

method	result
default	$\frac{g\left(\operatorname{arctanh}\left(\frac{\cos(fx+e)-\sin(fx+e)+1}{2(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)\sin(fx+e)-\operatorname{arctanh}\left(\frac{\cos(fx+e)+\sin(fx+e)+1}{2(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)\sin(fx+e)+2\sqrt{\frac{1}{\cos(fx+e)+1}}\cos(}{cf(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}$

[In] int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] g/c/f*(arctanh(1/2*(cos(f*x+e)-sin(f*x+e)+1)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2))*sin(f*x+e)-arctanh(1/2*(cos(f*x+e)+sin(f*x+e)+1)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2))*sin(f*x+e)+2*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+2*(1/(cos(f*x+e)+1))^(1/2))*(a*(sec(f*x+e)+1))^(1/2)*(g*sec(f*x+e))^(1/2)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.27

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \left[\frac{\sqrt{ag}\,g\log\left(\frac{ag\cos(fx+e)^3-7\,ag\cos(fx+e)^2+4\,\sqrt{ag}\left(\cos(fx+e)^2-2\,\cos(fx+\right.}{\cos(fx+e)^3+\cos(fx\cdot}\right.}{}\right.$$

$$-\frac{\sqrt{-ag}\,g\arctan\left(\frac{2\sqrt{-ag}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{ag\cos(fx+e)^2-ag\cos(fx+e)-2\,ag}\right)\sin(fx+e)-2\,g\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\,\mathrm{c}}{cf\sin(fx+e)}$$

[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, algorithm="fricas")

[Out] [1/2*(sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2 + 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 + cos(f*x + e)^2))*sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(c*f*sin(f*x + e)), -(sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x +

```
e)*sin(f*x + e)/(a*g*cos(f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g))*sin(f*x +
 e) - 2*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(
f*x + e))/(c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = -\frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}\sqrt{a\sec(e+fx)+a}}{\sec(e+fx)-1}\,dx}{c}$$

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e)),x)
```

```
[Out] -Integral((g*sec(e + f*x))**(3/2)*sqrt(a*sec(e + f*x) + a)/(sec(e + f*x) -
1), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 979 vs. $2(88) = 176$.

Time = 0.45 (sec) , antiderivative size = 979, normalized size of antiderivative = 9.41

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1
/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*sqrt(2)*g*cos(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 4*sqrt(2)*g*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e))) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*s
in(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) +
(g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(
2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (g*cos(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f*x
```

```
 + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e))) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)
*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) + (g*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + g)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin
(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e))) + 2))*sqrt(a)*sqrt(g)/((c*cos(1/2*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + c*sin(1/2*arctan2(sin(2*f*x + 2*e),
 cos(2*f*x + 2*e)))^2 - 2*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + c)*f)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \int -\frac{\sqrt{a\sec(fx+e)+a}(g\sec(fx+e))^{\frac{3}{2}}}{c\sec(fx+e)-c}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c-c\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,{\left(\frac{g}{\cos(e+f\,x)}\right)}^{3/2}}{c-\frac{c}{\cos(e+f\,x)}}\,dx$$

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c - c/cos(e + f*x)
),x)
```

```
[Out] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c - c/cos(e + f*x)
), x)
```

3.179 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 81

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\cot(e+fx)\sqrt{a+a\sec(e+fx)}}{acf}$$

```
[Out] -1/2*arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1
/2)/a^(1/2)+cot(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4049, 79, 65, 214}

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\tan(e+fx)\operatorname{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}$$

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -(Tan[e + f*x]/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x]))) - (ArcTan
h[Sqrt[c - c*Sec[e + f*x]]/(Sqrt[2]*Sqrt[c])]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c
]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{x}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} \\
&\quad + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{(a+ax)\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} - \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{2a-\frac{ax^2}{c}}\,dx, x, \sqrt{c-c\sec(e+fx)}\right)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} - \frac{\text{arctanh}\left(\frac{\sqrt{c-c\sec(e+fx)}}{\sqrt{2}\sqrt{c}}\right)\tan(e+fx)}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\cot\left(\frac{1}{2}(e+fx)\right)\left(-2+\sqrt{2}\arctan\left(\frac{\sqrt{-1+\sec(e+fx)}}{\sqrt{2}}\right)\sqrt{-1+\sec(e+fx)}\right)}{2cf\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
[Out] -1/2*(Cot[(e + f*x)/2]*(-2 + Sqrt[2]*ArcTan[Sqrt[-1 + Sec[e + f*x]]/Sqrt[2]
]*Sqrt[-1 + Sec[e + f*x]]))/(c*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\sqrt{a(\sec(fx+e)+1)}\left(\sqrt{2}\sqrt{-\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{\cot(fx+e)^2-2\csc(fx+e)\cot(fx+e)+\csc(fx+e)^2-1}\right)-2}{2cfa}$

```
[In] int(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
[Out] -1/2/c/f/a*(a*(sec(f*x+e)+1))^(1/2)*(2^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*ln(csc(f*x+e)-cot(f*x+e)+(cot(f*x+e)^2-2*csc(f*x+e)*cot(f*x+e)+csc(f*x
+e)^2-1)^(1/2))-2*cot(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.21

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \left[\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\cos(fx+e)\sin(fx+e)+3\cos(fx+e)^2+2\cos(fx+e)-1}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+4\sqrt{\frac{a\cos(fx-}{\cos(fx}}}{4\,acf\sin(fx+e)}\right.$$

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
 + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x
+ e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 4*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqr
t(2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)/(sqrt(a)*sin(f*x + e)))*sin(f*x + e) + 2*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{\sec^2(e+fx)}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

```
[In] integrate(sec(f*x+e)**2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral(sec(e + f*x)**2/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) - sqrt(a*s
ec(e + f*x) + a)), x)/c
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int -\frac{\sec(fx+e)^2}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] -integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c)),
x)
```

Giac [A] (verification not implemented)

none

Time = 0.82 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.63

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{\frac{\sqrt{2}\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2\right)}{\sqrt{-a}c\,\mathrm{sgn}(\cos(fx+e))}-\frac{4\sqrt{2}\sqrt{-a}}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{-a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-a\right)c\,\mathrm{sgn}(\cos(fx+e))}}{4f}$$

```
[In] integrate(sec(f*x+e)^2/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/4*(sqrt(2)*log((sqrt(-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2
*e)^2 + a))^2)/(sqrt(-a)*c*sgn(cos(f*x + e))) - 4*sqrt(2)*sqrt(-a)/(((sqrt(
-a)*tan(1/2*f*x + 1/2*e) - sqrt(-a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*c*sg
n(cos(f*x + e))))/f
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\int \frac{1}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c\,\cos\left(e+f\,x\right)-c\,\left(\frac{\cos(2\,e+2\,f\,x)}{2}+\frac{1}{2}\right)\right)}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))),x)
```

```
[Out] -int(1/((a + a/cos(e + f*x))^(1/2)*(c*cos(e + f*x) - c*(cos(2*e + 2*f*x)/2
+ 1/2))), x)
```

3.180 $\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 140

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{2\text{arcsinh}\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf} + \frac{\text{arctanh}\left(\frac{\sqrt{a}\sqrt{\sec(e+fx)}\sin(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{\csc(e+fx)\sqrt{a+a\sec(e+fx)}}{acf\sqrt{\sec(e+fx)}}$$

```
[Out] -2*arcsinh(a^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2))/c/f/a^(1/2)+1/2*arcta
nh(1/2*sin(f*x+e)*a^(1/2)*sec(f*x+e)^(1/2)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))/
c/f*2^(1/2)/a^(1/2)+csc(f*x+e)*(a+a*sec(f*x+e))^(1/2)/a/c/f/sec(f*x+e)^(1/2
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

$= \{4049, 100, 163, 65, 223, 209, 95, 211\}$

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{2\tan(e+fx)\arctan\left(\frac{\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\tan(e+fx)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{\sin(e+fx)\sec^{\frac{3}{2}}(e+fx)}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}$$

```
[In] Int[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),x]
```

```
[Out] -((Sec[e + f*x]^(3/2)*Sin[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[
e + f*x]))) + (2*ArcTan[(Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x
]]]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*
x]]) - (ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[Sec[e + f*x]])/Sqrt[c - c*Sec[e + f*x]
]]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(ac\tan(e+fx))\text{Subst}\left(\int \frac{x^{3/2}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{\frac{ac}{2}+acx}{\sqrt{x}(a+ax)\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{\sqrt{x}(a+ax)\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{1}{\sqrt{c-cx^2}}\,dx, x, \sqrt{\sec(e+fx)}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{1}{a+2acx^2}\,dx, x, \frac{\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{(2\tan(e+fx))\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{\sec^{\frac{3}{2}}(e+fx)\sin(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{2\arctan\left(\frac{\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 724 vs. 2(140) = 280.

Time = 11.25 (sec) , antiderivative size = 724, normalized size of antiderivative = 5.17

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$= \frac{\sec^{\frac{3}{2}}(e+fx)\sqrt{(1+\cos(e+fx))\sec(e+fx)}\sqrt{1+\sec(e+fx)}\left(-\frac{2\cot(e)}{f}+\frac{\csc\left(\frac{e}{2}\right)\csc\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{fx}{2}\right)}{f}+\frac{\sec\left(\frac{e}{2}\right)}{}\right.}{\sqrt{a(1+\sec(e+fx))}(c-c\sec(e+fx))}$$
$$+\frac{\cos(e+fx)\left(\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)-2\sqrt{2}\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}\sqrt{-1+\mathrm{se}}\right.\right.}{2f(1+\mathrm{co}}$$
$$+\frac{\cos(e+fx)\left(-8\log(1+\sec(e+fx))+8\log\left(\sqrt{\sec(e+fx)}+\sec^{\frac{3}{2}}(e+fx)+\sqrt{1+\sec(e+fx)}\sqrt{-}\right.\right.}{}$$

```
[In] Integrate[Sec[e + f*x]^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])
),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*Sqrt[(1 + Cos[e + f*x])*Sec[e + f*x]]*Sqrt[1 + Sec[e +
f*x]]*((-2*Cot[e])/f + (Csc[e/2]*Csc[e/2 + (f*x)/2]*Sin[(f*x)/2])/f + (Sec[
e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/f)*Sin[e/2 + (f*x)/2]^2)/(Sqrt[a*(1 +
 Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e + f*x]*(Log[1 - 2*Sec[e + f*
x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]
*Sqrt[-1 + Sec[e + f*x]^2]] - Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2
*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2
]])*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e/2 + (f*x)/2]^2
*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*Sqrt[2 - 2*Cos[e + f*x]^2]*Sqrt[1 -
Cos[e + f*x]^2]*Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x])) + (Cos[e +
 f*x]*(-8*Log[1 + Sec[e + f*x]] + 8*Log[Sqrt[Sec[e + f*x]] + Sec[e + f*x]^(
3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Sqrt[2]*(-Log[1
- 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 +
 Sec[e + f*x]]*Sqrt[-1 + Sec[e + f*x]^2]] + Log[1 - 2*Sec[e + f*x] - 3*Sec[
e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[-1 +
Sec[e + f*x]^2]]))*(1 + Sec[e + f*x])^(3/2)*Sqrt[-1 + Sec[e + f*x]^2]*Sin[e
/2 + (f*x)/2]^2*Sin[e + f*x])/(2*f*(1 + Cos[e + f*x])*(1 - Cos[e + f*x]^2)*
Sqrt[a*(1 + Sec[e + f*x])]*(c - c*Sec[e + f*x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(120) = 240.

Time = 5.05 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.82

method	result
default	$-\frac{\left(\sqrt{2}\,\arctan\left(\frac{\sin(fx+e)\sqrt{2}}{2(\cos(fx+e)+1)\sqrt{-\frac{1}{\cos(fx+e)+1}}}\right)\sin(fx+e)-2\sqrt{-\frac{1}{\cos(fx+e)+1}}\,\cos(fx+e)-2\arctan\left(\frac{-\cos(fx+e)+\sin(fx+e)-1}{2(\cos(fx+e)+1)\sqrt{-\frac{1}{\cos(fx+e)+1}}}\right.}{2cfa(\cos(f}$

```
[In] int(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/c/f/a*(2^(1/2)*arctan(1/2*sin(f*x+e)*2^(1/2)/(cos(f*x+e)+1)/(-1/(cos(f
*x+e)+1))^(1/2))*sin(f*x+e)-2*(-1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*arctan
(1/2*(-cos(f*x+e)+sin(f*x+e)-1)/(cos(f*x+e)+1)/(-1/(cos(f*x+e)+1))^(1/2))*s
in(f*x+e)-2*arctan(1/2*(cos(f*x+e)+sin(f*x+e)+1)/(cos(f*x+e)+1)/(-1/(cos(f*
x+e)+1))^(1/2))*sin(f*x+e)-2*(-1/(cos(f*x+e)+1))^(1/2))*(a*(sec(f*x+e)+1))^
(1/2)*sec(f*x+e)^(5/2)/(cos(f*x+e)+1)/(-1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^
2*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.30

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$

$$=\left[\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(fx+e)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\cos(fx+e)}\sin(fx+e)}{\sqrt{a}}-2\cos(fx+e)-3}{\cos(fx+e)^2+2\cos(fx+e)+1}\right)\sin(fx+e)+2\sqrt{a}\log\left(\frac{a\cos(fx+e)^3}{\cdots}\right.}{4\,acf\sin}\right.$$

$$-\frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(fx+e)}}{\sin(fx+e)}\right)\sin(fx+e)+2\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\cos(fx}}{a\cos(fx+e)^2-a\cos(fx-}\right.}{2\,acf\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*sqrt(a)*log(-(cos(f*x + e)^2 - 2*sqrt(2)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(cos(f*x + e))*sin(f*x + e)/sqrt(a) - 2*cos(f*x + e
) - 3)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*sin(f*x + e) + 2*sqrt(a)*log(
(a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - 2*cos(f*x + e)
)*sqrt(a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e)/sqrt(cos(f*x
 + e)) + 8*a)/(cos(f*x + e)^3 + cos(f*x + e)^2))*sin(f*x + e) + 4*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x + e)), -1
/2*(sqrt(2)*a*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt(-1/a)*sqrt(cos(f*x + e))/sin(f*x + e))*sin(f*x + e) + 2*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)
)*sin(f*x + e)/(a*cos(f*x + e)^2 - a*cos(f*x + e) - 2*a))*sin(f*x + e) - 2*
sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(cos(f*x + e)))/(a*c*f*sin(f*x
+ e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)**(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(120) = 240$.

Time = 0.45 (sec) , antiderivative size = 1310, normalized size of antiderivative = 9.36

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqr
t(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*log(2*cos(1/4*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*
```

```
x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2))*lo
g(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e))) + 2) + (sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*
sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*sin(1/2*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e))) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt
(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/
4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (cos(1/2*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e)))^2 - 2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) +
 1)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e))) + 1) + (cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2
*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1)*log(cos(1/4*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) + 1) - 4*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 4*cos(1/2*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e))) - 4*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))))/((sqrt(2)*
c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

Giac [F]

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$$
$$=\int -\frac{\sec\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)-c)}\,dx$$

```
[In] integrate(sec(f*x+e)^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-sec(f*x + e)^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e) - c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(e+fx)}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int \frac{\left(\frac{1}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((1/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
```

```
[Out] int((1/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

3.181 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 116

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{g^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf} + \frac{g\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{acf}$$

```
[Out] -1/2*g^(3/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2^(1/2)/(g*sec(f*x+e))^
(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2)+g*cot(f*x+e)*(g*sec(f*x+e
))^(1/2)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.29, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4049, 96, 95, 211}

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{g^{3/2}\tan(e+fx)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}} - \frac{g\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}$$

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),
x]
```

```
[Out] -((g*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c - c*
Sec[e + f*x]))) + (g^(3/2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(S
qrt[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[2]*Sqrt[c]*f*Sqrt[a +
 a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(acg\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{gx}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(ag^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{gx}(a+ax)\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$=-\frac{g\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}+\frac{(ag^2\tan(e+fx))\operatorname{Subst}\left(\int\frac{1}{ag+2acx^2}\,dx,x,\frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$=-\frac{g\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}+\frac{g^{3/2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 236 vs. $2(116)=232$.

Time = 3.92 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.03

$$\int\frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx=$$

$$-\frac{a\cos\left(\frac{1}{2}(e+fx)\right)(g\sec(e+fx))^{5/2}\sin^3\left(\frac{1}{2}(e+fx)\right)\left(-4-4\sec(e+fx)+\frac{\left(\log\left(1-2\sec(e+fx)-3\sec^2(e+fx)-2\sqrt{2}\right.\right.}{}\right.}{cfg(-1+\sec(e+fx)}$$

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f
*x])),x]
```

```
[Out] -((a*Cos[(e + f*x)/2]*(g*Sec[e + f*x])^(5/2)*Sin[(e + f*x)/2]^3*(-4 - 4*Sec
[e + f*x] + ((Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]*Sqrt[Se
c[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 - 2*Sec[e
+ f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f
*x]]*Sqrt[Tan[e + f*x]^2]])*Sqrt[Tan[e + f*x]^2])/Sqrt[Sec[(e + f*x)/2]^2])
)/(c*f*g*(-1 + Sec[e + f*x])^2*(a*(1 + Sec[e + f*x]))^(3/2)))
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

method	result
default	$\frac{g\left(\operatorname{arcsinh}(\cot(fx+e)-\csc(fx+e))\sqrt{2}\,\sin(fx+e)+2\sqrt{\frac{1}{\cos(fx+e)+1}}\,\cos(fx+e)+2\sqrt{\frac{1}{\cos(fx+e)+1}}\right)\sqrt{g\sec(fx+e)}\,\sqrt{a(\sec(fx+e)+1)}\,\text{co}}{2cfa(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}$

```
[In] int((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/2*g/c/f/a*(arcsinh(cot(f*x+e)-csc(f*x+e))*2^(1/2)*sin(f*x+e)+2*(1/(cos(f*
x+e)+1))^(1/2)*cos(f*x+e)+2*(1/(cos(f*x+e)+1))^(1/2))*(g*sec(f*x+e))^(1/2)*
(a*(sec(f*x+e)+1))^(1/2)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.84

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \left[\frac{\sqrt{2}ag\sqrt{\frac{g}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(fx-}{\cos(fx+e)^2+2\cos(fx+}\right.}{}\right.$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*g*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e)
+ a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos(f
*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))*
sin(f*x + e) + 4*g*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x +
 e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), 1/2*(sqrt(2)*a*g*sqrt(-g/a)*arctan
(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x
+ e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*g*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)
)]
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a\sec(e+fx)+a}\sec(e+fx)-\sqrt{a\sec(e+fx)+a}}\,dx}{c}$$

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] -Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*sec(e + f*x) + a)*sec(e + f*x) -
sqrt(a*sec(e + f*x) + a)), x)/c
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(97) = 194$.

Time = 0.42 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.62

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{\left(4\,g\cos\left(\frac{1}{4}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)\right)\right)\sin\left(\frac{1}{2}\,\mathrm{a}\right.\right.}{}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*g*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g*cos(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) - (g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), co
s(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2
 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + (g*cos(1/2
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + g*sin(1/2*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*g*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e))) + g)*log(cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
))^2 + sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*a
rctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 1) + 4*g*sin(1/4*arctan2(sin(
2*f*x + 2*e), cos(2*f*x + 2*e))))*sqrt(g)/((sqrt(2)*c*cos(1/2*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + sqrt(2)*c)*sqrt(a)*f)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int -\frac{(g\sec\left(fx+e\right))^{\frac{3}{2}}}{\sqrt{a\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)-c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(-(g*sec(f*x + e))^(3/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e)
 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
[Out] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

3.182 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 179

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{2g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}cf}+\frac{g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{a}cf}+\frac{g^2\cot(e+fx)\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{acf}$$

```
[Out] -2*g^(5/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec
(f*x+e))^(1/2))/c/f/a^(1/2)+1/2*g^(5/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x
+e)*2^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/c/f*2^(1/2)/a^(1/2
)+g^2*cot(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+a*sec(f*x+e))^(1/2)/a/c/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4049, 100, 163, 65, 223, 209, 95, 211}

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \frac{2g^{5/2}\tan(e+fx)\arctan\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g^{5/2}\tan(e+fx)\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)}{\sqrt{2}\sqrt{c}f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}-\frac{g^2\tan(e+fx)\sqrt{g\sec(e+fx)}}{f\sqrt{a\sec(e+fx)+a}(c-c\sec(e+fx))}$$

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f*x])),
x]
```

```
[Out] -((g^2*Sqrt[g*Sec[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*(c -
c*Sec[e + f*x]))) + (2*g^(5/2)*ArcTan[(Sqrt[c]*Sqrt[g*Sec[e + f*x]])/(Sqrt[
g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c]*f*Sqrt[a + a*Sec[e + f
*x]]*Sqrt[c - c*Sec[e + f*x]]) - (g^(5/2)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[g*Se
c[e + f*x]])/(Sqrt[g]*Sqrt[c - c*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[c]*f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4049

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
*c*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2), x], x, Csc
[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(acg\tan(e+fx))\text{Subst}\left(\int\frac{(gx)^{3/2}}{(a+ax)(c-cx)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} \\
&\quad+\frac{(g\tan(e+fx))\text{Subst}\left(\int\frac{\frac{1}{2}acg^2+acg^2x}{\sqrt{gx}(a+ax)\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{cf\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} \\
&\quad+\frac{(g^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{gx}\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} \\
&\quad-\frac{(ag^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{gx}(a+ax)\sqrt{c-cx}}\,dx, x, \sec(e+fx)\right)}{2f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{(2g^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{c-\frac{cx^2}{g}}}\,dx,x,\sqrt{g\sec(e+fx)}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{(ag^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{ag+2acx^2}\,dx,x,\frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} - \frac{g^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} + \frac{(2g^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1+\frac{cx^2}{g}}\,dx,x,\frac{\sqrt{g\sec(e+fx)}}{\sqrt{c-c\sec(e+fx)}}\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

$$= -\frac{g^2\sqrt{g\sec(e+fx)}\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))} + \frac{2g^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}} - \frac{g^{5/2}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{g\sec(e+fx)}}{\sqrt{g}\sqrt{c-c\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{2}\sqrt{c}f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 3.07 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.83

$$\int\frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = -\frac{(g\sec(e+fx))^{5/2}\sqrt{1+\sec(e+fx)}\sin^3(e+fx)\left(8\sqrt{\sec(e+fx)}\sqrt{1+\sec(e+fx)}+\left(16\log(1+\sec(e-\right.\right.}{}$$

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c - c*Sec[e + f
*x])),x]
```

```
[Out] -1/8*((g*Sec[e + f*x])^(5/2)*Sqrt[1 + Sec[e + f*x]]*Sin[e + f*x]^3*(8*Sqrt[
Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]] + (16*Log[1 + Sec[e + f*x]] - 16*Log[S
qrt[Sec[e + f*x]] + Sec[e + f*x]^(3/2) + Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e
+ f*x]^2]] + Sqrt[2]*(Log[1 - 2*Sec[e + f*x] - 3*Sec[e + f*x]^2 - 2*Sqrt[2]
```

```
*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]]*Sqrt[Tan[e + f*x]^2]] - Log[1 -
2*Sec[e + f*x] - 3*Sec[e + f*x]^2 + 2*Sqrt[2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + S
ec[e + f*x]]*Sqrt[Tan[e + f*x]^2]]))*Sqrt[Tan[e + f*x]^2]))/(c*f*(-1 + Cos[
e + f*x])*(1 + Cos[e + f*x])^2*(-1 + Sec[e + f*x])*Sec[e + f*x]^(5/2)*Sqrt[
a*(1 + Sec[e + f*x])])
```

Maple [A] (verified)

Time = 5.20 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.24

method	result
default	$-\frac{g^{2}\left(\operatorname{arcsinh}(\cot(fx+e)-\csc(fx+e))\sqrt{2}\,\sin(fx+e)-2\sqrt{\frac{1}{\cos(fx+e)+1}}\,\cos(fx+e)+2\,\operatorname{arctanh}\left(\frac{-\cos(fx+e)+\sin(fx+e)-1}{2(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)\sin(f}{2cfa(\cos(fx+e)+1)}$

```
[In] int((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/2/c/f/a*g^2*(arcsinh(cot(f*x+e)-csc(f*x+e))*2^(1/2)*sin(f*x+e)-2*(1/(cos
(f*x+e)+1))^(1/2)*cos(f*x+e)+2*arctanh(1/2*(-cos(f*x+e)+sin(f*x+e)-1)/(cos(
f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2))*sin(f*x+e)+2*arctanh(1/2*(cos(f*x+e)+si
n(f*x+e)+1)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2))*sin(f*x+e)-2*(1/(cos(f
*x+e)+1))^(1/2))*(a*(sec(f*x+e)+1))^(1/2)*(g*sec(f*x+e))^(1/2)/(cos(f*x+e)+
1)/(1/(cos(f*x+e)+1))^(1/2)*cot(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.18

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \left[\frac{\sqrt{2}ag^{2}\sqrt{\frac{g}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{\cos(fx+e)^{2}+2\cos(fx+e)}\right.}{}\right.$$

$$-\frac{\sqrt{2}ag^{2}\sqrt{-\frac{g}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{g}{a}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos(fx+e)}{g\sin(fx+e)}\right)\sin(fx+e)+2\,ag^{2}\sqrt{-\frac{g}{a}}\arctan\left(\frac{2\sqrt{-\frac{g}{a}}\sqrt{\frac{a\cos(f}{\cos(f}}}{g\cos}\right.}{2\,acf\sin(fx+e)}$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*a*g^2*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e)
 + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(
f*x + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
*sin(f*x + e) + 2*a*g^2*sqrt(g/a)*log((g*cos(f*x + e)^3 + 4*(cos(f*x + e)^2
 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g
/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g)/(cos(f*x + e)^3 + c
os(f*x + e)^2))*sin(f*x + e) + 4*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e)), -1/2*(sqrt(2)*a*
g^2*sqrt(-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e)))*sin(f*x + e) + 2*
a*g^2*sqrt(-g/a)*arctan(2*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e)
)*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(
f*x + e) - 2*g))*sin(f*x + e) - 2*g^2*sqrt((a*cos(f*x + e) + a)/cos(f*x + e
))*sqrt(g/cos(f*x + e))*cos(f*x + e))/(a*c*f*sin(f*x + e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(149) = 298$.

Time = 0.45 (sec) , antiderivative size = 1400, normalized size of antiderivative = 7.82

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] 1/2*(4*g^2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))*sin(1/2*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 4*g^2*cos(1/2*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 4*g^2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - (sqrt(
2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2
*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(
1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(
```

```
2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x +
2*e), cos(2*f*x + 2*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(
2*f*x + 2*e))) + 2) + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
)) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
)^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sqrt(2)*
cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2) - (sqrt(2)*g^2*cos(1/2*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sqrt(2)*g^2*sin(1/2*arctan2(si
n(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*g^2*cos(1/2*arctan2(sin(2*
f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*log(2*cos(1/4*arctan2(sin(2*f
*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2
*e))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + 2)
 + (sqrt(2)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sq
rt(2)*g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2
)*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + sqrt(2)*g^2)*l
og(2*cos(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arc
tan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e))) + 2) + (g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2
*f*x + 2*e)))^2 + g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^
2 - 2*g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g^2)*log(c
os(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + 2*sin(1/4*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e))) + 1) - (g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 + g^2*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 - 2*
g^2*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))) + g^2)*log(cos(1/4
*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)))^2 + sin(1/4*arctan2(sin(2*f*x
 + 2*e), cos(2*f*x + 2*e)))^2 - 2*sin(1/4*arctan2(sin(2*f*x + 2*e), cos(2*f
*x + 2*e))) + 1))*sqrt(g)/((sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos
(2*f*x + 2*e)))^2 + sqrt(2)*c*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e)))^2 - 2*sqrt(2)*c*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e))
) + sqrt(2)*c)*sqrt(a)*f)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int -\frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{a\sec(fx+e)+a}(c\sec(fx+e)-c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c-c*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(-(g*sec(f*x + e))^(5/2)/(sqrt(a*sec(f*x + e) + a)*(c*sec(f*x + e)
 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c-c\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c-\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
),x)
[Out] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))
), x)
```

3.183 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 46

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\log(\tan(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}$$

```
[Out] ln(tan(f*x+e))*tan(f*x+e)/f/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {4048, 2700, 29}

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx=\frac{\tan(e+fx)\log(\tan(e+fx))}{f\sqrt{a\sec(e+fx)+a}\sqrt{c-c\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]),x]
```

```
[Out] (Log[Tan[e + f*x]]*Tan[e + f*x])/(f*Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec
[e + f*x]])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4048

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(m_), x_Symbol] :> Dist[(
(-a)*c)^(m + 1/2)*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e
+ f*x]])), Int[(g*Csc[e + f*x])^p*Cot[e + f*x]^(2*m), x], x] /; FreeQ[{a, b
, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && Intege
rQ[m + 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tan(e+fx)\int\csc(e+fx)\sec(e+fx)\,dx}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{\tan(e+fx)\mathrm{Subst}\left(\int\frac{1}{x}\,dx,x,\tan(e+fx)\right)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\\
&= \frac{\log(\tan(e+fx))\tan(e+fx)}{f\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\begin{aligned}
&\int\frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx\\
&=\frac{4i\left(-1+e^{i(e+fx)}\right)\operatorname{arctanh}\left(e^{2i(e+fx)}\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}{\left(1+e^{i(e+fx)}\right)f\sqrt{a(1+\sec(e+fx))}\sqrt{c-c\sec(e+fx)}}
\end{aligned}$$

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c - c*Sec[e + f*x]]
),x]
```

```
[Out] ((4*I)*(-1 + E^(I*(e + f*x)))*ArcTanh[E^((2*I)*(e + f*x))]*Cos[(e + f*x)/2]
^2*Sec[e + f*x])/((1 + E^(I*(e + f*x)))*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c
 - c*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(42) = 84$.

Time = 2.58 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.20

method	result
default	$\frac{\sqrt{a(\sec(fx+e)+1)}\,(\ln(-\cot(fx+e)+\csc(fx+e)+1)+\ln(-\cot(fx+e)+\csc(fx+e)-1)-\ln(-\cot(fx+e)+\csc(fx+e)))(\cot(fx+e)-\csc}{fa\sqrt{-c(\sec(fx+e)-1)}}$
risch	$\frac{i(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)\ln(1+e^{2i(fx+e)})}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}(1+e^{2i(fx+e)})\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}f} - \frac{i(e^{i(fx+e)}+1)(e^{i(fx+e)}-1)\ln(e^{2i(fx+e)}-1)}{\sqrt{\frac{a(e^{i(fx+e)}+1)^2}{1+e^{2i(fx+e)}}}(1+e^{2i(fx+e)})\sqrt{\frac{c(e^{i(fx+e)}-1)^2}{1+e^{2i(fx+e)}}}f}$

```
[In] int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f/a*(a*(sec(f*x+e)+1))^(1/2)*(ln(-cot(f*x+e)+csc(f*x+e)+1)+ln(-cot(f*x+e)
+csc(f*x+e)-1)-ln(-cot(f*x+e)+csc(f*x+e)))/(-c*(sec(f*x+e)-1))^(1/2)*(cot(f
*x+e)-csc(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(42) = 84$.

Time = 0.34 (sec) , antiderivative size = 255, normalized size of antiderivative = 5.54

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \left[-\frac{\sqrt{-ac}\log\left(-\frac{8\left(\left(2\cos(fx+e)^3-\cos(fx+e)\right)\sqrt{-ac}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)-c}{\cos(fx+e)}}+\left(2ac\cos(fx+e)^4-2ac\cos(fx+e)^2+ac\right)\sin(fx+e\right.}{\left(\cos(fx+e)^4-\cos(fx+e)^2\right)\sin(fx+e)}}{2acf}\right.$$

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(-8*((2*cos(f*x + e)^3 - cos(f*x + e))*sqrt(-a*c)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f*x + e))
+ (2*a*c*cos(f*x + e)^4 - 2*a*c*cos(f*x + e)^2 + a*c)*sin(f*x + e))/((cos(f
*x + e)^4 - cos(f*x + e)^2)*sin(f*x + e)))/(a*c*f), sqrt(a*c)*arctan(sqrt(a
*c)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) - c)/cos(f
*x + e))*cos(f*x + e)/((2*a*c*cos(f*x + e)^2 - a*c)*sin(f*x + e)))/(a*c*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\sqrt{-c(\sec(e+fx)-1)}}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c-c*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(-c*(sec(e + f*x)
- 1))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = -\frac{\arctan(\sin(2fx+2e),\cos(2fx+2e)+1)-\arctan(\sin(2fx+2e),\cos(2fx+2e)-1)}{\sqrt{a}\sqrt{c}f}$$

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
[Out] -(arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) - arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) - 1))/(sqrt(a)*sqrt(c)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c-c*sec(f*x+e))^(1/2),x, alg
orithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c-c\sec(e+fx)}}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c-\frac{c}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c - c/cos(e + f*x))^(1/2)
), x)
```

3.184 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 65

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx=\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{c-d}\sqrt{d}f}$$

```
[Out] 2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(c-d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^
(1/2)/f/(c-d)^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {4052, 214}

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx=\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c-d}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c - d]*Sqrt[a + a*S
ec[e + f*x]])])/(Sqrt[c - d]*Sqrt[d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\text{integral} = -\frac{(2a)\text{Subst}\left(\int \frac{1}{ac-ad-dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}$$
$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c-d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{c-d}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx$$
$$= \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c-d}\sqrt{\cos(e+fx)}}\right)\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{c-d}\sqrt{d}f}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c - d*Sec[e + f*x]),x]
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c - d]*Sqrt[Cos[e
 + f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/
(Sqrt[c - d]*Sqrt[d]*f)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(51) = 102.

Time = 18.88 (sec) , antiderivative size = 402, normalized size of antiderivative = 6.18

method	result
default	$\left(\ln\left(-\frac{2\left(\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\sqrt{-\frac{2d}{c+d}}c+\sqrt{-\frac{2d}{c+d}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}d+\sqrt{(c+d)(c-d)}(-\cot(fx+e)+\csc(fx+e))-c-d\right)}{-c(-\cot(fx+e)+\csc(fx+e))-(-\cot(fx+e)+\csc(fx+e))d+\sqrt{(c+d)(c-d)}}\right.\right.$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/f/(-2*d/(c+d))^(1/2)/((c+d)*(c-d))^(1/2)*(ln(-2*(((1-cos(f*x+e))^2*csc(f*
x+e)^2-1)^(1/2)*(-2*d/(c+d))^(1/2)*c+(-2*d/(c+d))^(1/2)*((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c-d)/(-
c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))
-ln(-2*(-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(-2*d/(c+d))^(1/2)*c-(-2*d
/(c+d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)
*(-cot(f*x+e)+csc(f*x+e))+c+d)/(c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc
(f*x+e))*d+((c+d)*(c-d))^(1/2))))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(
-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(51) = 102$.

Time = 0.42 (sec) , antiderivative size = 357, normalized size of antiderivative = 5.49

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = \left[\frac{\sqrt{\frac{a}{cd-d^2}}\log\left(-\frac{(ac^2-8\,acd+8\,ad^2)\cos(fx+e)^3+ad^2+(ac^2-2\,acd)\cos(fx+e)^2+4\left((c^2d-3\,cd^2+2\,d^3)\cos(fx+e)^2+(cd^2-d^3)\cos(fx+e)\right.}{c^2\cos(fx+e)^3+(c^2-2\,cd)\cos(fx+e)^2+d^2-(2\,cd-d^2)\cos(fx}\right.}{2\,f}\right.$$
$$\left. -\frac{\sqrt{-\frac{a}{cd-d^2}}\arctan\left(\frac{2\,(cd-d^2)\sqrt{-\frac{a}{cd-d^2}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\cos(fx+e)\sin(fx+e)}{(ac-2\,ad)\cos(fx+e)^2+ad+(ac-ad)\cos(fx+e)}\right)}{f}\right]$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*sqrt(a/(c*d - d^2))*log(-((a*c^2 - 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 +
 a*d^2 + (a*c^2 - 2*a*c*d)*cos(f*x + e)^2 + 4*((c^2*d - 3*c*d^2 + 2*d^3)*co
s(f*x + e)^2 + (c*d^2 - d^3)*cos(f*x + e))*sqrt(a/(c*d - d^2))*sqrt((a*cos(
f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + (6*a*c*d - 7*a*d^2)*cos(f*x + e)
)/(c^2*cos(f*x + e)^3 + (c^2 - 2*c*d)*cos(f*x + e)^2 + d^2 - (2*c*d - d^2)*
cos(f*x + e)))/f, -sqrt(-a/(c*d - d^2))*arctan(2*(c*d - d^2)*sqrt(-a/(c*d -
 d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((
a*c - 2*a*d)*cos(f*x + e)^2 + a*d + (a*c - a*d)*cos(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{c-d\sec\left(e+fx\right)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c-d*sec(f*x+e)),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c - d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = \int -\frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)-c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
maxima")
[Out] -integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) - c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = \int -\frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)-c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c-d*sec(f*x+e)),x, algorithm="
giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c-d\sec(e+fx)}\,dx = -\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{d-c\,\cos\left(e+f\,x\right)}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c - d/cos(e + f*x))),x)
[Out] -int((a + a/cos(e + f*x))^(1/2)/(d - c*cos(e + f*x)), x)
```

3.185 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 29, antiderivative size = 236

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx \\
&= \frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad+\frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\tan(e+fx)}{30f} \\
&\quad+\frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(e+fx)\tan(e+fx)}{120f} \\
&\quad+\frac{a(12c^2+35cd+16d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60f} \\
&\quad+\frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f}
\end{aligned}
$$

```
[Out] 1/8*a*(8*c^4+16*c^3*d+24*c^2*d^2+12*c*d^3+3*d^4)*arctanh(sin(f*x+e))/f+1/30
*a*(12*c^4+95*c^3*d+112*c^2*d^2+80*c*d^3+16*d^4)*tan(f*x+e)/f+1/120*a*d*(24
*c^3+130*c^2*d+116*c*d^2+45*d^3)*sec(f*x+e)*tan(f*x+e)/f+1/60*a*(12*c^2+35*
c*d+16*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/20*a*(4*c+5*d)*(c+d*sec(f*x+e
))^3*tan(f*x+e)/f+1/5*a*(c+d*sec(f*x+e))^4*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx\\ &=\frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\,\text{arctanh}(\sin(e+fx))}{8f}\\ &\quad+\frac{a(12c^2+35cd+16d^2)\tan(e+fx)(c+d\sec(e+fx))^2}{60f}\\ &\quad+\frac{ad(24c^3+130c^2d+116cd^2+45d^3)\tan(e+fx)\sec(e+fx)}{120f}\\ &\quad+\frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\tan(e+fx)}{30f}\\ &\quad+\frac{a\tan(e+fx)(c+d\sec(e+fx))^4}{5f}+\frac{a(4c+5d)\tan(e+fx)(c+d\sec(e+fx))^3}{20f}\end{aligned}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f*x]]
)/(8*f) + (a*(12*c^4 + 95*c^3*d + 112*c^2*d^2 + 80*c*d^3 + 16*d^4)*Tan[e +
f*x])/(30*f) + (a*d*(24*c^3 + 130*c^2*d + 116*c*d^2 + 45*d^3)*Sec[e + f*x]*
Tan[e + f*x])/(120*f) + (a*(12*c^2 + 35*c*d + 16*d^2)*(c + d*Sec[e + f*x])^
2*Tan[e + f*x])/(60*f) + (a*(4*c + 5*d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(20*f) + (a*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f} \\
&\quad+\frac{1}{5}\int\sec(e+fx)(c+d\sec(e+fx))^3(a(5c+4d)+a(4c+5d)\sec(e+fx))\,dx \\
&= \frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f} \\
&\quad+\frac{1}{20}\int\sec(e+fx)(c+d\sec(e+fx))^2\left(a\left(20c^2+28cd+15d^2\right)\right. \\
&\qquad\left.+a\left(12c^2+35cd+16d^2\right)\sec(e+fx)\right)\,dx \\
&= \frac{a\left(12c^2+35cd+16d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{60f} \\
&\quad+\frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f} \\
&\quad+\frac{1}{60}\int\sec(e+fx)(c+d\sec(e+fx))\left(a\left(60c^3+108c^2d+115cd^2+32d^3\right)\right. \\
&\qquad\left.+a\left(24c^3+130c^2d+116cd^2+45d^3\right)\sec(e+fx)\right)\,dx
\end{aligned}$$

$$= \frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(e+fx)\tan(e+fx)}{120f} + \frac{a(12c^2+35cd+16d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f} + \frac{1}{120}\int \sec(e+fx)\left(15a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right) + 4a\left(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4\right)\sec(e+fx)\right)\,dx$$

$$= \frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(e+fx)\tan(e+fx)}{120f} + \frac{a(12c^2+35cd+16d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f} + \frac{1}{8}\left(a\left(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4\right)\right)\int \sec(e+fx)\,dx + \frac{1}{30}\left(a\left(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4\right)\right)\int \sec^2(e+fx)\,dx$$

$$= \frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(e+fx)\tan(e+fx)}{120f} + \frac{a(12c^2+35cd+16d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f} - \frac{(a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4))\operatorname{Subst}(\int 1\,dx, x, -\tan(e+fx))}{30f}$$

$$= \frac{a(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{a(12c^4+95c^3d+112c^2d^2+80cd^3+16d^4)\tan(e+fx)}{30f} + \frac{ad(24c^3+130c^2d+116cd^2+45d^3)\sec(e+fx)\tan(e+fx)}{120f} + \frac{a(12c^2+35cd+16d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{a(4c+5d)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{a(c+d\sec(e+fx))^4\tan(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$=\frac{a(15(8c^4+16c^3d+24c^2d^2+12cd^3+3d^4)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\,(120(c+d)^4+15d(16c^3+$$

[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]

[Out] (a*(15*(8*c^4 + 16*c^3*d + 24*c^2*d^2 + 12*c*d^3 + 3*d^4)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(120*(c + d)^4 + 15*d*(16*c^3 + 24*c^2*d + 12*c*d^2 + 3*d^3)*Sec[e + f*x] + 30*d^3*(4*c + d)*Sec[e + f*x]^3 + 80*d^2*(3*c^2 + 2*c*d + d^2)*Tan[e + f*x]^2 + 24*d^4*Tan[e + f*x]^4)))/(120*f)

Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00

method	result
parts	$\frac{(4acd^3+ad^4)\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{(6ac^2d^2+4acd^3)\left(-\frac{2}{3}-\frac{\sec}{}\right.}{f}$
derivativedivides	$a\,c^4\tan(fx+e)+4a\,c^3d\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-6a\,c^2d^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+4ac\,d^3$
default	$a\,c^4\tan(fx+e)+4a\,c^3d\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-6a\,c^2d^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+4ac\,d^3$
norman	$-\frac{a\left(8c^4+16c^3d+24c^2d^2+12c\,d^3+3d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}-\frac{a\left(8c^4+48c^3d+72c^2d^2+52c\,d^3+13d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{4a\left(45c^4+180c^3d+150\right.}{}$
parallelrisch	$2a\left(-5\left(2c^3d+3c^2d^2+\frac{3}{2}c\,d^3+\frac{3}{8}d^4+c^4\right)\left(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}+\cos(fx+e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+5\left(2c^3d+3c^2d^2+\right.$
risch	$ia\left(64d^4+120c^4-720c^2d^2\mathrm{e}^{7i(fx+e)}+960c\,d^3\mathrm{e}^{6i(fx+e)}+2880c^3d\,\mathrm{e}^{4i(fx+e)}-180c\,d^3\mathrm{e}^{9i(fx+e)}+480c^3d\,\mathrm{e}^{8i(fx+e)}+1440c^2\right.$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)

[Out] (4*a*c*d^3+a*d^4)/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))-(6*a*c^2*d^2+4*a*c*d^3)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(4*a*c^3*d+6*a*c^2*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+(a*c^4+4*a*c^3*d)/f*tan(f*x+e)+a*c^4/f*ln(sec(f*x+e)+tan(f*x+e))-a*d^4/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \frac{15\,(8\,ac^4+16\,ac^3d+24\,ac^2d^2+12\,acd^3+3\,ad^4)\cos{(fx+e)}^5\log{(\sin{(fx+e)}+1)}-15\,(8\,ac^4+16\,ac^3d}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(
f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2
+ 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*a*d^4
 + 8*(15*a*c^4 + 60*a*c^3*d + 60*a*c^2*d^2 + 40*a*c*d^3 + 8*a*d^4)*cos(f*x
+ e)^4 + 15*(16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*cos(f*x + e)
^3 + 16*(15*a*c^2*d^2 + 10*a*c*d^3 + 2*a*d^4)*cos(f*x + e)^2 + 30*(4*a*c*d^
3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx &= a\left(\int c^4\sec{(e+fx)}\,dx+\int c^4\sec^2{(e+fx)}\,dx+\int d^4\sec^5{(e+fx)}\,dx\right.\\ &\quad+\int d^4\sec^6{(e+fx)}\,dx+\int 4cd^3\sec^4{(e+fx)}\,dx+\int 4cd^3\sec^5{(e+fx)}\,dx\\ &\quad+\int 6c^2d^2\sec^3{(e+fx)}\,dx+\int 6c^2d^2\sec^4{(e+fx)}\,dx+\int 4c^3d\sec^2{(e+fx)}\,dx\\ &\quad\left.+\int 4c^3d\sec^3{(e+fx)}\,dx\right)\end{aligned}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(c**4*sec(e + f*x), x) + Integral(c**4*sec(e + f*x)**2, x) + Int
egral(d**4*sec(e + f*x)**5, x) + Integral(d**4*sec(e + f*x)**6, x) + Integr
al(4*c*d**3*sec(e + f*x)**4, x) + Integral(4*c*d**3*sec(e + f*x)**5, x) + I
ntegral(6*c**2*d**2*sec(e + f*x)**3, x) + Integral(6*c**2*d**2*sec(e + f*x)
**4, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(4*c**3*d*sec(e +
 f*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.61

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$

$$= \frac{480\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)ac^2d^2+320\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)acd^3+16\left(3\tan\left(fx+\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c^2*d^2 + 320*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15
*tan(f*x + e))*a*d^4 - 60*a*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) - 240*a*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
 e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*lo
g(sec(f*x + e) + tan(f*x + e)) + 240*a*c^4*tan(f*x + e) + 960*a*c^3*d*tan(f
*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(224) = 448.

Time = 0.39 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.40

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$

$$= \frac{15\left(8\,ac^4+16\,ac^3d+24\,ac^2d^2+12\,acd^3+3\,ad^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\left(8\,ac^4+16\,ac^3d+24\,a\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 + 12*a*c*d^3 + 3*a*d^4)*log(
abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(8*a*c^4 + 16*a*c^3*d + 24*a*c^2*d^2 +
12*a*c*d^3 + 3*a*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(120*a*c^4*tan
(1/2*f*x + 1/2*e)^9 + 240*a*c^3*d*tan(1/2*f*x + 1/2*e)^9 + 360*a*c^2*d^2*ta
```

```
n(1/2*f*x + 1/2*e)^9 + 180*a*c*d^3*tan(1/2*f*x + 1/2*e)^9 + 45*a*d^4*tan(1/
2*f*x + 1/2*e)^9 - 480*a*c^4*tan(1/2*f*x + 1/2*e)^7 - 1440*a*c^3*d*tan(1/2*
f*x + 1/2*e)^7 - 1200*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 1160*a*c*d^3*tan(1
/2*f*x + 1/2*e)^7 - 130*a*d^4*tan(1/2*f*x + 1/2*e)^7 + 720*a*c^4*tan(1/2*f*
x + 1/2*e)^5 + 2880*a*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 2400*a*c^2*d^2*tan(1/2
*f*x + 1/2*e)^5 + 1600*a*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 464*a*d^4*tan(1/2*f
*x + 1/2*e)^5 - 480*a*c^4*tan(1/2*f*x + 1/2*e)^3 - 2400*a*c^3*d*tan(1/2*f*x
 + 1/2*e)^3 - 2640*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 1400*a*c*d^3*tan(1/2*
f*x + 1/2*e)^3 - 190*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 120*a*c^4*tan(1/2*f*x +
 1/2*e) + 720*a*c^3*d*tan(1/2*f*x + 1/2*e) + 1080*a*c^2*d^2*tan(1/2*f*x + 1
/2*e) + 780*a*c*d^3*tan(1/2*f*x + 1/2*e) + 195*a*d^4*tan(1/2*f*x + 1/2*e))/
(tan(1/2*f*x + 1/2*e)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 17.12 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.53

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{2\,\left(4\,c^4+8\,c^3\,d+12\,c^2\,d^2+6\,c\,d^3+\frac{3\,d^4}{2}\right)}\right)\,\left(8\,c^4+16\,c^3\,d+24\,c^2\,d^2+12\,c\,d^3+3\,d^4\right)}{4\,f}$$
$$-\frac{\left(2\,a\,c^4+4\,a\,c^3\,d+6\,a\,c^2\,d^2+3\,a\,c\,d^3+\frac{3\,a\,d^4}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+\left(-8\,a\,c^4-24\,a\,c^3\,d-20\,a\,c^2\,d^2-\frac{58\,a\,c\,d^3}{3}\right.}{}$$

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (a*atanh((tan(e/2 + (f*x)/2)*(12*c*d^3 + 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*
d^2))/(2*(6*c*d^3 + 8*c^3*d + 4*c^4 + (3*d^4)/2 + 12*c^2*d^2)))*(12*c*d^3 +
 16*c^3*d + 8*c^4 + 3*d^4 + 24*c^2*d^2))/(4*f) - (tan(e/2 + (f*x)/2)^9*(2*a
*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*a*c*d^3 + 4*a*c^3*d) - tan(e/2 + (f*x)
/2)^7*(8*a*c^4 + (13*a*d^4)/6 + 20*a*c^2*d^2 + (58*a*c*d^3)/3 + 24*a*c^3*d)
 - tan(e/2 + (f*x)/2)^3*(8*a*c^4 + (19*a*d^4)/6 + 44*a*c^2*d^2 + (70*a*c*d^
3)/3 + 40*a*c^3*d) + tan(e/2 + (f*x)/2)^5*(12*a*c^4 + (116*a*d^4)/15 + 40*a
*c^2*d^2 + (80*a*c*d^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*(2*a*c^4 + (13
*a*d^4)/4 + 18*a*c^2*d^2 + 13*a*c*d^3 + 12*a*c^3*d))/(f*(5*tan(e/2 + (f*x)/
2)^2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x
)/2)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.186 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 171

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx &= \frac{a(8c^3+12c^2d+12cd^2+3d^3)\operatorname{arctanh}(\sin(e+fx))}{8f}\\ &+\frac{a(3c^3+16c^2d+12cd^2+4d^3)\tan(e+fx)}{6f}\\ &+\frac{ad(6c^2+20cd+9d^2)\sec(e+fx)\tan(e+fx)}{24f}\\ &+\frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f}\end{aligned}$$

```
[Out] 1/8*a*(8*c^3+12*c^2*d+12*c*d^2+3*d^3)*arctanh(sin(f*x+e))/f+1/6*a*(3*c^3+16
*c^2*d+12*c*d^2+4*d^3)*tan(f*x+e)/f+1/24*a*d*(6*c^2+20*c*d+9*d^2)*sec(f*x+e
)*tan(f*x+e)/f+1/12*a*(3*c+4*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/4*a*(c+d*
sec(f*x+e))^3*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$=\frac{a(8c^3+12c^2d+12cd^2+3d^3)\,\text{arctanh}(\sin(e+fx))}{8f}$$
$$+\frac{ad(6c^2+20cd+9d^2)\tan(e+fx)\sec(e+fx)}{24f}$$
$$+\frac{a(3c^3+16c^2d+12cd^2+4d^3)\tan(e+fx)}{6f}+\frac{a\tan(e+fx)(c+d\sec(e+fx))^3}{4f}$$
$$+\frac{a(3c+4d)\tan(e+fx)(c+d\sec(e+fx))^2}{12f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]])/(8*f) + (a*
(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3)*Tan[e + f*x])/(6*f) + (a*d*(6*c^2 + 2
0*c*d + 9*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + (a*(3*c + 4*d)*(c + d*Se
c[e + f*x])^2*Tan[e + f*x])/(12*f) + (a*(c + d*Sec[e + f*x])^3*Tan[e + f*x]
)/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad + \frac{1}{4}\int \sec(e+fx)(c+d\sec(e+fx))^2(a(4c+3d)+a(3c+4d)\sec(e+fx))\,dx \\
&= \frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad + \frac{1}{12}\int \sec(e+fx)(c+d\sec(e+fx))\left(a\left(12c^2+15cd+8d^2\right)\right. \\
&\qquad \left. + a\left(6c^2+20cd+9d^2\right)\sec(e+fx)\right)\,dx \\
&= \frac{ad\left(6c^2+20cd+9d^2\right)\sec(e+fx)\tan(e+fx)}{24f} \\
&\quad + \frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad + \frac{1}{24}\int \sec(e+fx)\left(3a\left(8c^3+12c^2d+12cd^2+3d^3\right)\right. \\
&\qquad \left. + 4a\left(3c^3+16c^2d+12cd^2+4d^3\right)\sec(e+fx)\right)\,dx \\
&= \frac{ad\left(6c^2+20cd+9d^2\right)\sec(e+fx)\tan(e+fx)}{24f} \\
&\quad + \frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad + \frac{1}{8}\left(a\left(8c^3+12c^2d+12cd^2+3d^3\right)\right)\int \sec(e+fx)\,dx \\
&\quad + \frac{1}{6}\left(a\left(3c^3+16c^2d+12cd^2+4d^3\right)\right)\int \sec^2(e+fx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a(8c^3+12c^2d+12cd^2+3d^3)\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{ad(6c^2+20cd+9d^2)\sec(e+fx)\tan(e+fx)}{24f} \\
&\quad + \frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad - \frac{(a(3c^3+16c^2d+12cd^2+4d^3))\operatorname{Subst}(\int 1\,dx, x, -\tan(e+fx))}{6f} \\
&= \frac{a(8c^3+12c^2d+12cd^2+3d^3)\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{a(3c^3+16c^2d+12cd^2+4d^3)\tan(e+fx)}{6f} \\
&\quad + \frac{ad(6c^2+20cd+9d^2)\sec(e+fx)\tan(e+fx)}{24f} \\
&\quad + \frac{a(3c+4d)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{a(c+d\sec(e+fx))^3\tan(e+fx)}{4f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.60

$$
\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx
$$
$$
= \frac{a(3(8c^3+12c^2d+12cd^2+3d^3)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\,(24(c+d)^3+9d(2c+d)^2\sec(e+fx)}{24f}
$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(3*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3)*ArcTanh[Sin[e + f*x]] + Tan[e +
 f*x]*(24*(c + d)^3 + 9*d*(2*c + d)^2*Sec[e + f*x] + 6*d^3*Sec[e + f*x]^3 +
 8*d^2*(3*c + d)*Tan[e + f*x]^2)))/(24*f)
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

method	result
parts	$-\frac{(3acd^2+ad^3)\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{(3ac^2d+3acd^2)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
derivativedivides	$\frac{ac^3\tan(fx+e)+3ac^2d\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-3acd^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+ad^3\left(\right.}{}$
default	$\frac{ac^3\tan(fx+e)+3ac^2d\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-3acd^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+ad^3\left(\right.}{}$
parallelrisch	$\frac{2\left(-2\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(\frac{3}{2}c^2d+\frac{3}{2}cd^2+\frac{3}{8}d^3+c^3\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+2\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right.\right.}{}$
norman	$\frac{-\frac{a(8c^3+12c^2d+12cd^2+3d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{a(8c^3+36c^2d+36cd^2+13d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}+\frac{a(72c^3+180c^2d+84cd^2+49d^3)\tan\left(\frac{fx}{2}\right.}{12f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$
risch	$\frac{ia\left(24c^3+48cd^2+16d^3+72c^2d+36cd^2e^{i(fx+e)}+192cd^2e^{2i(fx+e)}+36c^2de^{i(fx+e)}+72c^2de^{6i(fx+e)}-36c^2de^{5i(fx+e)}-3\right.}{}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] -(3*a*c*d^2+a*d^3)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(3*a*c^2*d+3*a*c*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+(a*c^3+3*a*c^2*d)/f*tan(f*x+e)+a*c^3/f*ln(sec(f*x+e)+tan(f*x+e))+a*d^3/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.23

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$=\frac{3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,ac^3+12\,ac^2d+12\,acd^2+\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x + e)^4*log(si
n(f*x + e) + 1) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) + 2*(6*a*d^3 + 8*(3*a*c^3 + 9*a*c^2*d + 6*a*c*
d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*a*c^2*d + 4*a*c*d^2 + a*d^3)*cos(f*x +
 e)^2 + 8*(3*a*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= a\left(\int c^3\sec(e+fx)\,dx + \int c^3\sec^2(e+fx)\,dx + \int d^3\sec^4(e+fx)\,dx + \int d^3\sec^5(e+fx)\,dx + \int 3cd^2\sec^3(e+fx)\,dx + \int 3cd^2\sec^4(e+fx)\,dx + \int 3c^2d\sec^2(e+fx)\,dx + \int 3c^2d\sec^3(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a*(Integral(c**3*sec(e + f*x), x) + Integral(c**3*sec(e + f*x)**2, x) + Int
egral(d**3*sec(e + f*x)**4, x) + Integral(d**3*sec(e + f*x)**5, x) + Integr
al(3*c*d**2*sec(e + f*x)**3, x) + Integral(3*c*d**2*sec(e + f*x)**4, x) + I
ntegral(3*c**2*d*sec(e + f*x)**2, x) + Integral(3*c**2*d*sec(e + f*x)**3, x
))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= \frac{48\left(\tan(fx+e)^3+3\tan(fx+e)\right)acd^2+16\left(\tan(fx+e)^3+3\tan(fx+e)\right)ad^3-3ad^3\left(\frac{2\left(3\sin(fx+e)^3-5\right.}{\sin(fx+e)^4-2\sin(\right.}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*c*d^2 + 16*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a*d^3 - 3*a*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(
f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*
x + e) - 1)) - 36*a*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin(f*x
 + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c^3*log
(sec(f*x + e) + tan(f*x + e)) + 48*a*c^3*tan(f*x + e) + 144*a*c^2*d*tan(f*x
 + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(161) = 322$.

Time = 0.36 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.22

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= \frac{3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(8\,ac^3+12\,ac^2d+12\,acd^2+3\,ad^3\right)\mathrm{l}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*log(abs(tan(1/2*f*x +
 1/2*e) + 1)) - 3*(8*a*c^3 + 12*a*c^2*d + 12*a*c*d^2 + 3*a*d^3)*log(abs(tan
(1/2*f*x + 1/2*e) - 1)) - 2*(24*a*c^3*tan(1/2*f*x + 1/2*e)^7 + 36*a*c^2*d*t
an(1/2*f*x + 1/2*e)^7 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 9*a*d^3*tan(1/2
*f*x + 1/2*e)^7 - 72*a*c^3*tan(1/2*f*x + 1/2*e)^5 - 180*a*c^2*d*tan(1/2*f*x
 + 1/2*e)^5 - 84*a*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 49*a*d^3*tan(1/2*f*x + 1/
2*e)^5 + 72*a*c^3*tan(1/2*f*x + 1/2*e)^3 + 252*a*c^2*d*tan(1/2*f*x + 1/2*e)
^3 + 156*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 31*a*d^3*tan(1/2*f*x + 1/2*e)^3 -
 24*a*c^3*tan(1/2*f*x + 1/2*e) - 108*a*c^2*d*tan(1/2*f*x + 1/2*e) - 108*a*c
*d^2*tan(1/2*f*x + 1/2*e) - 39*a*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1
/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 17.05 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.49

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$= \frac{\left(-2\,a\,c^3-3\,a\,c^2\,d-3\,a\,c\,d^2-\frac{3\,a\,d^3}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(6\,a\,c^3+15\,a\,c^2\,d+7\,a\,c\,d^2+\frac{49\,a\,d^3}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\right.}$$
$$+\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{2\,\left(4\,c^3+6\,c^2\,d+6\,c\,d^2+\frac{3\,d^3}{2}\right)}\right)\,\left(8\,c^3+12\,c^2\,d+12\,c\,d^2+3\,d^3\right)}{4\,f}$$

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(2*a*c^3 + (13*a*d^3)/4 + 9*a*c*d^2 + 9*a*c^2*d) - tan(
e/2 + (f*x)/2)^7*(2*a*c^3 + (3*a*d^3)/4 + 3*a*c*d^2 + 3*a*c^2*d) - tan(e/2
+ (f*x)/2)^3*(6*a*c^3 + (31*a*d^3)/12 + 13*a*c*d^2 + 21*a*c^2*d) + tan(e/2
```

```
+ (f*x)/2)^5*(6*a*c^3 + (49*a*d^3)/12 + 7*a*c*d^2 + 15*a*c^2*d))/(f*(6*tan(
e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/
2 + (f*x)/2)^8 + 1)) + (a*atanh((tan(e/2 + (f*x)/2)*(12*c*d^2 + 12*c^2*d +
8*c^3 + 3*d^3))/(2*(6*c*d^2 + 6*c^2*d + 4*c^3 + (3*d^3)/2)))*(12*c*d^2 + 12
*c^2*d + 8*c^3 + 3*d^3))/(4*f)
```

3.187 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 108

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx = \frac{a(2c^2+2cd+d^2)\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{2a(c^2+3cd+d^2)\tan(e+fx)}{3f} + \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f}+\frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f}$$

```
[Out] 1/2*a*(2*c^2+2*c*d+d^2)*arctanh(sin(f*x+e))/f+2/3*a*(c^2+3*c*d+d^2)*tan(f*x
+e)/f+1/6*a*d*(2*c+3*d)*sec(f*x+e)*tan(f*x+e)/f+1/3*a*(c+d*sec(f*x+e))^2*ta
n(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx = \frac{a(2c^2+2cd+d^2)\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{2a(c^2+3cd+d^2)\tan(e+fx)}{3f} + \frac{a\tan(e+fx)(c+d\sec(e+fx))^2}{3f}+\frac{ad(2c+3d)\tan(e+fx)\sec(e+fx)}{6f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a*(c^2 + 3*c*d +
 d^2)*Tan[e + f*x])/(3*f) + (a*d*(2*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(6*
f) + (a*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f} \\
&\quad + \frac{1}{3}\int \sec(e+fx)(c+d\sec(e+fx))(a(3c+2d)+a(2c+3d)\sec(e+fx))\,dx \\
&= \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f} + \frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f} \\
&\quad + \frac{1}{6}\int \sec(e+fx)\left(3a\left(2c^2+2cd+d^2\right)+4a\left(c^2+3cd+d^2\right)\sec(e+fx)\right)\,dx \\
&= \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f} + \frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f} \\
&\quad + \frac{1}{2}\left(a\left(2c^2+2cd+d^2\right)\right)\int \sec(e+fx)\,dx + \frac{1}{3}\left(2a\left(c^2+3cd+d^2\right)\right)\int \sec^2(e+fx)\,dx \\
&= \frac{a(2c^2+2cd+d^2)\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f} \\
&\quad + \frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f} - \frac{(2a(c^2+3cd+d^2))\,\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{3f} \\
&= \frac{a(2c^2+2cd+d^2)\operatorname{arctanh}(\sin(e+fx))}{2f} + \frac{2a(c^2+3cd+d^2)\tan(e+fx)}{3f} \\
&\quad + \frac{ad(2c+3d)\sec(e+fx)\tan(e+fx)}{6f} + \frac{a(c+d\sec(e+fx))^2\tan(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= \frac{a(3(2c^2+2cd+d^2)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(3d(2c+d)\sec(e+fx)+2(3(c+d)^2+d^2\tan^2(\right.}{6f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a*(3*(2*c^2 + 2*c*d + d^2)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*d*(2*c
+ d)*Sec[e + f*x] + 2*(3*(c + d)^2 + d^2*Tan[e + f*x]^2))))/(6*f)
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

method	result
parts	$\frac{(2acd+a\,d^2)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{(a\,c^2+2acd)\tan(fx+e)}{f}+\frac{a\,c^2\ln(\sec(fx+e)+\tan(fx\cdots}{f}$
derivativedivides	$\frac{a\,c^2\tan(fx+e)+2acd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,d^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+a\,c^2\ln(\sec(f\cdots}{f}$
default	$\frac{a\,c^2\tan(fx+e)+2acd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,d^2\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+a\,c^2\ln(\sec(f\cdots}{f}$
norman	$\frac{-\frac{a(2c^2+2cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{a(2c^2+6cd+3d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{4a(3c^2+6cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{a(2c^2+2cd+d^2)\ln\left(\tan\left(\cdots\right.\right.}{2f}$
parallelrisch	$\frac{2\left(-\frac{3\left(cd+\frac{1}{2}d^2+c^2\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3\left(cd+\frac{1}{2}d^2+c^2\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\cdots\right.}{2}\right.}{f(\cos(3fx+3e)+3\cos(fx+e))}$
risch	$-\frac{ia(6cd\,e^{5i(fx+e)}+3d^2e^{5i(fx+e)}-6c^2e^{4i(fx+e)}-12cd\,e^{4i(fx+e)}-12c^2e^{2i(fx+e)}-24cd\,e^{2i(fx+e)}-12d^2e^{2i(fx+e)}-6d\,e^{i(f\cdots}{3f\left(1+e^{2i(fx+e)}\right)^3}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2*a*c*d+a*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))
+(a*c^2+2*a*c*d)/f*tan(f*x+e)+a*c^2/f*ln(sec(f*x+e)+tan(f*x+e))-a*d^2/f*(-2
/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{3\left(2\,ac^2+2\,acd+ad^2\right)\cos\left(fx+e\right)^3\log\left(\sin\left(fx+e\right)+1\right)-3\left(2\,ac^2+2\,acd+ad^2\right)\cos\left(fx+e\right)^3\log\left(-\sin\cdots\right.}{12\,f\cos\cdots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
3*(2*a*c^2 + 2*a*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*
a*d^2 + 2*(3*a*c^2 + 6*a*c*d + 2*a*d^2)*cos(f*x + e)^2 + 3*(2*a*c*d + a*d^2
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= a\left(\int c^2\sec\left(e+fx\right)dx + \int c^2\sec^2\left(e+fx\right)dx + \int d^2\sec^3\left(e+fx\right)dx + \int d^2\sec^4\left(e+fx\right)dx + \int 2cd\sec^2\left(e+fx\right)dx + \int 2cd\sec^3\left(e+fx\right)dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(c**2*sec(e + f*x), x) + Integral(c**2*sec(e + f*x)**2, x) + Int
egral(d**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**4, x) + Integr
al(2*c*d*sec(e + f*x)**2, x) + Integral(2*c*d*sec(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.53

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= \frac{4\left(\tan\left(fx+e\right)^3 + 3\tan\left(fx+e\right)\right)ad^2 - 6\,acd\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1} - \log\left(\sin\left(fx+e\right)+1\right) + \log\left(\sin\left(fx+e\right) - \right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*d^2 - 6*a*c*d*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*a*
d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*a*c^2*tan(f
*x + e) + 24*a*c*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(100) = 200.

Time = 0.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.15

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= \frac{3\left(2\,ac^2 + 2\,acd + ad^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) + 1\right|\right) - 3\left(2\,ac^2 + 2\,acd + ad^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx + \frac{1}{2}\,e\right) - 1\right|\right)}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*a*c^2 + 2*a*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(
2*a*c^2 + 2*a*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(6*a*c^2*
tan(1/2*f*x + 1/2*e)^5 + 6*a*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*a*d^2*tan(1/2*f
*x + 1/2*e)^5 - 12*a*c^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*c*d*tan(1/2*f*x + 1/
2*e)^3 - 4*a*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*a*c^2*tan(1/2*f*x + 1/2*e) + 18
*a*c*d*tan(1/2*f*x + 1/2*e) + 9*a*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x +
1/2*e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.81

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)}{4\,c^2+4\,c\,d+2\,d^2}\right)\,\left(2\,c^2+2\,c\,d+d^2\right)}{f}$$
$$- \frac{\left(2\,a\,c^2+2\,a\,c\,d+a\,d^2\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-4\,a\,c^2-8\,a\,c\,d-\frac{4\,a\,d^2}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(2\,a\,c^2+6\,a\,c\,d+3\,a\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a*atanh((2*tan(e/2 + (f*x)/2)*(2*c*d + 2*c^2 + d^2))/(4*c*d + 4*c^2 + 2*d^
2))*(2*c*d + 2*c^2 + d^2))/f - (tan(e/2 + (f*x)/2)*(2*a*c^2 + 3*a*d^2 + 6*a
*c*d) + tan(e/2 + (f*x)/2)^5*(2*a*c^2 + a*d^2 + 2*a*c*d) - tan(e/2 + (f*x)/
2)^3*(4*a*c^2 + (4*a*d^2)/3 + 8*a*c*d))/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(
e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

3.188 $\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 56

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{a(2c+d)\text{arctanh}(\sin(e+fx))}{2f}+\frac{a(c+d)\tan(e+fx)}{f}+\frac{ad\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*a*(2*c+d)*arctanh(sin(f*x+e))/f+a*(c+d)*tan(f*x+e)/f+1/2*a*d*sec(f*x+e)
*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4082, 3872, 3855, 3852, 8}

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{a(2c+d)\text{arctanh}(\sin(e+fx))}{2f}+\frac{a(c+d)\tan(e+fx)}{f}+\frac{ad\tan(e+fx)\sec(e+fx)}{2f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*(2*c + d)*ArcTanh[Sin[e + f*x]])/(2*f) + (a*(c + d)*Tan[e + f*x])/f + (a
*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ad\sec(e+fx)\tan(e+fx)}{2f} + \frac{1}{2}\int \sec(e+fx)(a(2c+d)+2a(c+d)\sec(e+fx))\,dx \\
&= \frac{ad\sec(e+fx)\tan(e+fx)}{2f} + (a(c+d))\int \sec^2(e+fx)\,dx + \frac{1}{2}(a(2c+d))\int \sec(e+fx)\,dx \\
&= \frac{a(2c+d)\text{arctanh}(\sin(e+fx))}{2f} + \frac{ad\sec(e+fx)\tan(e+fx)}{2f} - \frac{(a(c+d))\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{f} \\
&= \frac{a(2c+d)\text{arctanh}(\sin(e+fx))}{2f} + \frac{a(c+d)\tan(e+fx)}{f} + \frac{ad\sec(e+fx)\tan(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{ac\text{arctanh}(\sin(e+fx))}{f} + \frac{ad\text{arctanh}(\sin(e+fx))}{2f} + \frac{ac\tan(e+fx)}{f} + \frac{ad\tan(e+fx)}{f} + \frac{ad\sec(e+fx)\tan(e+fx)}{2f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/f + (a*d*ArcTanh[Sin[e + f*x]])/(2*f) + (a*c*Ta
n[e + f*x])/f + (a*d*Tan[e + f*x])/f + (a*d*Sec[e + f*x]*Tan[e + f*x])/(2*f
)
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{ac\tan(fx+e)+ad\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)}{f}$
default	$\frac{ac\tan(fx+e)+ad\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)}{f}$
parts	$\frac{(ac+ad)\tan(fx+e)}{f}+\frac{ac\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{ad\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parallelrisch	$-\frac{\left(\left(c+\frac{d}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\left(c+\frac{d}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+(-c-d)\sin(2fx+2e}{f(1+\cos(2fx+2e))}$
norman	$\frac{\frac{a(2c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{a(2c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{a(2c+d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a(2c+d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2f}$
risch	$-\frac{ia\left(d\,e^{3i(fx+e)}-2\,e^{2i(fx+e)}c-2d\,e^{2i(fx+e)}-d\,e^{i(fx+e)}-2c-2d\right)}{f\left(1+e^{2i(fx+e)}\right)^2}-\frac{ac\ln\left(e^{i(fx+e)}-i\right)}{f}-\frac{a\ln\left(e^{i(fx+e)}-i\right)d}{2f}+ac\,\mathrm{l}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c*tan(f*x+e)+a*d*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*
x+e)))+a*c*ln(sec(f*x+e)+tan(f*x+e))+a*d*tan(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.71

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=\frac{(2\,ac+ad)\cos{(fx+e)}^2\log{(\sin{(fx+e)}+1)}-(2\,ac+ad)\cos{(fx+e)}^2\log{(-\sin{(fx+e)}+1)}+2\,(ad}{4\,f\cos{(fx+e)}^2}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*a*c + a*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + a*d)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a*d + 2*(a*c + a*d)*cos(f*x + e))*s
in(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=a\bigg(\int c\sec{(e+fx)}\,dx+\int c\sec^2{(e+fx)}\,dx+\int d\sec^2{(e+fx)}\,dx+\int d\sec^3{(e+fx)}\,dx\bigg)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] a*(Integral(c*sec(e + f*x), x) + Integral(c*sec(e + f*x)**2, x) + Integral(
d*sec(e + f*x)**2, x) + Integral(d*sec(e + f*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.57

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx=$$
$$-\frac{ad\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log{(\sin{(fx+e)}+1)}+\log{(\sin{(fx+e)}-1)}\right)-4\,ac\log{(\sec{(fx+e)}+\tan{(fx+e)})}}{4\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -1/4*(a*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*a*c*tan(f
*x + e) - 4*a*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(52) = 104$.

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.21

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$= \frac{(2\,ac+ad)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-(2\,ac+ad)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(2\,ac\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+ad}{}}{2\,f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a*c + a*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - (2*a*c + a*d)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(2*a*c*tan(1/2*f*x + 1/2*e)^3 + a*d*tan(1
/2*f*x + 1/2*e)^3 - 2*a*c*tan(1/2*f*x + 1/2*e) - 3*a*d*tan(1/2*f*x + 1/2*e)
)/(tan(1/2*f*x + 1/2*e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 14.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

$$\int \sec(e+fx)(a+a\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c+d)}{4\,c+2\,d}\right)\,(2\,c+d)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{3}\,(2\,a\,c+a\,d)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,a\,c+3\,a\,d)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{4}-2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{2}+1\right)}$$

```
[In] int(((a + a/cos(e + f*x))*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (a*atanh((2*tan(e/2 + (f*x)/2)*(2*c + d))/(4*c + 2*d))*(2*c + d))/f - (tan(
e/2 + (f*x)/2)^3*(2*a*c + a*d) - tan(e/2 + (f*x)/2)*(2*a*c + 3*a*d))/(f*(ta
n(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

3.189 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\operatorname{arctanh}(\sin(e+fx))}{df} - \frac{2a\sqrt{c-d}\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d\sqrt{c+d}f}$$

```
[Out] a*arctanh(sin(f*x+e))/d/f-2*a*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^
(1/2))*(c-d)^(1/2)/d/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4083, 3855, 3916, 2738, 214}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\operatorname{arctanh}(\sin(e+fx))}{df} - \frac{2a\sqrt{c-d}\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c+d}}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*ArcTanh[Sin[e + f*x]])/(d*f) - (2*a*Sqrt[c - d]*ArcTanh[(Sqrt[c - d]*Tan
[(e + f*x)/2])/Sqrt[c + d]])/(d*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\int \sec(e+fx)\,dx}{d} + \frac{(-ac+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{d} \\
&= \frac{a\text{arctanh}(\sin(e+fx))}{df} - \frac{(a(c-d))\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d^2} \\
&= \frac{a\text{arctanh}(\sin(e+fx))}{df} - \frac{(2a(c-d))\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2 f} \\
&= \frac{a\text{arctanh}(\sin(e+fx))}{df} - \frac{2a\sqrt{c-d}\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d\sqrt{c+d}f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\left(\frac{2(c-d)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{df}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*((2*(c - d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c
^2 - d^2] - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]]))/(d*f)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{4a\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4d}-\frac{(c-d)\,\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2d\sqrt{(c+d)(c-d)}}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4d}\right)}{f}$
default	$\frac{4a\left(-\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4d}-\frac{(c-d)\,\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2d\sqrt{(c+d)(c-d)}}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{4d}\right)}{f}$
risch	$\frac{\sqrt{(c+d)(c-d)}\,a\ln\left(e^{i(fx+e)}-\frac{i\sqrt{(c+d)(c-d)}-d}{c}\right)}{(c+d)fd}-\frac{\sqrt{(c+d)(c-d)}\,a\ln\left(e^{i(fx+e)}+\frac{i\sqrt{(c+d)(c-d)}+d}{c}\right)}{(c+d)fd}+\frac{a\ln(e^{i(fx+e)}+}{df}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a*(-1/4/d*ln(tan(1/2*f*x+1/2*e)-1)-1/2*(c-d)/d/((c+d)*(c-d))^(1/2)*arct
anh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))+1/4/d*ln(tan(1/2*f*x+1/2*
e)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$
$$= \left[\frac{a\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\,(c^2+cd+(cd+d^2)\cos(fx+e))\sqrt{\frac{c-d}{c+d}}\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)+a\log(\sin(fx}{2\,df}\right.$$
$$\left.-\frac{2\,a\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right)-a\log(\sin(fx+e)+1)+a\log(-\sin(fx+e)+1)}{2\,df}\right]$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [1/2*(a*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f
*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*
sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)
) + a*log(sin(f*x + e) + 1) - a*log(-sin(f*x + e) + 1))/(d*f), -1/2*(2*a*sq
rt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((
c - d)*sin(f*x + e))) - a*log(sin(f*x + e) + 1) + a*log(-sin(f*x + e) + 1))
/(d*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$
$$= a\left(\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx+\int \frac{\sec^2(e+fx)}{c+d\sec(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**
2/(c + d*sec(e + f*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(60) = 120$.

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.84

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d} + \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(ac-ad}{\sqrt{-c^2+d^2}\,d}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d - a*log(abs(tan(1/2*f*x + 1/2*e) -
1))/d + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - a*d)/(
sqrt(-c^2 + d^2)*d))/f
```

Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.83

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,(c+d)} + \frac{2\,a\,\left(\mathrm{atanh}\left(\frac{d^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c^3\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,d^2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-c^2\,d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(c^2-d^2)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}\,(d^2+c\,d)}\right)\,\sqrt{c^2-d^2}+c\,\mathrm{atanh}}{d\,f\,(c+d)}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (2*a*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*a*(atan
h((d^3*sin(e/2 + (f*x)/2) - c^3*sin(e/2 + (f*x)/2) + c*d^2*sin(e/2 + (f*x)/
2) - c^2*d*sin(e/2 + (f*x)/2) + c*sin(e/2 + (f*x)/2)*(c^2 - d^2))/(cos(e/2
+ (f*x)/2)*(c^2 - d^2)^(1/2)*(c*d + d^2)))*(c^2 - d^2)^(1/2) + c*atanh(sin(
e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/(d*f*(c + d))
```

3.190 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 79

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2a\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}(c+d)^{3/2}f} + \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))}$$

```
[Out] 2*a*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c+d)^(3/2)/f/(c-d)
^(1/2)+a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4088, 12, 3916, 2738, 214}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2a\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}(c+d)^{3/2}} + \frac{a\tan(e+fx)}{f(c+d)(c+d\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*(c +
 d)^(3/2)*f) + (a*Tan[e + f*x])/((c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} - \frac{\int \frac{a(c-d)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{-c^2+d^2} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{a\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c+d} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{a\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d(c+d)} \\
&= \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))} + \frac{(2a)\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d(c+d)f}
\end{aligned}$$

$$= \frac{2a\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}(c+d)^{3/2}f} + \frac{a\tan(e+fx)}{(c+d)f(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{a\left(-\frac{2\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{\sin(e+fx)}{d+c\cos(e+fx)}\right)}{(c+d)f}$$

[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]

[Out] (a*((-2*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + Sin[e + f*x]/(d + c*Cos[e + f*x])))/((c + d)*f)

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{4a\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}+\frac{\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{f}$
default	$\frac{4a\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}+\frac{\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{f}$
risch	$\frac{2ia\left(d\,e^{i(fx+e)}+c\right)}{cf(c+d)\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{a\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c+d)f}-\frac{a\ln\left(e^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}\,(c+d)f}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 4/f*a*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 4.52

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{(ac\cos(fx+e)+ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right) -}{2\left((c^4+c^3d-c^2d^2-cd^3)f\cos(fx+e)+(c^3d+c^2d^2-cd^3-d^4)f\right)}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*c*cos(f*x + e) + a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c
^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x
 + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(
a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*cos(f*x + e
) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f), ((a*c*cos(f*x + e) + a*d)*sqrt(-c^2
 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x
+ e))) + (a*c^2 - a*d^2)*sin(f*x + e))/((c^4 + c^3*d - c^2*d^2 - c*d^3)*f*c
os(f*x + e) + (c^3*d + c^2*d^2 - c*d^3 - d^4)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = a\left(\int \frac{\sec(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx + \int \frac{\sec^2(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**2)
, x) + Integral(sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f
*x)**2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.73

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a}{\sqrt{-c^2+d^2}(c+d)}+\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(c+d)}\right)}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a/(sqrt(-c^2 + d^2)
*(c + d)) + a*tan(1/2*f*x + 1/2*e)/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f
*x + 1/2*e)^2 - c - d)*(c + d)))/f
```

Mupad [B] (verification not implemented)

Time = 13.47 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(c+d\right)\,\left(\left(d-c\right)\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+c+d\right)} + \frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)}{f\,{\left(c+d\right)}^{3/2}\,\sqrt{c-d}}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*a*tan(e/2 + (f*x)/2))/(f*(c + d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
 + (2*a*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)
^(3/2)*(c - d)^(1/2))
```

3.191 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 131

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{a(2c-d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{5/2}f} + \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))}$$

```
[Out] a*(2*c-d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/2)/(
c+d)^(5/2)/f+1/2*a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^2+1/2*a*(c-2*d)*tan(
f*x+e)/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4088, 12, 3916, 2738, 214}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{a(2c-d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{5/2}} + \frac{a(c-2d)\tan(e+fx)}{2f(c-d)(c+d)^2(c+d\sec(e+fx))} + \frac{a\tan(e+fx)}{2f(c+d)(c+d\sec(e+fx))^2}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(2*c - d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^
(3/2)*(c + d)^(5/2)*f) + (a*Tan[e + f*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])
^2) + (a*(c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^2*f*(c + d*Sec[e + f*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\text{integral} = \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-2a(c-d)-a(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx}{2\left(c^2-d^2\right)}$$
$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))} + \frac{\int \frac{a(c-d)(2c-d)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{2\left(c^2-d^2\right)^2}$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))} + \frac{(a(2c-d))\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{2(c-d)(c+d)^2}$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))} + \frac{(a(2c-d))\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{2(c-d)d(c+d)^2}$$

$$= \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))} + \frac{(a(2c-d))\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)d(c+d)^2f}$$

$$= \frac{a(2c-d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{5/2}f} + \frac{a\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{a(c-2d)\tan(e+fx)}{2(c-d)(c+d)^2f(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{a(1+\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)\left(-2(2c-d)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx))^2+\sqrt{c^2-\right.}{4(c-d)(c+d)^2\sqrt{c^2-d^2}f(d+c\cos(e+fx))^2}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(-2*(2*c - d)*ArcTanh[((-c + d)*Ta
n[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^2 + Sqrt[c^2 - d^2]*(
(c - 2*d)*d + (2*c^2 - 2*c*d - d^2)*Cos[e + f*x])*Sin[e + f*x]))/(4*(c - d)
*(c + d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{4a\left(\frac{-\frac{(2c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{4\left(c^2+2cd+d^2\right)}+\frac{(2c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)(c-d)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}+\frac{(2c-d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{4\left(c^3+c^2d-c\,d^2-d^3\right)\sqrt{(c+d)(c-d)}}\right)}{f}$
default	$\frac{4a\left(\frac{-\frac{(2c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{4\left(c^2+2cd+d^2\right)}+\frac{(2c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)(c-d)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}+\frac{(2c-d)\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{4\left(c^3+c^2d-c\,d^2-d^3\right)\sqrt{(c+d)(c-d)}}\right)}{f}$
risch	$\frac{ia\left(-3c^3d\,e^{3i(fx+e)}+2c^2d^2e^{3i(fx+e)}+2c\,d^3e^{3i(fx+e)}-2c^4e^{2i(fx+e)}+2c^3d\,e^{2i(fx+e)}-3c^2d^2e^{2i(fx+e)}+4c\,d^3e^{2i(fx+e)}+2\right.}{c^2(-c^2+d^2)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)^2(c}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 4/f*a*((-1/4*(2*c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/4*(2*c-3*d)/(c+d)/(c-d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2+1/4*(2*c-d)/(c^3+c^2*d-c*d^2-d^3)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(118) = 236.

Time = 0.31 (sec) , antiderivative size = 736, normalized size of antiderivative = 5.62

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$
$$=\left[\frac{\left(2\,acd^2-ad^3+(2\,ac^3-ac^2d)\cos(fx+e)^2+2\,(2\,ac^2d-acd^2)\cos(fx+e)\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-}{}\right.}{4\left((c^7+c^6d-2\,c^5d^2-2\,c^4d^3+c^3d^4+c^2d^5)f\cos(fx+\right.}\right.$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*((2*a*c*d^2 - a*d^3 + (2*a*c^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c^6*d - 2*c^5*d^2 - 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6*d + c^5*d^2 - 2*

```
c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^5*d^2 + c^4*d^3
- 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f), 1/2*((2*a*c*d^2 - a*d^3 + (2*a*c
^3 - a*c^2*d)*cos(f*x + e)^2 + 2*(2*a*c^2*d - a*c*d^2)*cos(f*x + e))*sqrt(-
c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (a*c^3*d - 2*a*c^2*d^2 - a*c*d^3 + 2*a*d^4 + (2*a*c^4 - 2*a*c^3
*d - 3*a*c^2*d^2 + 2*a*c*d^3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/((c^7 + c
^6*d - 2*c^5*d^2 - 2*c^4*d^3 + c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^6
*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c
^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = a\left(\int \frac{\sec(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx + \int \frac{\sec^2(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f
*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**2/(c**3 + 3*c**
2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(118) = 236$.

Time = 0.36 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.01

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(2ac-ad)}{(c^3+c^2d-cd^2-d^3)\sqrt{-c^2+d^2}} - \frac{2ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3acd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+ad^2}{(c^3+c^2d-cd^2-d^3}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] ((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x
 + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*a*c - a*d)/((c^3
+ c^2*d - c*d^2 - d^3)*sqrt(-c^2 + d^2)) - (2*a*c^2*tan(1/2*f*x + 1/2*e)^3
- 3*a*c*d*tan(1/2*f*x + 1/2*e)^3 + a*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*c^2*t
an(1/2*f*x + 1/2*e) + a*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2*f*x + 1/
2*e))/((c^3 + c^2*d - c*d^2 - d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*
x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)(2c-d)}{f\,(c+d)^{5/2}\,(c-d)^{3/2}} - \frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(2ac-ad)}{(c+d)^2} - \frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2c-3d)}{(c+d)\,(c-d)}}{f\left(2cd-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(2c^2-2d^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(c^2-2cd+d^2)+c^2+d^2\right)}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] (a*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(2*c - d))/(f*(c
 + d)^(5/2)*(c - d)^(3/2)) - ((tan(e/2 + (f*x)/2)^3*(2*a*c - a*d))/(c + d)^
2 - (a*tan(e/2 + (f*x)/2)*(2*c - 3*d))/((c + d)*(c - d)))/(f*(2*c*d - tan(e
/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2)
+ c^2 + d^2))
```

3.192 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 189

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{a(2c^2-2cd+d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{7/2}f} + \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a(c-4d)(2c-d)\tan(e+fx)}{6(c-d)^2(c+d)^3f(c+d\sec(e+fx))}$$

```
[Out] a*(2*c^2-2*c*d+d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-
d)^(5/2)/(c+d)^(7/2)/f+1/3*a*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^3+1/6*a*(2
*c-3*d)*tan(f*x+e)/(c-d)/(c+d)^2/f/(c+d*sec(f*x+e))^2+1/6*a*(c-4*d)*(2*c-d)
*tan(f*x+e)/(c-d)^2/(c+d)^3/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used

$= \{4088, 12, 3916, 2738, 214\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{a(2c^2-2cd+d^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{7/2}} + \frac{a(c-4d)(2c-d)\tan(e+fx)}{6f(c-d)^2(c+d)^3(c+d\sec(e+fx))} + \frac{a(2c-3d)\tan(e+fx)}{6f(c-d)(c+d)^2(c+d\sec(e+fx))^2} + \frac{a\tan(e+fx)}{3f(c+d)(c+d\sec(e+fx))^3}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a*(2*c^2 - 2*c*d + d^2)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]
])/((c - d)^(5/2)*(c + d)^(7/2)*f) + (a*Tan[e + f*x])/(3*(c + d)*f*(c + d*S
ec[e + f*x])^3) + (a*(2*c - 3*d)*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c +
d*Sec[e + f*x])^2) + (a*(c - 4*d)*(2*c - d)*Tan[e + f*x])/(6*(c - d)^2*(c +
 d)^3*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
```

```
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{\int \frac{\sec(e+fx)(-3a(c-d)-2a(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx}{3\left(c^2-d^2\right)} \\
&= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} \\
&\quad + \frac{\int \frac{\sec(e+fx)(2a(3c-2d)(c-d)+a(2c-3d)(c-d)\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx}{6\left(c^2-d^2\right)^2} \\
&= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} \\
&\quad + \frac{a(c-4d)(2c-d)\tan(e+fx)}{6(c-d)^2(c+d)^3f(c+d\sec(e+fx))} - \frac{\int -\frac{3a(c-d)\left(2c^2-2cd+d^2\right)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{6\left(c^2-d^2\right)^3} \\
&= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} \\
&\quad + \frac{a(c-4d)(2c-d)\tan(e+fx)}{6(c-d)^2(c+d)^3f(c+d\sec(e+fx))} + \frac{\left(a\left(2c^2-2cd+d^2\right)\right)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{2(c-d)^2(c+d)^3} \\
&= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} \\
&\quad + \frac{a(c-4d)(2c-d)\tan(e+fx)}{6(c-d)^2(c+d)^3f(c+d\sec(e+fx))} + \frac{\left(a\left(2c^2-2cd+d^2\right)\right)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{2(c-d)^2d(c+d)^3} \\
&= \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} \\
&\quad + \frac{a(c-4d)(2c-d)\tan(e+fx)}{6(c-d)^2(c+d)^3f(c+d\sec(e+fx))} \\
&\quad + \frac{\left(a\left(2c^2-2cd+d^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(c-d)^2d(c+d)^3f} \\
&= \frac{a\left(2c^2-2cd+d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{7/2}f} + \frac{a\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} \\
&\quad + \frac{a(2c-3d)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a(c-4d)(2c-d)\tan(e+fx)}{6(c-d)^2(c+d)^3f(c+d\sec(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.31

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx =$$
$$-\frac{a(1+\cos(e+fx))\sec^2\left(\frac{1}{2}(e+fx)\right)\left(6(2c^2-2cd+d^2)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+c\cos(e+fx)\right)}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -1/12*(a*(1 + Cos[e + f*x])*Sec[(e + f*x)/2]^2*(6*(2*c^2 - 2*c*d + d^2)*Arc
Tanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x])^3 -
(Sqrt[c^2 - d^2]*(6*c^4 - 12*c^3*d + 2*c^2*d^2 - 15*c*d^3 + 10*d^4 + 6*d*(2
*c^3 - 7*c^2*d + 2*c*d^2 + d^3)*Cos[e + f*x] + (6*c^4 - 12*c^3*d - 2*c^2*d^
2 + 3*c*d^3 + 2*d^4)*Cos[2*(e + f*x)])*Sin[e + f*x])/2))/((c - d)^2*(c + d)
^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{4a\left(\frac{-\frac{(2c^2-2cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4(c^3+3c^2d+3c\,d^2+d^3)}+\frac{(3c^2-6cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3(c-d)(c^2+2cd+d^2)}-\frac{(2c^2-6cd+3d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)(c^2-2cd+d^2)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}+\frac{(2c^2-2cd+d^2)\operatorname{arctanh}\left(\right.}{4(c^5+c^4d-2c^3d^2-2c^2d^3+}\right.}{f}$
default	$\frac{4a\left(\frac{-\frac{(2c^2-2cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{4(c^3+3c^2d+3c\,d^2+d^3)}+\frac{(3c^2-6cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3(c-d)(c^2+2cd+d^2)}-\frac{(2c^2-6cd+3d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)(c^2-2cd+d^2)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}+\frac{(2c^2-2cd+d^2)\operatorname{arctanh}\left(\right.}{4(c^5+c^4d-2c^3d^2-2c^2d^3+}\right.}{f}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f*a*((-1/4*(2*c^2-2*c*d+d^2)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)
^5+1/3*(3*c^2-6*c*d+d^2)/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/4*(2*
c^2-6*c*d+3*d^2)/(c+d)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2
*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3+1/4*(2*c^2-2*c*d+d^2)/(c^5+c^4*d-2*c^
3*d^2-2*c^2*d^3+c*d^4+d^5)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/
2*e)/((c+d)*(c-d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(174) = 348.

Time = 0.36 (sec) , antiderivative size = 1278, normalized size of antiderivative = 6.76

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d + a*c^3*d^
2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*x + e)^2
+ 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^
5 - 4*a*d^6 + (6*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4*a*c^2*
d^4 - 3*a*c*d^5 - 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*
a*c^2*d^4 - 2*a*c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10 + c^9*d -
 3*c^8*d^2 - 3*c^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^7)*f*cos(f
*x + e)^3 + 3*(c^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d^5 + 3*c^4*
d^6 - c^3*d^7 - c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^
4 - 3*c^5*d^5 + 3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f*x + e) + (
c^7*d^3 + c^6*d^4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 -
 d^10)*f), 1/6*(3*(2*a*c^2*d^3 - 2*a*c*d^4 + a*d^5 + (2*a*c^5 - 2*a*c^4*d +
 a*c^3*d^2)*cos(f*x + e)^3 + 3*(2*a*c^4*d - 2*a*c^3*d^2 + a*c^2*d^3)*cos(f*
x + e)^2 + 3*(2*a*c^3*d^2 - 2*a*c^2*d^3 + a*c*d^4)*cos(f*x + e))*sqrt(-c^2
+ d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +
 e))) + (2*a*c^4*d^2 - 9*a*c^3*d^3 + 2*a*c^2*d^4 + 9*a*c*d^5 - 4*a*d^6 + (6
*a*c^6 - 12*a*c^5*d - 8*a*c^4*d^2 + 15*a*c^3*d^3 + 4*a*c^2*d^4 - 3*a*c*d^5
- 2*a*d^6)*cos(f*x + e)^2 + 3*(2*a*c^5*d - 7*a*c^4*d^2 + 8*a*c^2*d^4 - 2*a*
c*d^5 - a*d^6)*cos(f*x + e))*sin(f*x + e))/((c^10 + c^9*d - 3*c^8*d^2 - 3*c
^7*d^3 + 3*c^6*d^4 + 3*c^5*d^5 - c^4*d^6 - c^3*d^7)*f*cos(f*x + e)^3 + 3*(c
^9*d + c^8*d^2 - 3*c^7*d^3 - 3*c^6*d^4 + 3*c^5*d^5 + 3*c^4*d^6 - c^3*d^7 -
c^2*d^8)*f*cos(f*x + e)^2 + 3*(c^8*d^2 + c^7*d^3 - 3*c^6*d^4 - 3*c^5*d^5 +
3*c^4*d^6 + 3*c^3*d^7 - c^2*d^8 - c*d^9)*f*cos(f*x + e) + (c^7*d^3 + c^6*d^
4 - 3*c^5*d^5 - 3*c^4*d^6 + 3*c^3*d^7 + 3*c^2*d^8 - c*d^9 - d^10)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$$
$$= a\left(\int \frac{\sec(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx + \int \frac{\sec^2(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e
+ f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral
(sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**
2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(174) = 348$.

Time = 0.37 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.38

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx =$$
$$-\frac{\frac{3\left(2ac^2-2acd+ad^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^5+c^4d-2c^3d^2-2c^2d^3+cd^4+d^5)\sqrt{-c^2+d^2}} + \frac{6ac^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-18ac^3d\tan\left(\frac{1}{2}fx\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*a*c^2 - 2*a*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2
*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-
c^2 + d^2)))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c^2*d^3 + c*d^4 + d^5)*sqrt(-c^2
 + d^2)) + (6*a*c^4*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^3*d*tan(1/2*f*x + 1/2*e
)^5 + 21*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a*c*d^3*tan(1/2*f*x + 1/2*e)
^5 + 3*a*d^4*tan(1/2*f*x + 1/2*e)^5 - 12*a*c^4*tan(1/2*f*x + 1/2*e)^3 + 24*
a*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 8*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*
c*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*a*d^4*tan(1/2*f*x + 1/2*e)^3 + 6*a*c^4*tan
(1/2*f*x + 1/2*e) - 6*a*c^3*d*tan(1/2*f*x + 1/2*e) - 21*a*c^2*d^2*tan(1/2*f
*x + 1/2*e) + 9*a*d^4*tan(1/2*f*x + 1/2*e))/((c^5 + c^4*d - 2*c^3*d^2 - 2*c
^2*d^3 + c*d^4 + d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2
- c - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.70

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\left(2\,a\,c^2-2\,a\,c\,d+a\,d^2\right)}{(c+d)^3}+\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(2\,c^2-6\,c\,d+3\,d^2\right)}{(c+d)\left(c^2-2\,c\,d+d^2\right)}-\frac{4\,a\tan\left(\frac{e}{2}\right.}{}}{f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3\right)+3\,c\,d^2+\right.}$$
$$+\frac{a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2\,c-2\,d)\left(c^2-2\,c\,d+d^2\right)}{2\sqrt{c+d}\,(c-d)^{5/2}}\right)\left(2\,c^2-2\,c\,d+d^2\right)}{f\,(c+d)^{7/2}\,(c-d)^{5/2}}$$

```
[In] int((a + a/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(2*a*c^2 + a*d^2 - 2*a*c*d))/(c + d)^3 + (a*tan(e/2
+ (f*x)/2)*(2*c^2 - 6*c*d + 3*d^2))/((c + d)*(c^2 - 2*c*d + d^2)) - (4*a*ta
n(e/2 + (f*x)/2)^3*(3*c^2 - 6*c*d + d^2))/(3*(c + d)^2*(c - d)))/(f*(tan(e/
2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(
3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/
2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (a*atanh((tan(e/2 + (f*x
)/2)*(2*c - 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*(c - d)^(5/2)))*(2*c
^2 - 2*c*d + d^2))/(f*(c + d)^(7/2)*(c - d)^(5/2))
```

3.193 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 31, antiderivative size = 327

$$
\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx \\
&= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\operatorname{arctanh}(\sin(e+fx))}{16f} \\
&\quad - \frac{a^2(4c^5-48c^4d-311c^3d^2-448c^2d^3-288cd^4-64d^5)\tan(e+fx)}{60df} \\
&\quad - \frac{a^2(8c^4-96c^3d-438c^2d^2-464cd^3-165d^4)\sec(e+fx)\tan(e+fx)}{240f} \\
&\quad - \frac{a^2(4c^3-48c^2d-123cd^2-64d^3)\,(c+d\sec(e+fx))^2\tan(e+fx)}{120df} \\
&\quad - \frac{a^2(4c^2-48cd-55d^2)\,(c+d\sec(e+fx))^3\tan(e+fx)}{120df} \\
&\quad - \frac{a^2(c-12d)(c+d\sec(e+fx))^4\tan(e+fx)}{30df} + \frac{a^2(c+d\sec(e+fx))^5\tan(e+fx)}{6df}
\end{aligned}
$$

```
[Out] 1/16*a^2*(24*c^4+64*c^3*d+84*c^2*d^2+48*c*d^3+11*d^4)*arctanh(sin(f*x+e))/f
-1/60*a^2*(4*c^5-48*c^4*d-311*c^3*d^2-448*c^2*d^3-288*c*d^4-64*d^5)*tan(f*x
+e)/d/f-1/240*a^2*(8*c^4-96*c^3*d-438*c^2*d^2-464*c*d^3-165*d^4)*sec(f*x+e)
*tan(f*x+e)/f-1/120*a^2*(4*c^3-48*c^2*d-123*c*d^2-64*d^3)*(c+d*sec(f*x+e))^
2*tan(f*x+e)/d/f-1/120*a^2*(4*c^2-48*c*d-55*d^2)*(c+d*sec(f*x+e))^3*tan(f*x
+e)/d/f-1/30*a^2*(c-12*d)*(c+d*sec(f*x+e))^4*tan(f*x+e)/d/f+1/6*a^2*(c+d*se
c(f*x+e))^5*tan(f*x+e)/d/f
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4072, 102, 158, 152, 52, 65, 223, 209}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$$
$$= \frac{a^3(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)}{16f}$$
$$+\frac{(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{48f}$$
$$+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2\left(d(48c^2+32cd+19d^2)\sec(e+fx)+2(52c^3+56c^2d+48cd^2+9d^3)\right)}{120f}$$
$$+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^3}{6f}$$
$$+\frac{d(9c+2d)\tan(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^2}{30f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*Tan[e + f*x])/(16
*f) + (a^3*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*ArcTan[Sqrt
[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(8*f*Sqrt[a
- a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((24*c^4 + 64*c^3*d + 84*c^2*
d^2 + 48*c*d^3 + 11*d^4)*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(48*f) + (d
*(9*c + 2*d)*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3
0*f) + (d*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(6*f)
 + (d*(a + a*Sec[e + f*x])^2*(2*(52*c^3 + 56*c^2*d + 48*c*d^2 + 9*d^3) + d*
(48*c^2 + 32*c*d + 19*d^2)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{3/2}(c+dx)^4}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} + \frac{\tan(e+fx)\text{Subst}\left(\int\frac{(a+ax)^{3/2}(c+dx)^2\left(-a^2\left(6c^2+2cd+3d^2\right)-a^2d(9c+2d)x\right)}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{6f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} - \frac{\tan(e+fx)\text{Subst}\left(\int\frac{(a+ax)^{3/2}(c+dx)\left(a^4\left(30c^3+28c^2d+37cd^2+4d^3\right)+a^4d\left(48c^2+32cd+19d^2\right)x\right)}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{30a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\left(2\left(52c^3+56c^2d+48cd^2+9d^3\right)+d\left(48c^2+32cd+19d^2\right)\sec(e+fx)\right)\tan(}{120f} - \frac{\left(a^2\left(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4\right)\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{24f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\,(a^2+a^2\sec(e+fx))\tan(e+fx)}{48f} + \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\,(2(52c^3+56c^2d+48cd^2+9d^3)+d(48c^2+32cd+19d^2)\sec(e+fx))\tan}{120f} - \frac{(a^3(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx))\,\text{Subst}\left(\int\frac{\sqrt{a+ax}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{16f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)}{16f} + \frac{(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\,(a^2+a^2\sec(e+fx))\tan(e+fx)}{48f} + \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\,(2(52c^3+56c^2d+48cd^2+9d^3)+d(48c^2+32cd+19d^2)\sec(e+fx))\tan}{120f} - \frac{(a^4(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx\right.}{16f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)}{16f} + \frac{(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\,(a^2+a^2\sec(e+fx))\tan(e+fx)}{48f} + \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\,(2(52c^3+56c^2d+48cd^2+9d^3)+d(48c^2+32cd+19d^2)\sec(e+fx))\tan}{120f} + \frac{(a^3(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+\right.}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)}{16f} + \frac{(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{48f} + \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\left(2(52c^3+56c^2d+48cd^2+9d^3)+d(48c^2+32cd+19d^2)\sec(e+fx)\right)\tan(}{120f} + \frac{\left(a^3(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\tan(e+fx)}{16f} + \frac{a^3(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{48f} + \frac{d(9c+2d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\left(2(52c^3+56c^2d+48cd^2+9d^3)+d(48c^2+32cd+19d^2)\sec(e+fx)\right)\tan(}{120f}$$

Mathematica [A] (verified)

Time = 6.19 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.55

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = \frac{a^2(15(24c^4+64c^3d+84c^2d^2+48cd^3+11d^4)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(480(c+d)^4+15(8c^4+\right.}{}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(15*(24*c^4 + 64*c^3*d + 84*c^2*d^2 + 48*c*d^3 + 11*d^4)*ArcTanh[Sin[e
 + f*x]] + Tan[e + f*x]*(480*(c + d)^4 + 15*(8*c^4 + 64*c^3*d + 84*c^2*d^2
+ 48*c*d^3 + 11*d^4)*Sec[e + f*x] + 10*d^2*(36*c^2 + 48*c*d + 11*d^2)*Sec[e
 + f*x]^3 + 40*d^4*Sec[e + f*x]^5 + 320*d*(c + d)^3*Tan[e + f*x]^2 + 96*d^3
*(2*c + d)*Tan[e + f*x]^4)))/(240*f)
```

Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.10

method	result
parts	$-\frac{(4a^2d^3c+2a^2d^4)\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}+\frac{(2c^4a^2+4a^2c^3d)\tan(fx+e)}{f}+\frac{(6a^2c^2d^2+8a^2d}{}$
norman	$\frac{\frac{17a^2\left(24c^4+64c^3d+84c^2d^2+48c\,d^3+11d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{24f}-\frac{a^2\left(24c^4+64c^3d+84c^2d^2+48c\,d^3+11d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{8f}+\frac{a^2\left(40c^4+192\right.}{}}{}$
parallelrisch	$\frac{2a^2\left(-\frac{45\left(c^4+\frac{8}{3}c^3d+\frac{7}{2}c^2d^2+2c\,d^3+\frac{11}{24}d^4\right)\left(\frac{2\cos(4fx+4e)}{5}+\frac{2}{3}+\cos(2fx+2e)+\frac{\cos(6fx+6e)}{15}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{45\left(c^4+\frac{8}{3}c^3d\right.}{}\right.}{}$
derivativedivides	$\frac{c^4a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4a^2c^3d\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+6a^2c^2d^2\left(-\left(-\frac{\sec(fx+e)}{4}\right.\right.}{}$
default	$\frac{c^4a^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4a^2c^3d\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+6a^2c^2d^2\left(-\left(-\frac{\sec(fx+e)}{4}\right.\right.}{}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] -(4*a^2*c*d^3+2*a^2*d^4)/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f
*x+e)+(2*a^2*c^4+4*a^2*c^3*d)/f*tan(f*x+e)+(6*a^2*c^2*d^2+8*a^2*c*d^3+a^2*d
^4)/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan
(f*x+e)))-(4*a^2*c^3*d+12*a^2*c^2*d^2+4*a^2*c*d^3)/f*(-2/3-1/3*sec(f*x+e)^2
)*tan(f*x+e)+(a^2*c^4+8*a^2*c^3*d+6*a^2*c^2*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+
e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+a^2*d^4/f*(-(-1/6*sec(f*x+e)^5-5/24*sec(f
*x+e)^3-5/16*sec(f*x+e))*tan(f*x+e)+5/16*ln(sec(f*x+e)+tan(f*x+e)))+1/f*ln(
sec(f*x+e)+tan(f*x+e))*a^2*c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.18

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$$
$$=\frac{15\left(24\,a^2c^4+64\,a^2c^3d+84\,a^2c^2d^2+48\,a^2cd^3+11\,a^2d^4\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\left(24\,a^2c^4\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a
^2*d^4)*cos(f*x + e)^6*log(sin(f*x + e) + 1) - 15*(24*a^2*c^4 + 64*a^2*c^3*
d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^6*log(-sin(f*x
 + e) + 1) + 2*(40*a^2*d^4 + 32*(15*a^2*c^4 + 50*a^2*c^3*d + 60*a^2*c^2*d^2
 + 36*a^2*c*d^3 + 8*a^2*d^4)*cos(f*x + e)^5 + 15*(8*a^2*c^4 + 64*a^2*c^3*d
+ 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^4 + 64*(5*a^2*c^
3*d + 15*a^2*c^2*d^2 + 9*a^2*c*d^3 + 2*a^2*d^4)*cos(f*x + e)^3 + 10*(36*a^2
*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*cos(f*x + e)^2 + 96*(2*a^2*c*d^3 + a^
2*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = a^2\left(\int c^4\sec(e+fx)\,dx + \int 2c^4\sec^2(e+fx)\,dx + \int c^4\sec^3(e+fx)\,dx + \int d^4\sec^5(e+fx)\,dx + \int 2d^4\sec^6(e+fx)\,dx + \int d^4\sec^7(e+fx)\,dx + \int 4cd^3\sec^4(e+fx)\,dx + \int 8cd^3\sec^5(e+fx)\,dx + \int 4cd^3\sec^6(e+fx)\,dx + \int 6c^2d^2\sec^3(e+fx)\,dx + \int 12c^2d^2\sec^4(e+fx)\,dx + \int 6c^2d^2\sec^5(e+fx)\,dx + \int 4c^3d\sec^2(e+fx)\,dx + \int 8c^3d\sec^3(e+fx)\,dx + \int 4c^3d\sec^4(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(c**4*sec(e + f*x), x) + Integral(2*c**4*sec(e + f*x)**2, x)
+ Integral(c**4*sec(e + f*x)**3, x) + Integral(d**4*sec(e + f*x)**5, x) + I
ntegral(2*d**4*sec(e + f*x)**6, x) + Integral(d**4*sec(e + f*x)**7, x) + In
tegral(4*c*d**3*sec(e + f*x)**4, x) + Integral(8*c*d**3*sec(e + f*x)**5, x)
 + Integral(4*c*d**3*sec(e + f*x)**6, x) + Integral(6*c**2*d**2*sec(e + f*x
)**3, x) + Integral(12*c**2*d**2*sec(e + f*x)**4, x) + Integral(6*c**2*d**2
*sec(e + f*x)**5, x) + Integral(4*c**3*d*sec(e + f*x)**2, x) + Integral(8*c
**3*d*sec(e + f*x)**3, x) + Integral(4*c**3*d*sec(e + f*x)**4, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(313) = 626$.

Time = 0.22 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.09

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$$

$$= \frac{640\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^3d+1920\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^2d^2+128\left(3\,\tan\left(f\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] 1/480*(640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^3*d + 1920*(tan(f*x + e)
^3 + 3*tan(f*x + e))*a^2*c^2*d^2 + 128*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^
3 + 15*tan(f*x + e))*a^2*c*d^3 + 640*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*
c*d^3 + 64*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^2*d^4
 - 5*a^2*d^4*(2*(15*sin(f*x + e)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(
sin(f*x + e)^6 - 3*sin(f*x + e)^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x
+ e) + 1) + 15*log(sin(f*x + e) - 1)) - 180*a^2*c^2*d^2*(2*(3*sin(f*x + e)^
3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x
 + e) + 1) + 3*log(sin(f*x + e) - 1)) - 240*a^2*c*d^3*(2*(3*sin(f*x + e)^3
- 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x +
 e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a^2*d^4*(2*(3*sin(f*x + e)^3 - 5*s
in(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) +
 1) + 3*log(sin(f*x + e) - 1)) - 120*a^2*c^4*(2*sin(f*x + e)/(sin(f*x + e)^
2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 960*a^2*c^3*d*(2*
sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e
) - 1)) - 720*a^2*c^2*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*
x + e) + 1) + log(sin(f*x + e) - 1)) + 480*a^2*c^4*log(sec(f*x + e) + tan(f
*x + e)) + 960*a^2*c^4*tan(f*x + e) + 1920*a^2*c^3*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(313) = 626$.

Time = 0.41 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.25

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(24*a^2*c^4 + 64*a^2*c^3*d + 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a
^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(24*a^2*c^4 + 64*a^2*c^3*d
+ 84*a^2*c^2*d^2 + 48*a^2*c*d^3 + 11*a^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e)
- 1)) - 2*(360*a^2*c^4*tan(1/2*f*x + 1/2*e)^11 + 960*a^2*c^3*d*tan(1/2*f*x
+ 1/2*e)^11 + 1260*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^11 + 720*a^2*c*d^3*tan(
1/2*f*x + 1/2*e)^11 + 165*a^2*d^4*tan(1/2*f*x + 1/2*e)^11 - 2040*a^2*c^4*ta
n(1/2*f*x + 1/2*e)^9 - 5440*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^9 - 7140*a^2*c^2
*d^2*tan(1/2*f*x + 1/2*e)^9 - 4080*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^9 - 935*a
^2*d^4*tan(1/2*f*x + 1/2*e)^9 + 4560*a^2*c^4*tan(1/2*f*x + 1/2*e)^7 + 13440
*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^7 + 15480*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^
7 + 10272*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^7 + 1986*a^2*d^4*tan(1/2*f*x + 1/2
*e)^7 - 5040*a^2*c^4*tan(1/2*f*x + 1/2*e)^5 - 17280*a^2*c^3*d*tan(1/2*f*x +
 1/2*e)^5 - 19080*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 11232*a^2*c*d^3*tan(
1/2*f*x + 1/2*e)^5 - 3006*a^2*d^4*tan(1/2*f*x + 1/2*e)^5 + 2760*a^2*c^4*tan
(1/2*f*x + 1/2*e)^3 + 11200*a^2*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 13980*a^2*c^
2*d^2*tan(1/2*f*x + 1/2*e)^3 + 7440*a^2*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 1305
*a^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 600*a^2*c^4*tan(1/2*f*x + 1/2*e) - 2880*a
^2*c^3*d*tan(1/2*f*x + 1/2*e) - 4500*a^2*c^2*d^2*tan(1/2*f*x + 1/2*e) - 312
0*a^2*c*d^3*tan(1/2*f*x + 1/2*e) - 795*a^2*d^4*tan(1/2*f*x + 1/2*e))/(tan(1
/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.48

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^4\,dx$$

$$= \frac{\left(-3\,a^2\,c^4-8\,a^2\,c^3\,d-\frac{21\,a^2\,c^2\,d^2}{2}-6\,a^2\,c\,d^3-\frac{11\,a^2\,d^4}{8}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\left(17\,a^2\,c^4+\frac{136\,a^2\,c^3\,d}{3}+\frac{119\,a^2\,c^2\,d^2}{2}+3\right.}{}$$

$$+\frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(24\,c^4+64\,c^3\,d+84\,c^2\,d^2+48\,c\,d^3+11\,d^4\right)}{4\,\left(6\,c^4+16\,c^3\,d+21\,c^2\,d^2+12\,c\,d^3+\frac{11\,d^4}{4}\right)}\right)\,\left(24\,c^4+64\,c^3\,d+84\,c^2\,d^2+48\,c\,d^3+11\,d^4\right)}{8\,f}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(5*a^2*c^4 + (53*a^2*d^4)/8 + 26*a^2*c*d^3 + 24*a^2*c^3
*d + (75*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^11*(3*a^2*c^4 + (11*a^2*d^4)/
8 + 6*a^2*c*d^3 + 8*a^2*c^3*d + (21*a^2*c^2*d^2)/2) + tan(e/2 + (f*x)/2)^9*
(17*a^2*c^4 + (187*a^2*d^4)/24 + 34*a^2*c*d^3 + (136*a^2*c^3*d)/3 + (119*a^
2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^3*(23*a^2*c^4 + (87*a^2*d^4)/8 + 62*a^2*
c*d^3 + (280*a^2*c^3*d)/3 + (233*a^2*c^2*d^2)/2) - tan(e/2 + (f*x)/2)^7*(38
*a^2*c^4 + (331*a^2*d^4)/20 + (428*a^2*c*d^3)/5 + 112*a^2*c^3*d + 129*a^2*c
^2*d^2) + tan(e/2 + (f*x)/2)^5*(42*a^2*c^4 + (501*a^2*d^4)/20 + (468*a^2*c*
```

```
d^3)/5 + 144*a^2*c^3*d + 159*a^2*c^2*d^2))/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*
tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 -
6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (a^2*atanh((tan(e/2
 + (f*x)/2)*(48*c*d^3 + 64*c^3*d + 24*c^4 + 11*d^4 + 84*c^2*d^2))/(4*(12*c*
d^3 + 16*c^3*d + 6*c^4 + (11*d^4)/4 + 21*c^2*d^2)))*(48*c*d^3 + 64*c^3*d +
24*c^4 + 11*d^4 + 84*c^2*d^2))/(8*f)
```

3.194 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 31, antiderivative size = 242

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx\\&=\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\operatorname{arctanh}(\sin(e+fx))}{8f}\\&\quad-\frac{a^2(c^4-10c^3d-44c^2d^2-40cd^3-12d^4)\tan(e+fx)}{10df}\\&\quad-\frac{a^2(2c^3-20c^2d-57cd^2-30d^3)\sec(e+fx)\tan(e+fx)}{40f}\\&\quad-\frac{a^2\left(c^2-10cd-12d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{20df}\\&\quad-\frac{a^2(c-10d)(c+d\sec(e+fx))^3\tan(e+fx)}{20df}+\frac{a^2(c+d\sec(e+fx))^4\tan(e+fx)}{5df}\end{aligned}$$

```
[Out] 3/8*a^2*(2*c+d)*(2*c^2+3*c*d+2*d^2)*arctanh(sin(f*x+e))/f-1/10*a^2*(c^4-10*
c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)*tan(f*x+e)/d/f-1/40*a^2*(2*c^3-20*c^2*d-5
7*c*d^2-30*d^3)*sec(f*x+e)*tan(f*x+e)/f-1/20*a^2*(c^2-10*c*d-12*d^2)*(c+d*s
ec(f*x+e))^2*tan(f*x+e)/d/f-1/20*a^2*(c-10*d)*(c+d*sec(f*x+e))^3*tan(f*x+e)
/d/f+1/5*a^2*(c+d*sec(f*x+e))^4*tan(f*x+e)/d/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 102, 152, 52, 65, 223, 209}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$
$$= \frac{3a^3(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f}$$
$$+\frac{(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{8f}$$
$$+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2\left(2\left(8c^2+5cd+2d^2\right)+d(7c+2d)\sec(e+fx)\right)}{20f}$$
$$+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^2}{5f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (3*a^2*(2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*Tan[e + f*x])/(8*f) + (3*a^3*(2*c
+ d)*(2*c^2 + 3*c*d + 2*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Se
c[e + f*x])]]*Tan[e + f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e
+ f*x]]) + ((2*c + d)*(2*c^2 + 3*c*d + 2*d^2)*(a^2 + a^2*Sec[e + f*x])*Tan[
e + f*x])/(8*f) + (d*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2*Tan[e +
f*x])/(5*f) + (d*(a + a*Sec[e + f*x])^2*(2*(8*c^2 + 5*c*d + 2*d^2) + d*(7*c
 + 2*d)*Sec[e + f*x])*Tan[e + f*x])/(20*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)^3}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(a+ax)^{3/2}(c+dx)\left(-a^2\left(5c^2+2cd+2d^2\right)-a^2d(7c+2d)x\right)}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{d(a+a\sec(e+fx))^2\left(2\left(8c^2+5cd+2d^2\right)+d(7c+2d)\sec(e+fx)\right)\tan(e+fx)}{20f} - \frac{\left(a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c+d)\left(2c^2+3cd+2d^2\right)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{d(a+a\sec(e+fx))^2\left(2\left(8c^2+5cd+2d^2\right)+d(7c+2d)\sec(e+fx)\right)\tan(e+fx)}{20f} - \frac{\left(3a^3(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+ax}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d)\left(2c^2+3cd+2d^2\right)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{d(a+a\sec(e+fx))^2\left(2\left(8c^2+5cd+2d^2\right)+d(7c+2d)\sec(e+fx)\right)\tan(e+fx)}{20f} - \frac{\left(3a^4(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d)\left(2c^2+3cd+2d^2\right)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{d(a+a\sec(e+fx))^2\left(2\left(8c^2+5cd+2d^2\right)+d(7c+2d)\sec(e+fx)\right)\tan(e+fx)}{20f} + \frac{\left(3a^3(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{(2c+d)\left(2c^2+3cd+2d^2\right)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{d(a+a\sec(e+fx))^2\left(2\left(8c^2+5cd+2d^2\right)+d(7c+2d)\sec(e+fx)\right)\tan(e+fx)}{20f} + \frac{\left(3a^3(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{3a^2(2c+d)\left(2c^2+3cd+2d^2\right)\tan(e+fx)}{8f} + \frac{3a^3(2c+d)\left(2c^2+3cd+2d^2\right)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(2c+d)\left(2c^2+3cd+2d^2\right)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{8f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{5f} + \frac{d(a+a\sec(e+fx))^2\left(2\left(8c^2+5cd+2d^2\right)+d(7c+2d)\sec(e+fx)\right)\tan(e+fx)}{20f}$$

Mathematica [A] (verified)

Time = 3.45 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.58

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$
$$=\frac{a^2(15(4c^3+8c^2d+7cd^2+2d^3)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(5(4c^3+24c^2d+21cd^2+6d^3)\sec(e\right.}{40f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*(15*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3)*ArcTanh[Sin[e + f*x]] + Tan[e
+ f*x]*(5*(4*c^3 + 24*c^2*d + 21*c*d^2 + 6*d^3)*Sec[e + f*x] + 10*d^2*(3*c
+ 2*d)*Sec[e + f*x]^3 + 8*(10*(c + d)^3 + 5*d*(c + d)^2*Tan[e + f*x]^2 + d^
3*Tan[e + f*x]^4))))/(40*f)
```

Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.11

method	result
parts	$\frac{(3a^2cd^2+2a^2d^3)\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{(2a^2c^3+3a^2c^2d)\tan(fx}{f}$
norman	$\frac{\frac{7a^2\left(4c^3+8c^2d+7cd^2+2d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{2f}-\frac{3a^2\left(4c^3+8c^2d+7cd^2+2d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{4f}-\frac{8a^2\left(15c^3+35c^2d+25cd^2+9d^3\right)\tan\left(\frac{fx}{2}+\right.}{5f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^5}$
parallelrisch	$\frac{4\left(-\frac{15\left(c+\frac{d}{2}\right)\left(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}+\cos(fx+e)\right)\left(c^2+\frac{3}{2}cd+d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{15\left(c+\frac{d}{2}\right)\left(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+}{2}\right.}{}\right.}{}$
derivativedivides	$\frac{a^2c^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-3a^2c^2d\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+3a^2cd^2\left(-\left(-\frac{\sec(fx+e)^3}{4}\right.\right.}{}$
default	$\frac{a^2c^3\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-3a^2c^2d\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+3a^2cd^2\left(-\left(-\frac{\sec(fx+e)^3}{4}\right.\right.}{}$
risch	$-\frac{ia^2\left(-80c^3-160cd^2-48d^3-200c^2d-105cd^2e^{i(fx+e)}-800cd^2e^{2i(fx+e)}-330cd^2e^{3i(fx+e)}-480cd^2e^{6i(fx+e)}-120c^2\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] (3*a^2*c*d^2+2*a^2*d^3)/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3
/8*ln(sec(f*x+e)+tan(f*x+e)))+(2*a^2*c^3+3*a^2*c^2*d)/f*tan(f*x+e)-(3*a^2*c
^2*d+6*a^2*c*d^2+a^2*d^3)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(a^2*c^3+6*a
^2*c^2*d+3*a^2*c*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*
```

```
x+e))+1/f*ln(sec(f*x+e)+tan(f*x+e))*a^2*c^3-a^2*d^3/f*(-8/15-1/5*sec(f*x+e
)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.21

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$
$$=\frac{15\,(4\,a^2c^3+8\,a^2c^2d+7\,a^2cd^2+2\,a^2d^3)\cos{(fx+e)}^5\log{(\sin{(fx+e)}+1)}-15\,(4\,a^2c^3+8\,a^2c^2d+7\,a^2cd^2}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/80*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*cos(f*x + e)^5
*log(sin(f*x + e) + 1) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*
d^3)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(8*a^2*d^3 + 8*(10*a^2*c^3 +
 25*a^2*c^2*d + 20*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^4 + 5*(4*a^2*c^3 + 2
4*a^2*c^2*d + 21*a^2*c*d^2 + 6*a^2*d^3)*cos(f*x + e)^3 + 8*(5*a^2*c^2*d + 1
0*a^2*c*d^2 + 3*a^2*d^3)*cos(f*x + e)^2 + 10*(3*a^2*c*d^2 + 2*a^2*d^3)*cos(
f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$
$$=a^2\left(\int c^3\sec{(e+fx)}\,dx+\int 2c^3\sec^2{(e+fx)}\,dx+\int c^3\sec^3{(e+fx)}\,dx+\int d^3\sec^4{(e+fx)}\,dx+\int 2d^3\sec^5{(e+fx)}\,dx+\int d^3\sec^6{(e+fx)}\,dx+\int 3cd^2\sec^3{(e+fx)}\,dx+\int 6cd^2\sec^4{(e+fx)}\,dx+\int 3cd^2\sec^5{(e+fx)}\,dx+\int 3c^2d\sec^2{(e+fx)}\,dx+\int 6c^2d\sec^3{(e+fx)}\,dx+\int 3c^2d\sec^4{(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**2*(Integral(c**3*sec(e + f*x), x) + Integral(2*c**3*sec(e + f*x)**2, x)
+ Integral(c**3*sec(e + f*x)**3, x) + Integral(d**3*sec(e + f*x)**4, x) + I
ntegral(2*d**3*sec(e + f*x)**5, x) + Integral(d**3*sec(e + f*x)**6, x) + In
tegral(3*c*d**2*sec(e + f*x)**3, x) + Integral(6*c*d**2*sec(e + f*x)**4, x)
```

```
 + Integral(3*c*d**2*sec(e + f*x)**5, x) + Integral(3*c**2*d*sec(e + f*x)**
2, x) + Integral(6*c**2*d*sec(e + f*x)**3, x) + Integral(3*c**2*d*sec(e + f
*x)**4, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(230) = 460$.

Time = 0.22 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.94

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$

$$= \frac{240\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2c^2d+480\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)a^2cd^2+16\left(3\,\tan\left(fx-\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c^2*d + 480*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a^2*c*d^2 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 +
15*tan(f*x + e))*a^2*d^3 + 80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d^3 - 4
5*a^2*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(
f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 30*a
^2*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
 e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 60*a^2*c^
3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*
x + e) - 1)) - 360*a^2*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin
(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 180*a^2*c*d^2*(2*sin(f*x + e)/(si
n(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a^
2*c^3*log(sec(f*x + e) + tan(f*x + e)) + 480*a^2*c^3*tan(f*x + e) + 720*a^2
*c^2*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(230) = 460$.

Time = 0.39 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.09

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx$$

$$= \frac{15\left(4\,a^2c^3+8\,a^2c^2d+7\,a^2cd^2+2\,a^2d^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\left(4\,a^2c^3+8\,a^2c^2d+7\,a^2cd^2+2\,a\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/40*(15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^3)*log(abs(tan(1/
2*f*x + 1/2*e) + 1)) - 15*(4*a^2*c^3 + 8*a^2*c^2*d + 7*a^2*c*d^2 + 2*a^2*d^
3)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(60*a^2*c^3*tan(1/2*f*x + 1/2*e)^
9 + 120*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^9 + 105*a^2*c*d^2*tan(1/2*f*x + 1/2*
e)^9 + 30*a^2*d^3*tan(1/2*f*x + 1/2*e)^9 - 280*a^2*c^3*tan(1/2*f*x + 1/2*e)
^7 - 560*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^7 - 490*a^2*c*d^2*tan(1/2*f*x + 1/2
*e)^7 - 140*a^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 480*a^2*c^3*tan(1/2*f*x + 1/2*
e)^5 + 1120*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 800*a^2*c*d^2*tan(1/2*f*x +
1/2*e)^5 + 288*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 360*a^2*c^3*tan(1/2*f*x + 1
/2*e)^3 - 1040*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - 790*a^2*c*d^2*tan(1/2*f*x
 + 1/2*e)^3 - 180*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 100*a^2*c^3*tan(1/2*f*x
+ 1/2*e) + 360*a^2*c^2*d*tan(1/2*f*x + 1/2*e) + 375*a^2*c*d^2*tan(1/2*f*x +
 1/2*e) + 130*a^2*d^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^5)
/f
```

Mupad [B] (verification not implemented)

Time = 17.06 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.63

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\,dx = \frac{3\,a^2\,\mathrm{atanh}\left(\frac{3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c+d)\,(2\,c^2+3\,c\,d+2\,d^2)}{2\,\left(6\,c^3+12\,c^2\,d+\frac{21\,c\,d^2}{2}+3\,d^3\right)}\right)\,(2\,c+d)\,(2\,c^2+3\,c\,d+2\,d^2)}{4\,f} - \frac{\left(3\,a^2\,c^3+6\,a^2\,c^2\,d+\frac{21\,a^2\,c\,d^2}{4}+\frac{3\,a^2\,d^3}{2}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+\left(-14\,a^2\,c^3-28\,a^2\,c^2\,d-\frac{49\,a^2\,c\,d^2}{2}-7\,a^2\,d^3\right)\,\tan(\cdots}{f\,\left(\tan(\cdots\right.}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (3*a^2*atanh((3*tan(e/2 + (f*x)/2)*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2))/(2*((
21*c*d^2)/2 + 12*c^2*d + 6*c^3 + 3*d^3)))*(2*c + d)*(3*c*d + 2*c^2 + 2*d^2)
)/(4*f) - (tan(e/2 + (f*x)/2)^9*(3*a^2*c^3 + (3*a^2*d^3)/2 + (21*a^2*c*d^2)
/4 + 6*a^2*c^2*d) - tan(e/2 + (f*x)/2)^7*(14*a^2*c^3 + 7*a^2*d^3 + (49*a^2*
c*d^2)/2 + 28*a^2*c^2*d) - tan(e/2 + (f*x)/2)^3*(18*a^2*c^3 + 9*a^2*d^3 + (
79*a^2*c*d^2)/2 + 52*a^2*c^2*d) + tan(e/2 + (f*x)/2)^5*(24*a^2*c^3 + (72*a^
2*d^3)/5 + 40*a^2*c*d^2 + 56*a^2*c^2*d) + tan(e/2 + (f*x)/2)*(5*a^2*c^3 + (
13*a^2*d^3)/2 + (75*a^2*c*d^2)/4 + 18*a^2*c^2*d))/(f*(5*tan(e/2 + (f*x)/2)^
2 - 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2
)^8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.195 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 176

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx &= \frac{a^2(12c^2+16cd+7d^2)\operatorname{arctanh}(\sin(e+fx))}{8f} - \frac{a^2(c^3-8c^2d-20cd^2-8d^3)\tan(e+fx)}{6df} \\ &\quad - \frac{a^2(2c(c-8d)-21d^2)\sec(e+fx)\tan(e+fx)}{24f} \\ &\quad - \frac{a^2(c-8d)(c+d\sec(e+fx))^2\tan(e+fx)}{12df} + \frac{a^2(c+d\sec(e+fx))^3\tan(e+fx)}{4df}\end{aligned}$$

```
[Out] 1/8*a^2*(12*c^2+16*c*d+7*d^2)*arctanh(sin(f*x+e))/f-1/6*a^2*(c^3-8*c^2*d-20
*c*d^2-8*d^3)*tan(f*x+e)/d/f-1/24*a^2*(2*c*(c-8*d)-21*d^2)*sec(f*x+e)*tan(f
*x+e)/f-1/12*a^2*(c-8*d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/d/f+1/4*a^2*(c+d*sec
(f*x+e))^3*tan(f*x+e)/d/f
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used

$= \{4072, 92, 81, 52, 65, 223, 209\}$

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$
$$= \frac{a^3(12c^2+16cd+7d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{(12c^2+16cd+7d^2)\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{24f} + \frac{d(5c+2d)\tan(e+fx)(a\sec(e+fx)+a)^2}{12f} + \frac{d\tan(e+fx)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))}{4f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2 + 16*c*d + 7*d^2)*Tan[e + f*x])/(8*f) + (a^3*(12*c^2 + 16*c*d
+ 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e
+ f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (d*(5*c +
 2*d)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + ((12*c^2 + 16*c*d + 7*d
^2)*(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(24*f) + (d*(a + a*Sec[e + f*x])
^2*(c + d*Sec[e + f*x])*Tan[e + f*x])/(4*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{3/2}(c+dx)^2}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f}\\ &\quad+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(a+ax)^{3/2}\left(-a^2\left(4c^2+2cd+d^2\right)-a^2d(5c+2d)x\right)}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

$$= \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f} - \frac{\left(a^2(12c^2+16cd+7d^2)\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{12f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{(12c^2+16cd+7d^2)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f} - \frac{\left(a^3(12c^2+16cd+7d^2)\tan(e+fx)\right)\text{Subst}\left(\int\frac{\sqrt{a+ax}}{\sqrt{a-ax}}\,dx,x,\sec(e+fx)\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{(12c^2+16cd+7d^2)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f} - \frac{\left(a^4(12c^2+16cd+7d^2)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{(12c^2+16cd+7d^2)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f} + \frac{\left(a^3(12c^2+16cd+7d^2)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{(12c^2+16cd+7d^2)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f} + \frac{(a^3(12c^2+16cd+7d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^2(12c^2+16cd+7d^2)\tan(e+fx)}{8f} + \frac{a^3(12c^2+16cd+7d^2)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{d(5c+2d)(a+a\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{(12c^2+16cd+7d^2)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^2(c+d\sec(e+fx))\tan(e+fx)}{4f}$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.57

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$
$$= \frac{a^2(3(12c^2+16cd+7d^2)\,\text{arctanh}(\sin(e+fx)) + \tan(e+fx)\left(3(4c^2+16cd+7d^2)\sec(e+fx)+6d^2\sec^3(\right.}{24f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(3*(12*c^2 + 16*c*d + 7*d^2)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(3*(
4*c^2 + 16*c*d + 7*d^2)*Sec[e + f*x] + 6*d^2*Sec[e + f*x]^3 + 16*(c + d)*(3
*(c + d) + d*Tan[e + f*x]^2))))/(24*f)
```

Maple [A] (verified)

Time = 4.36 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

method	result
parts	$-\frac{(2a^2cd+2a^2d^2)\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{(2a^2c^2+2a^2cd)\tan(fx+e)}{f}+\frac{(a^2c^2+4a^2cd+a^2d^2)\left(\frac{\sec(fx+e)\tan(}{2}\right.}{f}$
parallelrisch	$\frac{4a^2\left(-\frac{3\left(c^2+\frac{4}{3}cd+\frac{7}{12}d^2\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{3\left(c^2+\frac{4}{3}cd+\frac{7}{12}d^2\right)\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2\right.}{2}\right.}{f(3+\cos(4}$
norman	$\frac{\frac{11a^2(12c^2+16cd+7d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}-\frac{a^2(12c^2+16cd+7d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{a^2(20c^2+48cd+25d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}-\frac{a^2(156c^2+272cd}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}$
derivativedivides	$\frac{a^2c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-2a^2cd\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+a^2d^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\,\mathrm{se}}{}\right.\right.}{}$
default	$\frac{a^2c^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-2a^2cd\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+a^2d^2\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\,\mathrm{se}}{}\right.\right.}{}$
risch	$-\frac{ia^2(12c^2e^{7i(fx+e)}+48cd\,e^{7i(fx+e)}+21d^2e^{7i(fx+e)}-48c^2e^{6i(fx+e)}-48cd\,e^{6i(fx+e)}+12c^2e^{5i(fx+e)}+48cd\,e^{5i(fx+e)}+45}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] -(2*a^2*c*d+2*a^2*d^2)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(2*a^2*c^2+2*a^
2*c*d)/f*tan(f*x+e)+(a^2*c^2+4*a^2*c*d+a^2*d^2)/f*(1/2*sec(f*x+e)*tan(f*x+e
)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+1/f*ln(sec(f*x+e)+tan(f*x+e))*a^2*c^2+a^2*
d^2/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan
(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.19

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$
$$=\frac{3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)\cos\left(fx+\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(sin(f*x +
e) + 1) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^4*log(-sin(f
```

x + e) + 1) + 2(6*a^2*d^2 + 16*(3*a^2*c^2 + 5*a^2*c*d + 2*a^2*d^2)*cos(f*
x + e)^3 + 3*(4*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*cos(f*x + e)^2 + 16*(a^2*
c*d + a^2*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)

Sympy [F]

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx\\&=a^2\left(\int c^2\sec{(e+fx)}\,dx+\int 2c^2\sec^2{(e+fx)}\,dx+\int c^2\sec^3{(e+fx)}\,dx\right.\\&\quad+\int d^2\sec^3{(e+fx)}\,dx+\int 2d^2\sec^4{(e+fx)}\,dx+\int d^2\sec^5{(e+fx)}\,dx\\&\quad\left.+\int 2cd\sec^2{(e+fx)}\,dx+\int 4cd\sec^3{(e+fx)}\,dx+\int 2cd\sec^4{(e+fx)}\,dx\right)\end{aligned}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e))**2,x)

[Out] a**2*(Integral(c**2*sec(e + f*x), x) + Integral(2*c**2*sec(e + f*x)**2, x)
+ Integral(c**2*sec(e + f*x)**3, x) + Integral(d**2*sec(e + f*x)**3, x) + I
ntegral(2*d**2*sec(e + f*x)**4, x) + Integral(d**2*sec(e + f*x)**5, x) + In
tegral(2*c*d*sec(e + f*x)**2, x) + Integral(4*c*d*sec(e + f*x)**3, x) + Int
egral(2*c*d*sec(e + f*x)**4, x))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.84

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$
$$=\frac{32\left(\tan{(fx+e)}^3+3\tan{(fx+e)}\right)a^2cd+32\left(\tan{(fx+e)}^3+3\tan{(fx+e)}\right)a^2d^2-3a^2d^2\left(\frac{2\left(3\sin(fx+e)\cdots\right.}{\sin(fx+e)^4-2\cdots}\right.}{}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")

[Out] 1/48*(32*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*c*d + 32*(tan(f*x + e)^3 + 3
*tan(f*x + e))*a^2*d^2 - 3*a^2*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(
sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(si
n(f*x + e) - 1)) - 12*a^2*c^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(si
n(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 48*a^2*c*d*(2*sin(f*x + e)/(sin(
f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 12*a^2*d

```
^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f
*x + e) - 1)) + 48*a^2*c^2*log(sec(f*x + e) + tan(f*x + e)) + 96*a^2*c^2*ta
n(f*x + e) + 96*a^2*c*d*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.82

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$
$$= \frac{3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(12\,a^2c^2+16\,a^2cd+7\,a^2d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\right.\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*log(abs(tan(1/2*f*x + 1/2*e)
+ 1)) - 3*(12*a^2*c^2 + 16*a^2*c*d + 7*a^2*d^2)*log(abs(tan(1/2*f*x + 1/2*e
) - 1)) - 2*(36*a^2*c^2*tan(1/2*f*x + 1/2*e)^7 + 48*a^2*c*d*tan(1/2*f*x + 1
/2*e)^7 + 21*a^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^2*tan(1/2*f*x + 1/2
*e)^5 - 176*a^2*c*d*tan(1/2*f*x + 1/2*e)^5 - 77*a^2*d^2*tan(1/2*f*x + 1/2*e
)^5 + 156*a^2*c^2*tan(1/2*f*x + 1/2*e)^3 + 272*a^2*c*d*tan(1/2*f*x + 1/2*e)
^3 + 83*a^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 60*a^2*c^2*tan(1/2*f*x + 1/2*e) -
144*a^2*c*d*tan(1/2*f*x + 1/2*e) - 75*a^2*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/
2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 17.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.35

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\,dx$$
$$= \frac{\left(-3\,a^2\,c^2-4\,a^2\,c\,d-\frac{7\,a^2\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(11\,a^2\,c^2+\frac{44\,a^2\,c\,d}{3}+\frac{77\,a^2\,d^2}{12}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-13\,a^2\,c^2-\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\right.}$$
$$+ \frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)}{2\,\left(6\,c^2+8\,c\,d+\frac{7\,d^2}{2}\right)}\right)\,\left(12\,c^2+16\,c\,d+7\,d^2\right)}{4\,f}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(5*a^2*c^2 + (25*a^2*d^2)/4 + 12*a^2*c*d) - tan(e/2 + (
f*x)/2)^7*(3*a^2*c^2 + (7*a^2*d^2)/4 + 4*a^2*c*d) + tan(e/2 + (f*x)/2)^5*(1
1*a^2*c^2 + (77*a^2*d^2)/12 + (44*a^2*c*d)/3) - tan(e/2 + (f*x)/2)^3*(13*a^
2*c^2 + (83*a^2*d^2)/12 + (68*a^2*c*d)/3))/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*t
an(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) +
 (a^2*atanh((tan(e/2 + (f*x)/2)*(16*c*d + 12*c^2 + 7*d^2))/(2*(8*c*d + 6*c^
2 + (7*d^2)/2)))*(16*c*d + 12*c^2 + 7*d^2))/(4*f)
```

3.196 $\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 103

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx = \frac{a^2(3c+2d)\text{arctanh}(\sin(e+fx))}{2f} + \frac{2a^2(3c+2d)\tan(e+fx)}{3f} + \frac{a^2(3c+2d)\sec(e+fx)\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f}$$

```
[Out] 1/2*a^2*(3*c+2*d)*arctanh(sin(f*x+e))/f+2/3*a^2*(3*c+2*d)*tan(f*x+e)/f+1/6*
a^2*(3*c+2*d)*sec(f*x+e)*tan(f*x+e)/f+1/3*d*(a+a*sec(f*x+e))^2*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4086, 3873, 3852, 8, 4131, 3855}

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx = \frac{a^2(3c+2d)\text{arctanh}(\sin(e+fx))}{2f} + \frac{2a^2(3c+2d)\tan(e+fx)}{3f} + \frac{a^2(3c+2d)\tan(e+fx)\sec(e+fx)}{6f} + \frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*(3*c + 2*d)*ArcTanh[Sin[e + f*x]])/(2*f) + (2*a^2*(3*c + 2*d)*Tan[e +
f*x])/(3*f) + (a^2*(3*c + 2*d)*Sec[e + f*x]*Tan[e + f*x])/(6*f) + (d*(a + a
*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f} + \frac{1}{3}(3c+2d)\int \sec(e+fx)(a+a\sec(e+fx))^2\,dx$$

$$= \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f} + \frac{1}{3}(3c+2d)\int \sec(e+fx)\left(a^2+a^2\sec^2(e+fx)\right)\,dx + \frac{1}{3}\left(2a^2(3c+2d)\right)\int \sec^2(e+fx)\,dx$$

$$= \frac{a^2(3c+2d)\sec(e+fx)\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f} + \frac{1}{2}\left(a^2(3c+2d)\right)\int \sec(e+fx)\,dx - \frac{(2a^2(3c+2d))\,\mathrm{Subst}(\int 1\,dx,x,-\tan(e+fx))}{3f}$$

$$= \frac{a^2(3c+2d)\mathrm{arctanh}(\sin(e+fx))}{2f} + \frac{2a^2(3c+2d)\tan(e+fx)}{3f} + \frac{a^2(3c+2d)\sec(e+fx)\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3f}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx = \frac{a^2((9c+6d)\mathrm{arctanh}(\sin(e+fx))+\tan(e+fx)\left(12(c+d)+3(c+2d)\sec(e+fx)+2d\tan^2(e+fx)\right))}{6f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]),x]
[Out] (a^2*((9*c + 6*d)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(12*(c + d) + 3*(c +
 2*d)*Sec[e + f*x] + 2*d*Tan[e + f*x]^2)))/(6*f)
```

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17

method	result
parts	$\frac{(a^2c+2a^2d)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{(2a^2c+a^2d)\tan(fx+e)}{f}+\frac{\ln(\sec(fx+e)+\tan(fx+e)}{f}$
derivativedivides	$\frac{a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2d\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2a^2c\tan(fx+e)+2a^2d\left(\frac{\sec(f}{}\right.}{f}$
default	$\frac{a^2c\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a^2d\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2a^2c\tan(fx+e)+2a^2d\left(\frac{\sec(f}{}\right.}{f}$
parallelrisch	$\frac{\left(-\frac{9\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\left(c+\frac{2d}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2}+\frac{9\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)\left(c+\frac{2d}{3}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2}+(c+2\right.}{f(\cos(3fx+3e)+3\cos(fx+e))}$
norman	$\frac{\frac{8a^2(3c+2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{a^2(3c+2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{a^2(5c+6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}-\frac{a^2(3c+2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{a^2(3c}{}$
risch	$-\frac{ia^2\left(3c\,e^{5i(fx+e)}+6d\,e^{5i(fx+e)}-12c\,e^{4i(fx+e)}-6d\,e^{4i(fx+e)}-24\,e^{2i(fx+e)}c-24d\,e^{2i(fx+e)}-3\,e^{i(fx+e)}c-6d\,e^{i(fx+e)}-\right.}{3f\left(1+e^{2i(fx+e)}\right)^3}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] (a^2*c+2*a^2*d)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+(2*a^2*c+a^2*d)/f*tan(f*x+e)+1/f*ln(sec(f*x+e)+tan(f*x+e))*a^2*c-a^2*d/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$=\frac{3\,(3\,a^2c+2\,a^2d)\cos{(fx+e)}^3\log{(\sin{(fx+e)}+1)}-3\,(3\,a^2c+2\,a^2d)\cos{(fx+e)}^3\log{(-\sin{(fx+e)}+}}{12\,f\cos{(fx+e)}}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="fricas")

[Out] 1/12*(3*(3*a^2*c + 2*a^2*d)*cos(f*x + e)^3*log(sin(f*x + e) + 1) - 3*(3*a^2*c + 2*a^2*d)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*a^2*d + 2*(6*a^2*c + 5*a^2*d)*cos(f*x + e)^2 + 3*(a^2*c + 2*a^2*d)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$= a^2\left(\int c\sec(e+fx)\,dx+\int 2c\sec^2(e+fx)\,dx+\int c\sec^3(e+fx)\,dx\right.$$
$$\left.+\int d\sec^2(e+fx)\,dx+\int 2d\sec^3(e+fx)\,dx+\int d\sec^4(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2*(c+d*sec(f*x+e)),x)
```

```
[Out] a**2*(Integral(c*sec(e + f*x), x) + Integral(2*c*sec(e + f*x)**2, x) + Inte
gral(c*sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**2, x) + Integral(2*d*
sec(e + f*x)**3, x) + Integral(d*sec(e + f*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.62

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$= \frac{4\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^2d-3a^2c\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^2*d - 3*a^2*c*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 6*a^
2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a^2*c*log(sec(f*x + e) + tan(f*x + e)) + 24*a^2*c*tan(f
*x + e) + 12*a^2*d*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.73

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$= \frac{3\left(3a^2c+2a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-3\left(3a^2c+2a^2d\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(9a^2c\tan\left(\frac{1}{2}f\right.\right.}{}}{6f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(3*a^2*c + 2*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(3*a^2*c
+ 2*a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(9*a^2*c*tan(1/2*f*x + 1/
2*e)^5 + 6*a^2*d*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c*tan(1/2*f*x + 1/2*e)^3 -
 16*a^2*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c*tan(1/2*f*x + 1/2*e) + 18*a^2*d
*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.56

$$\int \sec(e+fx)(a+a\sec(e+fx))^2(c+d\sec(e+fx))\,dx$$
$$=\frac{2\,a^2\,\mathrm{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{3\,c}{2}+d\right)}{6\,c+4\,d}\right)\,\left(\frac{3\,c}{2}+d\right)}{f}-\frac{\left(3\,a^2\,c+2\,a^2\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-8\,a^2\,c-\frac{16\,a^2\,d}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(5\,a^2\,c+6\,a^2\,d\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)}$$

```
[In] int(((a + a/cos(e + f*x))^2*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (2*a^2*atanh((4*tan(e/2 + (f*x)/2)*((3*c)/2 + d))/(6*c + 4*d))*((3*c)/2 + d
))/f - (tan(e/2 + (f*x)/2)*(5*a^2*c + 6*a^2*d) + tan(e/2 + (f*x)/2)^5*(3*a^
2*c + 2*a^2*d) - tan(e/2 + (f*x)/2)^3*(8*a^2*c + (16*a^2*d)/3))/(f*(3*tan(e
/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1))
```

3.197 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = -\frac{a^2(c-2d)\text{arctanh}(\sin(e+fx))}{d^2f} + \frac{2a^2(c-d)^{3/2}\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2\sqrt{c+d}f} + \frac{a^2\tan(e+fx)}{df}$$

```
[Out] -a^2*(c-2*d)*arctanh(sin(f*x+e))/d^2/f+2*a^2*(c-d)^(3/2)*arctanh((c-d)^(1/2
)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/d^2/f/(c+d)^(1/2)+a^2*tan(f*x+e)/d/f
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(95) = 190.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.19, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4072, 104, 163, 65, 223, 209, 95, 211}

$$\begin{aligned}&\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx\\&= -\frac{2a^3(c-2d)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\&\quad-\frac{2a^3(c-d)^{3/2}\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^2f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}+\frac{a^2\tan(e+fx)}{df}\end{aligned}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Tan[e + f*x])/(d*f) - (2*a^3*(c - 2*d)*ArcTan[Sqrt[a - a*Sec[e + f*x]]
/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^2*f*Sqrt[a - a*Sec[e + f*x]]*
Sqrt[a + a*Sec[e + f*x]]) - (2*a^3*(c - d)^(3/2)*ArcTan[(Sqrt[c + d]*Sqrt[a
 + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(
d^2*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a^2\tan(e+fx)}{df}+\frac{(a\tan(e+fx))\text{Subst}\left(\int\frac{-a^3d+a^3(c-2d)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{df\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a^2\tan(e+fx)}{df}+\frac{(a^4(c-2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad-\frac{(a^4(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a^2\tan(e+fx)}{df}-\frac{(2a^3(c-2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad-\frac{(2a^4(c-d)^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{ac-ad-(-ac-ad)x^2}\,dx,x,\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a^2\tan(e+fx)}{df}-\frac{2a^3(c-d)^{3/2}\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad-\frac{(2a^3(c-2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= \frac{a^2\tan(e+fx)}{df} - \frac{2a^3(c-2d)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{2a^3(c-d)^{3/2}\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^2\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.46

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$$

$$= \frac{a^2\cos(e+fx)(d+c\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^2\left((c-2d)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\mathrm{si}\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^2*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x]
)^2*((c - 2*d)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - (c - 2*d)*Log[Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]] - ((2*I)*(c - d)^2*ArcTan[((I*Cos[e] + Si
n[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos
[e] - I*Sin[e])^2])]*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I
*Sin[e])^2]) + (d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])) + (d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))))/(4*d^2*f*(c + d*Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{8a^2\left(-\frac{(-c^2+2cd-d^2)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{4d^2\sqrt{(c+d)(c-d)}}-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{(-c+2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^2}-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}\right.}{f}$
default	$\frac{8a^2\left(-\frac{(-c^2+2cd-d^2)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{4d^2\sqrt{(c+d)(c-d)}}-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{(-c+2d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^2}-\frac{1}{8d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}\right.}{f}$
risch	$\frac{2ia^2}{fd\left(1+e^{2i(fx+e)}\right)}+\frac{\sqrt{(c+d)(c-d)}\,a^2\ln\left(e^{i(fx+e)}+\frac{i\sqrt{(c+d)(c-d)}+d}{c}\right)c}{(c+d)f\,d^2}-\frac{\sqrt{(c+d)(c-d)}\,a^2\ln\left(e^{i(fx+e)}+\frac{i\sqrt{(c+d)(c-d)}}{c}\right.}{(c+d)fd}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/f*a^2*(-1/4/d^2*(-c^2+2*c*d-d^2)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/
2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))-1/8/d/(tan(1/2*f*x+1/2*e)+1)+1/8/d^2*(-c+
2*d)*ln(tan(1/2*f*x+1/2*e)+1)-1/8/d/(tan(1/2*f*x+1/2*e)-1)+1/8*(c-2*d)/d^2*
ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.19

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$$
$$=\left[\frac{2a^2d\sin(fx+e)-(a^2c-a^2d)\sqrt{\frac{c-d}{c+d}}\cos(fx+e)\log\left(\frac{2cd\cos(fx+e)-(c^2-2d^2)\cos(fx+e)^2-2(c^2+cd+(cd+d^2)\cos(fx\right.}{c^2\cos(fx+e)^2+2cd\cos(fx+e)+d^2}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*a^2*d*sin(f*x + e) - (a^2*c - a^2*d)*sqrt((c - d)/(c + d))*cos(f*x
 + e)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d
 + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^
2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (a^2*c - 2*a^2*d)*cos
(f*x + e)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(f
*x + e) + 1))/(d^2*f*cos(f*x + e)), 1/2*(2*a^2*d*sin(f*x + e) + 2*(a^2*c -
```

```
a^2*d)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c
 + d))/((c - d)*sin(f*x + e)))*cos(f*x + e) - (a^2*c - 2*a^2*d)*cos(f*x + e
)*log(sin(f*x + e) + 1) + (a^2*c - 2*a^2*d)*cos(f*x + e)*log(-sin(f*x + e)
+ 1))/(d^2*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = a^2\left(\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx + \int \frac{2\sec^2(e+fx)}{c+d\sec(e+fx)}\,dx + \int \frac{\sec^3(e+fx)}{c+d\sec(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(2*sec(e + f
*x)**2/(c + d*sec(e + f*x)), x) + Integral(sec(e + f*x)**3/(c + d*sec(e + f
*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(86) = 172$.

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.06

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx =$$

$$-\frac{\frac{2\,a^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)d}+\frac{(a^2c-2\,a^2d)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d^2}-\frac{(a^2c-2\,a^2d)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d^2}+\frac{2\,(a^2c^2-2\,a^2cd+a^2d^2)\left(\pi\left\lfloor\frac{f\cdots}{\cdots}\right.\right.}{\cdots}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -(2*a^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*d) + (a^2*c - 2*
a^2*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^2 - (a^2*c - 2*a^2*d)*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/d^2 + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) -
d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/(sqrt(-c^2 + d^2)*d^2))/f
```

Mupad [B] (verification not implemented)

Time = 14.26 (sec) , antiderivative size = 529, normalized size of antiderivative = 5.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{c+d\sec(e+fx)}\,dx$$

$$=\frac{2\,a^2\left(\frac{\sin(e+f\,x)}{2}+2\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}$$

$$+\frac{2\,a^2\left(\frac{c\,\sin(e+f\,x)}{2}+c\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\right)}{d\,f\,\cos\left(e+f\,x\right)\,\left(c+d\right)}$$

$$-\frac{2\,a^2\left(c^2\,\cos\left(e+f\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)+\cos\left(e+f\,x\right)\,\mathrm{atan}\left(\frac{\left(2\,c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(c^4-2\,c^3\,d+2\,c\,d^3-d^4\right)^{3/2}-2\,c^5\,\sin\left(\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (2*a^2*(sin(e + f*x)/2 + 2*cos(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 +
(f*x)/2))))/(f*cos(e + f*x)*(c + d)) + (2*a^2*((c*sin(e + f*x))/2 + c*cos(e
 + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2))))/(d*f*cos(e + f*x)*(c
 + d)) - (2*a^2*(c^2*cos(e + f*x)*atanh(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/
2)) + cos(e + f*x)*atan(((2*c*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 -
```

```
 d^4)^(3/2) - 2*c^5*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2
) + 5*d^5*sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) - c*d^4*
sin(e/2 + (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) + 4*c^4*d*sin(e/2
+ (f*x)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) - 9*c^2*d^3*sin(e/2 + (f*x
)/2)*(2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2) + 3*c^3*d^2*sin(e/2 + (f*x)/2)*(
2*c*d^3 - 2*c^3*d + c^4 - d^4)^(1/2))*1i)/(d*cos(e/2 + (f*x)/2)*(c + d)*(8*
c*d^4 + 3*c^4*d - 5*d^5 + 2*c^2*d^3 - 8*c^3*d^2)))*((c + d)*(c - d)^3)^(1/2
)*1i))/(d^2*f*cos(e + f*x)*(c + d))
```

3.198 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 117

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2} dx = \frac{a^2\text{arctanh}(\sin(e+fx))}{d^2f} - \frac{2a^2\sqrt{c-d}(c+2d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^2(c+d)^{3/2}f} - \frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))}$$

```
[Out] a^2*arctanh(sin(f*x+e))/d^2/f-2*a^2*(c+2*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x
+1/2*e)/(c+d)^(1/2))*(c-d)^(1/2)/d^2/(c+d)^(3/2)/f-a^2*(c-d)*tan(f*x+e)/d/(
c+d)/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4072, 100, 163, 65, 223, 209, 95, 211}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2} dx = \frac{2a^3\sqrt{c-d}(c+2d)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^2f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^3\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{a^2(c-d)\tan(e+fx)}{df(c+d)(c+d\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(d^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^3*Sq
rt[c - d]*(c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^2*(c + d)^(3/2)*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^2*(c - d)*Tan[e + f*x])/(d*(
c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} \\
&\quad + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{-2a^3d-a^3(c+d)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{d(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\left(a^4\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{\left(a\left(-2a^3d^2+a^3c(c+d)\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{d^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(2a^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(2a(-2a^3d^2+a^3c(c+d))\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{d^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\sqrt{c-d}(c+2d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(2a^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^3\sqrt{c-d}(c+2d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^2(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^2(c-d)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.44 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.67

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{a^2(d+c\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^2\left(-\left((d+c\cos(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*(1 + Sec[e + f*x])^2*(-((d + c
*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (d + c*Cos[e + f
*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (2*(c^2 + c*d - 2*d^2)*ArcT
an[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^
```

```
2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x])*(I*Cos[e] + Sin
[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((c - d)*d*(d
*Sin[e] - c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2
]))))/(4*d^2*f*(c + d*Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{8a^2\left(\frac{(c-d)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(c+2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)\sqrt{(c+d)(c-d)}}\right)}{4d^2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^2}-\frac{\ln\left(\mathrm{ta}\right.}{}}{f}$
default	$\frac{8a^2\left(\frac{(c-d)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(c+2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)\sqrt{(c+d)(c-d)}}\right)}{4d^2}+\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{8d^2}-\frac{\ln\left(\mathrm{ta}\right.}{}}{f}$
risch	$-\frac{2ia^2(c-d)\left(d\,e^{i(fx+e)}+c\right)}{df(c+d)c\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{\sqrt{(c+d)(c-d)}\,a^2\ln\left(e^{i(fx+e)}-\frac{i\sqrt{(c+d)(c-d)}-d}{c}\right)c}{(c+d)^2f\,d^2}+\frac{2\sqrt{(c+d)(c-d)}\,a^2\ln\left(\right.}{(}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*a^2*(1/4*(c-d)/d^2*(d/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-
tan(1/2*f*x+1/2*e)^2*d-c-d)-(c+2*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)
*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))+1/8/d^2*ln(tan(1/2*f*x+1/2*e)+1)-
1/8/d^2*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(108) = 216$.

Time = 0.42 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.85

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{(a^2cd + 2\,a^2d^2 + (a^2c^2 + 2\,a^2cd)\cos(fx+e))\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2\,cd\cos(fx+e) - (c^2-2\,d^2)\cos(fx+e)^2 - 2\,(c^2+cd+(cd+d^2)\cos}{c^2\cos(fx+e)^2 + 2\,cd\cos(fx+e)-}\right.}{}\right.$$
$$-\frac{2\,(a^2cd + 2\,a^2d^2 + (a^2c^2 + 2\,a^2cd)\cos(fx+e))\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right) - (a^2cd + a^2d^2}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e))*sqrt((c -
d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2
 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c
^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (a^2*c*d + a^2
*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(sin(f*x + e) + 1) - (a^2*c*d +
 a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(a^
2*c*d - a^2*d^2)*sin(f*x + e))/((c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 +
 d^4)*f), -1/2*(2*(a^2*c*d + 2*a^2*d^2 + (a^2*c^2 + 2*a^2*c*d)*cos(f*x + e)
)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d)
)/((c - d)*sin(f*x + e))) - (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*cos(f*
x + e))*log(sin(f*x + e) + 1) + (a^2*c*d + a^2*d^2 + (a^2*c^2 + a^2*c*d)*co
s(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^2*c*d - a^2*d^2)*sin(f*x + e))/((
c^2*d^2 + c*d^3)*f*cos(f*x + e) + (c*d^3 + d^4)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = a^2\left(\int \frac{\sec(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx + \int \frac{2\sec^2(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx + \int \frac{\sec^3(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)*
*2), x) + Integral(2*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(
e + f*x)**2), x) + Integral(sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) + d*
*2*sec(e + f*x)**2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(108) = 216$.

Time = 0.36 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \frac{\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d^2} - \frac{a^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d^2} + \frac{2\left(a^2c^2+a^2cd-2a^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(cd^2+d^3)\sqrt{-c^2+d^2}}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (a^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^2 - a^2*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/d^2 + 2*(a^2*c^2 + a^2*c*d - 2*a^2*d^2)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
1/2*e))/sqrt(-c^2 + d^2)))/((c*d^2 + d^3)*sqrt(-c^2 + d^2)) + 2*(a^2*c*tan(
1/2*f*x + 1/2*e) - a^2*d*tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^2 -
 d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c*d + d^2)))/f
```

Mupad [B] (verification not implemented)

Time = 16.42 (sec) , antiderivative size = 2563, normalized size of antiderivative = 21.91

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (a^2*atan(((a^2*((a^2*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2
*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*a^2*tan(e/2 + (f*x
)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d
^2 + (32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*
d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2))*1i)/d^2 - (a^2*((a^2*((32*
(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2
*c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 +
 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 - (32*tan(e/2 + (f*x)/2)
*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*
d^3 + d^4 + c^2*d^2))*1i)/d^2)/((64*(2*a^6*d^4 - a^6*c^4 - 5*a^6*c*d^3 + a^
6*c^3*d + 3*a^6*c^2*d^2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((a^2*((32*(3*a^
2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^
4 + d^5 + c^2*d^3) - (32*a^2*tan(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^
5*d^4))/(d^2*(2*c*d^3 + d^4 + c^2*d^2))))/d^2 + (32*tan(e/2 + (f*x)/2)*(2*a
^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 +
 d^4 + c^2*d^2)))/d^2 + (a^2*((a^2*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^
2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (32*a^2*t
an(e/2 + (f*x)/2)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/(d^2*(2*c*d^3 + d^4 +
c^2*d^2))))/d^2 - (32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d
^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2)))/d^2))*2i)/(d
^2*f) + (a^2*atan(((a^2*((32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*
a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) + (a^2*
((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c
^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) - (32*a^2*
tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6
 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*
d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))*((c + d)^3*(c - d))^(1/2)*(c
 + 2*d)*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) + (a^2*((32*tan(e/2 + (f*
x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/
(2*c*d^3 + d^4 + c^2*d^2) - (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(
3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*
c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/
2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*
(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d
^2))*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*1i)/(3*c*d^4 + d^5 + 3*c^2*d^3 + c
^3*d^2))/((64*(2*a^6*d^4 - a^6*c^4 - 5*a^6*c*d^3 + a^6*c^3*d + 3*a^6*c^2*d^
```

```
2))/(2*c*d^4 + d^5 + c^2*d^3) + (a^2*((32*tan(e/2 + (f*x)/2)*(2*a^4*c^5 - 5
*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*c^3*d^2))/(2*c*d^3 + d^4 + c^2
*d^2) + (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*((32*(3*a^2*d^8 - 2*a^2*c*
d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^
3) - (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*(2*c*d^
8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 + c^2*d^2)*(3*c*d^4 + d^5 + 3*c
^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))*((c + d)^3*(c -
 d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2) - (a^2*((32*tan
(e/2 + (f*x)/2)*(2*a^4*c^5 - 5*a^4*d^5 + 9*a^4*c*d^4 + a^4*c^2*d^3 - 7*a^4*
c^3*d^2))/(2*c*d^3 + d^4 + c^2*d^2) - (a^2*((c + d)^3*(c - d))^(1/2)*(c + 2
*d)*((32*(3*a^2*d^8 - 2*a^2*c*d^7 - 4*a^2*c^2*d^6 + 2*a^2*c^3*d^5 + a^2*c^4
*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (32*a^2*tan(e/2 + (f*x)/2)*((c + d)^3*(c
 - d))^(1/2)*(c + 2*d)*(2*c*d^8 - 4*c^3*d^6 + 2*c^5*d^4))/((2*c*d^3 + d^4 +
 c^2*d^2)*(3*c*d^4 + d^5 + 3*c^2*d^3 + c^3*d^2))))/(3*c*d^4 + d^5 + 3*c^2*d
^3 + c^3*d^2))*((c + d)^3*(c - d))^(1/2)*(c + 2*d))/(3*c*d^4 + d^5 + 3*c^2*
d^3 + c^3*d^2)))*((c + d)^3*(c - d))^(1/2)*(c + 2*d)*2i)/(f*(3*c*d^4 + d^5
+ 3*c^2*d^3 + c^3*d^2)) - (2*a^2*tan(e/2 + (f*x)/2)*(c - d))/(d*f*(c + d)*(
c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
```

3.199 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 130

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = \frac{3a^2\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}(c+d)^{5/2}f} + \frac{(a^2+a^2\sec(e+fx))\tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2\tan(e+fx)}{2(c+d)^2f(c+d\sec(e+fx))}$$

```
[Out] 3*a^2*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c+d)^(5/2)/f/(c-
d)^(1/2)+1/2*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^2+3/2
*a^2*tan(f*x+e)/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4072, 96, 95, 211}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$
$$= -\frac{3a^3\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f\sqrt{c-d}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{3a^2\tan(e+fx)}{2f(c+d)^2(c+d\sec(e+fx))} + \frac{\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{2f(c+d)(c+d\sec(e+fx))^2}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (-3*a^3*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
 a*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c - d]*(c + d)^(5/2)*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((a^2 + a^2*Sec[e + f*x])*Tan[e + f
*x])/(2*(c + d)*f*(c + d*Sec[e + f*x])^2) + (3*a^2*Tan[e + f*x])/(2*(c + d)
^2*f*(c + d*Sec[e + f*x]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(a^2 + a^2 \sec(e+fx)) \tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} - \frac{(3a^3 \tan(e+fx)) \operatorname{Subst}\left(\int \frac{\sqrt{a+ax}}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(a^2 + a^2 \sec(e+fx)) \tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2 \tan(e+fx)}{2(c+d)^2 f(c+d\sec(e+fx))} - \frac{(3a^4 \tan(e+fx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2(c+d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(a^2 + a^2 \sec(e+fx)) \tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2 \tan(e+fx)}{2(c+d)^2 f(c+d\sec(e+fx))} - \frac{(3a^4 \tan(e+fx)) \operatorname{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{(c+d)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{3a^3 \arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c-d}(c+d)^{5/2} f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a^2 + a^2 \sec(e+fx)) \tan(e+fx)}{2(c+d)f(c+d\sec(e+fx))^2} + \frac{3a^2 \tan(e+fx)}{2(c+d)^2 f(c+d\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.92

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{a^2(d+c\cos(e+fx))\sec^4\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(1+\sec(e+fx))^2\left(-\frac{6i\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(\cdots}}\right)}{\sqrt{c^2-d^2}\sqrt{\cdots}}\cdots\right.}{8(c+d\cdots}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^4*Sec[e + f*x]*(1 + Sec[e + f*x]
)^2*(((-6*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f
*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*x]
)^2*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((
c - d)*(c + d)*Sec[e]*(-(d*Sin[e]) + c*Sin[f*x]))/c^2 + ((d + c*Cos[e + f*x
])*Sec[e]*((c^2 - 4*c*d - 2*d^2)*Sin[e] + c*(4*c + d)*Sin[f*x]))/c^2))/(8*(
c + d)^2*f*(c + d*Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{8a^2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}+\frac{-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{8(c+d)\sqrt{(c+d)(c-d)}}}{c+d}\right)}{f}$
default	$\frac{8a^2\left(\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}+\frac{-\frac{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{8(c+d)\sqrt{(c+d)(c-d)}}}{c+d}\right)}{f}$
risch	$\frac{ia^2\left(-c^3e^{3i(fx+e)}+4c^2d\,e^{3i(fx+e)}+2c\,d^2e^{3i(fx+e)}+4c^3e^{2i(fx+e)}+c^2d\,e^{2i(fx+e)}+8c\,d^2e^{2i(fx+e)}+2d^3e^{2i(fx+e)}+c^3e^{i(fx+\cdots}\right.}{c^2(c+d)^2f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)^2}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 8/f*a^2*(1/4*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2+3/4/(c+d)*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(117) = 234.

Time = 0.31 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.78

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx$$
$$=\left[\frac{3\left(a^2c^2\cos\left(fx+e\right)^2+2\,a^2cd\cos\left(fx+e\right)+a^2d^2\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\sqrt{c^2-d^2}(d\cos\cdots}{c^2\cos(fx+e)^2+2\,cd\cos(fx+\cdots}\right.}{4\left((c^6+2\,c^5d-2\,c^3d^3-c^2d^4)f\cos\left(fx+e\right)^2+2\,(c^5\cdots\right.}\right.$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e) + a^2*d^2)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^

```
2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)
^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*
a^2*d^3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))*sin
(f*x + e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 + 2*(c^5
*d + 2*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^3*d^3 -
 2*c*d^5 - d^6)*f), 1/2*(3*(a^2*c^2*cos(f*x + e)^2 + 2*a^2*c*d*cos(f*x + e)
 + a^2*d^2)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/
((c^2 - d^2)*sin(f*x + e))) + (a^2*c^3 + 4*a^2*c^2*d - a^2*c*d^2 - 4*a^2*d^
3 + (4*a^2*c^3 + a^2*c^2*d - 4*a^2*c*d^2 - a^2*d^3)*cos(f*x + e))*sin(f*x +
 e))/((c^6 + 2*c^5*d - 2*c^3*d^3 - c^2*d^4)*f*cos(f*x + e)^2 + 2*(c^5*d + 2
*c^4*d^2 - 2*c^2*d^4 - c*d^5)*f*cos(f*x + e) + (c^4*d^2 + 2*c^3*d^3 - 2*c*d
^5 - d^6)*f)]
```

Sympy [F]

$$\begin{aligned}&\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx\\&=a^2\left(\int \frac{\sec{(e+fx)}}{c^3+3c^2d\sec{(e+fx)}+3cd^2\sec^2{(e+fx)}+d^3\sec^3{(e+fx)}}\,dx\right.\\&\quad+\int \frac{2\sec^2{(e+fx)}}{c^3+3c^2d\sec{(e+fx)}+3cd^2\sec^2{(e+fx)}+d^3\sec^3{(e+fx)}}\,dx\\&\quad\left.+\int \frac{\sec^3{(e+fx)}}{c^3+3c^2d\sec{(e+fx)}+3cd^2\sec^2{(e+fx)}+d^3\sec^3{(e+fx)}}\,dx\right)\end{aligned}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(2*sec(e + f*x)**2/(c**3 +
3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x
) + Integral(sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**3), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see 'assume?' for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx = -\frac{\frac{3\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^2}{(c^2+2cd+d^2)\sqrt{-c^2+d^2}}+\frac{3a^2c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-3a^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5a^2c\tan\left(\frac{1}{2}\cdots\right.}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\cdots\right.}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -(3*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^2/((c^2 + 2*c*d +
 d^2)*sqrt(-c^2 + d^2)) + (3*a^2*c*tan(1/2*f*x + 1/2*e)^3 - 3*a^2*d*tan(1/2
*f*x + 1/2*e)^3 - 5*a^2*c*tan(1/2*f*x + 1/2*e) - 5*a^2*d*tan(1/2*f*x + 1/2*
e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2*(c^2 +
 2*c*d + d^2)))/f
```

Mupad [B] (verification not implemented)

Time = 15.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^3}\,dx &= \frac{\frac{5a^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+d}-\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(a^2c-a^2d)}{(c+d)^2}}{f\left(2cd-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(2c^2-2d^2)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(c^2-2cd+d^2)+c^2+d^2\right)}\\&\quad+\frac{3a^2\operatorname{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)}{f\,(c+d)^{5/2}\sqrt{c-d}}\end{aligned}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] ((5*a^2*tan(e/2 + (f*x)/2))/(c + d) - (3*tan(e/2 + (f*x)/2)^3*(a^2*c - a^2*
d))/(c + d)^2)/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 +
 (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) + (3*a^2*atanh((tan(e/2 + (f*
x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)^(5/2)*(c - d)^(1/2))
```

3.200 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 213

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \frac{a^2(3c-2d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{7/2}f} - \frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)(a^2+a^2\sec(e+fx))\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a^2(3c-2d)\tan(e+fx)}{2(c-d)(c+d)^3f(c+d\sec(e+fx))}$$

```
[Out] a^2*(3*c-2*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/
2)/(c+d)^(7/2)/f-1/3*d*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(f
*x+e))^3+1/6*(3*c-2*d)*(a^2+a^2*sec(f*x+e))*tan(f*x+e)/(c-d)/(c+d)^2/f/(c+d
*sec(f*x+e))^2+1/2*a^2*(3*c-2*d)*tan(f*x+e)/(c-d)/(c+d)^3/f/(c+d*sec(f*x+e)
)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used

$= \{4072, 98, 96, 95, 211\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$
$$= -\frac{a^3(3c-2d)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{3/2}(c+d)^{7/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{a^2(3c-2d)\tan(e+fx)}{2f(c-d)(c+d)^3(c+d\sec(e+fx))}$$
$$+\frac{(3c-2d)\tan(e+fx)\left(a^2\sec(e+fx)+a^2\right)}{6f(c-d)(c+d)^2(c+d\sec(e+fx))^2}-\frac{d\tan(e+fx)(a\sec(e+fx)+a)^2}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] -((a^3*(3*c - 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(3/2)*(c + d)^(7/2)*f*
Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])) - (d*(a + a*Sec[e + f*x
])^2*Tan[e + f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((3*c - 2*d)*
(a^2 + a^2*Sec[e + f*x])*Tan[e + f*x])/(6*(c - d)*(c + d)^2*f*(c + d*Sec[e
+ f*x])^2) + (a^2*(3*c - 2*d)*Tan[e + f*x])/(2*(c - d)*(c + d)^3*f*(c + d*S
ec[e + f*x]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^4}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} - \frac{(a^2(3c-2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{3\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} - \frac{(a^3(3c-2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{\sqrt{a+ax}}{\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{2(c+d)\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^2\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3} + \frac{(3c-2d)\left(a^2+a^2\sec(e+fx)\right)\tan(e+fx)}{6(c-d)(c+d)^2f(c+d\sec(e+fx))^2} + \frac{a^2(3c-2d)\tan(e+fx)}{2(c-d)(c+d)^3f(c+d\sec(e+fx))} - \frac{(a^4(3c-2d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2(c+d)^2\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a + a\sec(e + fx))^2 \tan(e + fx)}{3(c^2 - d^2) f(c + d\sec(e + fx))^3} + \frac{(3c - 2d)(a^2 + a^2\sec(e + fx))\tan(e + fx)}{6(c - d)(c + d)^2 f(c + d\sec(e + fx))^2} + \frac{a^2(3c - 2d)\tan(e + fx)}{2(c - d)(c + d)^3 f(c + d\sec(e + fx))} - \frac{(a^4(3c - 2d)\tan(e + fx))\,\text{Subst}\left(\int \frac{1}{ac - ad - (-ac - ad)x^2}\,dx, x, \frac{\sqrt{a + a\sec(e + fx)}}{\sqrt{a - a\sec(e + fx)}}\right)}{(c + d)^2(c^2 - d^2) f\sqrt{a - a\sec(e + fx)}\sqrt{a + a\sec(e + fx)}}$$

$$= -\frac{a^3(3c - 2d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e + fx)}{(c - d)^{3/2}(c + d)^{7/2} f\sqrt{a - a\sec(e + fx)}\sqrt{a + a\sec(e + fx)}} - \frac{d(a + a\sec(e + fx))^2 \tan(e + fx)}{3(c^2 - d^2) f(c + d\sec(e + fx))^3} + \frac{(3c - 2d)(a^2 + a^2\sec(e + fx))\tan(e + fx)}{6(c - d)(c + d)^2 f(c + d\sec(e + fx))^2} + \frac{a^2(3c - 2d)\tan(e + fx)}{2(c - d)(c + d)^3 f(c + d\sec(e + fx))}$$

Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e + fx)(a + a\sec(e + fx))^2}{(c + d\sec(e + fx))^4}\,dx = \frac{a^2(c - d)^2\left(24(3c - 2d)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d + c\cos(e + fx))^3 - 2\sqrt{c^2 - d^2}(12c^3 - 5c^2d + 6cd^2 -}{24(-c + d)^3(c + d)^3\sqrt{c^2 -}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^2*(c - d)^2*(24*(3*c - 2*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2
 - d^2]]*(d + c*Cos[e + f*x])^3 - 2*Sqrt[c^2 - d^2]*(12*c^3 - 5*c^2*d + 6*c
*d^2 - 22*d^3 + 6*(c^3 + 6*c^2*d - 7*c*d^2 - 2*d^3)*Cos[e + f*x] + (12*c^3
- 7*c^2*d - 6*c*d^2 - 2*d^3)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(24*(-c + d)^
3*(c + d)^3*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{8a^2\left(-\frac{\frac{(3c-2d)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{8c^3+24c^2d+24c\,d^2+8d^3}-\frac{(3c-2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3\left(c^2+2cd+d^2\right)}+\frac{(5c-6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)(c-d)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}+\frac{(3c-2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{8\left(c^4+2c^3d-2c\,d^3-d^4\right)\sqrt{(c+d)(c-d)}}\right)}{f}$
default	$\frac{8a^2\left(-\frac{\frac{(3c-2d)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{8c^3+24c^2d+24c\,d^2+8d^3}-\frac{(3c-2d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3\left(c^2+2cd+d^2\right)}+\frac{(5c-6d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{8(c+d)(c-d)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}+\frac{(3c-2d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{8\left(c^4+2c^3d-2c\,d^3-d^4\right)\sqrt{(c+d)(c-d)}}\right)}{f}$
risch	$\frac{ia^2\left(7c^5d+2c^3d^3+6c^4d^2-12c^6+12c\,d^5\mathrm{e}^{2i(fx+e)}+24c^3d^3\mathrm{e}^{4i(fx+e)}+36c^2d^4\mathrm{e}^{4i(fx+e)}+12c\,d^5\mathrm{e}^{4i(fx+e)}-72c^5d\,\mathrm{e}^{3i(fx+e)}\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*a^2*(-(1/8*(3*c-2*d)*(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)
^5-1/3*(3*c-2*d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/8*(5*c-6*d)/(c+d)/(
c-d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d
)^3+1/8*(3*c-2*d)/(c^4+2*c^3*d-2*c*d^3-d^4)/((c+d)*(c-d))^(1/2)*arctanh((c-
d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(198) = 396$.

Time = 0.34 (sec) , antiderivative size = 1234, normalized size of antiderivative = 5.79

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^4 - 2*a^2*c^3*d)*cos(f*x + e)^
3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x + e)^2 + 3*(3*a^2*c^2*d^2 - 2*a
^2*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*
d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^2*c^4
*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*a^2*c^
5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a^2*d^5)
*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^2*d^3
+ 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c^8*d - c^
7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)^3 + 3*(c^
8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^2*d^7)*f*co
```

```
s(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6 + 2*c
^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^3*d^6
 - c^2*d^7 + 2*c*d^8 + d^9)*f), 1/6*(3*(3*a^2*c*d^3 - 2*a^2*d^4 + (3*a^2*c^
4 - 2*a^2*c^3*d)*cos(f*x + e)^3 + 3*(3*a^2*c^3*d - 2*a^2*c^2*d^2)*cos(f*x +
 e)^2 + 3*(3*a^2*c^2*d^2 - 2*a^2*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arct
an(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (a^
2*c^4*d + 6*a^2*c^3*d^2 - 11*a^2*c^2*d^3 - 6*a^2*c*d^4 + 10*a^2*d^5 + (12*a
^2*c^5 - 7*a^2*c^4*d - 18*a^2*c^3*d^2 + 5*a^2*c^2*d^3 + 6*a^2*c*d^4 + 2*a^2
*d^5)*cos(f*x + e)^2 + 3*(a^2*c^5 + 6*a^2*c^4*d - 8*a^2*c^3*d^2 - 8*a^2*c^2
*d^3 + 7*a^2*c*d^4 + 2*a^2*d^5)*cos(f*x + e))*sin(f*x + e))/((c^9 + 2*c^8*d
 - c^7*d^2 - 4*c^6*d^3 - c^5*d^4 + 2*c^4*d^5 + c^3*d^6)*f*cos(f*x + e)^3 +
3*(c^8*d + 2*c^7*d^2 - c^6*d^3 - 4*c^5*d^4 - c^4*d^5 + 2*c^3*d^6 + c^2*d^7)
*f*cos(f*x + e)^2 + 3*(c^7*d^2 + 2*c^6*d^3 - c^5*d^4 - 4*c^4*d^5 - c^3*d^6
+ 2*c^2*d^7 + c*d^8)*f*cos(f*x + e) + (c^6*d^3 + 2*c^5*d^4 - c^4*d^5 - 4*c^
3*d^6 - c^2*d^7 + 2*c*d^8 + d^9)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$
$$= a^2\left(\int \frac{\sec(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx + \int \frac{2\sec^2(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx + \int \frac{\sec^3(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec
(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integ
ral(2*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f
*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(se
c(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2 +
 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(198) = 396$.

Time = 0.41 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.89

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx = -\frac{\frac{3\,(3\,a^2c-2\,a^2d)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4+2\,c^3d-2\,cd^3-d^4)\sqrt{-c^2+d^2}}+\frac{9\,a^2c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-24\,a^2c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(3*a^2*c - 2*a^2*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d
) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^
2)))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*sqrt(-c^2 + d^2)) + (9*a^2*c^3*tan(1/
2*f*x + 1/2*e)^5 - 24*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 21*a^2*c*d^2*tan(1
/2*f*x + 1/2*e)^5 - 6*a^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 24*a^2*c^3*tan(1/2*f
*x + 1/2*e)^3 + 16*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 24*a^2*c*d^2*tan(1/2*
f*x + 1/2*e)^3 - 16*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^2*c^3*tan(1/2*f*x
 + 1/2*e) + 12*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - 21*a^2*c*d^2*tan(1/2*f*x +
1/2*e) - 18*a^2*d^3*tan(1/2*f*x + 1/2*e))/((c^4 + 2*c^3*d - 2*c*d^3 - d^4)*
(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,\left(3\,a^2\,c^2-5\,a^2\,c\,d+2\,a^2\,d^2\right)}{(c+d)^3}-\frac{8\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(3\,a^2\,c-2\,a^2\,d\right)}{3\,(c+d)^2}+\frac{a^2\,\mathrm{t}}{}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,\left(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,\left(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3\right)+3\,c\,d^2+\right.}$$

$$+\frac{2\,a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)\,\left(\frac{3\,c}{2}-d\right)}{f\,{\left(c+d\right)}^{7/2}\,{\left(c-d\right)}^{3/2}}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(3*a^2*c^2 + 2*a^2*d^2 - 5*a^2*c*d))/(c + d)^3 - (8*
tan(e/2 + (f*x)/2)^3*(3*a^2*c - 2*a^2*d))/(3*(c + d)^2) + (a^2*tan(e/2 + (f
*x)/2)*(5*c - 6*d))/((c + d)*(c - d)))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 -
3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3
- 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 -
3*c^2*d + c^3 - d^3))) + (2*a^2*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c
 + d)^(1/2))*((3*c)/2 - d))/(f*(c + d)^(7/2)*(c - d)^(3/2))
```

3.201 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 276

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \frac{a^2(12c^2-16cd+7d^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4(c-d)^{5/2}(c+d)^{9/2}f} - \frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24(c-d)d(c+d)^3f(c+d\sec(e+fx))^2} + \frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\tan(e+fx)}{24(c-d)^2d(c+d)^4f(c+d\sec(e+fx))}$$

```
[Out] 1/4*a^2*(12*c^2-16*c*d+7*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^
(1/2))/(c-d)^(5/2)/(c+d)^(9/2)/f-1/4*a^2*(c-d)*tan(f*x+e)/d/(c+d)/f/(c+d*se
c(f*x+e))^4+1/12*a^2*(c+8*d)*tan(f*x+e)/d/(c+d)^2/f/(c+d*sec(f*x+e))^3+1/24
*a^2*(2*c^2+16*c*d-21*d^2)*tan(f*x+e)/(c-d)/d/(c+d)^3/f/(c+d*sec(f*x+e))^2+
1/24*a^2*(2*c^3+16*c^2*d-59*c*d^2+32*d^3)*tan(f*x+e)/(c-d)^2/d/(c+d)^4/f/(c
+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4072, 100, 156, 12, 95, 211}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$$
$$= -\frac{a^3(12c^2-16cd+7d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{4f(c-d)^{5/2}(c+d)^{9/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24df(c-d)(c+d)^3(c+d\sec(e+fx))^2} + \frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\tan(e+fx)}{24df(c-d)^2(c+d)^4(c+d\sec(e+fx))} + \frac{a^2(c+8d)\tan(e+fx)}{12df(c+d)^2(c+d\sec(e+fx))^3} - \frac{a^2(c-d)\tan(e+fx)}{4df(c+d)(c+d\sec(e+fx))^4}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] -1/4*(a^3*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(5/2)
*(c + d)^(9/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (a^2*
(c - d)*Tan[e + f*x])/(4*d*(c + d)*f*(c + d*Sec[e + f*x])^4) + (a^2*(c + 8*
d)*Tan[e + f*x])/(12*d*(c + d)^2*f*(c + d*Sec[e + f*x])^3) + (a^2*(2*c^2 +
16*c*d - 21*d^2)*Tan[e + f*x])/(24*(c - d)*d*(c + d)^3*f*(c + d*Sec[e + f*x
])^2) + (a^2*(2*c^3 + 16*c^2*d - 59*c*d^2 + 32*d^3)*Tan[e + f*x])/(24*(c -
d)^2*d*(c + d)^4*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^5}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4}\\
&\quad+\frac{(a\tan(e+fx))\text{Subst}\left(\int\frac{-8a^3d-a^3(c+7d)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)^4}\,dx,x,\sec(e+fx)\right)}{4d(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4}+\frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3}\\
&\quad+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{-21a^5(c-d)d-2a^5(c-d)(c+8d)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{12ad(c+d)\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24(c-d)d(c+d)^3f(c+d\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{-2a^7(19c-16d)(c-d)d+a^7(c-d)(21d^2-2c(c+8d))x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{24a^3d(c+d)\left(c^2-d^2\right)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24(c-d)d(c+d)^3f(c+d\sec(e+fx))^2} + \frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\tan(e+fx)}{24(c-d)^2d(c+d)^4f(c+d\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int -\frac{3a^9(c-d)d(12c^2-16cd+7d^2)}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{24a^5d(c+d)\left(c^2-d^2\right)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24(c-d)d(c+d)^3f(c+d\sec(e+fx))^2} + \frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\tan(e+fx)}{24(c-d)^2d(c+d)^4f(c+d\sec(e+fx))} - \frac{\left(a^4(c-d)\left(12c^2-16cd+7d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{8(c+d)\left(c^2-d^2\right)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24(c-d)d(c+d)^3f(c+d\sec(e+fx))^2} + \frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\tan(e+fx)}{24(c-d)^2d(c+d)^4f(c+d\sec(e+fx))} - \frac{\left(a^4(c-d)\left(12c^2-16cd+7d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{4(c+d)\left(c^2-d^2\right)^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^3(12c^2-16cd+7d^2)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{4(c-d)^{5/2}(c+d)^{9/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{a^2(c-d)\tan(e+fx)}{4d(c+d)f(c+d\sec(e+fx))^4} + \frac{a^2(c+8d)\tan(e+fx)}{12d(c+d)^2f(c+d\sec(e+fx))^3} + \frac{a^2(2c^2+16cd-21d^2)\tan(e+fx)}{24(c-d)d(c+d)^3f(c+d\sec(e+fx))^2} + \frac{a^2(2c^3+16c^2d-59cd^2+32d^3)\tan(e+fx)}{24(c-d)^2d(c+d)^4f(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 8.01 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.17

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \frac{a^2\left(-\frac{24(12c^2-16cd+7d^2)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{(24c^5+192c^4d-446c^3d^2+128c^2d^3-148cd^4+160d^5+(144c^5-172c^4d+208c^3\ldots}{}\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^2)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (a^2*((-24*(12*c^2 - 16*c*d + 7*d^2)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sq
rt[c^2 - d^2]])/Sqrt[c^2 - d^2] + ((24*c^5 + 192*c^4*d - 446*c^3*d^2 + 128*
c^2*d^3 - 148*c*d^4 + 160*d^5 + (144*c^5 - 172*c^4*d + 208*c^3*d^2 - 785*c^
2*d^3 + 368*c*d^4 + 102*d^5)*Cos[e + f*x] + 2*(12*c^5 + 96*c^4*d - 227*c^3*
d^2 + 32*c^2*d^3 + 44*c*d^4 + 16*d^5)*Cos[2*(e + f*x)] + 48*c^5*Cos[3*(e +
f*x)] - 68*c^4*d*Cos[3*(e + f*x)] - 16*c^3*d^2*Cos[3*(e + f*x)] + 5*c^2*d^3
*Cos[3*(e + f*x)] + 16*c*d^4*Cos[3*(e + f*x)] + 6*d^5*Cos[3*(e + f*x)])*Sin
[e + f*x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)^2*(c + d)^4*f)
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{8a^2\left(-\frac{\frac{(12c^2-16cd+7d^2)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{32c^4+128c^3d+192c^2d^2+128c\,d^3+32d^4}-\frac{11(12c^2-16cd+7d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{96(c^3+3c^2d+3c\,d^2+d^3)}+\frac{(156c^2-272cd+83d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{96(c-d)(c^2+2cd+d^2)}-\frac{(20c^2-}{32}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^4}\right.}{f}$
default	$\frac{8a^2\left(-\frac{\frac{(12c^2-16cd+7d^2)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{32c^4+128c^3d+192c^2d^2+128c\,d^3+32d^4}-\frac{11(12c^2-16cd+7d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{96(c^3+3c^2d+3c\,d^2+d^3)}+\frac{(156c^2-272cd+83d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{96(c-d)(c^2+2cd+d^2)}-\frac{(20c^2-}{32}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^4}\right.}{f}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)
```

```
[Out] 8/f*a^2*(-(1/32*(12*c^2-16*c*d+7*d^2)*(c-d)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+
d^4)*tan(1/2*f*x+1/2*e)^7-11/96*(12*c^2-16*c*d+7*d^2)/(c^3+3*c^2*d+3*c*d^2+
d^3)*tan(1/2*f*x+1/2*e)^5+1/96*(156*c^2-272*c*d+83*d^2)/(c-d)/(c^2+2*c*d+d^
2)*tan(1/2*f*x+1/2*e)^3-1/32*(20*c^2-48*c*d+25*d^2)/(c+d)/(c^2-2*c*d+d^2)*t
an(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^4+1/
32*(12*c^2-16*c*d+7*d^2)/(c^6+2*c^5*d-c^4*d^2-4*c^3*d^3-c^2*d^4+2*c*d^5+d^6
)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(257) = 514$.

Time = 0.40 (sec) , antiderivative size = 1908, normalized size of antiderivative = 6.91

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] [1/48*(3*(12*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2*
c^5*d + 7*a^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 +
7*a^2*c^3*d^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a^2*
c^2*d^4)*cos(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)
*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(
f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 -
d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^2*c^5*d^2 +
16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^
7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*a^2*c^5*d^2 + 73*a^2*c^4*d^3 + 32*a^2*c
```

```
^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6 - 6*a^2*d^7)*cos(f*x + e)^3 + (12*a^2*c
^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2 - 64*a^2*c^4*d^3 + 271*a^2*c^3*d^4 - 16
*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2*d^7)*cos(f*x + e)^2 + (8*a^2*c^6*d + 6
4*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2
*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*sin(f*x + e))/((c^12 + 2*c^11*d - 2*c^10
*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6*d^6 - 2*c^5*d^7 - c^4*d^8)*f*cos(f*x +
 e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c^9*d^3 - 6*c^8*d^4 + 6*c^6*d^6 + 2*c^5*
d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x + e)^3 + 6*(c^10*d^2 + 2*c^9*d^3 - 2*c
^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c^4*d^8 - 2*c^3*d^9 - c^2*d^10)*f*cos(f*
x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 - 2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c
^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(f*x + e) + (c^8*d^4 + 2*c^7*d^5 - 2*c^6
*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2*d^10 - 2*c*d^11 - d^12)*f), 1/24*(3*(1
2*a^2*c^2*d^4 - 16*a^2*c*d^5 + 7*a^2*d^6 + (12*a^2*c^6 - 16*a^2*c^5*d + 7*a
^2*c^4*d^2)*cos(f*x + e)^4 + 4*(12*a^2*c^5*d - 16*a^2*c^4*d^2 + 7*a^2*c^3*d
^3)*cos(f*x + e)^3 + 6*(12*a^2*c^4*d^2 - 16*a^2*c^3*d^3 + 7*a^2*c^2*d^4)*co
s(f*x + e)^2 + 4*(12*a^2*c^3*d^3 - 16*a^2*c^2*d^4 + 7*a^2*c*d^5)*cos(f*x +
e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 -
d^2)*sin(f*x + e))) + (2*a^2*c^5*d^2 + 16*a^2*c^4*d^3 - 61*a^2*c^3*d^4 + 16
*a^2*c^2*d^5 + 59*a^2*c*d^6 - 32*a^2*d^7 + (48*a^2*c^7 - 68*a^2*c^6*d - 64*
a^2*c^5*d^2 + 73*a^2*c^4*d^3 + 32*a^2*c^3*d^4 + a^2*c^2*d^5 - 16*a^2*c*d^6
- 6*a^2*d^7)*cos(f*x + e)^3 + (12*a^2*c^7 + 96*a^2*c^6*d - 239*a^2*c^5*d^2
- 64*a^2*c^4*d^3 + 271*a^2*c^3*d^4 - 16*a^2*c^2*d^5 - 44*a^2*c*d^6 - 16*a^2
*d^7)*cos(f*x + e)^2 + (8*a^2*c^6*d + 64*a^2*c^5*d^2 - 208*a^2*c^4*d^3 + 16
*a^2*c^3*d^4 + 221*a^2*c^2*d^5 - 80*a^2*c*d^6 - 21*a^2*d^7)*cos(f*x + e))*s
in(f*x + e))/((c^12 + 2*c^11*d - 2*c^10*d^2 - 6*c^9*d^3 + 6*c^7*d^5 + 2*c^6
*d^6 - 2*c^5*d^7 - c^4*d^8)*f*cos(f*x + e)^4 + 4*(c^11*d + 2*c^10*d^2 - 2*c
^9*d^3 - 6*c^8*d^4 + 6*c^6*d^6 + 2*c^5*d^7 - 2*c^4*d^8 - c^3*d^9)*f*cos(f*x
 + e)^3 + 6*(c^10*d^2 + 2*c^9*d^3 - 2*c^8*d^4 - 6*c^7*d^5 + 6*c^5*d^7 + 2*c
^4*d^8 - 2*c^3*d^9 - c^2*d^10)*f*cos(f*x + e)^2 + 4*(c^9*d^3 + 2*c^8*d^4 -
2*c^7*d^5 - 6*c^6*d^6 + 6*c^4*d^8 + 2*c^3*d^9 - 2*c^2*d^10 - c*d^11)*f*cos(
f*x + e) + (c^8*d^4 + 2*c^7*d^5 - 2*c^6*d^6 - 6*c^5*d^7 + 6*c^3*d^9 + 2*c^2
*d^10 - 2*c*d^11 - d^12)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$$
$$=a^2\Bigg(\int \frac{\sec(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(}$$
$$+\int \frac{2\sec^2(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e}$$
$$+\int \frac{\sec^3(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**5,x)
```

```
[Out] a**2*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*se
c(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d
**5*sec(e + f*x)**5), x) + Integral(2*sec(e + f*x)**2/(c**5 + 5*c**4*d*sec(
e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*
c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(sec(e + f*x)*
*3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d
**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(257) = 514$.

Time = 0.51 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.57

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx = \frac{\frac{3\,(12\,a^2c^2-16\,a^2cd+7\,a^2d^2)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^6+2\,c^5d-c^4d^2-4\,c^3d^3-c^2d^4+2\,cd^5+d^6)\sqrt{-c^2+d^2}}-\frac{36\,a^2c^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-156\,a^2c^4 \cdots}{\cdots}}{\cdots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/12*(3*(12*a^2*c^2 - 16*a^2*c*d + 7*a^2*d^2)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/
2*e))/sqrt(-c^2 + d^2)))/((c^6 + 2*c^5*d - c^4*d^2 - 4*c^3*d^3 - c^2*d^4 +
2*c*d^5 + d^6)*sqrt(-c^2 + d^2)) - (36*a^2*c^5*tan(1/2*f*x + 1/2*e)^7 - 156
*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^7 + 273*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^7
```

```
- 243*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^7 + 111*a^2*c*d^4*tan(1/2*f*x + 1/2*
e)^7 - 21*a^2*d^5*tan(1/2*f*x + 1/2*e)^7 - 132*a^2*c^5*tan(1/2*f*x + 1/2*e)
^5 + 308*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^5 - 121*a^2*c^3*d^2*tan(1/2*f*x + 1
/2*e)^5 - 231*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 253*a^2*c*d^4*tan(1/2*f*
x + 1/2*e)^5 - 77*a^2*d^5*tan(1/2*f*x + 1/2*e)^5 + 156*a^2*c^5*tan(1/2*f*x
+ 1/2*e)^3 - 116*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 - 345*a^2*c^3*d^2*tan(1/2
*f*x + 1/2*e)^3 + 199*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 189*a^2*c*d^4*ta
n(1/2*f*x + 1/2*e)^3 - 83*a^2*d^5*tan(1/2*f*x + 1/2*e)^3 - 60*a^2*c^5*tan(1
/2*f*x + 1/2*e) - 36*a^2*c^4*d*tan(1/2*f*x + 1/2*e) + 177*a^2*c^3*d^2*tan(1
/2*f*x + 1/2*e) + 147*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 81*a^2*c*d^4*tan(1
/2*f*x + 1/2*e) - 75*a^2*d^5*tan(1/2*f*x + 1/2*e))/((c^6 + 2*c^5*d - c^4*d^
2 - 4*c^3*d^3 - c^2*d^4 + 2*c*d^5 + d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(
1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.59

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^2}{(c+d\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{11\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(12\,a^2\,c^2-16\,a^2\,c\,d+7\,a^2\,d^2)}{12\,(c+d)^3}-\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7\,(12\,a^2\,c^3-28\,a^2\,c^2\,d+2\cdots}{4\,(c+d)^4}\cdots}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(6\,c^4-12\,c^2\,d^2+6\,d^4)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(-4\,c^4-8\,c^3\,d+8\,c\,d^3+4\,d^4)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6\cdots\right.}$$
$$+\frac{a^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c-2\,d)\,(c^2-2\,c\,d+d^2)}{2\,\sqrt{c+d}\,(c-d)^{5/2}}\right)\,(12\,c^2-16\,c\,d+7\,d^2)}{4\,f\,(c+d)^{9/2}\,(c-d)^{5/2}}$$

```
[In] int((a + a/cos(e + f*x))^2/(cos(e + f*x)*(c + d/cos(e + f*x))^5),x)
```

```
[Out] ((11*tan(e/2 + (f*x)/2)^5*(12*a^2*c^2 + 7*a^2*d^2 - 16*a^2*c*d))/(12*(c + d
)^3) - (tan(e/2 + (f*x)/2)^7*(12*a^2*c^3 - 7*a^2*d^3 + 23*a^2*c*d^2 - 28*a^
2*c^2*d))/(4*(c + d)^4) - (a^2*tan(e/2 + (f*x)/2)^3*(156*c^2 - 272*c*d + 83
*d^2))/(12*(c + d)^2*(c - d)) + (a^2*tan(e/2 + (f*x)/2)*(20*c^2 - 48*c*d +
25*d^2))/(4*(c + d)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f*x)/2)^4*(6*c^4 +
 6*d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/2)^2*(8*c*d^3 - 8*c^3*d - 4*c^4 + 4*
d^4) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 - 8*c^3*d + 4*c^4 - 4*d^4) + tan(e/2 +
 (f*x)/2)^8*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2) + 4*c*d^3 + 4*c^3*d
 + c^4 + d^4 + 6*c^2*d^2)) + (a^2*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(c^
2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*(c - d)^(5/2)))*(12*c^2 - 16*c*d + 7*d^2
))/(4*f*(c + d)^(9/2)*(c - d)^(5/2))
```

3.202 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 31, antiderivative size = 288

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx \\
&= \frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\,\mathrm{arctanh}(\sin(e+fx))}{16f} \\
&\quad+\frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f} \\
&\quad+\frac{(40c^3+90c^2d+78cd^2+23d^3)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{48f} \\
&\quad+\frac{a(3c+8d)(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2\tan(e+fx)}{30f} \\
&\quad+\frac{a(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3\tan(e+fx)}{6f} \\
&\quad+\frac{a(a+a\sec(e+fx))^2\,(2(4c^3+74c^2d+66cd^2+21d^3)+d(6c^2+62cd+31d^2)\sec(e+fx))\tan(e+fx)}{120f}
\end{aligned}$$

```
[Out] 1/16*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)*arctanh(sin(f*x+e))/f+1/16*a^3*(
40*c^3+90*c^2*d+78*c*d^2+23*d^3)*tan(f*x+e)/f+1/48*(40*c^3+90*c^2*d+78*c*d^
2+23*d^3)*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/f+1/30*a*(3*c+8*d)*(a+a*sec(f*x+e
))^2*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/6*a*(a+a*sec(f*x+e))^2*(c+d*sec(f*x+
e))^3*tan(f*x+e)/f+1/120*a*(a+a*sec(f*x+e))^2*(8*c^3+148*c^2*d+132*c*d^2+42
*d^3+d*(6*c^2+62*c*d+31*d^2)*sec(f*x+e))*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4072, 102, 152, 52, 65, 223, 209}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$$
$$= \frac{a^4(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f}$$
$$+\frac{(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{48f}$$
$$+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3\left(70c^2+4d(8c+3d)\sec(e+fx)+54cd+19d^2\right)}{120f}$$
$$+\frac{a(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)(a\sec(e+fx)+a)^2}{120f}$$
$$+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))^2}{6f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Tan[e + f*x])/(16*f) + (a^4*(4
0*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[
a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(8*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a +
 a*Sec[e + f*x]]) + (a*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*(a + a*Sec[e
 + f*x])^2*Tan[e + f*x])/(120*f) + ((40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)
*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(48*f) + (d*(a + a*Sec[e + f*x])^3*
(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(6*f) + (d*(a + a*Sec[e + f*x])^3*(70*
c^2 + 54*c*d + 19*d^2 + 4*d*(8*c + 3*d)*Sec[e + f*x])*Tan[e + f*x])/(120*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
```

```
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c+dx)^3}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx)}{6f} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(a+ax)^{5/2}(c+dx)\left(-a^2\left(6c^2+3cd+2d^2\right)-a^2d(8c+3d)x\right)}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{6f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^3\left(70c^2+54cd+19d^2+4d(8c+3d)\sec(e+fx)\right)\tan(e+fx)}{120f} - \frac{\left(a^2\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{40f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a\left(40c^3+90c^2d+78cd^2+23d^3\right)(a+a\sec(e+fx))^2\tan(e+fx)}{120f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^3\left(70c^2+54cd+19d^2+4d(8c+3d)\sec(e+fx)\right)\tan(e+fx)}{120f} - \frac{\left(a^3\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{24f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a\left(40c^3+90c^2d+78cd^2+23d^3\right)(a+a\sec(e+fx))^2\tan(e+fx)}{120f} + \frac{\left(40c^3+90c^2d+78cd^2+23d^3\right)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{48f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^3\left(70c^2+54cd+19d^2+4d(8c+3d)\sec(e+fx)\right)\tan(e+fx)}{120f} - \frac{\left(a^4\left(40c^3+90c^2d+78cd^2+23d^3\right)\tan(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+ax}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{16f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^3(40c^3 + 90c^2d + 78cd^2 + 23d^3)\tan(e + fx)}{16f} + \frac{a(40c^3 + 90c^2d + 78cd^2 + 23d^3)\,(a + a\sec(e + fx))^2\tan(e + fx)}{120f} + \frac{(40c^3 + 90c^2d + 78cd^2 + 23d^3)\,(a^3 + a^3\sec(e + fx))\tan(e + fx)}{48f} + \frac{d(a + a\sec(e + fx))^3(c + d\sec(e + fx))^2\tan(e + fx)}{6f} + \frac{d(a + a\sec(e + fx))^3\,(70c^2 + 54cd + 19d^2 + 4d(8c + 3d)\sec(e + fx))\tan(e + fx)}{120f} - \frac{(a^5(40c^3 + 90c^2d + 78cd^2 + 23d^3)\tan(e + fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e + fx)\right)}{16f\sqrt{a - a\sec(e + fx)}\sqrt{a + a\sec(e + fx)}}$$

$$= \frac{a^3(40c^3 + 90c^2d + 78cd^2 + 23d^3)\tan(e + fx)}{16f} + \frac{a(40c^3 + 90c^2d + 78cd^2 + 23d^3)\,(a + a\sec(e + fx))^2\tan(e + fx)}{120f} + \frac{(40c^3 + 90c^2d + 78cd^2 + 23d^3)\,(a^3 + a^3\sec(e + fx))\tan(e + fx)}{48f} + \frac{d(a + a\sec(e + fx))^3(c + d\sec(e + fx))^2\tan(e + fx)}{6f} + \frac{d(a + a\sec(e + fx))^3\,(70c^2 + 54cd + 19d^2 + 4d(8c + 3d)\sec(e + fx))\tan(e + fx)}{120f} + \frac{(a^4(40c^3 + 90c^2d + 78cd^2 + 23d^3)\tan(e + fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a - a\sec(e + fx)}\right)}{8f\sqrt{a - a\sec(e + fx)}\sqrt{a + a\sec(e + fx)}}$$

$$= \frac{a^3(40c^3 + 90c^2d + 78cd^2 + 23d^3)\tan(e + fx)}{16f} + \frac{a(40c^3 + 90c^2d + 78cd^2 + 23d^3)\,(a + a\sec(e + fx))^2\tan(e + fx)}{120f} + \frac{(40c^3 + 90c^2d + 78cd^2 + 23d^3)\,(a^3 + a^3\sec(e + fx))\tan(e + fx)}{48f} + \frac{d(a + a\sec(e + fx))^3(c + d\sec(e + fx))^2\tan(e + fx)}{6f} + \frac{d(a + a\sec(e + fx))^3\,(70c^2 + 54cd + 19d^2 + 4d(8c + 3d)\sec(e + fx))\tan(e + fx)}{120f} + \frac{(a^4(40c^3 + 90c^2d + 78cd^2 + 23d^3)\tan(e + fx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{8f\sqrt{a - a\sec(e + fx)}\sqrt{a + a\sec(e + fx)}}$$

$$= \frac{a^3(40c^3+90c^2d+78cd^2+23d^3)\tan(e+fx)}{16f} + \frac{a^4(40c^3+90c^2d+78cd^2+23d^3)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a(40c^3+90c^2d+78cd^2+23d^3)(a+a\sec(e+fx))^2\tan(e+fx)}{120f} + \frac{(40c^3+90c^2d+78cd^2+23d^3)(a^3+a^3\sec(e+fx))\tan(e+fx)}{48f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\tan(e+fx)}{6f} + \frac{d(a+a\sec(e+fx))^3(70c^2+54cd+19d^2+4d(8c+3d)\sec(e+fx))\tan(e+fx)}{120f}$$

Mathematica [A] (verified)

Time = 6.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.59

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx = \frac{a^3(15(40c^3+90c^2d+78cd^2+23d^3)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)(15(24c^3+90c^2d+78cd^2+23d$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(15*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*ArcTanh[Sin[e + f*x]] + Ta
n[e + f*x]*(15*(24*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3)*Sec[e + f*x] + 10*d*
(18*c^2 + 54*c*d + 23*d^2)*Sec[e + f*x]^3 + 40*d^3*Sec[e + f*x]^5 + 16*(c +
 d)*(60*(c + d)^2 + 5*(c^2 + 8*c*d + 7*d^2)*Tan[e + f*x]^2 + 9*d^2*Tan[e +
f*x]^4))))/(240*f)
```

Maple [A] (verified)

Time = 6.19 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.23

method	result
norman	$-\frac{33a^3(40c^3+90c^2d+78c\,d^2+23d^3)\tan(\frac{fx}{2}+\frac{e}{2})^7}{20f}+\frac{17a^3(40c^3+90c^2d+78c\,d^2+23d^3)\tan(\frac{fx}{2}+\frac{e}{2})^9}{24f}-\frac{a^3(40c^3+90c^2d+78c\,d^2+23d^3}{8f}$
parallelrisch	$6\left(-\frac{25\left(\frac{2\cos(4fx+4e)}{5}+\frac{2}{3}+\cos(2fx+2e)+\frac{\cos(6fx+6e)}{15}\right)\left(c^3+\frac{9}{4}c^2d+\frac{39}{20}c\,d^2+\frac{23}{40}d^3\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{4}+\frac{25\left(\frac{2\cos(4fx+4e)}{5}+\frac{2}{3}+c\right.}{}\right.$
parts	$-\frac{(3a^3c\,d^2+3a^3d^3)\left(-\frac{8}{15}-\frac{\sec(fx+e)^4}{5}-\frac{4\sec(fx+e)^2}{15}\right)\tan(fx+e)}{f}+\frac{(3c^3a^3+3a^3c^2d)\tan(fx+e)}{f}+\frac{(3a^3c^2d+9a^3c\,d^2}{}$
derivativedivides	$\frac{-c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+3a^3c^2d\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-}{}$
default	$\frac{-c^3a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+3a^3c^2d\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-}{}$
risch	$-\frac{ia^3\left(-880c^3-1824c\,d^2-544d^3-2160c^2d-1170c\,d^2e^{i(fx+e)}-10944c\,d^2e^{2i(fx+e)}-5670c\,d^2e^{3i(fx+e)}-18240c\,d^2e^{6i(fx+}\right.}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] (-33/20*a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)/f*tan(1/2*f*x+1/2*e)^7+17/24*
a^3*(40*c^3+90*c^2*d+78*c*d^2+23*d^3)/f*tan(1/2*f*x+1/2*e)^9-1/8*a^3*(40*c^
3+90*c^2*d+78*c*d^2+23*d^3)/f*tan(1/2*f*x+1/2*e)^11+1/8*a^3*(88*c^3+294*c^2
*d+306*c*d^2+105*d^3)/f*tan(1/2*f*x+1/2*e)+3/20*a^3*(520*c^3+1250*c^2*d+998
*c*d^2+323*d^3)/f*tan(1/2*f*x+1/2*e)^5-1/24*a^3*(1112*c^3+3078*c^2*d+2514*c
*d^2+633*d^3)/f*tan(1/2*f*x+1/2*e)^3)/(tan(1/2*f*x+1/2*e)^2-1)^6-1/16*a^3*(
40*c^3+90*c^2*d+78*c*d^2+23*d^3)/f*ln(tan(1/2*f*x+1/2*e)-1)+1/16*a^3*(40*c^
3+90*c^2*d+78*c*d^2+23*d^3)/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.17

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$$
$$=\frac{15\left(40\,a^3c^3+90\,a^3c^2d+78\,a^3cd^2+23\,a^3d^3\right)\cos\left(fx+e\right)^6\log\left(\sin\left(fx+e\right)+1\right)-15\left(40\,a^3c^3+90\,a^3c^2d+\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x +
 e)^6*log(sin(f*x + e) + 1) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2
```

```
+ 23*a^3*d^3)*cos(f*x + e)^6*log(-sin(f*x + e) + 1) + 2*(40*a^3*d^3 + 16*(5
5*a^3*c^3 + 135*a^3*c^2*d + 114*a^3*c*d^2 + 34*a^3*d^3)*cos(f*x + e)^5 + 15
*(24*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^4 + 1
6*(5*a^3*c^3 + 45*a^3*c^2*d + 57*a^3*c*d^2 + 17*a^3*d^3)*cos(f*x + e)^3 + 1
0*(18*a^3*c^2*d + 54*a^3*c*d^2 + 23*a^3*d^3)*cos(f*x + e)^2 + 144*(a^3*c*d^
2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^6)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx = a^3\left(\int c^3\sec(e+fx)\,dx + \int 3c^3\sec^2(e+fx)\,dx + \int 3c^3\sec^3(e+fx)\,dx + \int c^3\sec^4(e+fx)\,dx + \int d^3\sec^4(e+fx)\,dx + \int 3d^3\sec^5(e+fx)\,dx + \int 3d^3\sec^6(e+fx)\,dx + \int d^3\sec^7(e+fx)\,dx + \int 3cd^2\sec^3(e+fx)\,dx + \int 9cd^2\sec^4(e+fx)\,dx + \int 9cd^2\sec^5(e+fx)\,dx + \int 3cd^2\sec^6(e+fx)\,dx + \int 3c^2d\sec^2(e+fx)\,dx + \int 9c^2d\sec^3(e+fx)\,dx + \int 9c^2d\sec^4(e+fx)\,dx + \int 3c^2d\sec^5(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(c**3*sec(e + f*x), x) + Integral(3*c**3*sec(e + f*x)**2, x)
+ Integral(3*c**3*sec(e + f*x)**3, x) + Integral(c**3*sec(e + f*x)**4, x) +
 Integral(d**3*sec(e + f*x)**4, x) + Integral(3*d**3*sec(e + f*x)**5, x) +
Integral(3*d**3*sec(e + f*x)**6, x) + Integral(d**3*sec(e + f*x)**7, x) + I
ntegral(3*c*d**2*sec(e + f*x)**3, x) + Integral(9*c*d**2*sec(e + f*x)**4, x
) + Integral(9*c*d**2*sec(e + f*x)**5, x) + Integral(3*c*d**2*sec(e + f*x)*
*6, x) + Integral(3*c**2*d*sec(e + f*x)**2, x) + Integral(9*c**2*d*sec(e +
f*x)**3, x) + Integral(9*c**2*d*sec(e + f*x)**4, x) + Integral(3*c**2*d*sec
(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(273) = 546$.

Time = 0.24 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.43

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$$

$$= \frac{160\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^3c^3+1440\left(\tan\left(fx+e\right)^3+3\tan\left(fx+e\right)\right)a^3c^2d+96\left(3\tan\left(fx+\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/480*(160*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^3 + 1440*(tan(f*x + e)^3
 + 3*tan(f*x + e))*a^3*c^2*d + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 1
5*tan(f*x + e))*a^3*c*d^2 + 1440*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c*d^
2 + 96*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*tan(f*x + e))*a^3*d^3 + 1
60*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^3 - 5*a^3*d^3*(2*(15*sin(f*x + e
)^5 - 40*sin(f*x + e)^3 + 33*sin(f*x + e))/(sin(f*x + e)^6 - 3*sin(f*x + e)
^4 + 3*sin(f*x + e)^2 - 1) - 15*log(sin(f*x + e) + 1) + 15*log(sin(f*x + e)
 - 1)) - 90*a^3*c^2*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^
4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) -
1)) - 270*a^3*c*d^2*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4
- 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)
) - 90*a^3*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*s
in(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 3
60*a^3*c^3*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + l
og(sin(f*x + e) - 1)) - 1080*a^3*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1)
 - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 360*a^3*c*d^2*(2*sin(f*
x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)
) + 480*a^3*c^3*log(sec(f*x + e) + tan(f*x + e)) + 1440*a^3*c^3*tan(f*x + e
) + 1440*a^3*c^2*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(273) = 546$.

Time = 0.44 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.03

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$$

$$= \frac{15\left(40\,a^3c^3+90\,a^3c^2d+78\,a^3cd^2+23\,a^3d^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\left(40\,a^3c^3+90\,a^3c^2d+78\,a^3cd\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 + 23*a^3*d^3)*log(abs(t
an(1/2*f*x + 1/2*e) + 1)) - 15*(40*a^3*c^3 + 90*a^3*c^2*d + 78*a^3*c*d^2 +
23*a^3*d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(600*a^3*c^3*tan(1/2*f*x
 + 1/2*e)^11 + 1350*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^11 + 1170*a^3*c*d^2*tan(
1/2*f*x + 1/2*e)^11 + 345*a^3*d^3*tan(1/2*f*x + 1/2*e)^11 - 3400*a^3*c^3*ta
n(1/2*f*x + 1/2*e)^9 - 7650*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^9 - 6630*a^3*c*d
^2*tan(1/2*f*x + 1/2*e)^9 - 1955*a^3*d^3*tan(1/2*f*x + 1/2*e)^9 + 7920*a^3*
c^3*tan(1/2*f*x + 1/2*e)^7 + 17820*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^7 + 15444
*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 4554*a^3*d^3*tan(1/2*f*x + 1/2*e)^7 - 9
360*a^3*c^3*tan(1/2*f*x + 1/2*e)^5 - 22500*a^3*c^2*d*tan(1/2*f*x + 1/2*e)^5
 - 17964*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 5814*a^3*d^3*tan(1/2*f*x + 1/2*
e)^5 + 5560*a^3*c^3*tan(1/2*f*x + 1/2*e)^3 + 15390*a^3*c^2*d*tan(1/2*f*x +
1/2*e)^3 + 12570*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 3165*a^3*d^3*tan(1/2*f*
x + 1/2*e)^3 - 1320*a^3*c^3*tan(1/2*f*x + 1/2*e) - 4410*a^3*c^2*d*tan(1/2*f
*x + 1/2*e) - 4590*a^3*c*d^2*tan(1/2*f*x + 1/2*e) - 1575*a^3*d^3*tan(1/2*f*
x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^6)/f
```

Mupad [B] (verification not implemented)

Time = 17.05 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.43

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3\,dx$$
$$= \frac{a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(40\,c^3+90\,c^2\,d+78\,c\,d^2+23\,d^3\right)}{4\,\left(10\,c^3+\frac{45\,c^2\,d}{2}+\frac{39\,c\,d^2}{2}+\frac{23\,d^3}{4}\right)}\right)\,\left(40\,c^3+90\,c^2\,d+78\,c\,d^2+23\,d^3\right)}{8\,f} - \frac{\left(5\,a^3\,c^3+\frac{45\,a^3\,c^2\,d}{4}+\frac{39\,a^3\,c\,d^2}{4}+\frac{23\,a^3\,d^3}{8}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{11}+\left(-\frac{85\,a^3\,c^3}{3}-\frac{255\,a^3\,c^2\,d}{4}-\frac{221\,a^3\,c\,d^2}{4}-\frac{391\,a^3\,d^3}{24}\right)\,\mathrm{ta}}{}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (a^3*atanh((tan(e/2 + (f*x)/2)*(78*c*d^2 + 90*c^2*d + 40*c^3 + 23*d^3))/(4*
((39*c*d^2)/2 + (45*c^2*d)/2 + 10*c^3 + (23*d^3)/4)))*(78*c*d^2 + 90*c^2*d
+ 40*c^3 + 23*d^3))/(8*f) - (tan(e/2 + (f*x)/2)^11*(5*a^3*c^3 + (23*a^3*d^3
)/8 + (39*a^3*c*d^2)/4 + (45*a^3*c^2*d)/4) - tan(e/2 + (f*x)/2)^9*((85*a^3*
c^3)/3 + (391*a^3*d^3)/24 + (221*a^3*c*d^2)/4 + (255*a^3*c^2*d)/4) + tan(e/
2 + (f*x)/2)^3*((139*a^3*c^3)/3 + (211*a^3*d^3)/8 + (419*a^3*c*d^2)/4 + (51
3*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^7*(66*a^3*c^3 + (759*a^3*d^3)/20 + (12
87*a^3*c*d^2)/10 + (297*a^3*c^2*d)/2) - tan(e/2 + (f*x)/2)^5*(78*a^3*c^3 +
(969*a^3*d^3)/20 + (1497*a^3*c*d^2)/10 + (375*a^3*c^2*d)/2) - tan(e/2 + (f*
x)/2)*(11*a^3*c^3 + (105*a^3*d^3)/8 + (153*a^3*c*d^2)/4 + (147*a^3*c^2*d)/4
```

```
))/(f*(15*tan(e/2 + (f*x)/2)^4 - 6*tan(e/2 + (f*x)/2)^2 - 20*tan(e/2 + (f*x
)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 - 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x
)/2)^12 + 1))
```

3.203 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 257

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx \\
&= \frac{a^3(20c^2+30cd+13d^2)\,\text{arctanh}(\sin(e+fx))}{8f} \\
&\quad + \frac{a^3(2c^4-15c^3d+72c^2d^2+180cd^3+76d^4)\tan(e+fx)}{30d^2f} \\
&\quad + \frac{a^3(4c^3-30c^2d+146cd^2+195d^3)\sec(e+fx)\tan(e+fx)}{120df} \\
&\quad + \frac{a^3(2c^2-15cd+76d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60d^2f} \\
&\quad - \frac{a^3(2c-11d)(c+d\sec(e+fx))^3\tan(e+fx)}{20d^2f} \\
&\quad + \frac{(a^3+a^3\sec(e+fx))\,(c+d\sec(e+fx))^3\tan(e+fx)}{5df}
\end{aligned}$$

```
[Out] 1/8*a^3*(20*c^2+30*c*d+13*d^2)*arctanh(sin(f*x+e))/f+1/30*a^3*(2*c^4-15*c^3
*d+72*c^2*d^2+180*c*d^3+76*d^4)*tan(f*x+e)/d^2/f+1/120*a^3*(4*c^3-30*c^2*d+
146*c*d^2+195*d^3)*sec(f*x+e)*tan(f*x+e)/d/f+1/60*a^3*(2*c^2-15*c*d+76*d^2)
*(c+d*sec(f*x+e))^2*tan(f*x+e)/d^2/f-1/20*a^3*(2*c-11*d)*(c+d*sec(f*x+e))^3
*tan(f*x+e)/d^2/f+1/5*(a^3+a^3*sec(f*x+e))*(c+d*sec(f*x+e))^3*tan(f*x+e)/d/
f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4072, 92, 81, 52, 65, 223, 209}

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$
$$= \frac{a^4(20c^2+30cd+13d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{(20c^2+30cd+13d^2)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{24f} + \frac{a(20c^2+30cd+13d^2)\tan(e+fx)(a\sec(e+fx)+a)^2}{60f} + \frac{3d(2c+d)\tan(e+fx)(a\sec(e+fx)+a)^3}{20f} + \frac{d\tan(e+fx)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))}{5f}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(20*c^2 + 30*c*d + 13*d^2)*Tan[e + f*x])/(8*f) + (a^4*(20*c^2 + 30*c*d
 + 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[
e + f*x])/(4*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a*(20*
c^2 + 30*c*d + 13*d^2)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + (3*d*(
2*c + d)*(a + a*Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + ((20*c^2 + 30*c*d +
13*d^2)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(24*f) + (d*(a + a*Sec[e + f
*x])^3*(c + d*Sec[e + f*x])*Tan[e + f*x])/(5*f)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^{5/2}(c+dx)^2}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(a+ax)^{5/2}(-a^2(5c^2+3cd+d^2)-3a^2d(2c+d)x)}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{5f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f} - \frac{(a^2(20c^2+30cd+13d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{20f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a(20c^2+30cd+13d^2)\,(a+a\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f} - \frac{(a^3(20c^2+30cd+13d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{12f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a(20c^2+30cd+13d^2)\,(a+a\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{(20c^2+30cd+13d^2)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f} - \frac{(a^4(20c^2+30cd+13d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{\sqrt{a-ax}}\,dx, x, \sec(e+fx)\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{a(20c^2+30cd+13d^2)(a+a\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{(20c^2+30cd+13d^2)(a^3+a^3\sec(e+fx))\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f} - \frac{(a^5(20c^2+30cd+13d^2)\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{8f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{a(20c^2+30cd+13d^2)(a+a\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{(20c^2+30cd+13d^2)(a^3+a^3\sec(e+fx))\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f} + \frac{(a^4(20c^2+30cd+13d^2)\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{a(20c^2+30cd+13d^2)(a+a\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{(20c^2+30cd+13d^2)(a^3+a^3\sec(e+fx))\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f} + \frac{(a^4(20c^2+30cd+13d^2)\tan(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^3(20c^2+30cd+13d^2)\tan(e+fx)}{8f} + \frac{a^4(20c^2+30cd+13d^2)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{a(20c^2+30cd+13d^2)\,(a+a\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{3d(2c+d)(a+a\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{(20c^2+30cd+13d^2)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{24f} + \frac{d(a+a\sec(e+fx))^3(c+d\sec(e+fx))\tan(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.51

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx = \frac{a^3(15(20c^2+30cd+13d^2)\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\,(15(12c^2+30cd+13d^2)\sec(e+fx)+30d}{120f}$$

```
[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*(15*(20*c^2 + 30*c*d + 13*d^2)*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(1
5*(12*c^2 + 30*c*d + 13*d^2)*Sec[e + f*x] + 30*d*(2*c + 3*d)*Sec[e + f*x]^3
 + 8*(60*(c + d)^2 + 5*(c^2 + 6*c*d + 5*d^2)*Tan[e + f*x]^2 + 3*d^2*Tan[e +
 f*x]^4))))/(120*f)
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00

method	result
norman	$\frac{-\frac{32a^3(20c^2+30cd+13d^2)\tan(\frac{fx}{2}+\frac{e}{2})^5}{15f}+\frac{7a^3(20c^2+30cd+13d^2)\tan(\frac{fx}{2}+\frac{e}{2})^7}{6f}-\frac{a^3(20c^2+30cd+13d^2)\tan(\frac{fx}{2}+\frac{e}{2})^9}{4f}-\frac{a^3(44c^2}{}}{(\tan(\frac{fx}{2}+\frac{e}{2})^2-1)^5}$
parts	$\frac{(2a^3cd+3a^3d^2)\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}+\frac{(3c^2a^3+2a^3cd)\tan(fx+e}{f}$
parallelrisch	$\frac{26a^3\left(-\frac{75\left(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}+\cos(fx+e)\right)(c^2+\frac{3}{2}cd+\frac{13}{20}d^2)\ln(\tan(\frac{fx}{2}+\frac{e}{2})-1)}{26}+\frac{75\left(\frac{\cos(5fx+5e)}{10}+\frac{\cos(3fx+3e)}{2}+\mathrm{co}\right.}{}\right.}{}$
derivativedivides	$\frac{-c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2a^3cd\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-}{}$
default	$\frac{-c^2a^3\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+2a^3cd\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)-}{}$
risch	$-\frac{ia^3(-440c^2-4800cd\,e^{4i(fx+e)}-3360cd\,e^{2i(fx+e)}-450d\,e^{i(fx+e)}c-720cd-304d^2-1520d^2e^{2i(fx+e)}-195d^2e^{i(fx+e)}-}{}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] (-32/15*a^3*(20*c^2+30*c*d+13*d^2)/f*tan(1/2*f*x+1/2*e)^5+7/6*a^3*(20*c^2+3
0*c*d+13*d^2)/f*tan(1/2*f*x+1/2*e)^7-1/4*a^3*(20*c^2+30*c*d+13*d^2)/f*tan(1
/2*f*x+1/2*e)^9-1/4*a^3*(44*c^2+98*c*d+51*d^2)/f*tan(1/2*f*x+1/2*e)+1/6*a^3
*(212*c^2+366*c*d+133*d^2)/f*tan(1/2*f*x+1/2*e)^3)/(tan(1/2*f*x+1/2*e)^2-1)
^5-1/8*a^3*(20*c^2+30*c*d+13*d^2)/f*ln(tan(1/2*f*x+1/2*e)-1)+1/8*a^3*(20*c^
2+30*c*d+13*d^2)/f*ln(tan(1/2*f*x+1/2*e)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.95

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$
$$=\frac{15\,(20\,a^3c^2+30\,a^3cd+13\,a^3d^2)\cos(fx+e)^5\log(\sin(fx+e)+1)-15\,(20\,a^3c^2+30\,a^3cd+13\,a^3d^2)\cos}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(sin(f*x
 + e) + 1) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^5*log(-
sin(f*x + e) + 1) + 2*(24*a^3*d^2 + 8*(55*a^3*c^2 + 90*a^3*c*d + 38*a^3*d^2
)*cos(f*x + e)^4 + 15*(12*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*cos(f*x + e)^3
 + 8*(5*a^3*c^2 + 30*a^3*c*d + 19*a^3*d^2)*cos(f*x + e)^2 + 30*(2*a^3*c*d +
 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$
$$= a^3\left(\int c^2\sec(e+fx)\,dx + \int 3c^2\sec^2(e+fx)\,dx + \int 3c^2\sec^3(e+fx)\,dx + \int c^2\sec^4(e+fx)\,dx + \int d^2\sec^3(e+fx)\,dx + \int 3d^2\sec^4(e+fx)\,dx + \int 3d^2\sec^5(e+fx)\,dx + \int d^2\sec^6(e+fx)\,dx + \int 2cd\sec^2(e+fx)\,dx + \int 6cd\sec^3(e+fx)\,dx + \int 6cd\sec^4(e+fx)\,dx + \int 2cd\sec^5(e+fx)\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(c**2*sec(e + f*x), x) + Integral(3*c**2*sec(e + f*x)**2, x)
+ Integral(3*c**2*sec(e + f*x)**3, x) + Integral(c**2*sec(e + f*x)**4, x) +
 Integral(d**2*sec(e + f*x)**3, x) + Integral(3*d**2*sec(e + f*x)**4, x) +
Integral(3*d**2*sec(e + f*x)**5, x) + Integral(d**2*sec(e + f*x)**6, x) + I
ntegral(2*c*d*sec(e + f*x)**2, x) + Integral(6*c*d*sec(e + f*x)**3, x) + In
tegral(6*c*d*sec(e + f*x)**4, x) + Integral(2*c*d*sec(e + f*x)**5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.79

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$
$$= \frac{80\left(\tan(fx+e)^3 + 3\tan(fx+e)\right)a^3c^2 + 480\left(\tan(fx+e)^3 + 3\tan(fx+e)\right)a^3cd + 16\left(3\tan(fx+e)^5\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c^2 + 480*(tan(f*x + e)^3 +
 3*tan(f*x + e))*a^3*c*d + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15*ta
n(f*x + e))*a^3*d^2 + 240*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*d^2 - 30*a^
3*c*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x +
e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 45*a^3*d^2
*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2
+ 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 180*a^3*c^2*(2*
sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e
) - 1)) - 360*a^3*c*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
e) + 1) + log(sin(f*x + e) - 1)) - 60*a^3*d^2*(2*sin(f*x + e)/(sin(f*x + e)
^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a^3*c^2*log(
sec(f*x + e) + tan(f*x + e)) + 720*a^3*c^2*tan(f*x + e) + 480*a^3*c*d*tan(f
*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.46

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$

$$= \frac{15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-15\left(20\,a^3c^2+30\,a^3cd+13\,a^3d^2\right)\log\left(\left|\tan\left(\right.\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*
e) + 1)) - 15*(20*a^3*c^2 + 30*a^3*c*d + 13*a^3*d^2)*log(abs(tan(1/2*f*x +
1/2*e) - 1)) - 2*(300*a^3*c^2*tan(1/2*f*x + 1/2*e)^9 + 450*a^3*c*d*tan(1/2*
f*x + 1/2*e)^9 + 195*a^3*d^2*tan(1/2*f*x + 1/2*e)^9 - 1400*a^3*c^2*tan(1/2*
f*x + 1/2*e)^7 - 2100*a^3*c*d*tan(1/2*f*x + 1/2*e)^7 - 910*a^3*d^2*tan(1/2*
f*x + 1/2*e)^7 + 2560*a^3*c^2*tan(1/2*f*x + 1/2*e)^5 + 3840*a^3*c*d*tan(1/2
*f*x + 1/2*e)^5 + 1664*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 2120*a^3*c^2*tan(1/
2*f*x + 1/2*e)^3 - 3660*a^3*c*d*tan(1/2*f*x + 1/2*e)^3 - 1330*a^3*d^2*tan(1
/2*f*x + 1/2*e)^3 + 660*a^3*c^2*tan(1/2*f*x + 1/2*e) + 1470*a^3*c*d*tan(1/2
*f*x + 1/2*e) + 765*a^3*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 -
 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.12

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2\,dx$$
$$=\frac{a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(20\,c^2+30\,c\,d+13\,d^2)}{2\,\left(10\,c^2+15\,c\,d+\frac{13\,d^2}{2}\right)}\right)\,(20\,c^2+30\,c\,d+13\,d^2)}{4\,f}$$
$$-\frac{\left(5\,a^3\,c^2+\frac{15\,a^3\,c\,d}{2}+\frac{13\,a^3\,d^2}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9+\left(-\frac{70\,a^3\,c^2}{3}-35\,a^3\,c\,d-\frac{91\,a^3\,d^2}{6}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(\frac{128\,a^3\,c^2}{3}+\right.}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^{10}-5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8\right.}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (a^3*atanh((tan(e/2 + (f*x)/2)*(30*c*d + 20*c^2 + 13*d^2))/(2*(15*c*d + 10*
c^2 + (13*d^2)/2)))*(30*c*d + 20*c^2 + 13*d^2))/(4*f) - (tan(e/2 + (f*x)/2)
*(11*a^3*c^2 + (51*a^3*d^2)/4 + (49*a^3*c*d)/2) + tan(e/2 + (f*x)/2)^9*(5*a
^3*c^2 + (13*a^3*d^2)/4 + (15*a^3*c*d)/2) - tan(e/2 + (f*x)/2)^7*((70*a^3*c
^2)/3 + (91*a^3*d^2)/6 + 35*a^3*c*d) - tan(e/2 + (f*x)/2)^3*((106*a^3*c^2)/
3 + (133*a^3*d^2)/6 + 61*a^3*c*d) + tan(e/2 + (f*x)/2)^5*((128*a^3*c^2)/3 +
 (416*a^3*d^2)/15 + 64*a^3*c*d))/(f*(5*tan(e/2 + (f*x)/2)^2 - 10*tan(e/2 +
(f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (
f*x)/2)^10 - 1))
```

3.204 $\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 29, antiderivative size = 125

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx &= \frac{5a^3(4c+3d)\text{arctanh}(\sin(e+fx))}{8f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}\\&+\frac{3a^3(4c+3d)\sec(e+fx)\tan(e+fx)}{8f}\\&+\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f}+\frac{a^3(4c+3d)\tan^3(e+fx)}{12f}\end{aligned}$$

```
[Out] 5/8*a^3*(4*c+3*d)*arctanh(sin(f*x+e))/f+a^3*(4*c+3*d)*tan(f*x+e)/f+3/8*a^3*
(4*c+3*d)*sec(f*x+e)*tan(f*x+e)/f+1/4*d*(a+a*sec(f*x+e))^3*tan(f*x+e)/f+1/1
2*a^3*(4*c+3*d)*tan(f*x+e)^3/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4086, 3876, 3855, 3852, 8, 3853}

$$\begin{aligned}&\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx\\&=\frac{5a^3(4c+3d)\text{arctanh}(\sin(e+fx))}{8f}+\frac{a^3(4c+3d)\tan^3(e+fx)}{12f}+\frac{a^3(4c+3d)\tan(e+fx)}{f}\\&+\frac{3a^3(4c+3d)\tan(e+fx)\sec(e+fx)}{8f}+\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3}{4f}\end{aligned}$$

```
[In] Int[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]
```

```
[Out] (5*a^3*(4*c + 3*d)*ArcTanh[Sin[e + f*x]])/(8*f) + (a^3*(4*c + 3*d)*Tan[e +
f*x])/f + (3*a^3*(4*c + 3*d)*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (d*(a + a*S
ec[e + f*x])^3*Tan[e + f*x])/(4*f) + (a^3*(4*c + 3*d)*Tan[e + f*x]^3)/(12*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{1}{4}(4c+3d)\int \sec(e+fx)(a+a\sec(e+fx))^3\,dx \\
&= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{1}{4}(4c+3d)\int \left(a^3\sec(e+fx) \right.\\
&\qquad \left. +3a^3\sec^2(e+fx)+3a^3\sec^3(e+fx)+a^3\sec^4(e+fx)\right)\,dx \\
&= \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{1}{4}\left(a^3(4c+3d)\right)\int \sec(e+fx)\,dx \\
&\quad + \frac{1}{4}\left(a^3(4c+3d)\right)\int \sec^4(e+fx)\,dx + \frac{1}{4}\left(3a^3(4c+3d)\right)\int \sec^2(e+fx)\,dx \\
&\quad + \frac{1}{4}\left(3a^3(4c+3d)\right)\int \sec^3(e+fx)\,dx \\
&= \frac{a^3(4c+3d)\text{arctanh}(\sin(e+fx))}{4f} + \frac{3a^3(4c+3d)\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{1}{8}\left(3a^3(4c+3d)\right)\int \sec(e+fx)\,dx \\
&\quad - \frac{\left(a^3(4c+3d)\right)\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(e+fx)\right)}{4f} \\
&\quad - \frac{\left(3a^3(4c+3d)\right)\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{4f} \\
&= \frac{5a^3(4c+3d)\text{arctanh}(\sin(e+fx))}{8f} + \frac{a^3(4c+3d)\tan(e+fx)}{f} \\
&\quad + \frac{3a^3(4c+3d)\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4f} + \frac{a^3(4c+3d)\tan^3(e+fx)}{12f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.38

$$\begin{aligned}
&\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx \\
&= \frac{5a^3c\text{arctanh}(\sin(e+fx))}{2f} + \frac{15a^3d\text{arctanh}(\sin(e+fx))}{8f} + \frac{4a^3c\tan(e+fx)}{f} \\
&\quad + \frac{4a^3d\tan(e+fx)}{f} + \frac{3a^3c\sec(e+fx)\tan(e+fx)}{2f} + \frac{15a^3d\sec(e+fx)\tan(e+fx)}{8f} \\
&\quad + \frac{a^3d\sec^3(e+fx)\tan(e+fx)}{4f} + \frac{a^3c\tan^3(e+fx)}{3f} + \frac{a^3d\tan^3(e+fx)}{f}
\end{aligned}$$

[In] Integrate[Sec[e + f*x]*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]),x]

[Out] (5*a^3*c*ArcTanh[Sin[e + f*x]])/(2*f) + (15*a^3*d*ArcTanh[Sin[e + f*x]])/(8*f) + (4*a^3*c*Tan[e + f*x])/f + (4*a^3*d*Tan[e + f*x])/f + (3*a^3*c*Sec[e + f*x]*Tan[e + f*x])/(2*f) + (15*a^3*d*Sec[e + f*x]*Tan[e + f*x])/(8*f) + (a^3*d*Sec[e + f*x]^3*Tan[e + f*x])/(4*f) + (a^3*c*Tan[e + f*x]^3)/(3*f) + (a^3*d*Tan[e + f*x]^3)/f

Maple [A] (verified)

Time = 3.98 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

method	result
norman	$\frac{-\frac{73a^3(4c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12f}+\frac{55a^3(4c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{12f}-\frac{5a^3(4c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{a^3(49d+44c)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^4}-\frac{5a^3(4c+3d}{}$
parallelrisch	$\frac{26\left(-\frac{15\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(c+\frac{3d}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{13}+\frac{15\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(c+\frac{3d}{4}\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right.\right.}{13}\right.}{3f(3+\cos(4fx+4e)+4\cos(2}$
parts	$-\frac{(a^3c+3a^3d)\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{(3a^3c+a^3d)\tan(fx+e)}{f}+\frac{(3a^3c+3a^3d)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(}{}\right.}{f}$
derivativedivides	$\frac{-a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+a^3d\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+3a^3c}{}$
default	$\frac{-a^3c\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)+a^3d\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)+3a^3c}{}$
risch	$-\frac{ia^3\left(36c\,e^{7i(fx+e)}+45d\,e^{7i(fx+e)}-72c\,e^{6i(fx+e)}-24d\,e^{6i(fx+e)}+36c\,e^{5i(fx+e)}+69d\,e^{5i(fx+e)}-264c\,e^{4i(fx+e)}-216d\,e^{4}\right.}{12f\left(1+e^{2i(fx\right.}}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] (-73/12*a^3*(4*c+3*d)/f*tan(1/2*f*x+1/2*e)^3+55/12*a^3*(4*c+3*d)/f*tan(1/2*f*x+1/2*e)^5-5/4*a^3*(4*c+3*d)/f*tan(1/2*f*x+1/2*e)^7+1/4*a^3*(49*d+44*c)/f*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2-1)^4-5/8*a^3*(4*c+3*d)/f*ln(tan(1/2*f*x+1/2*e)-1)+5/8*a^3*(4*c+3*d)/f*ln(tan(1/2*f*x+1/2*e)+1)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.29

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$
$$= \frac{15\,(4\,a^3c+3\,a^3d)\cos{(fx+e)}^4\log{(\sin{(fx+e)}+1)}-15\,(4\,a^3c+3\,a^3d)\cos{(fx+e)}^4\log{(-\sin{(fx+e)}}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*(4*a^3*c + 3*a^3*d)*cos(f*x + e)^4*log(sin(f*x + e) + 1) - 15*(4*a
^3*c + 3*a^3*d)*cos(f*x + e)^4*log(-sin(f*x + e) + 1) + 2*(6*a^3*d + 8*(11*
a^3*c + 9*a^3*d)*cos(f*x + e)^3 + 9*(4*a^3*c + 5*a^3*d)*cos(f*x + e)^2 + 8*
(a^3*c + 3*a^3*d)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4)
```

Sympy [F]

$$\begin{aligned}\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx\\ = a^3\bigg(\int c\sec{(e+fx)}\,dx+\int 3c\sec^2{(e+fx)}\,dx+\int 3c\sec^3{(e+fx)}\,dx\\ +\int c\sec^4{(e+fx)}\,dx+\int d\sec^2{(e+fx)}\,dx+\int 3d\sec^3{(e+fx)}\,dx\\ +\int 3d\sec^4{(e+fx)}\,dx+\int d\sec^5{(e+fx)}\,dx\bigg)\end{aligned}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3*(c+d*sec(f*x+e)),x)
```

```
[Out] a**3*(Integral(c*sec(e + f*x), x) + Integral(3*c*sec(e + f*x)**2, x) + Inte
gral(3*c*sec(e + f*x)**3, x) + Integral(c*sec(e + f*x)**4, x) + Integral(d*
sec(e + f*x)**2, x) + Integral(3*d*sec(e + f*x)**3, x) + Integral(3*d*sec(e
 + f*x)**4, x) + Integral(d*sec(e + f*x)**5, x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(117) = 234.

Time = 0.23 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.10

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$
$$= \frac{16\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3c+48\left(\tan(fx+e)^3+3\tan(fx+e)\right)a^3d-3a^3d\left(\frac{2\left(3\sin(fx+e)^3-5\sin(fx+e)\right)}{\sin(fx+e)^4-2\sin(fx+e)^2+1}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a^3*c + 48*(tan(f*x + e)^3 + 3*t
an(f*x + e))*a^3*d - 3*a^3*d*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*
x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x
+ e) - 1)) - 36*a^3*c*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
e) + 1) + log(sin(f*x + e) - 1)) - 36*a^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2
 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a^3*c*log(sec(f
*x + e) + tan(f*x + e)) + 144*a^3*c*tan(f*x + e) + 48*a^3*d*tan(f*x + e))/f
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.70

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$
$$= \frac{15\left(4a^3c+3a^3d\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)-15\left(4a^3c+3a^3d\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)-\frac{2\left(60a^3c\tan\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3*(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/24*(15*(4*a^3*c + 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(4*a^3
*c + 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(60*a^3*c*tan(1/2*f*x
+ 1/2*e)^7 + 45*a^3*d*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c*tan(1/2*f*x + 1/2*
e)^5 - 165*a^3*d*tan(1/2*f*x + 1/2*e)^5 + 292*a^3*c*tan(1/2*f*x + 1/2*e)^3
+ 219*a^3*d*tan(1/2*f*x + 1/2*e)^3 - 132*a^3*c*tan(1/2*f*x + 1/2*e) - 147*a
^3*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.62

$$\int \sec(e+fx)(a+a\sec(e+fx))^3(c+d\sec(e+fx))\,dx$$
$$=\frac{\left(-5\,a^3\,c-\frac{15\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^7+\left(\frac{55\,a^3\,c}{3}+\frac{55\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-\frac{73\,a^3\,c}{3}-\frac{73\,a^3\,d}{4}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^8-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-4\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+1\right.}$$
$$+\frac{5\,a^3\,\mathrm{atanh}\left(\frac{5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(4\,c+3\,d)}{2\,\left(10\,c+\frac{15\,d}{2}\right)}\right)\,(4\,c+3\,d)}{4\,f}$$

```
[In] int(((a + a/cos(e + f*x))^3*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(11*a^3*c + (49*a^3*d)/4) - tan(e/2 + (f*x)/2)^7*(5*a^3
*c + (15*a^3*d)/4) + tan(e/2 + (f*x)/2)^5*((55*a^3*c)/3 + (55*a^3*d)/4) - t
an(e/2 + (f*x)/2)^3*((73*a^3*c)/3 + (73*a^3*d)/4))/(f*(6*tan(e/2 + (f*x)/2)
^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8
 + 1)) + (5*a^3*atanh((5*tan(e/2 + (f*x)/2)*(4*c + 3*d))/(2*(10*c + (15*d)/
2)))*(4*c + 3*d))/(4*f)
```

3.205 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = \frac{a^3\operatorname{arctanh}(\sin(e+fx))}{2df} + \frac{a^3(c^2-3cd+3d^2)\operatorname{arctanh}(\sin(e+fx))}{d^3f} - \frac{2a^3(c-d)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3\sqrt{c+d}f} - \frac{a^3(c-3d)\tan(e+fx)}{d^2f} + \frac{a^3\sec(e+fx)\tan(e+fx)}{2df}$$

```
[Out] 1/2*a^3*arctanh(sin(f*x+e))/d/f+a^3*(c^2-3*c*d+3*d^2)*arctanh(sin(f*x+e))/d
^3/f-2*a^3*(c-d)^(5/2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/
d^3/f/(c+d)^(1/2)-a^3*(c-3*d)*tan(f*x+e)/d^2/f+1/2*a^3*sec(f*x+e)*tan(f*x+e
)/d/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.68, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used

$= \{4072, 104, 159, 163, 65, 223, 209, 95, 211\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = \frac{a^4(2c^2-6cd+7d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^4(c-d)^{5/2}\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^3f\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{2df}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

```
[Out] -1/2*(a^3*(2*c - 5*d)*Tan[e + f*x])/(d^2*f) + (a^4*(2*c^2 - 6*c*d + 7*d^2)*
ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(
d^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (2*a^4*(c - d)^(
5/2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*
Sec[e + f*x]])]*Tan[e + f*x])/(d^3*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]]) + ((a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(2*d*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2 \tan(e+fx)\right) \text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)} \, dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a+ax}\left(-a^3(c+2d)+a^3(2c-5d)x\right)}{\sqrt{a-ax}(c+dx)} \, dx, x, \sec(e+fx)\right)}{2df\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^5d(c+2d)+a^5(2c^2-6cd+7d^2)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)} \, dx, x, \sec(e+fx)\right)}{2d^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{\left(a^5(c-d)^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)} \, dx, x, \sec(e+fx)\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{\left(a^5(2c^2-6cd+7d^2)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}} \, dx, x, \sec(e+fx)\right)}{2d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{\left(2a^5(c-d)^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2} \, dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^4(2c^2-6cd+7d^2)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}} \, dx, x, \sqrt{a-a\sec(e+fx)}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{2a^4(c-d)^{5/2}\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2df} + \frac{\left(a^4(2c^2-6cd+7d^2)\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{a^3(2c-5d)\tan(e+fx)}{2d^2f} + \frac{a^4(2c^2-6cd+7d^2)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{2a^4(c-d)^{5/2}\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(a^3+a^3\sec(e+fx))\tan(e+fx)}{2df}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.74

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$$

$$= \frac{a^3\cos^2(e+fx)(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left(-2(2c^2-6cd+7d^2)\log\left(\cos\left(\frac{1}{2}(e\right.\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x]),x]
```

```
[Out] (a^3*Cos[e + f*x]^2*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*
x])^3*(-2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
+ 2*(2*c^2 - 6*c*d + 7*d^2)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (8*(
c - d)^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/
2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(I*Cos[e] + Sin[e]))/(S
qrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) + d^2/(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])) - d^2/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2 - (4*(c - 3*d)*d*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]))))/(32*d^3*f*(c + d*Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{8d^3\sqrt{(c+d)(c-d)}}+\frac{1}{32d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{5d-2c}{32d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{\left(-2c^2+6cd-7\right.}{}\right.}{f}$
default	$\frac{16a^3\left(-\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{8d^3\sqrt{(c+d)(c-d)}}+\frac{1}{32d\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{5d-2c}{32d^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{\left(-2c^2+6cd-7\right.}{}\right.}{f}$
risch	$-\frac{ia^3\left(d\,e^{3i(fx+e)}+2\,e^{2i(fx+e)}c-6d\,e^{2i(fx+e)}-d\,e^{i(fx+e)}+2c-6d\right)}{f\,d^2\left(1+e^{2i(fx+e)}\right)^2}+\frac{\sqrt{(c+d)(c-d)}\,a^3\ln\left(e^{i(fx+e)}-\frac{i\sqrt{(c+d)(c-d)}-d}{c}\right.}{(c+d)f\,d^3}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/f*a^3*(-1/8*(c^3-3*c^2*d+3*c*d^2-d^3)/d^3/((c+d)*(c-d))^(1/2)*arctanh((c
-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))+1/32/d/(tan(1/2*f*x+1/2*e)-1)^2
-1/32*(5*d-2*c)/d^2/(tan(1/2*f*x+1/2*e)-1)+1/32/d^3*(-2*c^2+6*c*d-7*d^2)*ln
(tan(1/2*f*x+1/2*e)-1)-1/32/d/(tan(1/2*f*x+1/2*e)+1)^2-1/32*(5*d-2*c)/d^2/(
tan(1/2*f*x+1/2*e)+1)+1/32*(2*c^2-6*c*d+7*d^2)/d^3*ln(tan(1/2*f*x+1/2*e)+1)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 532, normalized size of antiderivative = 3.48

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$$

$$= \left[\frac{2\left(a^3c^2-2\,a^3cd+a^3d^2\right)\sqrt{\frac{c-d}{c+d}}\cos\left(fx+e\right)^2\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2-2\left(c^2+cd+(cd+d^2)\cos(fx+e)\right)\sqrt{\cdot}}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right.}{}\right.$$

$$-\frac{4\left(a^3c^2-2\,a^3cd+a^3d^2\right)\sqrt{-\frac{c-d}{c+d}}\arctan\left(-\frac{(d\cos(fx+e)+c)\sqrt{-\frac{c-d}{c+d}}}{(c-d)\sin(fx+e)}\right)\cos\left(fx+e\right)^2-\left(2\,a^3c^2-6\,a^3cd+7\,a^3\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/4*(2*(a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt((c - d)/(c + d))*cos(f*x + e)^
2*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (
c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/
(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (2*a^3*c^2 - 6*a^3*c*d +
 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a^3*c^2 - 6*a^3*c*d +
 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(a^3*d^2 - 2*(a^3*c*d
 - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f*x + e)^2), -1/4*(4*(
a^3*c^2 - 2*a^3*c*d + a^3*d^2)*sqrt(-(c - d)/(c + d))*arctan(-(d*cos(f*x +
e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e)))*cos(f*x + e)^2 - (2*
a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(sin(f*x + e) + 1) + (2*
a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*
(a^3*d^2 - 2*(a^3*c*d - 3*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/(d^3*f*cos(f
*x + e)^2)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = a^3\left(\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx + \int \frac{3\sec^2(e+fx)}{c+d\sec(e+fx)}\,dx + \int \frac{3\sec^3(e+fx)}{c+d\sec(e+fx)}\,dx + \int \frac{\sec^4(e+fx)}{c+d\sec(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f
*x)**2/(c + d*sec(e + f*x)), x) + Integral(3*sec(e + f*x)**3/(c + d*sec(e +
 f*x)), x) + Integral(sec(e + f*x)**4/(c + d*sec(e + f*x)), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(140) = 280$.

Time = 0.37 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx$$

$$= \frac{\frac{(2a^3c^2-6a^3cd+7a^3d^2)\log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)+1|)}{d^3} - \frac{(2a^3c^2-6a^3cd+7a^3d^2)\log(|\tan(\frac{1}{2}fx+\frac{1}{2}e)-1|)}{d^3} + \frac{4(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\Big(}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))
/d^3 - (2*a^3*c^2 - 6*a^3*c*d + 7*a^3*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1
))/d^3 + 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(pi*floor(1/2*(f
*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1
/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/(sqrt(-c^2 + d^2)*d^3) + 2*(2*a^3*c*tan
(1/2*f*x + 1/2*e)^3 - 5*a^3*d*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*c*tan(1/2*f*x
+ 1/2*e) + 7*a^3*d*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*d^
2))/f
```

Mupad [B] (verification not implemented)

Time = 14.36 (sec) , antiderivative size = 1902, normalized size of antiderivative = 12.43

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (atanh((18824*a^9*c^2*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (
16680*a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/
d^4 - 11560*a^9*c*d) - (16680*a^9*c^3*tan(e/2 + (f*x)/2))/(2968*a^9*d^3 - 1
6680*a^9*c^3 - 11560*a^9*c*d^2 + 18824*a^9*c^2*d + (8608*a^9*c^4)/d - (2480
*a^9*c^5)/d^2 + (320*a^9*c^6)/d^3) + (8608*a^9*c^4*tan(e/2 + (f*x)/2))/(860
8*a^9*c^4 + 2968*a^9*d^4 - 11560*a^9*c*d^3 - 16680*a^9*c^3*d + 18824*a^9*c^
2*d^2 - (2480*a^9*c^5)/d + (320*a^9*c^6)/d^2) - (2480*a^9*c^5*tan(e/2 + (f*
x)/2))/(2968*a^9*d^5 - 2480*a^9*c^5 - 11560*a^9*c*d^4 + 8608*a^9*c^4*d + 18
824*a^9*c^2*d^3 - 16680*a^9*c^3*d^2 + (320*a^9*c^6)/d) + (320*a^9*c^6*tan(e
/2 + (f*x)/2))/(320*a^9*c^6 + 2968*a^9*d^6 - 11560*a^9*c*d^5 - 2480*a^9*c^5
*d + 18824*a^9*c^2*d^4 - 16680*a^9*c^3*d^3 + 8608*a^9*c^4*d^2) + (2968*a^9*
d^2*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (16680*a^9*c^3)/d +
```

```
 (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/d^4 - 11560*a^9*c*
d) - (11560*a^9*c*d*tan(e/2 + (f*x)/2))/(18824*a^9*c^2 + 2968*a^9*d^2 - (16
680*a^9*c^3)/d + (8608*a^9*c^4)/d^2 - (2480*a^9*c^5)/d^3 + (320*a^9*c^6)/d^
4 - 11560*a^9*c*d))*(2*a^3*c^2 + 7*a^3*d^2 - 6*a^3*c*d))/(d^3*f) - ((tan(e/
2 + (f*x)/2)*(2*a^3*c - 7*a^3*d))/d^2 - (a^3*tan(e/2 + (f*x)/2)^3*(2*c - 5*
d))/d^2)/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1)) - (a^3*ata
n(((a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^
6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 -
492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 + (a^3*((c + d)*(c - d)^5)^(1/2)*((
8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^4
*d^6))/d^6 - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8 -
 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d)))*1i)/(d^3*(c + d))
+ (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6
*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 - 4
92*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 - (a^3*((c + d)*(c - d)^5)^(1/2)*((8
*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^4*
d^6))/d^6 + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8 -
16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d)))*1i)/(d^3*(c + d)))/
((16*(4*a^9*c^8 + 35*a^9*d^8 - 219*a^9*c*d^7 - 42*a^9*c^7*d + 592*a^9*c^2*d
^6 - 904*a^9*c^3*d^5 + 855*a^9*c^4*d^4 - 515*a^9*c^5*d^3 + 194*a^9*c^6*d^2)
)/d^6 - (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 -
53*a^6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d
^4 - 492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 + (a^3*((c + d)*(c - d)^5)^(1/
2)*((8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^
3*c^4*d^6))/d^6 - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*
d^8 - 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d))))/(d^3*(c + d)
) + (a^3*((c + d)*(c - d)^5)^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a
^6*d^7 + 259*a^6*c*d^6 - 64*a^6*c^6*d - 547*a^6*c^2*d^5 + 657*a^6*c^3*d^4 -
 492*a^6*c^4*d^3 + 232*a^6*c^5*d^2))/d^4 - (a^3*((c + d)*(c - d)^5)^(1/2)*(
(8*(18*a^3*d^10 - 46*a^3*c*d^9 + 42*a^3*c^2*d^8 - 18*a^3*c^3*d^7 + 4*a^3*c^
4*d^6))/d^6 + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)*(c - d)^5)^(1/2)*(8*c*d^8
- 16*c^2*d^7 + 8*c^3*d^6))/(d^7*(c + d))))/(d^3*(c + d))))/(d^3*(c + d))))*
((c + d)*(c - d)^5)^(1/2)*2i)/(d^3*f*(c + d))
```

3.206 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 161

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = -\frac{a^3(2c-3d)\text{arctanh}(\sin(e+fx))}{d^3f} + \frac{2a^3(c-d)^{3/2}(2c+3d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3(c+d)^{3/2}f} + \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))}$$

```
[Out] -a^3*(2*c-3*d)*arctanh(sin(f*x+e))/d^3/f+2*a^3*(c-d)^(3/2)*(2*c+3*d)*arctan
h((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/d^3/(c+d)^(3/2)/f+2*a^3*c*tan
(f*x+e)/d^2/(c+d)/f-(c-d)*(a^3+a^3*sec(f*x+e))*tan(f*x+e)/d/(c+d)/f/(c+d*se
c(f*x+e))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.70, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used

$= \{4072, 100, 159, 163, 65, 223, 209, 95, 211\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$$
$$= -\frac{2a^4(2c-3d)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{2a^4(c-d)^{3/2}(2c+3d)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^3f(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{2a^3c\tan(e+fx)}{d^2f(c+d)} - \frac{(c-d)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{df(c+d)(c+d\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*a^3*c*Tan[e + f*x])/(d^2*(c + d)*f) - (2*a^4*(2*c - 3*d)*ArcTan[Sqrt[a -
 a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(d^3*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (2*a^4*(c - d)^(3/2)*(2*c + 3*d
)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec
[e + f*x]])]*Tan[e + f*x])/(d^3*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sq
rt[a + a*Sec[e + f*x]]) - ((c - d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(
d*(c + d)*f*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a+ax}(a^3(c-3d)-2a^3cx)}{\sqrt{a-ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{d(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{-a^5(c-3d)d-a^5(2c-3d)(c+d)x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{d^2(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} + \frac{(a^5(2c-3d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(a^5(c-d)^2(2c+3d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{d^3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{(c-d)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{(2a^4(2c-3d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(2a^5(c-d)^2(2c+3d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{d^3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{2a^4(c-d)^{3/2}(2c+3d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)\,(a^3+a^3\sec(e+fx))\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))} - \frac{(2a^4(2c-3d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^3c\tan(e+fx)}{d^2(c+d)f} - \frac{2a^4(2c-3d)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$- \frac{2a^4(c-d)^{3/2}(2c+3d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$- \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{d(c+d)f(c+d\sec(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.18 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.83

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{a^3\cos(e+fx)(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left((2c-3d)(d+c\cos(e+fx))\log\left(\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (a^3*Cos[e + f*x]*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x]
)^3*((2*c - 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/
2]] + (-2*c + 3*d)*(d + c*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x
)/2]] - ((2*I)*(c - d)^2*(2*c + 3*d)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e]
+ (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^
2])]*(d + c*Cos[e + f*x])*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqr
t[(Cos[e] - I*Sin[e])^2]) + ((c - d)^2*d*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c
+ d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])) + (d*(d + c*Cos[e + f*x])
*Sin[(f*x)/2])/((Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
) + (d*(d + c*Cos[e + f*x])*Sin[(f*x)/2])/((Cos[e/2] + Sin[e/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]))))/(8*d^3*f*(c + d*Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\left(c^2-2cd+d^2\right)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(2c+3d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{4d^3}-\frac{1}{16d^2\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f}$
default	$\frac{16a^3\left(-\frac{\left(c^2-2cd+d^2\right)\left(\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(2c+3d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{4d^3}-\frac{1}{16d^2\left(\tan\left(\frac{fx}{2}\right.\right.}\right.}{f}$
risch	$\frac{2ia^3\left(c^2d\,e^{3i(fx+e)}-2c\,d^2e^{3i(fx+e)}+d^3e^{3i(fx+e)}+2c^3e^{2i(fx+e)}-c^2d\,e^{2i(fx+e)}+c\,d^2e^{2i(fx+e)}+3c^2d\,e^{i(fx+e)}+d^3e^{i(fx+e)}\right.}{f\,d^2\left(1+e^{2i(fx+e)}\right)(c+d)c\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 16/f*a^3*(-1/4*(c^2-2*c*d+d^2)/d^3*(1/2*d/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2
*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)-1/2*(2*c+3*d)/(c+d)/((c+d)*(c-d
))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))-1/16/d^2/(t
an(1/2*f*x+1/2*e)-1)+1/16*(2*c-3*d)/d^3*ln(tan(1/2*f*x+1/2*e)-1)-1/16/d^2/(
tan(1/2*f*x+1/2*e)+1)+1/16/d^3*(-2*c+3*d)*ln(tan(1/2*f*x+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(152) = 304$.

Time = 0.56 (sec) , antiderivative size = 859, normalized size of antiderivative = 5.34

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx$$

$$= \left[-\frac{\left(\left(2a^3c^3+a^3c^2d-3a^3cd^2\right)\cos\left(fx+e\right)^2+\left(2a^3c^2d+a^3cd^2-3a^3d^3\right)\cos\left(fx+e\right)\right)\sqrt{\frac{c-d}{c+d}}\log\left(\frac{2cd\cos(fx\cdots}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*(((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d
 + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*co
```

```
s(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(
f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
e)^2 + 2*c*d*cos(f*x + e) + d^2)) + ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*
cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(si
n(f*x + e) + 1) - ((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (
2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) -
 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))
*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*
x + e)), 1/2*(2*((2*a^3*c^3 + a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*
a^3*c^2*d + a^3*c*d^2 - 3*a^3*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c + d))*arc
tan(-(d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e))) -
((2*a^3*c^3 - a^3*c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*
c*d^2 - 3*a^3*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) + ((2*a^3*c^3 - a^3*
c^2*d - 3*a^3*c*d^2)*cos(f*x + e)^2 + (2*a^3*c^2*d - a^3*c*d^2 - 3*a^3*d^3)
*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(a^3*c*d^2 + a^3*d^3 + (2*a^3*c^2
*d - a^3*c*d^2 + a^3*d^3)*cos(f*x + e))*sin(f*x + e))/((c^2*d^3 + c*d^4)*f*
cos(f*x + e)^2 + (c*d^4 + d^5)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = a^3\left(\int \frac{\sec(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx + \int \frac{3\sec^2(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx + \int \frac{3\sec^3(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx + \int \frac{\sec^4(e+fx)}{c^2+2cd\sec(e+fx)+d^2\sec^2(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)*
*2), x) + Integral(3*sec(e + f*x)**2/(c**2 + 2*c*d*sec(e + f*x) + d**2*sec(
e + f*x)**2), x) + Integral(3*sec(e + f*x)**3/(c**2 + 2*c*d*sec(e + f*x) +
d**2*sec(e + f*x)**2), x) + Integral(sec(e + f*x)**4/(c**2 + 2*c*d*sec(e +
f*x) + d**2*sec(e + f*x)**2), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(152) = 304.

Time = 0.37 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = -\frac{\frac{2\,(2\,a^3c^3-a^3c^2d-4\,a^3cd^2+3\,a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(cd^3+d^4)\sqrt{-c^2+d^2}}+\frac{4\left(a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-a^3\cdots\right.}{\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-\cdots\right.}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(2*a^3*c^3 - a^3*c^2*d - 4*a^3*c*d^2 + 3*a^3*d^3)*(pi*floor(1/2*(f*x +
e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*
x + 1/2*e))/sqrt(-c^2 + d^2)))/((c*d^3 + d^4)*sqrt(-c^2 + d^2)) + 4*(a^3*c^
2*tan(1/2*f*x + 1/2*e)^3 - a^3*c*d*tan(1/2*f*x + 1/2*e)^3 - a^3*c^2*tan(1/2
*f*x + 1/2*e) - a^3*d^2*tan(1/2*f*x + 1/2*e))/((c*tan(1/2*f*x + 1/2*e)^4 -
d*tan(1/2*f*x + 1/2*e)^4 - 2*c*tan(1/2*f*x + 1/2*e)^2 + c + d)*(c*d^2 + d^3
)) + (2*a^3*c - 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^3 - (2*a^3*c
- 3*a^3*d)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/d^3)/f
```

Mupad [B] (verification not implemented)

Time = 16.91 (sec) , antiderivative size = 3135, normalized size of antiderivative = 19.47

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (a^3*atan(((a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d
^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*
c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10
- 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^
8 + c^2*d^6) - (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 +
 c^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d)*1
i)/d^3 + (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6
 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^
5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 -
4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8
+ c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c
^5*d^6))/(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d)*1i)
/d^3)/((128*(4*a^9*c^7 - 9*a^9*c*d^6 - 16*a^9*c^6*d + 36*a^9*c^2*d^5 - 50*a
^9*c^3*d^4 + 20*a^9*c^4*d^3 + 15*a^9*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6) +
(a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6
*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(
2*c*d^5 + d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2
*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6
) - (64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/
(d^3*(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d))/d^3 - (a^3*
((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*
d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d
^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9
+ 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (
64*a^3*tan(e/2 + (f*x)/2)*(2*c - 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/(d^3*
(2*c*d^5 + d^6 + c^2*d^4)))*(2*c - 3*d))/d^3)*(2*c - 3*d))/d^3))*(2*c - 3*d
)*2i)/(d^3*f) - ((4*tan(e/2 + (f*x)/2)^3*(a^3*c^2 - a^3*c*d))/(d^2*(c + d))
 - (4*a^3*tan(e/2 + (f*x)/2)*(c^2 + d^2))/(d^2*(c + d)))/(f*(c + d + tan(e/
2 + (f*x)/2)^4*(c - d) - 2*c*tan(e/2 + (f*x)/2)^2)) + (a^3*atan(((a^3*((64*
tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*d - 1
6*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 +
d^6 + c^2*d^4) + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a
^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (64*a^
3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^
3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c
^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d
```

```
^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*1i)/(3*c*d^5 + d^6 +
 3*c^2*d^4 + c^3*d^3) + (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7
 + 27*a^6*c*d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c
^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 -
3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/
(2*c*d^7 + d^8 + c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3
)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d
^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*
c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2
)*(2*c + 3*d)*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))/((128*(4*a^9*c^7 -
 9*a^9*c*d^6 - 16*a^9*c^6*d + 36*a^9*c^2*d^5 - 50*a^9*c^3*d^4 + 20*a^9*c^4*
d^3 + 15*a^9*c^5*d^2))/(2*c*d^7 + d^8 + c^2*d^6) + (a^3*((64*tan(e/2 + (f*x
)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 - 12*a^6*c^6*d - 16*a^6*c^2*d^5
- 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*d^2))/(2*c*d^5 + d^6 + c^2*d^4)
 + (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a
^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) - (64*a^3*tan(e/2 + (f
*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d)*(c*d^10 - 2*c^3*d^8 + c^5*d^
6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c
+ d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))
*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*
d^3) - (a^3*((64*tan(e/2 + (f*x)/2)*(4*a^6*c^7 - 9*a^6*d^7 + 27*a^6*c*d^6 -
 12*a^6*c^6*d - 16*a^6*c^2*d^5 - 24*a^6*c^3*d^4 + 29*a^6*c^4*d^3 + a^6*c^5*
d^2))/(2*c*d^5 + d^6 + c^2*d^4) - (a^3*((64*(3*a^3*d^11 - 3*a^3*c*d^10 - 4*
a^3*c^2*d^9 + 4*a^3*c^3*d^8 + a^3*c^4*d^7 - a^3*c^5*d^6))/(2*c*d^7 + d^8 +
c^2*d^6) + (64*a^3*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*
d)*(c*d^10 - 2*c^3*d^8 + c^5*d^6))/((2*c*d^5 + d^6 + c^2*d^4)*(3*c*d^5 + d^
6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3*c*d^
5 + d^6 + 3*c^2*d^4 + c^3*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*d))/(3
*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(2*c + 3*
d)*2i)/(f*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))
```

3.207 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 188

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx &= \frac{a^3\text{arctanh}(\sin(e+fx))}{d^3f} - \frac{a^3\sqrt{c-d}(2c^2+6cd+7d^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{d^3(c+d)^{5/2}f} \\ &\quad - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))}\end{aligned}$$

```
[Out] a^3*arctanh(sin(f*x+e))/d^3/f-a^3*(2*c^2+6*c*d+7*d^2)*arctanh((c-d)^(1/2)*t
an(1/2*f*x+1/2*e)/(c+d)^(1/2))*(c-d)^(1/2)/d^3/(c+d)^(5/2)/f-1/2*(c-d)*(a^3
+a^3*sec(f*x+e))*tan(f*x+e)/d/(c+d)/f/(c+d*sec(f*x+e))^2-1/2*a^3*(c-d)*(2*c
+5*d)*tan(f*x+e)/d^2/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.60, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {4072, 100, 154, 163, 65, 223, 209, 95, 211}

$$\begin{aligned}&\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx\\ &=\frac{a^4\sqrt{c-d}(2c^2+6cd+7d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{d^3f(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\ &\quad+\frac{2a^4\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\ &\quad-\frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2f(c+d)^2(c+d\sec(e+fx))}-\frac{(c-d)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{2df(c+d)(c+d\sec(e+fx))^2}\end{aligned}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (2*a^4*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(d^3*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a^4*Sqrt
[c - d]*(2*c^2 + 6*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]
])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(d^3*(c + d)^(5/2)
*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - ((c - d)*(a^3 + a^3
*Sec[e + f*x])*Tan[e + f*x])/(2*d*(c + d)*f*(c + d*Sec[e + f*x])^2) - (a^3*
(c - d)*(2*c + 5*d)*Tan[e + f*x])/(2*d^2*(c + d)^2*f*(c + d*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
```

```
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} + \frac{(a\tan(e+fx))\text{Subst}\left(\int \frac{\sqrt{a+ax}\,(a^3(c-5d)-2a^3(c+d)x)}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{2d(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{-a^5d(c+7d)-2a^5(c+d)^2x}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2d^2(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))} - \frac{(a^5\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^5(2c(c+d)^2-d^2(c+7d))\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2d^3(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))} + \frac{(2a^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{\left(a^5(2c(c+d)^2-d^2(c+7d))\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{d^3(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{a^4\sqrt{c-d}(2c^2+6cd+7d^2)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))} + \frac{(2a^4\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{d^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2a^4 \arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{d^3 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{a^4\sqrt{c-d}(2c^2+6cd+7d^2)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{d^3(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$- \frac{(c-d)\left(a^3+a^3\sec(e+fx)\right)\tan(e+fx)}{2d(c+d)f(c+d\sec(e+fx))^2} - \frac{a^3(c-d)(2c+5d)\tan(e+fx)}{2d^2(c+d)^2f(c+d\sec(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.33 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.09

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{a^3(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)(1+\sec(e+fx))^3\left(-4(d+c\cos(e+fx))^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*(1 + Sec[e + f*x])^3*(-4*(d +
c*Cos[e + f*x])^2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*(d + c*Cos[e
 + f*x])^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (4*(2*c^3 + 4*c^2*d +
 c*d^2 - 7*d^3)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan
[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f
*x])^2*(I*Cos[e] + Sin[e]))/((c + d)^2*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin
[e])^2]) + ((c - d)*d*Sec[e]*((2*c^4 + 6*c^3*d + 5*c^2*d^2 + 12*c*d^3 + 2*d
^4)*Sin[e] - c*(d*(7*c^2 + 18*c*d + 2*d^2)*Sin[f*x] - d*(c^2 + 6*c*d + 2*d^
2)*Sin[2*e + f*x] + c*(2*c^2 + 6*c*d + d^2)*Sin[e + 2*f*x])))/(c^2*(c + d)^
2)))/(32*d^3*f*(c + d*Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{16a^3\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16d^3}+\frac{(c-d)\left(\frac{\frac{d\left(2c^2+3cd-5d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c^2+4cd+2d^2}-\frac{d(2c+7d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{\left(2c^2+6cd+7d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tan}{\sqrt{(c+}}\right.}{2\left(c^2+2cd+d^2\right)\sqrt{(c+d)(c-}}\right.}{8d^3}\right.}{f}$
default	$\frac{16a^3\left(\frac{\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{16d^3}+\frac{(c-d)\left(\frac{\frac{d\left(2c^2+3cd-5d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2c^2+4cd+2d^2}-\frac{d(2c+7d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{\left(2c^2+6cd+7d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tan}{\sqrt{(c+}}\right.}{2\left(c^2+2cd+d^2\right)\sqrt{(c+d)(c-}}\right.}{8d^3}\right.}{f}$
risch	$\frac{ia^3\left(-c^4d\,e^{3i(fx+e)}-5c^3d^2e^{3i(fx+e)}+4c^2d^3e^{3i(fx+e)}+2c\,d^4e^{3i(fx+e)}-2c^5e^{2i(fx+e)}-4c^4d\,e^{2i(fx+e)}+c^3d^2e^{2i(fx+e)}-7c^2\right.}{c^2d^2(c+d)^2f(}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 16/f*a^3*(1/16/d^3*ln(tan(1/2*f*x+1/2*e)+1)+1/8*(c-d)/d^3*((1/2*d*(2*c^2+3*c*d-5*d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1/2*d*(2*c+7*d)/(c+d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-1/2*(2*c^2+6*c*d+7*d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))-1/16/d^3*ln(tan(1/2*f*x+1/2*e)-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(175) = 350$.

Time = 0.60 (sec) , antiderivative size = 1176, normalized size of antiderivative = 6.26

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (2*a^3*c^4 + 6*a^3*c^3*d + 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d + 6*a^3*c^2*d^2 + 7*a^3*c*d^3)*cos(f*x + e))*sqrt((c - d)/(c + d))*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*(c^2 + c*d + (c*d + d^2)*cos(f*x + e))*sqrt((c - d)/(c + d))*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3

```
*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3
)*cos(f*x + e))*log(sin(f*x + e) + 1) - 2*(a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*
d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d +
 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) - 2*(3*a^3
*c^2*d^2 + 3*a^3*c*d^3 - 6*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^2 - 5*a^3*c
*d^3 - a^3*d^4)*cos(f*x + e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5
)*f*cos(f*x + e)^2 + 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^2*
d^5 + 2*c*d^6 + d^7)*f), -1/2*((2*a^3*c^2*d^2 + 6*a^3*c*d^3 + 7*a^3*d^4 + (
2*a^3*c^4 + 6*a^3*c^3*d + 7*a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a^3*c^3*d +
6*a^3*c^2*d^2 + 7*a^3*c*d^3)*cos(f*x + e))*sqrt(-(c - d)/(c + d))*arctan(-(
d*cos(f*x + e) + c)*sqrt(-(c - d)/(c + d))/((c - d)*sin(f*x + e))) - (a^3*c
^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c^3*d + a^3*c^2*d^2)*cos(
f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d^3)*cos(f*x + e))*log(si
n(f*x + e) + 1) + (a^3*c^2*d^2 + 2*a^3*c*d^3 + a^3*d^4 + (a^3*c^4 + 2*a^3*c
^3*d + a^3*c^2*d^2)*cos(f*x + e)^2 + 2*(a^3*c^3*d + 2*a^3*c^2*d^2 + a^3*c*d
^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + (3*a^3*c^2*d^2 + 3*a^3*c*d^3 - 6
*a^3*d^4 + (2*a^3*c^3*d + 4*a^3*c^2*d^2 - 5*a^3*c*d^3 - a^3*d^4)*cos(f*x +
e))*sin(f*x + e))/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*f*cos(f*x + e)^2 + 2*(c^
3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(f*x + e) + (c^2*d^5 + 2*c*d^6 + d^7)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = a^3\left(\int \frac{\sec(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx + \int \frac{3\sec^2(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx + \int \frac{3\sec^3(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx + \int \frac{\sec^4(e+fx)}{c^3+3c^2d\sec(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^3(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e
+ f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(3*sec(e + f*x)**2/(c**3 +
3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3), x
) + Integral(3*sec(e + f*x)**3/(c**3 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec
(e + f*x)**2 + d**3*sec(e + f*x)**3), x) + Integral(sec(e + f*x)**4/(c**3 +
 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**2 + d**3*sec(e + f*x)**3),
x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(175) = 350$.

Time = 0.42 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.00

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \frac{\frac{a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{d^3} - \frac{a^3\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{d^3} + \frac{(2a^3c^3+4a^3c^2d+a^3cd^2-7a^3d^3)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx\cdots\right.}{}\right.\right.}{(c^2d^3+2cd^4+d^5)\sqrt{-c^2+d^2}}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] (a^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d^3 - a^3*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/d^3 + (2*a^3*c^3 + 4*a^3*c^2*d + a^3*c*d^2 - 7*a^3*d^3)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) -
d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(
-c^2 + d^2)) + (2*a^3*c^3*tan(1/2*f*x + 1/2*e)^3 + a^3*c^2*d*tan(1/2*f*x +
1/2*e)^3 - 8*a^3*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 5*a^3*d^3*tan(1/2*f*x + 1/2
*e)^3 - 2*a^3*c^3*tan(1/2*f*x + 1/2*e) - 7*a^3*c^2*d*tan(1/2*f*x + 1/2*e) +
 2*a^3*c*d^2*tan(1/2*f*x + 1/2*e) + 7*a^3*d^3*tan(1/2*f*x + 1/2*e))/((c^2*d
^2 + 2*c*d^3 + d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 -
c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 20.18 (sec) , antiderivative size = 4131, normalized size of antiderivative = 21.97

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] - ((a^3*tan(e/2 + (f*x)/2)*(5*c*d + 2*c^2 - 7*d^2))/(d^2*(c + d)) - (a^3*ta
n(e/2 + (f*x)/2)^3*(c^2*d - 8*c*d^2 + 2*c^3 + 5*d^3))/(d^2*(c + d)^2))/(f*(
2*c*d - tan(e/2 + (f*x)/2)^2*(2*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 -
2*c*d + d^2) + c^2 + d^2)) - (a^3*atan(((a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*
c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^
3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3
*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 -
 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^
9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*a^3*tan(e/2 + (f*x)/2)*(8*
c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8
*c^7*d^6))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3)*1
i)/d^3 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6
 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*
c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) - (a^3*((8*(18*
a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^
8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^
7 + c^4*d^6) + (8*a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^
10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/(d^3*(4*c*d^7 + d^8
+ 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3)*1i)/d^3)/((16*(4*a^9*c^6 - 35*a^
9*d^6 + 61*a^9*c*d^5 + 10*a^9*c^5*d + 5*a^9*c^2*d^4 - 35*a^9*c^3*d^3 - 10*a
^9*c^4*d^2))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (a^3*((8*
tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d +
53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7
 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*
c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7
 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*
a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 -
8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^
3*d^5 + c^4*d^4))))/d^3))/d^3 + (a^3*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53
*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 -
52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c
^4*d^4) - (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*
c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10
 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (8*a^3*tan(e/2 + (f*x)/2)*(8*c*d^12 +
 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6
))/(d^3*(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4))))/d^3))/d^3))*2i
```

)/(d^3*f) - (a^3*atan(((a^3*((c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2
)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d^5 -
23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c^2*d^
6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*a^3*c
^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6*d^6)
)/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*a^3*tan(e/2 + (f*
x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*(8*c*d^12 + 16*c^
2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6))/((4
*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5*c*d^7 + d^8 + 10*c^2*d^6
 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c - d))^(1/2)*(3*c*d + c
^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5
d^3))(3*c*d + c^2 + (7*d^2)/2)*1i)/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d
^5 + 5*c^4*d^4 + c^5*d^3) + (a^3*((c + d)^5*(c - d))^(1/2)*((8*tan(e/2 + (f
x)/2)(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^6*d + 53*a^6*c^2*d
^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4*c*d^7 + d^8 + 6*c
^2*d^6 + 4*c^3*d^5 + c^4*d^4) - (a^3*((8*(18*a^3*d^12 + 10*a^3*c*d^11 - 32*
a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c^5*d^7 + 4*a^3*c^6
*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (8*a^3*tan(e/2
+ (f*x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2)*(8*c*d^12 +
16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^6*d^7 + 8*c^7*d^6)
)/((4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5*c*d^7 + d^8 + 10*c^
2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c - d))^(1/2)*(3*c*
d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4
+ c^5*d^3))*(3*c*d + c^2 + (7*d^2)/2)*1i)/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*
c^3*d^5 + 5*c^4*d^4 + c^5*d^3))/((16*(4*a^9*c^6 - 35*a^9*d^6 + 61*a^9*c*d^5
 + 10*a^9*c^5*d + 5*a^9*c^2*d^4 - 35*a^9*c^3*d^3 - 10*a^9*c^4*d^2))/(4*c*d^
9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (a^3*((c + d)^5*(c - d))^(1/2
)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*c^
6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/(4
*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) + (a^3*((8*(18*a^3*d^12 + 1
0*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3*c
^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6)
 - (8*a^3*tan(e/2 + (f*x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*d^
2)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*c^
6*d^7 + 8*c^7*d^6))/((4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5*c
*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c
 - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^
3*d^5 + 5*c^4*d^4 + c^5*d^3))*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 1
0*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3) + (a^3*((c + d)^5*(c - d))^(1
/2)*((8*tan(e/2 + (f*x)/2)*(8*a^6*c^7 - 53*a^6*d^7 + 59*a^6*c*d^6 + 16*a^6*
c^6*d + 53*a^6*c^2*d^5 - 23*a^6*c^3*d^4 - 52*a^6*c^4*d^3 - 8*a^6*c^5*d^2))/
(4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4) - (a^3*((8*(18*a^3*d^12 +
 10*a^3*c*d^11 - 32*a^3*c^2*d^10 - 20*a^3*c^3*d^9 + 10*a^3*c^4*d^8 + 10*a^3
*c^5*d^7 + 4*a^3*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^
6) + (8*a^3*tan(e/2 + (f*x)/2)*((c + d)^5*(c - d))^(1/2)*(3*c*d + c^2 + (7*

```
d^2)/2)*(8*c*d^12 + 16*c^2*d^11 - 8*c^3*d^10 - 32*c^4*d^9 - 8*c^5*d^8 + 16*
c^6*d^7 + 8*c^7*d^6))/((4*c*d^7 + d^8 + 6*c^2*d^6 + 4*c^3*d^5 + c^4*d^4)*(5
*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*
(c - d))^(1/2)*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*
c^3*d^5 + 5*c^4*d^4 + c^5*d^3))*(3*c*d + c^2 + (7*d^2)/2))/(5*c*d^7 + d^8 +
 10*c^2*d^6 + 10*c^3*d^5 + 5*c^4*d^4 + c^5*d^3)))*((c + d)^5*(c - d))^(1/2)
*(3*c*d + c^2 + (7*d^2)/2)*2i)/(f*(5*c*d^7 + d^8 + 10*c^2*d^6 + 10*c^3*d^5
+ 5*c^4*d^4 + c^5*d^3))
```

3.208 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 178

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}(c+d)^{7/2}f} + \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3} - \frac{5a^3(c-d)\tan(e+fx)}{6d(c+d)^2f(c+d\sec(e+fx))^2} + \frac{5a^3(c+4d)\tan(e+fx)}{6d(c+d)^3f(c+d\sec(e+fx))}$$

```
[Out] 5*a^3*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c+d)^(7/2)/f/(c-
d)^(1/2)+1/3*a*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c+d)/f/(c+d*sec(f*x+e))^3-5/6
*a^3*(c-d)*tan(f*x+e)/d/(c+d)^2/f/(c+d*sec(f*x+e))^2+5/6*a^3*(c+4*d)*tan(f*
x+e)/d/(c+d)^3/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used

$= \{4072, 96, 95, 211\}$

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$
$$= -\frac{5a^4\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f\sqrt{c-d}(c+d)^{7/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{5a^3\tan(e+fx)}{2f(c+d)^3(c+d\sec(e+fx))} + \frac{5\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{6f(c+d)^2(c+d\sec(e+fx))^2} + \frac{a\tan(e+fx)(a\sec(e+fx)+a)^2}{3f(c+d)(c+d\sec(e+fx))^3}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (-5*a^4*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a -
 a*Sec[e + f*x]])]*Tan[e + f*x])/(Sqrt[c - d]*(c + d)^(7/2)*f*Sqrt[a - a*Se
c[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (a*(a + a*Sec[e + f*x])^2*Tan[e + f
*x])/(3*(c + d)*f*(c + d*Sec[e + f*x])^3) + (5*(a^3 + a^3*Sec[e + f*x])*Tan
[e + f*x])/(6*(c + d)^2*f*(c + d*Sec[e + f*x])^2) + (5*a^3*Tan[e + f*x])/(2
*(c + d)^3*f*(c + d*Sec[e + f*x]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
```

```
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^4}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3}\\
&\quad -\frac{(5a^3\tan(e+fx))\,\text{Subst}\left(\int\frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{3(c+d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3}+\frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2}\\
&\quad -\frac{(5a^4\tan(e+fx))\,\text{Subst}\left(\int\frac{\sqrt{a+ax}}{\sqrt{a-ax}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{2(c+d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3}\\
&\quad +\frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2}+\frac{5a^3\tan(e+fx)}{2(c+d)^3f(c+d\sec(e+fx))}\\
&\quad -\frac{(5a^5\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2(c+d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3}\\
&\quad +\frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2}+\frac{5a^3\tan(e+fx)}{2(c+d)^3f(c+d\sec(e+fx))}\\
&\quad -\frac{(5a^5\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{ac-ad-(-ac-ad)x^2}\,dx,x,\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{(c+d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= -\frac{5a^4\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{\sqrt{c-d}(c+d)^{7/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&\quad +\frac{a(a+a\sec(e+fx))^2\tan(e+fx)}{3(c+d)f(c+d\sec(e+fx))^3}\\
&\quad +\frac{5(a^3+a^3\sec(e+fx))\tan(e+fx)}{6(c+d)^2f(c+d\sec(e+fx))^2}+\frac{5a^3\tan(e+fx)}{2(c+d)^3f(c+d\sec(e+fx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.88 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.24

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{a^3(d+c\cos(e+fx))\sec^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)(1+\sec(e+fx))^3\left(-\frac{120i\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\right.}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin}}\right.}{\sqrt{c^2-d^2}\sqrt{}}\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (a^3*(d + c*Cos[e + f*x])*Sec[(e + f*x)/2]^6*Sec[e + f*x]*(1 + Sec[e + f*x]
)^3*(((-120*I)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[
(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*(d + c*Cos[e + f*
x])^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]) +
(c*Sec[e]*(6*(8*c^4 + 6*c^3*d + 30*c^2*d^2 + 9*c*d^3 + 2*d^4)*Sin[f*x] - 3*
(6*c^4 - 3*c^3*d + 30*c^2*d^2 + 18*c*d^3 + 4*d^4)*Sin[2*e + f*x] + c*(3*(3*
c^3 + 38*c^2*d + 12*c*d^2 + 2*d^3)*Sin[e + 2*f*x] + 3*(3*c^3 - 6*c^2*d - 6*
c*d^2 - 2*d^3)*Sin[3*e + 2*f*x] + c*(22*c^2 + 9*c*d + 2*d^2)*Sin[2*e + 3*f*
x])) - 2*d*(66*c^4 + 27*c^3*d + 50*c^2*d^2 + 18*c*d^3 + 4*d^4)*Tan[e])/c^3)
)/(192*(c + d)^3*f*(c + d*Sec[e + f*x])^4)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{16a^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{6(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}-\frac{5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{3\left(-\frac{}{2(c+d)\left(\mathrm{ta}\right.}\right.}{}\right)}{6(}\right)}{f}$
default	$\frac{16a^3\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{6(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}-\frac{5\left(-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{3\left(-\frac{}{2(c+d)\left(\mathrm{ta}\right.}\right.}{}\right)}{6(}\right)}{f}$
risch	$\frac{ia^3(22c^5+132c^4d\,e^{3i(fx+e)}+54c^3d^2e^{3i(fx+e)}+100c^2d^3e^{3i(fx+e)}+36c\,d^4e^{3i(fx+e)}+36c^4d\,e^{2i(fx+e)}+180c^3d^2e^{2i(fx+e)}+}{}$

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)

[Out] 16/f*a^3*(-1/6*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^3-5/6/(c+d)*(-1/4*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-3/4/(c+d)*(-1/2*tan(1/2*f*x+1/2*e)/(c+d)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)+1/2/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(163) = 326$.

Time = 0.34 (sec) , antiderivative size = 1012, normalized size of antiderivative = 5.69

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$
$$= \left[\frac{15\left(a^3c^3\cos\left(fx+e\right)^3+3\,a^3c^2d\cos\left(fx+e\right)^2+3\,a^3cd^2\cos\left(fx+e\right)+a^3d^3\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(}{}\right.}{12\left(\left(c^8+3\,c^7d+2\,c^6d^2-2\,c^5d^3-3\,c^4d^4-c^3d^5\right)f\cos\right.}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*d*cos(f*x + e)^2 + 3*a^3*c*d^
2*cos(f*x + e) + a^3*d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 -
2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e)
 + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3
*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a^3*c^
4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e)^2
+ 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e))*sin
(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^3*d^5)*f
*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3*c^3*d^5
- c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 - 2*c^3*d^
5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^5 -
2*c^2*d^6 - 3*c*d^7 - d^8)*f), 1/6*(15*(a^3*c^3*cos(f*x + e)^3 + 3*a^3*c^2*
d*cos(f*x + e)^2 + 3*a^3*c*d^2*cos(f*x + e) + a^3*d^3)*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2
*a^3*c^4 + 9*a^3*c^3*d + 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 22*a^3*d^4 + (22*a^
3*c^4 + 9*a^3*c^3*d - 20*a^3*c^2*d^2 - 9*a^3*c*d^3 - 2*a^3*d^4)*cos(f*x + e
)^2 + 3*(3*a^3*c^4 + 16*a^3*c^3*d - 16*a^3*c*d^3 - 3*a^3*d^4)*cos(f*x + e))
*sin(f*x + e))/((c^8 + 3*c^7*d + 2*c^6*d^2 - 2*c^5*d^3 - 3*c^4*d^4 - c^3*d^
5)*f*cos(f*x + e)^3 + 3*(c^7*d + 3*c^6*d^2 + 2*c^5*d^3 - 2*c^4*d^4 - 3*c^3*
d^5 - c^2*d^6)*f*cos(f*x + e)^2 + 3*(c^6*d^2 + 3*c^5*d^3 + 2*c^4*d^4 - 2*c^
3*d^5 - 3*c^2*d^6 - c*d^7)*f*cos(f*x + e) + (c^5*d^3 + 3*c^4*d^4 + 2*c^3*d^
5 - 2*c^2*d^6 - 3*c*d^7 - d^8)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$
$$= a^3\left(\int \frac{\sec(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right.$$
$$+\int \frac{3\sec^2(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx$$
$$+\int \frac{3\sec^3(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx$$
$$\left.+\int \frac{\sec^4(e+fx)}{c^4+4c^3d\sec(e+fx)+6c^2d^2\sec^2(e+fx)+4cd^3\sec^3(e+fx)+d^4\sec^4(e+fx)}\,dx\right)$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**4,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec
(e + f*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integ
ral(3*sec(e + f*x)**2/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f
*x)**2 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(3*
sec(e + f*x)**3/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2
 + 4*c*d**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x) + Integral(sec(e +
f*x)**4/(c**4 + 4*c**3*d*sec(e + f*x) + 6*c**2*d**2*sec(e + f*x)**2 + 4*c*d
**3*sec(e + f*x)**3 + d**4*sec(e + f*x)**4), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.72

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx =$$
$$-\frac{\frac{15\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)a^3}{(c^3+3\,c^2d+3\,cd^2+d^3)\sqrt{-c^2+d^2}}+\frac{15\,a^3c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-30\,a^3cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+15\,a^3\cdots}{\cdots}}{\cdots}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(15*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1
/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*a^3/((c^3 + 3*
c^2*d + 3*c*d^2 + d^3)*sqrt(-c^2 + d^2)) + (15*a^3*c^2*tan(1/2*f*x + 1/2*e)
^5 - 30*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 + 15*a^3*d^2*tan(1/2*f*x + 1/2*e)^5
- 40*a^3*c^2*tan(1/2*f*x + 1/2*e)^3 + 40*a^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 3
3*a^3*c^2*tan(1/2*f*x + 1/2*e) + 66*a^3*c*d*tan(1/2*f*x + 1/2*e) + 33*a^3*d
^2*tan(1/2*f*x + 1/2*e))/((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*(c*tan(1/2*f*x +
1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 17.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.48

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^4}\,dx$$
$$=\frac{\frac{5\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(a^3\,c^2-2\,a^3\,c\,d+a^3\,d^2)}{(c+d)^3}+\frac{11\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{c+d}-\frac{40\tan\left(\frac{e}{2}+\cdots\right.}{3\cdots}}{f\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(-3\,c^3-3\,c^2\,d+3\,c\,d^2+3\,d^3)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(-3\,c^3+3\,c^2\,d+3\,c\,d^2-3\,d^3)+3\,c\,d^2\cdots\right.}$$
$$+\frac{5\,a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)}{f\,(c+d)^{7/2}\,\sqrt{c-d}}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((5*tan(e/2 + (f*x)/2)^5*(a^3*c^2 + a^3*d^2 - 2*a^3*c*d))/(c + d)^3 + (11*a
^3*tan(e/2 + (f*x)/2))/(c + d) - (40*tan(e/2 + (f*x)/2)^3*(a^3*c - a^3*d))/
(3*(c + d)^2))/(f*(tan(e/2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3)
 - tan(e/2 + (f*x)/2)^4*(3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c
^2*d + c^3 + d^3 - tan(e/2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) +
 (5*a^3*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2)))/(f*(c + d)
^(7/2)*(c - d)^(1/2))
```

3.209 $\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 266

$$\begin{aligned}\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx &= \frac{5a^3(4c-3d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{4(c-d)^{3/2}(c+d)^{9/2}f} \\ &- \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} \\ &+ \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3} \\ &- \frac{5a^3(4c-3d)\tan(e+fx)}{24d(c+d)^3f(c+d\sec(e+fx))^2} \\ &+ \frac{5a^3(4c-3d)(c+4d)\tan(e+fx)}{24(c-d)d(c+d)^4f(c+d\sec(e+fx))}\end{aligned}$$

```
[Out] 5/4*a^3*(4*c-3*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)
^(3/2)/(c+d)^(9/2)/f-1/4*d*(a+a*sec(f*x+e))^3*tan(f*x+e)/(c^2-d^2)/f/(c+d*s
ec(f*x+e))^4+1/12*a*(4*c-3*d)*(a+a*sec(f*x+e))^2*tan(f*x+e)/(c-d)/(c+d)^2/f
/(c+d*sec(f*x+e))^3-5/24*a^3*(4*c-3*d)*tan(f*x+e)/d/(c+d)^3/f/(c+d*sec(f*x+
e))^2+5/24*a^3*(4*c-3*d)*(c+4*d)*tan(f*x+e)/(c-d)/d/(c+d)^4/f/(c+d*sec(f*x+
e))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4072, 98, 96, 95, 211}

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$
$$= -\frac{5a^4(4c-3d)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{4f(c-d)^{3/2}(c+d)^{9/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{5a^3(4c-3d)\tan(e+fx)}{8f(c-d)(c+d)^4(c+d\sec(e+fx))}+\frac{5(4c-3d)\tan(e+fx)\left(a^3\sec(e+fx)+a^3\right)}{24f(c-d)(c+d)^3(c+d\sec(e+fx))^2}$$
$$-\frac{d\tan(e+fx)(a\sec(e+fx)+a)^3}{4f\left(c^2-d^2\right)(c+d\sec(e+fx))^4}+\frac{a(4c-3d)\tan(e+fx)(a\sec(e+fx)+a)^2}{12f(c-d)(c+d)^2(c+d\sec(e+fx))^3}$$

```
[In] Int[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]
```

```
[Out] (-5*a^4*(4*c - 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c -
 d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(4*(c - d)^(3/2)*(c + d)^(9/2)
*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*(a + a*Sec[e + f
*x])^3*Tan[e + f*x])/(4*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^4) + (a*(4*c - 3
*d)*(a + a*Sec[e + f*x])^2*Tan[e + f*x])/(12*(c - d)*(c + d)^2*f*(c + d*Sec
[e + f*x])^3) + (5*(4*c - 3*d)*(a^3 + a^3*Sec[e + f*x])*Tan[e + f*x])/(24*(
c - d)*(c + d)^3*f*(c + d*Sec[e + f*x])^2) + (5*a^3*(4*c - 3*d)*Tan[e + f*x
])/(8*(c - d)*(c + d)^4*f*(c + d*Sec[e + f*x]))
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(a^2 \tan(e+fx)\right) \text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^5}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} \\
&\quad -\frac{\left(a^2(4c-3d)\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{5/2}}{\sqrt{a-ax}(c+dx)^4}\,dx, x, \sec(e+fx)\right)}{4\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4\left(c^2-d^2\right)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3} \\
&\quad -\frac{\left(5a^3(4c-3d)\tan(e+fx)\right)\text{Subst}\left(\int \frac{(a+ax)^{3/2}}{\sqrt{a-ax}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{12(c+d)\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3} + \frac{5(4c-3d)(a^3+a^3\sec(e+fx))\tan(e+fx)}{24(c-d)(c+d)^3f(c+d\sec(e+fx))^2} - \frac{(5a^4(4c-3d)\tan(e+fx))\,\text{Subst}\left(\int \frac{\sqrt{a+ax}}{\sqrt{a-ax}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{8(c+d)^2(c^2-d^2)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3} + \frac{5(4c-3d)(a^3+a^3\sec(e+fx))\tan(e+fx)}{24(c-d)(c+d)^3f(c+d\sec(e+fx))^2} + \frac{5a^3(4c-3d)\tan(e+fx)}{8(c-d)(c+d)^4f(c+d\sec(e+fx))} - \frac{(5a^5(4c-3d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{8(c+d)^3(c^2-d^2)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3} + \frac{5(4c-3d)(a^3+a^3\sec(e+fx))\tan(e+fx)}{24(c-d)(c+d)^3f(c+d\sec(e+fx))^2} + \frac{5a^3(4c-3d)\tan(e+fx)}{8(c-d)(c+d)^4f(c+d\sec(e+fx))} - \frac{(5a^5(4c-3d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{4(c+d)^3(c^2-d^2)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{5a^4(4c-3d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{4(c-d)^{3/2}(c+d)^{9/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d(a+a\sec(e+fx))^3\tan(e+fx)}{4(c^2-d^2)f(c+d\sec(e+fx))^4} + \frac{a(4c-3d)(a+a\sec(e+fx))^2\tan(e+fx)}{12(c-d)(c+d)^2f(c+d\sec(e+fx))^3} + \frac{5(4c-3d)(a^3+a^3\sec(e+fx))\tan(e+fx)}{24(c-d)(c+d)^3f(c+d\sec(e+fx))^2} + \frac{5a^3(4c-3d)\tan(e+fx)}{8(c-d)(c+d)^4f(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 6.45 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.03

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \frac{a^3\left(-\frac{120(4c-3d)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} - \frac{(-72c^4-478c^3d+336c^2d^2+28cd^3+336d^4+(-296c^4-84c^3d-577c^2d^2+984cd^3+198\ldots}{\ldots}\right.}{\ldots}$$

[In] Integrate[(Sec[e + f*x]*(a + a*Sec[e + f*x])^3)/(c + d*Sec[e + f*x])^5,x]

[Out] (a^3*((-120*(4*c - 3*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/Sqrt[c^2 - d^2] - ((-72*c^4 - 478*c^3*d + 336*c^2*d^2 + 28*c*d^3 + 336*d
^4 + (-296*c^4 - 84*c^3*d - 577*c^2*d^2 + 984*c*d^3 + 198*d^4)*Cos[e + f*x]
 + (-72*c^4 - 470*c^3*d + 384*c^2*d^2 + 200*c*d^3 + 48*d^4)*Cos[2*(e + f*x)
] - 88*c^4*Cos[3*(e + f*x)] + 36*c^3*d*Cos[3*(e + f*x)] + 37*c^2*d^2*Cos[3*
(e + f*x)] + 24*c*d^3*Cos[3*(e + f*x)] + 6*d^4*Cos[3*(e + f*x)])*Sin[e + f*
x])/(d + c*Cos[e + f*x])^4))/(96*(c - d)*(c + d)^4*f)

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{16a^3\left(\frac{-\frac{5(4c-3d)\left(c^2-2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{64\left(c^4+4c^3d+6c^2d^2+4c\,d^3+d^4\right)}+\frac{55(c-d)(4c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{192\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{73(4c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{192\left(c^2+2cd+d^2\right)}+\frac{(44c-49d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64(c+d)(c-d)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^4}\right.}{f}$
default	$\frac{16a^3\left(\frac{-\frac{5(4c-3d)\left(c^2-2cd+d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{64\left(c^4+4c^3d+6c^2d^2+4c\,d^3+d^4\right)}+\frac{55(c-d)(4c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{192\left(c^3+3c^2d+3c\,d^2+d^3\right)}-\frac{73(4c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{192\left(c^2+2cd+d^2\right)}+\frac{(44c-49d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{64(c+d)(c-d)}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^4}\right.}{f}$
risch	Expression too large to display

[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x,method=_RETURNVERBOS
E)

[Out] 16/f*a^3*((-5/64*(4*c-3*d)*(c^2-2*c*d+d^2)/(c^4+4*c^3*d+6*c^2*d^2+4*c*d^3+d
^4)*tan(1/2*f*x+1/2*e)^7+55/192*(c-d)*(4*c-3*d)/(c^3+3*c^2*d+3*c*d^2+d^3)*t
an(1/2*f*x+1/2*e)^5-73/192*(4*c-3*d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1
/64*(44*c-49*d)/(c+d)/(c-d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan
(1/2*f*x+1/2*e)^2*d-c-d)^4+5/64*(4*c-3*d)/(c^5+3*c^4*d+2*c^3*d^2-2*c^2*d^3-
3*c*d^4-d^5)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c
-d))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(247) = 494$.

Time = 0.36 (sec) , antiderivative size = 1714, normalized size of antiderivative = 6.44

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="fr
icas")
```

```
[Out] [1/48*(15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e)
^4 + 4*(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 - 3*
a^3*c^2*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x + e))
*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2
*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(
f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*a^3*c^5*d + 12*a^3*c^4*d^2 +
 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a^3*c^6
- 36*a^3*c^5*d - 125*a^3*c^4*d^2 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4 + 24*a^3
*c*d^5 + 6*a^3*d^6)*cos(f*x + e)^3 + (36*a^3*c^6 + 235*a^3*c^5*d - 228*a^3*
c^4*d^2 - 335*a^3*c^3*d^3 + 168*a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*a^3*d^6)*c
os(f*x + e)^2 + (8*a^3*c^6 + 48*a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d
^3 - 217*a^3*c^2*d^4 + 228*a^3*c*d^5 + 45*a^3*d^6)*cos(f*x + e))*sin(f*x +
e))/((c^11 + 3*c^10*d + c^9*d^2 - 5*c^8*d^3 - 5*c^7*d^4 + c^6*d^5 + 3*c^5*d
^6 + c^4*d^7)*f*cos(f*x + e)^4 + 4*(c^10*d + 3*c^9*d^2 + c^8*d^3 - 5*c^7*d^
4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*d^7 + c^3*d^8)*f*cos(f*x + e)^3 + 6*(c^9*d^
2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*d^5 - 5*c^5*d^6 + c^4*d^7 + 3*c^3*d^8 + c^2
*d^9)*f*cos(f*x + e)^2 + 4*(c^8*d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c
^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c*d^10)*f*cos(f*x + e) + (c^7*d^4 + 3*c^6*d^
5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f), 1/24*(
15*(4*a^3*c*d^4 - 3*a^3*d^5 + (4*a^3*c^5 - 3*a^3*c^4*d)*cos(f*x + e)^4 + 4*
(4*a^3*c^4*d - 3*a^3*c^3*d^2)*cos(f*x + e)^3 + 6*(4*a^3*c^3*d^2 - 3*a^3*c^2
*d^3)*cos(f*x + e)^2 + 4*(4*a^3*c^2*d^3 - 3*a^3*c*d^4)*cos(f*x + e))*sqrt(-
c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f
*x + e))) + (2*a^3*c^5*d + 12*a^3*c^4*d^2 + 41*a^3*c^3*d^3 - 84*a^3*c^2*d^4
 - 43*a^3*c*d^5 + 72*a^3*d^6 + (88*a^3*c^6 - 36*a^3*c^5*d - 125*a^3*c^4*d^2
 + 12*a^3*c^3*d^3 + 31*a^3*c^2*d^4 + 24*a^3*c*d^5 + 6*a^3*d^6)*cos(f*x + e)
^3 + (36*a^3*c^6 + 235*a^3*c^5*d - 228*a^3*c^4*d^2 - 335*a^3*c^3*d^3 + 168*
a^3*c^2*d^4 + 100*a^3*c*d^5 + 24*a^3*d^6)*cos(f*x + e)^2 + (8*a^3*c^6 + 48*
a^3*c^5*d + 164*a^3*c^4*d^2 - 276*a^3*c^3*d^3 - 217*a^3*c^2*d^4 + 228*a^3*c
*d^5 + 45*a^3*d^6)*cos(f*x + e))*sin(f*x + e))/((c^11 + 3*c^10*d + c^9*d^2
- 5*c^8*d^3 - 5*c^7*d^4 + c^6*d^5 + 3*c^5*d^6 + c^4*d^7)*f*cos(f*x + e)^4 +
 4*(c^10*d + 3*c^9*d^2 + c^8*d^3 - 5*c^7*d^4 - 5*c^6*d^5 + c^5*d^6 + 3*c^4*
d^7 + c^3*d^8)*f*cos(f*x + e)^3 + 6*(c^9*d^2 + 3*c^8*d^3 + c^7*d^4 - 5*c^6*
d^5 - 5*c^5*d^6 + c^4*d^7 + 3*c^3*d^8 + c^2*d^9)*f*cos(f*x + e)^2 + 4*(c^8*
```

```
d^3 + 3*c^7*d^4 + c^6*d^5 - 5*c^5*d^6 - 5*c^4*d^7 + c^3*d^8 + 3*c^2*d^9 + c
*d^10)*f*cos(f*x + e) + (c^7*d^4 + 3*c^6*d^5 + c^5*d^6 - 5*c^4*d^7 - 5*c^3*
d^8 + c^2*d^9 + 3*c*d^10 + d^11)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$
$$= a^3\left(\int \frac{\sec(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e\right.$$
$$+\int \frac{3\sec^2(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e-}$$
$$+\int \frac{3\sec^3(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e-}$$
$$+\int \frac{\sec^4(e+fx)}{c^5+5c^4d\sec(e+fx)+10c^3d^2\sec^2(e+fx)+10c^2d^3\sec^3(e+fx)+5cd^4\sec^4(e+fx)+d^5\sec^5(e-}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**5,x)
```

```
[Out] a**3*(Integral(sec(e + f*x)/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*se
c(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d
**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x)**2/(c**5 + 5*c**4*d*sec(
e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*
c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x) + Integral(3*sec(e + f*x
)**3/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*sec(e + f*x)**2 + 10*c**2
*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 + d**5*sec(e + f*x)**5), x
) + Integral(sec(e + f*x)**4/(c**5 + 5*c**4*d*sec(e + f*x) + 10*c**3*d**2*s
ec(e + f*x)**2 + 10*c**2*d**3*sec(e + f*x)**3 + 5*c*d**4*sec(e + f*x)**4 +
d**5*sec(e + f*x)**5), x))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(247) = 494$.

Time = 0.48 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{15\,(4\,a^3c-3\,a^3d)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^5+3\,c^4d+2\,c^3d^2-2\,c^2d^3-3\,cd^4-d^5)\sqrt{-c^2+d^2}} - \frac{60\,a^3c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^7-225\,a^3c^3d\tan\left(\frac{1}{2}\,f\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^5,x, algorithm="gi
ac")
```

```
[Out] 1/12*(15*(4*a^3*c - 3*a^3*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2
*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
 d^2)))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*sqrt(-c^2
+ d^2)) - (60*a^3*c^4*tan(1/2*f*x + 1/2*e)^7 - 225*a^3*c^3*d*tan(1/2*f*x +
1/2*e)^7 + 315*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 195*a^3*c*d^3*tan(1/2*f
*x + 1/2*e)^7 + 45*a^3*d^4*tan(1/2*f*x + 1/2*e)^7 - 220*a^3*c^4*tan(1/2*f*x
 + 1/2*e)^5 + 385*a^3*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 55*a^3*c^2*d^2*tan(1/2
*f*x + 1/2*e)^5 - 385*a^3*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 165*a^3*d^4*tan(1/
2*f*x + 1/2*e)^5 + 292*a^3*c^4*tan(1/2*f*x + 1/2*e)^3 + 73*a^3*c^3*d*tan(1/
2*f*x + 1/2*e)^3 - 511*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 73*a^3*c*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 219*a^3*d^4*tan(1/2*f*x + 1/2*e)^3 - 132*a^3*c^4*tan
(1/2*f*x + 1/2*e) - 249*a^3*c^3*d*tan(1/2*f*x + 1/2*e) + 45*a^3*c^2*d^2*tan
(1/2*f*x + 1/2*e) + 309*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + 147*a^3*d^4*tan(1/
2*f*x + 1/2*e))/((c^5 + 3*c^4*d + 2*c^3*d^2 - 2*c^2*d^3 - 3*c*d^4 - d^5)*(c
*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^4))/f
```

Mupad [B] (verification not implemented)

Time = 17.09 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.45

$$\int \frac{\sec(e+fx)(a+a\sec(e+fx))^3}{(c+d\sec(e+fx))^5}\,dx$$
$$= \frac{\frac{55\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\,(4\,a^3\,c^2-7\,a^3\,c\,d+3\,a^3\,d^2)}{12\,(c+d)^3} - \frac{73\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\,(4\,a^3\,c-3\,a^3}{12\,(c+d)^2}}{f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\,(6\,c^4-12\,c^2\,d^2+6\,d^4)+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,(-4\,c^4-8\,c^3\,d+8\,c\,d^3+4\,d^4)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6\right.}$$
$$+ \frac{5\,a^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c-d}}{\sqrt{c+d}}\right)\,(4\,c-3\,d)}{4\,f\,(c+d)^{9/2}\,(c-d)^{3/2}}$$

```
[In] int((a + a/cos(e + f*x))^3/(cos(e + f*x)*(c + d/cos(e + f*x))^5),x)
```

```
[Out] ((55*tan(e/2 + (f*x)/2)^5*(4*a^3*c^2 + 3*a^3*d^2 - 7*a^3*c*d))/(12*(c + d)^
3) - (73*tan(e/2 + (f*x)/2)^3*(4*a^3*c - 3*a^3*d))/(12*(c + d)^2) - (5*tan(
e/2 + (f*x)/2)^7*(4*a^3*c^3 - 3*a^3*d^3 + 10*a^3*c*d^2 - 11*a^3*c^2*d))/(4*
(c + d)^4) + (a^3*tan(e/2 + (f*x)/2)*(44*c - 49*d))/(4*(c + d)*(c - d)))/(f
*(tan(e/2 + (f*x)/2)^4*(6*c^4 + 6*d^4 - 12*c^2*d^2) + tan(e/2 + (f*x)/2)^2*
(8*c*d^3 - 8*c^3*d - 4*c^4 + 4*d^4) - tan(e/2 + (f*x)/2)^6*(8*c*d^3 - 8*c^3
*d + 4*c^4 - 4*d^4) + tan(e/2 + (f*x)/2)^8*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 +
 6*c^2*d^2) + 4*c*d^3 + 4*c^3*d + c^4 + d^4 + 6*c^2*d^2)) + (5*a^3*atanh((t
an(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(4*c - 3*d))/(4*f*(c + d)^(
9/2)*(c - d)^(3/2))
```

3.210 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 183

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{d(8c^3-12c^2d+12cd^2-3d^3)\operatorname{arctanh}(\sin(e+fx))}{2af} - \frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d(4(3c^3-16c^2d+12cd^2-4d^3)+d(6c^2-20cd+9d^2)\sec(e+fx))\tan(e+fx)}{6af}$$

```
[Out] 1/2*d*(8*c^3-12*c^2*d+12*c*d^2-3*d^3)*arctanh(sin(f*x+e))/a/f-1/3*(3*c-4*d)
*d*(c+d*sec(f*x+e))^2*tan(f*x+e)/a/f+(c-d)*(c+d*sec(f*x+e))^3*tan(f*x+e)/f/
(a+a*sec(f*x+e))-1/6*d*(12*c^3-64*c^2*d+48*c*d^2-16*d^3+d*(6*c^2-20*c*d+9*d
^2)*sec(f*x+e))*tan(f*x+e)/a/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used

$= \{4072, 100, 158, 152, 65, 223, 209\}$

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$= \frac{d(8c^3-12c^2d+12cd^2-3d^3)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$-\frac{d\tan(e+fx)\left(d(6c^2-20cd+9d^2)\sec(e+fx)+4(3c^3-16c^2d+12cd^2-4d^3)\right)}{6af}$$
$$+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^3}{f(a\sec(e+fx)+a)}-\frac{d(3c-4d)\tan(e+fx)(c+d\sec(e+fx))^2}{3af}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d*(8*c^3 - 12*c^2*d + 12*c*d^2 - 3*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a
+ a*Sec[e + f*x]]) - ((3*c - 4*d)*d*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3
*a*f) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(f*(a + a*Sec[e + f*x
])) - (d*(4*(3*c^3 - 16*c^2*d + 12*c*d^2 - 4*d^3) + d*(6*c^2 - 20*c*d + 9*d
^2)*Sec[e + f*x])*Tan[e + f*x])/(6*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
```

```
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))}+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)^2(-a^2(4c-3d)d+a^2(3c-4d)dx)}{\sqrt{a-ax}\sqrt{a+ax}}dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=-\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af}+\frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)(a^4d(12c^2-15cd+8d^2)-a^4d(6c^2-20cd+9d^2)x)}{\sqrt{a-ax}\sqrt{a+ax}}dx,x,\sec(e+fx)\right)}{3a^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=-\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af}+\frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{d(4(3c^3-16c^2d+12cd^2-4d^3)+d(6c^2-20cd+9d^2)\sec(e+fx))\tan(e+fx)}{6af}-\frac{(ad(8c^3-12c^2d+12cd^2-3d^3)\tan(e+fx))\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}dx,x,\sec(e+fx)\right)}{2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=-\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af}+\frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{d(4(3c^3-16c^2d+12cd^2-4d^3)+d(6c^2-20cd+9d^2)\sec(e+fx))\tan(e+fx)}{6af}+\frac{(d(8c^3-12c^2d+12cd^2-3d^3)\tan(e+fx))\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}dx,x,\sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=-\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af}+\frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{d(4(3c^3-16c^2d+12cd^2-4d^3)+d(6c^2-20cd+9d^2)\sec(e+fx))\tan(e+fx)}{6af}+\frac{(d(8c^3-12c^2d+12cd^2-3d^3)\tan(e+fx))\text{Subst}\left(\int\frac{1}{1+x^2}dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{d(8c^3-12c^2d+12cd^2-3d^3)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}-\frac{(3c-4d)d(c+d\sec(e+fx))^2\tan(e+fx)}{3af}+\frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{f(a+a\sec(e+fx))}-\frac{d(4(3c^3-16c^2d+12cd^2-4d^3)+d(6c^2-20cd+9d^2)\sec(e+fx))\tan(e+fx)}{6af}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1243 vs. 2(183) = 366.

Time = 7.85 (sec) , antiderivative size = 1243, normalized size of antiderivative = 6.79

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$= \frac{(-8c^3d+12c^2d^2-12cd^3+3d^4)\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(c+d\sec(e}{f(d+c\cos(e+fx))^4(a+a\sec(e+fx))}$$
$$+\frac{(8c^3d-12c^2d^2+12cd^3-3d^4)\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(c+d\sec(e}{f(d+c\cos(e+fx))^4(a+a\sec(e+fx))}$$
$$+\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sec\left(\frac{e}{2}\right)\sec(e)(c+d\sec(e+fx))^4\left(-18c^4\sin\left(\frac{fx}{2}\right)+72c^3d\sin\left(\frac{fx}{2}\right)-36c^2d^2\sin\left(\frac{fx}{2}\right)+24c\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x]),x]
```

```
[Out] ((-8*c^3*d + 12*c^2*d^2 - 12*c*d^3 + 3*d^4)*Cos[e/2 + (f*x)/2]^2*Cos[e + f*
x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^4)/(
f*(d + c*Cos[e + f*x])^4*(a + a*Sec[e + f*x])) + ((8*c^3*d - 12*c^2*d^2 + 1
2*c*d^3 - 3*d^4)*Cos[e/2 + (f*x)/2]^2*Cos[e + f*x]^3*Log[Cos[e/2 + (f*x)/2]
 + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^4)/(f*(d + c*Cos[e + f*x])^4*(a
 + a*Sec[e + f*x])) + (Cos[e/2 + (f*x)/2]*Sec[e/2]*Sec[e]*(c + d*Sec[e + f*
x])^4*(-18*c^4*Sin[(f*x)/2] + 72*c^3*d*Sin[(f*x)/2] - 36*c^2*d^2*Sin[(f*x)/
2] + 24*c*d^3*Sin[(f*x)/2] + 6*d^4*Sin[(f*x)/2] + 18*c^4*Sin[(3*f*x)/2] - 7
2*c^3*d*Sin[(3*f*x)/2] + 180*c^2*d^2*Sin[(3*f*x)/2] - 108*c*d^3*Sin[(3*f*x)
/2] + 39*d^4*Sin[(3*f*x)/2] - 72*c^2*d^2*Sin[e - (f*x)/2] + 48*c*d^3*Sin[e
- (f*x)/2] - 24*d^4*Sin[e - (f*x)/2] - 36*c^2*d^2*Sin[e + (f*x)/2] + 24*c*d
^3*Sin[e + (f*x)/2] - 6*d^4*Sin[e + (f*x)/2] - 18*c^4*Sin[2*e + (f*x)/2] +
72*c^3*d*Sin[2*e + (f*x)/2] - 144*c^2*d^2*Sin[2*e + (f*x)/2] + 96*c*d^3*Sin
[2*e + (f*x)/2] - 24*d^4*Sin[2*e + (f*x)/2] + 72*c^2*d^2*Sin[e + (3*f*x)/2]
 - 36*c*d^3*Sin[e + (3*f*x)/2] + 21*d^4*Sin[e + (3*f*x)/2] + 18*c^4*Sin[2*e
 + (3*f*x)/2] - 72*c^3*d*Sin[2*e + (3*f*x)/2] + 72*c^2*d^2*Sin[2*e + (3*f*x
)/2] - 36*c*d^3*Sin[2*e + (3*f*x)/2] + 9*d^4*Sin[2*e + (3*f*x)/2] - 36*c^2*
d^2*Sin[3*e + (3*f*x)/2] + 36*c*d^3*Sin[3*e + (3*f*x)/2] - 9*d^4*Sin[3*e +
(3*f*x)/2] + 36*c^2*d^2*Sin[e + (5*f*x)/2] - 12*c*d^3*Sin[e + (5*f*x)/2] +
7*d^4*Sin[e + (5*f*x)/2] - 6*c^4*Sin[2*e + (5*f*x)/2] + 24*c^3*d*Sin[2*e +
(5*f*x)/2] + 12*c*d^3*Sin[2*e + (5*f*x)/2] + d^4*Sin[2*e + (5*f*x)/2] + 12*
c*d^3*Sin[3*e + (5*f*x)/2] - 3*d^4*Sin[3*e + (5*f*x)/2] - 6*c^4*Sin[4*e + (
5*f*x)/2] + 24*c^3*d*Sin[4*e + (5*f*x)/2] - 36*c^2*d^2*Sin[4*e + (5*f*x)/2]
 + 36*c*d^3*Sin[4*e + (5*f*x)/2] - 9*d^4*Sin[4*e + (5*f*x)/2] + 6*c^4*Sin[2
*e + (7*f*x)/2] - 24*c^3*d*Sin[2*e + (7*f*x)/2] + 72*c^2*d^2*Sin[2*e + (7*f
*x)/2] - 48*c*d^3*Sin[2*e + (7*f*x)/2] + 16*d^4*Sin[2*e + (7*f*x)/2] + 36*c
^2*d^2*Sin[3*e + (7*f*x)/2] - 24*c*d^3*Sin[3*e + (7*f*x)/2] + 10*d^4*Sin[3*
e + (7*f*x)/2] + 6*c^4*Sin[4*e + (7*f*x)/2] - 24*c^3*d*Sin[4*e + (7*f*x)/2]
```

```
 + 36*c^2*d^2*Sin[4*e + (7*f*x)/2] - 24*c*d^3*Sin[4*e + (7*f*x)/2] + 6*d^4*
Sin[4*e + (7*f*x)/2]))/(48*f*(d + c*Cos[e + f*x])^4*(a + a*Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.50

method	result
parallelrisch	$\frac{-12\left(c^3-\frac{3}{2}c^2d+\frac{3}{2}c\,d^2-\frac{3}{8}d^3\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+12\left(c^3-\frac{3}{2}c^2d+\frac{3}{2}c\,d^2-\frac{3}{8}d^3\right)\left(\cos(fx+e)+\right.}{}$
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^4-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^3d+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2d^2-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c\,d^3+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^4-\frac{d^4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{d\left(8c^3\right.}{}}{}$
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^4-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^3d+6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2d^2-4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c\,d^3+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^4-\frac{d^4}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{d\left(8c^3\right.}{}}{}$
norman	$\frac{\frac{\left(c^4-4c^3d+6c^2d^2-4c\,d^3+d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{af}+\frac{\left(c^4-4c^3d+18c^2d^2-8c\,d^3+4d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{\left(4c^4-16c^3d+36c^2d^2-28c\,d^3+9d^4\right)\tan}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$
risch	$\frac{i\left(36c^2d^2e^{6i(fx+e)}+144c^2d^2e^{4i(fx+e)}-108c\,d^3e^{2i(fx+e)}-72c^3d\,e^{2i(fx+e)}-24c^3d\,e^{6i(fx+e)}-36c\,d^3e^{5i(fx+e)}-96c\,d^3e^{4i(f}\right.}{}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 3*(-4*(c^3-3/2*c^2*d+3/2*c*d^2-3/8*d^3)*(cos(f*x+e)+1/3*cos(3*f*x+3*e))*d*l
n(tan(1/2*f*x+1/2*e)-1)+4*(c^3-3/2*c^2*d+3/2*c*d^2-3/8*d^3)*(cos(f*x+e)+1/3
*cos(3*f*x+3*e))*d*ln(tan(1/2*f*x+1/2*e)+1)+tan(1/2*f*x+1/2*e)*((1/3*c^4-4/
3*c^3*d+4*c^2*d^2-8/3*c*d^3+8/9*d^4)*cos(3*f*x+3*e)+(4*c^2*d^2-4/3*c*d^3+7/
9*d^4)*cos(2*f*x+2*e)+(c^4-4*c^3*d+12*c^2*d^2-16/3*c*d^3+22/9*d^4)*cos(f*x+
e)+4*c^2*d^2-4/3*c*d^3+11/9*d^4))/a/f/(cos(3*f*x+3*e)+3*cos(f*x+e))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx$$
$$=\frac{3\left(\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+\left(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3\right)\log\left(\sin\left(\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*
d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) -
3*((8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^4 + (8*c^3*d - 12
*c^2*d^2 + 12*c*d^3 - 3*d^4)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*
d^4 + 2*(3*c^4 - 12*c^3*d + 36*c^2*d^2 - 24*c*d^3 + 8*d^4)*cos(f*x + e)^3 +
 (36*c^2*d^2 - 12*c*d^3 + 7*d^4)*cos(f*x + e)^2 + (12*c*d^3 - d^4)*cos(f*x
+ e))*sin(f*x + e))/(a*f*cos(f*x + e)^4 + a*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c^4\sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{d^4\sec^5(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{4cd^3\sec^4(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{6c^2d^2\sec^3(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{4c^3d\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**4*sec(e +
f*x)**5/(sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(sec(e +
 f*x) + 1), x) + Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x) + 1), x
) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(174) = 348$.

Time = 0.22 (sec) , antiderivative size = 596, normalized size of antiderivative = 3.26

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{d^4\left(\frac{2\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1} - \frac{16\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{15\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a - \frac{3a\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3a\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{a\sin(fx+e)^6}{(\cos(fx+e)+1)^6}} - \frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a} + \frac{9\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a} + \frac{6\sin(fx+e)}{a(\cos(fx+e)+1)}\right) - 12}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(d^4*(2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 16*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a - 3*a*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 3*a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - a*sin(
f*x + e)^6/(cos(f*x + e) + 1)^6) - 9*log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a + 9*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 6*sin(f*x + e)/(a*(co
s(f*x + e) + 1))) - 12*c*d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*
```

```
x + e)^3/(cos(f*x + e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e
) + 1) + 1)/a + 3*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x +
e)/(a*(cos(f*x + e) + 1))) - 36*c^2*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a
- a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)
/(a*(cos(f*x + e) + 1))) + 24*c^3*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) +
1)/a - log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*
x + e) + 1))) + 6*c^4*sin(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{\frac{3\,(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{3\,(8\,c^3d-12\,c^2d^2+12\,cd^3-3\,d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}+\frac{6\,(c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-4}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*x + 1/2*
e) + 1))/a - 3*(8*c^3*d - 12*c^2*d^2 + 12*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*
x + 1/2*e) - 1))/a + 6*(c^4*tan(1/2*f*x + 1/2*e) - 4*c^3*d*tan(1/2*f*x + 1/
2*e) + 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 4*c*d^3*tan(1/2*f*x + 1/2*e) + d^4*
tan(1/2*f*x + 1/2*e))/a - 2*(36*c^2*d^2*tan(1/2*f*x + 1/2*e)^5 - 36*c*d^3*t
an(1/2*f*x + 1/2*e)^5 + 15*d^4*tan(1/2*f*x + 1/2*e)^5 - 72*c^2*d^2*tan(1/2*
f*x + 1/2*e)^3 + 48*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 16*d^4*tan(1/2*f*x + 1/2
*e)^3 + 36*c^2*d^2*tan(1/2*f*x + 1/2*e) - 12*c*d^3*tan(1/2*f*x + 1/2*e) + 9
*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a))/f
```

Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.15

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+a\sec(e+fx)}\,dx = \frac{(12\,c^2\,d^2-12\,c\,d^3+5\,d^4)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-24\,c^2\,d^2+16\,c\,d^3-\frac{16\,d^4}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+(12\,c^2\,d^2-4\,c\,d^3+}{f\,\left(-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6+3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4-3\,a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a\right)} + \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(c-d)^4}{a\,f}+\frac{d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(8\,c^3-12\,c^2\,d+12\,c\,d^2-3\,d^3)}{a\,f}$$

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3*d^4 - 4*c*d^3 + 12*c^2*d^2) + tan(e/2 + (f*x)/2)^5*(
5*d^4 - 12*c*d^3 + 12*c^2*d^2) - tan(e/2 + (f*x)/2)^3*((16*d^4)/3 - 16*c*d^
3 + 24*c^2*d^2))/(f*(a - 3*a*tan(e/2 + (f*x)/2)^2 + 3*a*tan(e/2 + (f*x)/2)^
4 - a*tan(e/2 + (f*x)/2)^6)) + (tan(e/2 + (f*x)/2)*(c - d)^4)/(a*f) + (d*at
anh(tan(e/2 + (f*x)/2))*(12*c*d^2 - 12*c^2*d + 8*c^3 - 3*d^3))/(a*f)
```

3.211 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 117

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx &= \frac{3d(2c^2-2cd+d^2)\,\mathrm{arctanh}(\sin(e+fx))}{2af}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}\\ &\quad-\frac{d(4(c^2-3cd+d^2)+(2c-3d)d\sec(e+fx))\tan(e+fx)}{2af}\end{aligned}$$

```
[Out] 3/2*d*(2*c^2-2*c*d+d^2)*arctanh(sin(f*x+e))/a/f+(c-d)*(c+d*sec(f*x+e))^2*ta
n(f*x+e)/f/(a+a*sec(f*x+e))-1/2*d*(4*c^2-12*c*d+4*d^2+(2*c-3*d)*d*sec(f*x+e
))*tan(f*x+e)/a/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 100, 152, 65, 223, 209}

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx &= \frac{3d(2c^2-2cd+d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\ &\quad-\frac{d\tan(e+fx)\left(4(c^2-3cd+d^2)+d(2c-3d)\sec(e+fx)\right)}{2af}\\ &\quad+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^2}{f(a\sec(e+fx)+a)}\end{aligned}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[
e + f*x])]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*
x]]) + ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x]
)) - (d*(4*(c^2 - 3*c*d + d^2) + (2*c - 3*d)*d*Sec[e + f*x])*Tan[e + f*x])/
(2*a*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(c+dx)^3}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}\\
&\quad+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)(-a^2(3c-2d)d+a^2(2c-3d)dx)}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}\\
&\quad-\frac{d(4(c^2-3cd+d^2)+(2c-3d)d\sec(e+fx))\tan(e+fx)}{2af}\\
&\quad-\frac{(3ad(2c^2-2cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))}\\
&\quad-\frac{d(4(c^2-3cd+d^2)+(2c-3d)d\sec(e+fx))\tan(e+fx)}{2af}\\
&\quad+\frac{(3d(2c^2-2cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d(4(c^2-3cd+d^2)+(2c-3d)d\sec(e+fx))\tan(e+fx)}{2af} + \frac{(3d(2c^2-2cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{3d(2c^2-2cd+d^2)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{d(4(c^2-3cd+d^2)+(2c-3d)d\sec(e+fx))\tan(e+fx)}{2af}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int\frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{3d(2c^2-2cd+d^2)\,\text{arctanh}(\sin(e+fx))+2(c-d)^3\tan\left(\frac{1}{2}(e+fx)\right)+d^2(6c-2d+d\sec(e+fx))\tan(e}{2af}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (3*d*(2*c^2 - 2*c*d + d^2)*ArcTanh[Sin[e + f*x]] + 2*(c - d)^3*Tan[(e + f*x
)/2] + d^2*(6*c - 2*d + d*Sec[e + f*x])*Tan[e + f*x])/(2*a*f)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.55

method	result
parallelrisch	$\frac{-3(c^2-cd+\frac{1}{2}d^2)(1+\cos(2fx+2e))d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+3(c^2-cd+\frac{1}{2}d^2)(1+\cos(2fx+2e))d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\tan\left(\right.}{af(1+\cos(2fx+2e))}$
derivativedivides	$\frac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c\,d^2-d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{3d\left(2c^2-2cd+d^2\right)\ln\left(\tan\left(\frac{fx}{2}\right.\right.}{2}}{fa}$
default	$\frac{c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c\,d^2-d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{d^3}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{3d\left(2c^2-2cd+d^2\right)\ln\left(\tan\left(\frac{fx}{2}\right.\right.}{2}}{fa}$
norman	$\frac{\frac{\left(c^3-3c^2d+3c\,d^2-d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}+\frac{\left(3c^3-9c^2d+21c\,d^2-7d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}-\frac{3\left(c^3-3c^2d+5c\,d^2-2d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}-\frac{\left(c^3-3c^2d\right.}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
risch	$\frac{i\left(2c^3e^{4i(fx+e)}-6c^2d\,e^{4i(fx+e)}+6c\,d^2e^{4i(fx+e)}-3d^3e^{4i(fx+e)}+6c\,d^2e^{3i(fx+e)}-3d^3e^{3i(fx+e)}+4c^3e^{2i(fx+e)}-12c^2d\,e^{2i(fx\right.}{fa\left(e^{i(fx+e)}+1\right)\left(1+e^{2i(fx+e)}\right)^2}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] (-3*(c^2-c*d+1/2*d^2)*(1+cos(2*f*x+2*e))*d*ln(tan(1/2*f*x+1/2*e)-1)+3*(c^2-
c*d+1/2*d^2)*(1+cos(2*f*x+2*e))*d*ln(tan(1/2*f*x+1/2*e)+1)+tan(1/2*f*x+1/2*
e)*((c^3-3*c^2*d+6*c*d^2-2*d^3)*cos(2*f*x+2*e)+(6*c*d^2-d^3)*cos(f*x+e)+c^3
-3*c^2*d+6*c*d^2-d^3))/a/f/(1+cos(2*f*x+2*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.85

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$$
$$= \frac{3\left((2\,c^2d-2\,cd^2+d^3)\cos\left(fx+e\right)^3+(2\,c^2d-2\,cd^2+d^3)\cos\left(fx+e\right)^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left((2\,c^2d\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/4*(3*((2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3
)*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - 3*((2*c^2*d - 2*c*d^2 + d^3)*cos(
f*x + e)^3 + (2*c^2*d - 2*c*d^2 + d^3)*cos(f*x + e)^2)*log(-sin(f*x + e) +
1) + 2*(d^3 + 2*(c^3 - 3*c^2*d + 6*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (6*c*d^2
 - d^3)*cos(f*x + e))*sin(f*x + e))/(a*f*cos(f*x + e)^3 + a*f*cos(f*x + e)^
2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c^3\sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{d^3\sec^4(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{3cd^2\sec^3(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{3c^2d\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**3*sec(e +
f*x)**4/(sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(sec(e +
 f*x) + 1), x) + Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x) + 1), x))/
a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(114) = 228.

Time = 0.22 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.32

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = -\frac{d^3\left(\frac{2\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-\frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a-\frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}+\frac{2\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+6\,cd^2\Big(\log\Big(}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(d^3*(2*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3)/(a - 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4) - 3*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a + 3*l
og(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a + 2*sin(f*x + e)/(a*(cos(f*x + e)
 + 1))) + 6*c*d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x
 + e)/(cos(f*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1)
)) - 6*c^2*d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c^3*s
in(f*x + e)/(a*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.87

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx$$
$$= \frac{\frac{3\left(2\,c^2d-2\,cd^2+d^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a}-\frac{3\left(2\,c^2d-2\,cd^2+d^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a}+\frac{2\left(c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+3\right.}{a}}{2\,f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*(3*(2*c^2*d - 2*c*d^2 + d^3)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - 3*(
2*c^2*d - 2*c*d^2 + d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a + 2*(c^3*tan(
1/2*f*x + 1/2*e) - 3*c^2*d*tan(1/2*f*x + 1/2*e) + 3*c*d^2*tan(1/2*f*x + 1/2
*e) - d^3*tan(1/2*f*x + 1/2*e))/a - 2*(6*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*d
^3*tan(1/2*f*x + 1/2*e)^3 - 6*c*d^2*tan(1/2*f*x + 1/2*e) + d^3*tan(1/2*f*x
+ 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^2*a))/f
```

Mupad [B] (verification not implemented)

Time = 13.68 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+a\sec(e+fx)}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(6\,c\,d^2-d^3\right)-{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,\left(6\,c\,d^2-3\,d^3\right)}{f\,\left(a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,a\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+a\right)} + \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,{\left(c-d\right)}^3}{a\,f} + \frac{3\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,c^2-2\,c\,d+d^2\right)}{a\,f}$$

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(6*c*d^2 - d^3) - tan(e/2 + (f*x)/2)^3*(6*c*d^2 - 3*d^3
))/(f*(a - 2*a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^4)) + (tan(e/2 +
 (f*x)/2)*(c - d)^3)/(a*f) + (3*d*atanh(tan(e/2 + (f*x)/2))*(2*c^2 - 2*c*d
+ d^2))/(a*f)
```

3.212 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 68

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{(2c-d)d\,\text{arctanh}(\sin(e+fx))}{af} + \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] (2*c-d)*d*arctanh(sin(f*x+e))/a/f+d^2*tan(f*x+e)/a/f+(c-d)^2*tan(f*x+e)/f/(
a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 91, 81, 65, 223, 209}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{2d(2c-d)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{(c-d)^2\tan(e+fx)}{f(a\sec(e+fx)+a)} + \frac{d^2\tan(e+fx)}{af}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d^2*Tan[e + f*x])/(a*f) + ((c - d)^2*Tan[e + f*x])/(f*(a + a*Sec[e + f*x])
) + (2*(2*c - d)*d*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{(c+dx)^2}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int\frac{a^3(2c-d)d+a^3d^2x}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} - \frac{(a(2c-d)d\tan(e+fx))\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(2(2c-d)d\tan(e+fx))\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{(2(2c-d)d\tan(e+fx))\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2\tan(e+fx)}{af} + \frac{(c-d)^2\tan(e+fx)}{f(a+a\sec(e+fx))} + \frac{2(2c-d)d\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 237 vs. 2(68) = 136.

Time = 2.66 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.49

$$\int\frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\cos(e+fx)(c+d\sec(e+fx))^2\left((c-d)^2\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+d\cos\left(\frac{1}{2}(e+fx)\right)\left(-((2c-\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x]),x]
[Out] (2*Cos[(e + f*x)/2]*Cos[e + f*x]*(c + d*Sec[e + f*x])^2*((c - d)^2*Sec[e/2]
*Sin[(f*x)/2] + d*Cos[(e + f*x)/2]*(-((2*c - d)*(Log[Cos[(e + f*x)/2] - Sin
```

```
[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + (d*Sin[f*x])/
((Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))))/(a*f*(d + c*Cos[e + f*x])
^2*(1 + Sec[e + f*x]))
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.51

method	result
parallelrisch	$\frac{-2\left(c-\frac{d}{2}\right)\cos(fx+e)d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+2\left(c-\frac{d}{2}\right)\cos(fx+e)d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left((c^2-2cd+2d^2)\cos\right.}{af\cos(fx+e)}$
derivativedivides	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cd+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2-\frac{d^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-d(2c-d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\frac{d^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+d(2c}{fa}$
default	$\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cd+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2-\frac{d^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-d(2c-d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)-\frac{d^2}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}+d(2c}{fa}$
norman	$\frac{\frac{(c^2-2cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{af}+\frac{(c^2-2cd+3d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{2(c^2-2cd+2d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}+\frac{d(2c-d)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}-$
risch	$\frac{2i(c^2e^{2i(fx+e)}-2cd\,e^{2i(fx+e)}+d^2e^{2i(fx+e)}+d^2e^{i(fx+e)}+c^2-2cd+2d^2)}{fa(1+e^{2i(fx+e)})(e^{i(fx+e)}+1)}+\frac{2d\ln(e^{i(fx+e)}+i)c}{af}-\frac{d^2\ln(e^{i(fx+e)}+i)}{af}-$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*(c-1/2*d)*cos(f*x+e)*d*ln(tan(1/2*f*x+1/2*e)-1)+2*(c-1/2*d)*cos(f*x+e)*
d*ln(tan(1/2*f*x+1/2*e)+1)+tan(1/2*f*x+1/2*e)*((c^2-2*c*d+2*d^2)*cos(f*x+e)
+d^2))/a/f/cos(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.28

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$$
$$=\frac{\left((2\,cd-d^2)\cos\left(fx+e\right)^2+(2\,cd-d^2)\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right)+1\right)-\left((2\,cd-d^2)\cos\left(fx+e\right)^2+(\right.}{2\left(af\cos\left(fx+e\right)^2+\right.}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] 1/2*(((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*log(sin(f*
x + e) + 1) - ((2*c*d - d^2)*cos(f*x + e)^2 + (2*c*d - d^2)*cos(f*x + e))*l
og(-sin(f*x + e) + 1) + 2*(d^2 + (c^2 - 2*c*d + 2*d^2)*cos(f*x + e))*sin(f*
x + e))/(a*f*cos(f*x + e)^2 + a*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c^2\sec(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{d^2\sec^3(e+fx)}{\sec(e+fx)+1}\,dx + \int \frac{2cd\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e)),x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d**2*sec(e +
f*x)**3/(sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e + f*
x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(68) = 136$.

Time = 0.22 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.28

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = -\frac{d^2\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a} - \frac{2\sin(fx+e)}{\left(a-\frac{a\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)} - \frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right) - 2\,cd\left(\frac{\log\left(\frac{\sin(f}{\cos(f}\right.}{}\right.}{f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -(d^2*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f
*x + e) + 1) - 1)/a - 2*sin(f*x + e)/((a - a*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*(cos(f*x + e) + 1)) - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - 2*c*d*(
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) - c^2*sin(f*x + e)/(a*(
cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.00

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx$$

$$= \frac{\frac{(2\,cd-d^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a} - \frac{(2\,cd-d^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a} - \frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a} + \frac{c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-2\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right.}{a}}{f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((2*c*d - d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - (2*c*d - d^2)*log(abs
(tan(1/2*f*x + 1/2*e) - 1))/a - 2*d^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x +
1/2*e)^2 - 1)*a) + (c^2*tan(1/2*f*x + 1/2*e) - 2*c*d*tan(1/2*f*x + 1/2*e) +
 d^2*tan(1/2*f*x + 1/2*e))/a)/f
```

Mupad [B] (verification not implemented)

Time = 13.53 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+a\sec(e+fx)}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(c-d)^2}{a\,f} + \frac{2\,d^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a-a\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\right)} + \frac{2\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(2\,c-d)}{a\,f}$$

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c - d)^2)/(a*f) + (2*d^2*tan(e/2 + (f*x)/2))/(f*(a - a
*tan(e/2 + (f*x)/2)^2)) + (2*d*atanh(tan(e/2 + (f*x)/2))*(2*c - d))/(a*f)
```

3.213 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 43

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{d\text{arctanh}(\sin(e+fx))}{af}+\frac{(c-d)\tan(e+fx)}{f(a+a\sec(e+fx))}$$

```
[Out] d*arctanh(sin(f*x+e))/a/f+(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4083, 3855, 3879}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{d\text{arctanh}(\sin(e+fx))}{af}+\frac{(c-d)\tan(e+fx)}{f(a\sec(e+fx)+a)}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(a*f) + ((c - d)*Tan[e + f*x])/(f*(a + a*Sec[e +
f*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
```

```
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (c-d)\int \frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx + \frac{d\int \sec(e+fx)\,dx}{a} \\ &= \frac{d\operatorname{arctanh}(\sin(e+fx))}{af} + \frac{(c-d)\tan(e+fx)}{f(a+a\sec(e+fx))}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 109 vs. $2(43) = 86$.

Time = 0.90 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.53

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(d\cos\left(\frac{1}{2}(e+fx)\right)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\mathrm{s}\right.\right.\right.}{af(1+\cos(e+fx))}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x]),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d*Cos[(e + f*x)/2]*(-Log[Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Sec[e/2]*Sin[
(f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

method	result	size
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(c-d)}{af}$	53
derivativedivides	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d}{fa}$	61
default	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d+\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d}{fa}$	61
risch	$\frac{2ic}{fa(e^{i(fx+e)}+1)}-\frac{2id}{fa(e^{i(fx+e)}+1)}+\frac{d\ln(e^{i(fx+e)}+i)}{af}-\frac{d\ln(e^{i(fx+e)}-i)}{af}$	91
norman	$\frac{\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{af}-\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1}+\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{af}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{af}$	105

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] (-ln(tan(1/2*f*x+1/2*e)-1)*d+ln(tan(1/2*f*x+1/2*e)+1)*d+tan(1/2*f*x+1/2*e)*
(c-d))/a/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{(d\cos(fx+e)+d)\log(\sin(fx+e)+1)-(d\cos(fx+e)+d)\log(-\sin(fx+e)+1)+2(c-d)\sin(}{2(af\cos(fx+e)+af)}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="fricas
")
[Out] 1/2*((d*cos(f*x + e) + d)*log(sin(f*x + e) + 1) - (d*cos(f*x + e) + d)*log(
-sin(f*x + e) + 1) + 2*(c - d)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{\int \frac{c\sec(e+fx)}{\sec(e+fx)+1}\,dx+\int \frac{d\sec^2(e+fx)}{\sec(e+fx)+1}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x)
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x) + 1), x) + Integral(d*sec(e + f*x)**
2/(sec(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(43) = 86$.

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{d\left(\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a}-\frac{\sin(fx+e)}{a(\cos(fx+e)+1)}\right)+\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}}{f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] (d*(log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a - log(sin(f*x + e)/(cos(f*x
+ e) + 1) - 1)/a - sin(f*x + e)/(a*(cos(f*x + e) + 1))) + c*sin(f*x + e)/(a
*(cos(f*x + e) + 1)))/f
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a}-\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a}+\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{a}}{f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a - d*log(abs(tan(1/2*f*x + 1/2*e) -
1))/a + (c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/a)/f
```

Mupad [B] (verification not implemented)

Time = 13.87 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+a\sec(e+fx)}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(c-d)}{a\,f}+\frac{2\,d\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a\,f}$$

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(c - d))/(a*f) + (2*d*atanh(tan(e/2 + (f*x)/2)))/(a*f)
```

3.214 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 83

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = -\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a(c-d)^{3/2}\sqrt{c+d}f} + \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))}$$

```
[Out] -2*d*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a/(c-d)^(3/2)/f/(c
+d)^(1/2)+tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4072, 98, 95, 211}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{2d\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{3/2}\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{\tan(e+fx)}{f(c-d)(a\sec(e+fx)+a)}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/((c - d)*f*(a + a*Sec[e + f*x])) + (2*d*ArcTan[(Sqrt[c + d]*Sq
rt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x
])/((c - d)^(3/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
 f*x]])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))}+\frac{(ad\tan(e+fx))\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))}\\
&\quad+\frac{(2ad\tan(e+fx))\text{Subst}\left(\int\frac{1}{ac-ad-(-ac-ad)x^2}\,dx,x,\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}$$

$$= \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))} + \frac{2d\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{(c-d)^{3/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.93

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(\frac{2d\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos\left(\frac{1}{2}(e+fx)\right)(i\cos(e)+\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)\right)}{a(c-d)f(1+\cos(e+fx))}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*((2*d*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*C
os[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e
 + f*x)/2]*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2
]) + Sec[e/2]*Sin[(f*x)/2]))/(a*(c - d)*f*(1 + Cos[e + f*x]))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c-d}-\frac{2d\,\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)\sqrt{(c+d)(c-d)}}}{fa}$	74
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c-d}-\frac{2d\,\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)\sqrt{(c+d)(c-d)}}}{fa}$	74
risch	$\frac{2i}{fa(c-d)\left(e^{i(fx+e)}+1\right)}+\frac{d\ln\left(e^{i(fx+e)}+\frac{-ic^2+id^2+\sqrt{c^2-d^2}\,d}{c\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}\,(c-d)fa}-\frac{d\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c-d)fa}$	188

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(tan(1/2*f*x+1/2*e)/(c-d)-2*d/(c-d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)
*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 353, normalized size of antiderivative = 4.25

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \left[-\frac{\sqrt{c^2-d^2}(d\cos(fx+e)+d)\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)-}{2\left((ac^3-ac^2d-acd^2+ad^3)f\cos(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f\right)} -\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+d)\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)-(c^2-d^2)\sin(fx+e)}{(ac^3-ac^2d-acd^2+ad^3)f\cos(fx+e)+(ac^3-ac^2d-acd^2+ad^3)f}\right]$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*(sqrt(c^2 - d^2)*(d*cos(f*x + e) + d)*log((2*c*d*cos(f*x + e) - (c^2
- 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x +
e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^2
 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) +
 (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f), -(sqrt(-c^2 + d^2)*(d*cos(f*x + e)
 + d)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x +
e))) - (c^2 - d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos
(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{c\sec(e+fx)+c+d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x) + c + d*sec(e + f*x)**2 + d*sec(e + f
*x)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{\frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d}{(ac-ad)\sqrt{-c^2+d^2}}+\frac{\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{ac-ad}}{f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x
 + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d/((a*c - a*d)*sqrt(
-c^2 + d^2)) + tan(1/2*f*x + 1/2*e)/(a*c - a*d))/f
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af\,(c-d)}-\frac{2\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{fx}{2}\right)c^2-2\,\sin\left(\frac{e}{2}+\frac{fx}{2}\right)c\,d+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)d^2}{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{c+d}\,(c-d)^{3/2}}\right)}{af\,\sqrt{c+d}\,(c-d)^{3/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)) - (2*d*atanh((c^2*sin(e/2 + (f*x)/2) + d^2
*sin(e/2 + (f*x)/2) - 2*c*d*sin(e/2 + (f*x)/2))/(cos(e/2 + (f*x)/2)*(c + d)
^(1/2)*(c - d)^(3/2))))/(a*f*(c + d)^(1/2)*(c - d)^(3/2))
```

3.215 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 145

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = -\frac{2d(2c+d)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a(c-d)^{5/2}(c+d)^{3/2}f} + \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))}$$

```
[Out] -2*d*(2*c+d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a/(c-d)^(5
/2)/(c+d)^(3/2)/f+(c+2*d)*tan(f*x+e)/(c-d)^2/(c+d)/f/(a+a*sec(f*x+e))-d*tan
(f*x+e)/(c^2-d^2)/f/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 105, 157, 12, 95, 211}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \frac{2d(2c+d)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{5/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)(c+d\sec(e+fx))} + \frac{(c+2d)\tan(e+fx)}{f(c-d)^2(c+d)(a\sec(e+fx)+a)}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 2*d)*Tan[e + f*x])/((c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])) + (2*d*
(2*c + d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a
 - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(5/2)*(c + d)^(3/2)*f*Sqrt[a -
a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*Tan[e + f*x])/((c^2 - d^2)*f
*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^2(c+d)-a^2dx}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^4d(2c+d)}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{a^3(c-d)\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))} + \frac{(ad(2c+d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{(c-d)\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))} + \frac{(2ad(2c+d)\tan(e+fx))\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{(c-d)\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+2d)\tan(e+fx)}{(c-d)^2(c+d)f(a+a\sec(e+fx))} + \frac{2d(2c+d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{(c-d)^{5/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.97

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^3(e+fx)\left(\frac{2d(2c+d)\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos\left(\frac{1}{2}\right.}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e)}}\right.}{a(c-d)^2f(1+\sec(e+}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^3*((2*d*(2*c + d)*Arc
Tan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c
^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x
])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2
]) + (d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (d^2*Cos[(e + f*x)/2]*(-(
d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e
/2]))))/(a*(c - d)^2*f*(1 + Sec[e + f*x])*(c + d*Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^{2}-2cd+d^{2}}+\frac{4d\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}-\frac{(2c+d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^{2}}}{fa}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^{2}-2cd+d^{2}}+\frac{4d\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)}-\frac{(2c+d)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2(c+d)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^{2}}}{fa}$
risch	$\frac{2i\left(c^{3}e^{2i(fx+e)}+c^{2}d\,e^{2i(fx+e)}+d^{3}e^{2i(fx+e)}+2c^{2}d\,e^{i(fx+e)}+3c\,d^{2}e^{i(fx+e)}+d^{3}e^{i(fx+e)}+c^{3}+c^{2}d+c\,d^{2}\right)}{\left(e^{i(fx+e)}+1\right)\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)f(c-d)^{2}ac(c+d)}+\frac{2d\ln\left(e^{i(fx+e}\right.}{\sqrt{c^{2}-d}}$

[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

```
[Out] 1/f/a*(tan(1/2*f*x+1/2*e)/(c^2-2*c*d+d^2)+4*d/(c-d)^2*(-1/2*d/(c+d)*tan(1/2
*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)-1/2*(2*c+d)
/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(136) = 272$.

Time = 0.32 (sec) , antiderivative size = 691, normalized size of antiderivative = 4.77

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$$
$$= \left[\frac{\left(2\,cd^2+d^3+\left(2\,c^2d+cd^2\right)\cos\left(fx+e\right)^2+\left(2\,c^2d+3\,cd^2+d^3\right)\cos\left(fx+e\right)\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2}{}\right.}{2\left(\left(ac^6-ac^5d-2\,ac^4d^2+2\,ac^3d^3+ac^2d^4-acd^5\right)f\cos\left(fx+e\right)^2+(a\right.}\right.$$
$$-\frac{\left(2\,cd^2+d^3+\left(2\,c^2d+cd^2\right)\cos\left(fx+e\right)^2+\left(2\,c^2d+3\,cd^2+d^3\right)\cos\left(fx+e\right)\right)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+}}{(c^2-}\right.}{\left(ac^6-ac^5d-2\,ac^4d^2+2\,ac^3d^3+ac^2d^4-acd^5\right)f\cos\left(fx+e\right)^2+\left(ac^6-3\,ac^4d^2+3\,ac\right.}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d^
2 + d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d
^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) +
```

```
2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(c^3*d +
2*c^2*d^2 - c*d^3 - 2*d^4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f
*x + e))/((a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^
5)*f*cos(f*x + e)^2 + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x
 + e) + (a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)
*f), -((2*c*d^2 + d^3 + (2*c^2*d + c*d^2)*cos(f*x + e)^2 + (2*c^2*d + 3*c*d
^2 + d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*
x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (c^3*d + 2*c^2*d^2 - c*d^3 - 2*d^
4 + (c^4 + c^3*d - c*d^3 - d^4)*cos(f*x + e))*sin(f*x + e))/((a*c^6 - a*c^5
*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*x + e)^2 + (a
*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f*cos(f*x + e) + (a*c^5*d - a*c^4
*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \frac{\int \frac{\sec(e+fx)}{c^2\sec(e+fx)+c^2+2cd\sec^2(e+fx)+2cd\sec(e+fx)+d^2\sec^3(e+fx)+d^2\sec^2(e+fx)}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**2 + 2
*c*d*sec(e + f*x) + d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.52

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = -\frac{\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{(ac^3-ac^2d-acd^2+ad^3)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\right.}{\sqrt{-c^2+d^2}}\right.\right.}{(ac^3-ac^2d-acd^2+ad^3)\sqrt{-c^2+d^2}}}{f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -(2*d^2*tan(1/2*f*x + 1/2*e)/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*(c*tan(1/
2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) - 2*(pi*floor(1/2*(f*
x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/
2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(2*c*d + d^2)/((a*c^3 - a*c^2*d - a*c*d^
2 + a*d^3)*sqrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^2 - 2*a*c*d + a*d^
2))/f
```

Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{a\,f\,(c-d)^2} - \frac{2\,d^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,(c+d)\,\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(a\,c^3-3\,a\,c^2\,d+3\,a\,c\,d^2-a\,d^3)-a\,d^3-a\,c^3+a\,c\,d^2+a\,c^2\,d\right)} - \frac{2\,d\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(2\,c-2\,d)\,(a\,c^2-2\,a\,c\,d+a\,d^2)}{2\,a\,\sqrt{c+d}\,(c-d)^{5/2}}\right)\,(2\,c+d)}{a\,f\,(c+d)^{3/2}\,(c-d)^{5/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)^2) - (2*d^2*tan(e/2 + (f*x)/2))/(f*(c + d)*
(tan(e/2 + (f*x)/2)^2*(a*c^3 - a*d^3 + 3*a*c*d^2 - 3*a*c^2*d) - a*d^3 - a*c
^3 + a*c*d^2 + a*c^2*d)) - (2*d*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(a*c^
2 + a*d^2 - 2*a*c*d))/(2*a*(c + d)^(1/2)*(c - d)^(5/2)))*(2*c + d))/(a*f*(c
 + d)^(3/2)*(c - d)^(5/2))
```

3.216 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 207

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = -\frac{3d(2c^2+2cd+d^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a(c-d)^{7/2}(c+d)^{5/2}f} + \frac{d(2c+3d)\tan(e+fx)}{2a(c-d)^2(c+d)f(c+d\sec(e+fx))^2} + \frac{\tan(e+fx)}{(c-d)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} + \frac{d(2c+d)(c+4d)\tan(e+fx)}{2a(c-d)^3(c+d)^2f(c+d\sec(e+fx))}$$

```
[Out] -3*d*(2*c^2+2*c*d+d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/
a/(c-d)^(7/2)/(c+d)^(5/2)/f+1/2*d*(2*c+3*d)*tan(f*x+e)/a/(c-d)^2/(c+d)/f/(c
+d*sec(f*x+e))^2+tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^2+1/2
*d*(2*c+d)*(c+4*d)*tan(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used

$= \{4072, 105, 156, 157, 12, 95, 211\}$

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{3d(2c^2+2cd+d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{f(c-d)^{7/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{d(4c+d)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(a\sec(e+fx)+a)(c+d\sec(e+fx))} - \frac{d\tan(e+fx)}{2f\left(c^2-d^2\right)(a\sec(e+fx)+a)(c+d\sec(e+fx))^2} + \frac{(2c+d)(c+4d)\tan(e+fx)}{2f(c-d)^3(c+d)^2(a\sec(e+fx)+a)}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c + d)*(c + 4*d)*Tan[e + f*x])/(2*(c - d)^3*(c + d)^2*f*(a + a*Sec[e +
f*x])) + (3*d*(2*c^2 + 2*c*d + d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/((c - d)^(7/2)
*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) - (d*Ta
n[e + f*x])/(2*(c^2 - d^2)*f*(a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^2) -
 (d*(4*c + d)*Tan[e + f*x])/(2*(c^2 - d^2)^2*f*(a + a*Sec[e + f*x])*(c + d*
Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)^3}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\ &= -\frac{d\tan(e+fx)}{2\,(c^2-d^2)\,f(a+a\sec(e+fx))(c+d\sec(e+fx))^2}\\ &\quad -\frac{\tan(e+fx)\text{Subst}\left(\int\frac{a^2(2c+d)-2a^2dx}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)^2}\,dx,x,\sec(e+fx)\right)}{2\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\end{aligned}$$

$$= -\frac{d\tan(e+fx)}{2(c^2-d^2) f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{d(4c+d)\tan(e+fx)}{2(c^2-d^2)^2 f(a+a\sec(e+fx))(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^4(c+d)(2c+3d)-a^4d(4c+d)x}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2a^2(c^2-d^2)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2 f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{2(c^2-d^2) f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{d(4c+d)\tan(e+fx)}{2(c^2-d^2)^2 f(a+a\sec(e+fx))(c+d\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{3a^6d(2c^2+2cd+d^2)}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2a^5(c-d)(c^2-d^2)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2 f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{2(c^2-d^2) f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{d(4c+d)\tan(e+fx)}{2(c^2-d^2)^2 f(a+a\sec(e+fx))(c+d\sec(e+fx))} + \frac{(3ad(2c^2+2cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2(c-d)(c^2-d^2)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2 f(a+a\sec(e+fx))} - \frac{d\tan(e+fx)}{2(c^2-d^2) f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{d(4c+d)\tan(e+fx)}{2(c^2-d^2)^2 f(a+a\sec(e+fx))(c+d\sec(e+fx))} + \frac{(3ad(2c^2+2cd+d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{(c-d)(c^2-d^2)^2 f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c+d)(c+4d)\tan(e+fx)}{2(c-d)^3(c+d)^2f(a+a\sec(e+fx))} + \frac{3d(2c^2+2cd+d^2)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{(c-d)^{7/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d\tan(e+fx)}{2(c^2-d^2)f(a+a\sec(e+fx))(c+d\sec(e+fx))^2} - \frac{d(4c+d)\tan(e+fx)}{2(c^2-d^2)^2f(a+a\sec(e+fx))(c+d\sec(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.45 (sec) , antiderivative size = 1422, normalized size of antiderivative = 6.87

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{(2c^2+2cd+d^2)\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec^4(e+fx)\left(-\frac{6id\arctan\left(\sec\left(\frac{fx}{2}\right)\left(\frac{\cos(e)}{\sqrt{c^2-d^2}\sqrt{\cos(2e)-i\sin(2e)}}-\cdots\right.\right.}{\sqrt{c^2-d\cdots}}\right.}{(-c+d)^3(c\cdots} + \frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))\sec\left(\frac{e}{2}\right)\sec(e)\sec^4(e+fx)\left(8c^5d\sin\left(\frac{fx}{2}\right)+10c^4d^2\sin\left(\frac{fx}{2}\right)-11c^3d^3\sin\cdots\right.}{\cdots}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 2*c*d + d^2)*Cos[e/2 + (f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e +
f*x]^4*(((-6*I)*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e
] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]])
)*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*
Sqrt[Cos[2*e] - I*Sin[2*e]]) - (6*d*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 -
 d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2
*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(
Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^3*(c + d)^2*(a +
 a*Sec[e + f*x])*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d + c*Cos[e
 + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^4*(8*c^5*d*Sin[(f*x)/2] + 10*c^4*d^2*
Sin[(f*x)/2] - 11*c^3*d^3*Sin[(f*x)/2] - 17*c^2*d^4*Sin[(f*x)/2] - 2*c*d^5*
Sin[(f*x)/2] + 2*d^6*Sin[(f*x)/2] - 8*c^5*d*Sin[(3*f*x)/2] - 22*c^4*d^2*Sin
[(3*f*x)/2] - 27*c^3*d^3*Sin[(3*f*x)/2] - 5*c^2*d^4*Sin[(3*f*x)/2] + 2*c*d^
5*Sin[(3*f*x)/2] + 4*c^6*Sin[e - (f*x)/2] + 8*c^5*d*Sin[e - (f*x)/2] + 18*c
^4*d^2*Sin[e - (f*x)/2] + 35*c^3*d^3*Sin[e - (f*x)/2] + 25*c^2*d^4*Sin[e -
(f*x)/2] + 2*c*d^5*Sin[e - (f*x)/2] - 2*d^6*Sin[e - (f*x)/2] - 4*c^6*Sin[e
+ (f*x)/2] - 8*c^5*d*Sin[e + (f*x)/2] - 6*c^4*d^2*Sin[e + (f*x)/2] - 7*c^3*
d^3*Sin[e + (f*x)/2] + 5*c^2*d^4*Sin[e + (f*x)/2] + 2*c*d^5*Sin[e + (f*x)/2
```

```
] - 2*d^6*Sin[e + (f*x)/2] + 8*c^5*d*Sin[2*e + (f*x)/2] + 22*c^4*d^2*Sin[2*
e + (f*x)/2] + 17*c^3*d^3*Sin[2*e + (f*x)/2] + 13*c^2*d^4*Sin[2*e + (f*x)/2
] + 2*c*d^5*Sin[2*e + (f*x)/2] - 2*d^6*Sin[2*e + (f*x)/2] + 2*c^6*Sin[e + (
3*f*x)/2] + 4*c^5*d*Sin[e + (3*f*x)/2] - 4*c^4*d^2*Sin[e + (3*f*x)/2] - 19*
c^3*d^3*Sin[e + (3*f*x)/2] - 5*c^2*d^4*Sin[e + (3*f*x)/2] + 2*c*d^5*Sin[e +
 (3*f*x)/2] - 8*c^5*d*Sin[2*e + (3*f*x)/2] - 16*c^4*d^2*Sin[2*e + (3*f*x)/2
] - c^3*d^3*Sin[2*e + (3*f*x)/2] + 2*c^2*d^4*Sin[2*e + (3*f*x)/2] - 2*c*d^5
*Sin[2*e + (3*f*x)/2] + 2*c^6*Sin[3*e + (3*f*x)/2] + 4*c^5*d*Sin[3*e + (3*f
*x)/2] + 2*c^4*d^2*Sin[3*e + (3*f*x)/2] + 7*c^3*d^3*Sin[3*e + (3*f*x)/2] +
2*c^2*d^4*Sin[3*e + (3*f*x)/2] - 2*c*d^5*Sin[3*e + (3*f*x)/2] - 2*c^6*Sin[e
 + (5*f*x)/2] - 4*c^5*d*Sin[e + (5*f*x)/2] - 8*c^4*d^2*Sin[e + (5*f*x)/2] -
 2*c^3*d^3*Sin[e + (5*f*x)/2] + c^2*d^4*Sin[e + (5*f*x)/2] - 6*c^4*d^2*Sin[
2*e + (5*f*x)/2] - 2*c^3*d^3*Sin[2*e + (5*f*x)/2] + c^2*d^4*Sin[2*e + (5*f*
x)/2] - 2*c^6*Sin[3*e + (5*f*x)/2] - 4*c^5*d*Sin[3*e + (5*f*x)/2] - 2*c^4*d
^2*Sin[3*e + (5*f*x)/2]))/(8*c^2*(-c + d)^3*(c + d)^2*f*(a + a*Sec[e + f*x]
)*(c + d*Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^3-3c^2d+3c\,d^2-d^3}+\frac{2d\left(\frac{-\frac{3d\left(2c^2-cd-d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2\left(c^2+2cd+d^2\right)}+\frac{d(6c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c+2d}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{3\left(2c^2+2cd+d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2\left(c^2+2cd+d^2\right)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^3}}{fa}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{c^3-3c^2d+3c\,d^2-d^3}+\frac{2d\left(\frac{-\frac{3d\left(2c^2-cd-d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{2\left(c^2+2cd+d^2\right)}+\frac{d(6c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2c+2d}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^2}-\frac{3\left(2c^2+2cd+d^2\right)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{2\left(c^2+2cd+d^2\right)\sqrt{(c+d)(c-d)}}\right)}{(c-d)^3}}{fa}$
risch	$i\left(2d^6e^{2i(fx+e)}+c^2d^4-4c^5d-2c^3d^3-8c^4d^2-2c^6-2c\,d^5e^{2i(fx+e)}-7c^3d^3e^{4i(fx+e)}-2c^2d^4e^{4i(fx+e)}+2c\,d^5e^{4i(fx+e)}-8c^5d\right.$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f/a*(tan(1/2*f*x+1/2*e)/(c^3-3*c^2*d+3*c*d^2-d^3)+2*d/(c-d)^3*((-3/2*d*(2
*c^2-c*d-d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*d*(6*c+d)/(c+d)*tan(
1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-3/2*(
2*c^2+2*c*d+d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*
f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(194) = 388$.

Time = 0.36 (sec) , antiderivative size = 1331, normalized size of antiderivative = 6.43

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] [-1/4*(3*(2*c^2*d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f
*x + e)^3 + (2*c^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (4
*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*
c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(
f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f
*x + e) + d^2)) - 2*(2*c^4*d^2 + 9*c^3*d^3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 +
(2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3 - 9*c^2*d^4 - 2*c*d^5 + d^6)*cos(f
*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6
)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a*c^8*d - 3*a*c^7*d^2 + 3*a*c^6*d^3
 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6 + a*c^2*d^7)*f*cos(f*x + e)^3 + (a
*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^3 + 9*a*c^5*d^4 + 3*a*c^4*d^5 - 7*
a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(f*x + e)^2 + (2*a*c^8*d - a*c^7*d^
2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a
*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a
*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f), -1/2*(3*(2*c^2*
d^3 + 2*c*d^4 + d^5 + (2*c^4*d + 2*c^3*d^2 + c^2*d^3)*cos(f*x + e)^3 + (2*c
^4*d + 6*c^3*d^2 + 5*c^2*d^3 + 2*c*d^4)*cos(f*x + e)^2 + (4*c^3*d^2 + 6*c^2
*d^3 + 4*c*d^4 + d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^
2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^4*d^2 + 9*c^3*d^
3 + 2*c^2*d^4 - 9*c*d^5 - 4*d^6 + (2*c^6 + 4*c^5*d + 6*c^4*d^2 - 2*c^3*d^3
- 9*c^2*d^4 - 2*c*d^5 + d^6)*cos(f*x + e)^2 + (4*c^5*d + 14*c^4*d^2 + 7*c^3
*d^3 - 13*c^2*d^4 - 11*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^9 - a
*c^8*d - 3*a*c^7*d^2 + 3*a*c^6*d^3 + 3*a*c^5*d^4 - 3*a*c^4*d^5 - a*c^3*d^6
+ a*c^2*d^7)*f*cos(f*x + e)^3 + (a*c^9 + a*c^8*d - 5*a*c^7*d^2 - 3*a*c^6*d^
3 + 9*a*c^5*d^4 + 3*a*c^4*d^5 - 7*a*c^3*d^6 - a*c^2*d^7 + 2*a*c*d^8)*f*cos(
f*x + e)^2 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4*d
^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e) + (a*c^7*d
^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*
c*d^8 + a*d^9)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \frac{\int \frac{\sec(e+fx)}{c^3\sec(e+fx)+c^3+3c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^3(e+fx)+3cd^2\sec^2(e+fx)+d^3\sec^4(e+fx)+d^3\sec^3(e+fx)}\,dx}{a}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**2
+ 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e + f*x)*
*2 + d**3*sec(e + f*x)**4 + d**3*sec(e + f*x)**3), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.75

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = -\frac{\frac{3\,(2\,c^2d+2\,cd^2+d^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(ac^5-ac^4d-2\,ac^3d^2+2\,ac^2d^3+acd^4-ad^5)\sqrt{-c^2+d^2}} - \frac{\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{ac^3-3\,ac^2d+3\,acd^2-ad^3} + \frac{6\,c^2d^2\tan(}{}}{f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] -(3*(2*c^2*d + 2*c*d^2 + d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c +
2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2
+ d^2)))/((a*c^5 - a*c^4*d - 2*a*c^3*d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*s
qrt(-c^2 + d^2)) - tan(1/2*f*x + 1/2*e)/(a*c^3 - 3*a*c^2*d + 3*a*c*d^2 - a*
d^3) + (6*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*c*d^3*tan(1/2*f*x + 1/2*e)^3 -
 3*d^4*tan(1/2*f*x + 1/2*e)^3 - 6*c^2*d^2*tan(1/2*f*x + 1/2*e) - 7*c*d^3*ta
n(1/2*f*x + 1/2*e) - d^4*tan(1/2*f*x + 1/2*e))/((a*c^5 - a*c^4*d - 2*a*c^3*
d^2 + 2*a*c^2*d^3 + a*c*d^4 - a*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*
f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.83

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))(c+d\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a\,f\,(c-d)^3}$$

$$-\frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(d^3+6\,c\,d^2)}{c+d}+\frac{3\tan\left(\cdots\right.}{\cdots}}{f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,(2\,a\,c^5-6\,a\,c^4\,d+4\,a\,c^3\,d^2+4\,a\,c^2\,d^3-6\,a\,c\,d^4+2\,a\,d^5)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\,(a\,c^5-5\,a\cdots\right.}$$

$$+\frac{d\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^4-4\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^3\,d+6\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2\,d^2-4\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c\,d^3+1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^4}{\sqrt{c+d}\,(c-d)^{7/2}}\right)\,(2\,c^2+2\,c\,d+d^2\cdots}{a\,f\,(c+d)^{5/2}\,(c-d)^{7/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))*(c + d/cos(e + f*x))^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)/(a*f*(c - d)^3) - ((tan(e/2 + (f*x)/2)*(6*c*d^2 + d^3))/
(c + d) + (3*tan(e/2 + (f*x)/2)^3*(c*d^3 + d^4 - 2*c^2*d^2))/(c + d)^2)/(f*
(tan(e/2 + (f*x)/2)^2*(2*a*c^5 + 2*a*d^5 + 4*a*c^2*d^3 + 4*a*c^3*d^2 - 6*a*
c*d^4 - 6*a*c^4*d) - tan(e/2 + (f*x)/2)^4*(a*c^5 - a*d^5 - 10*a*c^2*d^3 + 1
0*a*c^3*d^2 + 5*a*c*d^4 - 5*a*c^4*d) - a*c^5 + a*d^5 - 2*a*c^2*d^3 + 2*a*c^
3*d^2 - a*c*d^4 + a*c^4*d)) + (d*atan((c^4*tan(e/2 + (f*x)/2)*1i + d^4*tan(
e/2 + (f*x)/2)*1i - c*d^3*tan(e/2 + (f*x)/2)*4i - c^3*d*tan(e/2 + (f*x)/2)*
4i + c^2*d^2*tan(e/2 + (f*x)/2)*6i)/((c + d)^(1/2)*(c - d)^(7/2)))*(2*c*d +
 2*c^2 + d^2)*3i)/(a*f*(c + d)^(5/2)*(c - d)^(7/2))
```

3.217 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 258

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{5(2c-d)d^2(2c^2-3cd+2d^2)\operatorname{arctanh}(\sin(e+fx))}{2a^2f} - \frac{d(c^2+10cd-12d^2)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)+d(2c^3+20c^2d-57cd^2+30d^3)\sec(e+fx))\tan(e+fx)}{6a^2f}$$

```
[Out] 5/2*(2*c-d)*d^2*(2*c^2-3*c*d+2*d^2)*arctanh(sin(f*x+e))/a^2/f-1/3*d*(c^2+10
*c*d-12*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/a^2/f+1/3*(c-d)*(c+10*d)*(c+d*se
c(f*x+e))^3*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+1/3*(c-d)*(c+d*sec(f*x+e))^4*
tan(f*x+e)/f/(a+a*sec(f*x+e))^2-1/6*d*(4*c^4+40*c^3*d-176*c^2*d^2+160*c*d^3
-48*d^4+d*(2*c^3+20*c^2*d-57*c*d^2+30*d^3)*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4072, 100, 155, 158, 152, 65, 223, 209}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2} dx$$
$$= -\frac{d(c^2+10cd-12d^2)\tan(e+fx)(c+d\sec(e+fx))^2}{3a^2f}$$
$$-\frac{d\tan(e+fx)\left(d(2c^3+20c^2d-57cd^2+30d^3)\sec(e+fx)+4(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)\right)}{6a^2f}$$
$$+\frac{(c-d)(c+10d)\tan(e+fx)(c+d\sec(e+fx))^3}{3f(a^2\sec(e+fx)+a^2)}$$
$$+\frac{5d^2(2c-d)\left(2c^2-3cd+2d^2\right)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^4}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (5*(2*c - d)*d^2*(2*c^2 - 3*c*d + 2*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sq
rt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[
a + a*Sec[e + f*x]]) - (d*(c^2 + 10*c*d - 12*d^2)*(c + d*Sec[e + f*x])^2*Ta
n[e + f*x])/(3*a^2*f) + ((c - d)*(c + 10*d)*(c + d*Sec[e + f*x])^3*Tan[e +
f*x])/(3*f*(a^2 + a^2*Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^4*Tan[
e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) - (d*(4*(c^4 + 10*c^3*d - 44*c^2*d^2
 + 40*c*d^3 - 12*d^4) + d*(2*c^3 + 20*c^2*d - 57*c*d^2 + 30*d^3)*Sec[e + f*
x])*Tan[e + f*x])/(6*a^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^5}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)^3(-a^2(c^2+6cd-4d^2)+3a^2(c-2d)dx)}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)^2(-3a^4(11c-10d)d^2+3a^4d(c^2+10cd-12d^2)x)}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{3a^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(c^2+10cd-12d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)(3a^6d^2(31c^2-50cd+24d^2)-3a^6d(2c^3+20c^2d-57cd^2+30d^3)x)}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{9a^6f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(c^2+10cd-12d^2)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)+d(2c^3+20c^2d-57cd^2+30d^3)\sec(e+fx))\tan(e+}{6a^2f} - \frac{(5(2c-d)d^2(2c^2-3cd+2d^2)\tan(e+fx))\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}dx,x,\sec(e+fx)\right)}{2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(c^2+10cd-12d^2)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)+d(2c^3+20c^2d-57cd^2+30d^3)\sec(e+fx))\tan(e+}{6a^2f} + \frac{(5(2c-d)d^2(2c^2-3cd+2d^2)\tan(e+fx))\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}dx,x,\sqrt{a-a\sec(e+fx)}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(c^2+10cd-12d^2)(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)+d(2c^3+20c^2d-57cd^2+30d^3)\sec(e+fx))\tan(e+}{6a^2f} + \frac{(5(2c-d)d^2(2c^2-3cd+2d^2)\tan(e+fx))\text{Subst}\left(\int\frac{1}{1+x^2}dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{5(2c-d)d^2(2c^2-3cd+2d^2)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d(c^2+10cd-12d^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{3a^2f} + \frac{(c-d)(c+10d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^4+10c^3d-44c^2d^2+40cd^3-12d^4)+d(2c^3+20c^2d-57cd^2+30d^3)\sec(e+fx))\tan(e+}{6a^2f}$$

Mathematica [A] (verified)

Time = 7.42 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.73

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{240d^2(-4c^3+8c^2d-7cd^2+2d^3)\cos^4\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^2,x]
[Out] (240*d^2*(-4*c^3 + 8*c^2*d - 7*c*d^2 + 2*d^3)*Cos[(e + f*x)/2]^4*(Log[Cos[(
e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])
 + 2*Cos[(e + f*x)/2]*(6*c^5 + 15*c^4*d - 120*c^3*d^2 + 420*c^2*d^3 - 300*c
*d^4 + 104*d^5 + (6*c^5 + 60*c^4*d - 300*c^3*d^2 + 840*c^2*d^3 - 585*c*d^4
+ 190*d^5)*Cos[e + f*x] + 4*(2*c^5 + 5*c^4*d - 40*c^3*d^2 + 130*c^2*d^3 - 9
5*c*d^4 + 30*d^5)*Cos[2*(e + f*x)] + 2*c^5*Cos[3*(e + f*x)] + 20*c^4*d*Cos[
3*(e + f*x)] - 100*c^3*d^2*Cos[3*(e + f*x)] + 280*c^2*d^3*Cos[3*(e + f*x)]
- 215*c*d^4*Cos[3*(e + f*x)] + 66*d^5*Cos[3*(e + f*x)] + 2*c^5*Cos[4*(e + f
*x)] + 5*c^4*d*Cos[4*(e + f*x)] - 40*c^3*d^2*Cos[4*(e + f*x)] + 100*c^2*d^3
*Cos[4*(e + f*x)] - 80*c*d^4*Cos[4*(e + f*x)] + 24*d^5*Cos[4*(e + f*x)])*Se
c[e + f*x]^3*Sin[(e + f*x)/2])/(24*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.48

method	result
parallelrisch	$\frac{-360\left(c-\frac{d}{2}\right)\left(c^2-\frac{3}{2}cd+d^2\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+360\left(c-\frac{d}{2}\right)\left(c^2-\frac{3}{2}cd+d^2\right)\left(\cos(fx+e)+\frac{\cos}{}\right.}{}$
derivativedivides	$\frac{-\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{5c^4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{10c^3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{10c^2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{5c\,d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+c}{}$
default	$\frac{-\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{5c^4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{10c^3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{10c^2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{5c\,d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+c}{}$
norman	$\frac{-\frac{\left(c^5-5c^4d+10c^3d^2-10c^2d^3+5c\,d^4-d^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{6af}-\frac{\left(c^5+5c^4d-30c^3d^2+90c^2d^3-65c\,d^4+21d^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{10\left(2c^5+5c^4d-\right.}{}}{}$
risch	Expression too large to display

[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)

[Out] 1/12*(-360*(c-1/2*d)*(c^2-3/2*c*d+d^2)*(cos(f*x+e)+1/3*cos(3*f*x+3*e))*d^2*
ln(tan(1/2*f*x+1/2*e)-1)+360*(c-1/2*d)*(c^2-3/2*c*d+d^2)*(cos(f*x+e)+1/3*co
s(3*f*x+3*e))*d^2*ln(tan(1/2*f*x+1/2*e)+1)+2*sec(1/2*f*x+1/2*e)^2*((c^5+33*
d^5+10*c^4*d-50*c^3*d^2+140*c^2*d^3-215/2*c*d^4)*cos(3*f*x+3*e)+(4*c^5+10*c
^4*d-80*c^3*d^2+260*c^2*d^3-190*c*d^4+60*d^5)*cos(2*f*x+2*e)+(c^5+12*d^5+5/
2*c^4*d-20*c^3*d^2+50*c^2*d^3-40*c*d^4)*cos(4*f*x+4*e)+(95*d^5-150*c^3*d^2+
3*c^5+30*c^4*d+420*c^2*d^3-585/2*c*d^4)*cos(f*x+e)+52*d^5+3*c^5+15/2*c^4*d-
60*c^3*d^2+210*c^2*d^3-150*c*d^4)*tan(1/2*f*x+1/2*e))/f/a^2/(cos(3*f*x+3*e)
+3*cos(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.77

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \frac{15\left(\left(4\,c^3d^2-8\,c^2d^3+7\,cd^4-2\,d^5\right)\cos\left(fx+e\right)^5+2\left(4\,c^3d^2-8\,c^2d^3+7\,cd^4-2\,d^5\right)\cos\left(fx+e\right)^4+\left(4\,c^3d^2\right.\right.}{}$$

[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")

[Out] 1/12*(15*((4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c
^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d
^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((4*c^3*d^
2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*cos(f*x + e)^5 + 2*(4*c^3*d^2 - 8*c^2*d^3

```
+ 7*c*d^4 - 2*d^5)*cos(f*x + e)^4 + (4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^
5)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(2*d^5 + 2*(2*c^5 + 5*c^4*d -
 40*c^3*d^2 + 100*c^2*d^3 - 80*c*d^4 + 24*d^5)*cos(f*x + e)^4 + (2*c^5 + 20
*c^4*d - 100*c^3*d^2 + 280*c^2*d^3 - 215*c*d^4 + 66*d^5)*cos(f*x + e)^3 + 6
*(10*c^2*d^3 - 5*c*d^4 + 2*d^5)*cos(f*x + e)^2 + (15*c*d^4 - 2*d^5)*cos(f*x
 + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^5 + 2*a^2*f*cos(f*x + e)^4 + a^2*f
*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\int \frac{c^5\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^5\sec^6(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{5cd^4\sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{10c^2d^3\sec^4(e}{\sec^2(e+fx)+2\sec(}}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**5*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**5*sec(e + f*x)**6/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(5*c*d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(10*c**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) +
 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e
+ f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*se
c(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(247) = 494$.

Time = 0.24 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.99

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(d^5*(4*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 20*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 15*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^2 - 3*a^2*sin(f*x
 + e)^2/(cos(f*x + e) + 1)^2 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 -
a^2*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (27*sin(f*x + e)/(cos(f*x + e) +
 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 30*log(sin(f*x + e)/(cos(f
*x + e) + 1) + 1)/a^2 + 30*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) -
5*c*d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x +
```

```
e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*
x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) + 1)
 + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) + 10*c^2*d^3*(
(15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/
a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)
/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 10*c^3*d^2*((9*sin(f*x + e)/(co
s(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x
+ e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) -
1)/a^2) + 5*c^4*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(
f*x + e) + 1)^3)/a^2 + c^5*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e
)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(247) = 494$.

Time = 0.44 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.96

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\frac{15\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2} - \frac{15\left(4c^3d^2-8c^2d^3+7cd^4-2d^5\right)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2} - \frac{2\left(60c^2d^3\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*log(abs(tan(1/2*f*x + 1/2
*e) + 1))/a^2 - 15*(4*c^3*d^2 - 8*c^2*d^3 + 7*c*d^4 - 2*d^5)*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a^2 - 2*(60*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 75*c*d^4*
tan(1/2*f*x + 1/2*e)^5 + 30*d^5*tan(1/2*f*x + 1/2*e)^5 - 120*c^2*d^3*tan(1/
2*f*x + 1/2*e)^3 + 120*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 40*d^5*tan(1/2*f*x +
1/2*e)^3 + 60*c^2*d^3*tan(1/2*f*x + 1/2*e) - 45*c*d^4*tan(1/2*f*x + 1/2*e)
+ 18*d^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a^2) - (a^4*
c^5*tan(1/2*f*x + 1/2*e)^3 - 5*a^4*c^4*d*tan(1/2*f*x + 1/2*e)^3 + 10*a^4*c^
3*d^2*tan(1/2*f*x + 1/2*e)^3 - 10*a^4*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 5*a^
4*c*d^4*tan(1/2*f*x + 1/2*e)^3 - a^4*d^5*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^5
*tan(1/2*f*x + 1/2*e) - 15*a^4*c^4*d*tan(1/2*f*x + 1/2*e) + 90*a^4*c^3*d^2*
tan(1/2*f*x + 1/2*e) - 150*a^4*c^2*d^3*tan(1/2*f*x + 1/2*e) + 105*a^4*c*d^4
*tan(1/2*f*x + 1/2*e) - 27*a^4*d^5*tan(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.55 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{5\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(2\,c-d)\,\left(2\,c^2-3\,c\,d+2\,d^2\right)}{a^2\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{2\,(c-d)^5}{a^2}-\frac{5\,(c+d)\,(c-d)^4}{2\,a^2}\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(c-d)^5}{6\,a^2\,f} - \frac{\left(20\,c^2\,d^3-25\,c\,d^4+10\,d^5\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-40\,c^2\,d^3+40\,c\,d^4-\frac{40\,d^5}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+\left(20\,c^2\,d^3-15\right.}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)}$$

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (5*d^2*atanh(tan(e/2 + (f*x)/2))*(2*c - d)*(2*c^2 - 3*c*d + 2*d^2))/(a^2*f)
 - (tan(e/2 + (f*x)/2)*((2*(c - d)^5)/a^2 - (5*(c + d)*(c - d)^4)/(2*a^2)))
/f - (tan(e/2 + (f*x)/2)^3*(c - d)^5)/(6*a^2*f) - (tan(e/2 + (f*x)/2)*(6*d^
5 - 15*c*d^4 + 20*c^2*d^3) + tan(e/2 + (f*x)/2)^5*(10*d^5 - 25*c*d^4 + 20*c
^2*d^3) - tan(e/2 + (f*x)/2)^3*((40*d^5)/3 - 40*c*d^4 + 40*c^2*d^3))/(f*(3*
a^2*tan(e/2 + (f*x)/2)^2 - 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)
/2)^6 - a^2))
```

3.218 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 193

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx \\
&= \frac{d^2(12c^2-16cd+7d^2)\operatorname{arctanh}(\sin(e+fx))}{2a^2f} \\
&\quad + \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} \\
&\quad - \frac{d(4(c^3+8c^2d-20cd^2+8d^3)+d(2c^2+16cd-21d^2)\sec(e+fx))\tan(e+fx)}{6a^2f}
\end{aligned}$$

```
[Out] 1/2*d^2*(12*c^2-16*c*d+7*d^2)*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)*(c+8*d)*(
c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))+1/3*(c-d)*(c+d*sec(f*x+
e))^3*tan(f*x+e)/f/(a+a*sec(f*x+e))^2-1/6*d*(4*c^3+32*c^2*d-80*c*d^2+32*d^3
+d*(2*c^2+16*c*d-21*d^2)*sec(f*x+e))*tan(f*x+e)/a^2/f
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used

= {4072, 100, 155, 152, 65, 223, 209}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= -\frac{d\tan(e+fx)\left(d(2c^2+16cd-21d^2)\sec(e+fx)+4(c^3+8c^2d-20cd^2+8d^3)\right)}{6a^2f}$$
$$+ \frac{(c-d)(c+8d)\tan(e+fx)(c+d\sec(e+fx))^2}{3f\left(a^2\sec(e+fx)+a^2\right)}$$
$$+ \frac{d^2(12c^2-16cd+7d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+ \frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^3}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*(12*c^2 - 16*c*d + 7*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 +
Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[
e + f*x]]) + ((c - d)*(c + 8*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(
a^2 + a^2*Sec[e + f*x])) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(3
*f*(a + a*Sec[e + f*x])^2) - (d*(4*(c^3 + 8*c^2*d - 20*c*d^2 + 8*d^3) + d*(
2*c^2 + 16*c*d - 21*d^2)*Sec[e + f*x])*Tan[e + f*x])/(6*a^2*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c+dx)^4}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)^2(-a^2(c^2+5cd-3d^2)+a^2(2c-5d)dx)}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx,x,\sec(e+fx)\right)}{3af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)(-a^4(19c-16d)d^2+a^4d(2c^2+16cd-21d^2)x)}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{3a^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^3+8c^2d-20cd^2+8d^3)+d(2c^2+16cd-21d^2)\sec(e+fx))\tan(e+fx)}{6a^2f} - \frac{(d^2(12c^2-16cd+7d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^3+8c^2d-20cd^2+8d^3)+d(2c^2+16cd-21d^2)\sec(e+fx))\tan(e+fx)}{6a^2f} + \frac{(d^2(12c^2-16cd+7d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^3+8c^2d-20cd^2+8d^3)+d(2c^2+16cd-21d^2)\sec(e+fx))\tan(e+fx)}{6a^2f} + \frac{(d^2(12c^2-16cd+7d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^2(12c^2-16cd+7d^2)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+8d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{d(4(c^3+8c^2d-20cd^2+8d^3)+d(2c^2+16cd-21d^2)\sec(e+fx))\tan(e+fx)}{6a^2f}$$

Mathematica [A] (verified)

Time = 5.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.61

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{-24d^2(12c^2-16cd+7d^2)\cos^4\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx\right)\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (-24*d^2*(12*c^2 - 16*c*d + 7*d^2)*Cos[(e + f*x)/2]^4*(Log[Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[(e
 + f*x)/2]*(2*c^4 + 16*c^3*d - 60*c^2*d^2 + 112*c*d^3 - 37*d^4 + 6*(c^4 + 2
*c^3*d - 12*c^2*d^2 + 28*c*d^3 - 10*d^4)*Cos[e + f*x] + (2*c^4 + 16*c^3*d -
 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*Cos[2*(e + f*x)] + 2*c^4*Cos[3*(e + f*x)]
 + 4*c^3*d*Cos[3*(e + f*x)] - 24*c^2*d^2*Cos[3*(e + f*x)] + 40*c*d^3*Cos[3*
(e + f*x)] - 16*d^4*Cos[3*(e + f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(12*
a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.42

method	result
parallelrisch	$\frac{-12(1+\cos(2fx+2e))(c^2-\frac{4}{3}cd+\frac{7}{12}d^2)d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+12(1+\cos(2fx+2e))(c^2-\frac{4}{3}cd+\frac{7}{12}d^2)d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}{}$
derivativedivides	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^4}{3}+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^3d}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2d^2+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c\,d^3}{3}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3d^4}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^4+4\tan\left(\right.}{}$
default	$\frac{-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^4}{3}+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^3d}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2d^2+\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c\,d^3}{3}-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3d^4}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^4+4\tan\left(\right.}{}$
norman	$\frac{-\frac{\left(c^4-4c^3d+6c^2d^2-4c\,d^3+d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{6af}+\frac{\left(c^4+4c^3d-18c^2d^2+36c\,d^3-13d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}-\frac{\left(3c^4+4c^3d-30c^2d^2+44c\,d^3-18d\right.}{af}}{}$
risch	$\frac{i\left(-36c^2d^2e^{6i(fx+e)}-120c^2d^2e^{4i(fx+e)}+256c\,d^3e^{2i(fx+e)}+16c^3d\,e^{2i(fx+e)}+144c\,d^3e^{5i(fx+e)}+224c\,d^3e^{4i(fx+e)}-108c^2d\right.}{}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2*(-12*(1+cos(2*f*x+2*e))*(c^2-4/3*c*d+7/12*d^2)*d^2*ln(tan(1/2*f*x+1/2*e
)-1)+12*(1+cos(2*f*x+2*e))*(c^2-4/3*c*d+7/12*d^2)*d^2*ln(tan(1/2*f*x+1/2*e)
+1)+sec(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2*e)*((1/3*c^4+8/3*c^3*d-10*c^2*d^2+
```

```
56/3*c*d^3-43/6*d^4)*cos(2*f*x+2*e)+(1/3*c^4+2/3*c^3*d-4*c^2*d^2+20/3*c*d^3
-8/3*d^4)*cos(3*f*x+3*e)+(c^4+2*c^3*d-12*c^2*d^2+28*c*d^3-10*d^4)*cos(f*x+e
)+1/3*c^4+8/3*c^3*d-10*c^2*d^2+56/3*c*d^3-37/6*d^4))/f/a^2/(1+cos(2*f*x+2*e
))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.87

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{3\left((12\,c^2d^2-16\,cd^3+7\,d^4)\cos\left(fx+e\right)^4+2\,(12\,c^2d^2-16\,cd^3+7\,d^4)\cos\left(fx+e\right)^3+(12\,c^2d^2-16\,cd^3+\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^4 + 2*(12*c^2*d^2 - 1
6*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x +
 e)^2)*log(sin(f*x + e) + 1) - 3*((12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x +
 e)^4 + 2*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*cos(f*x + e)^3 + (12*c^2*d^2 - 16
*c*d^3 + 7*d^4)*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(3*d^4 + 4*(c^4
+ 2*c^3*d - 12*c^2*d^2 + 20*c*d^3 - 8*d^4)*cos(f*x + e)^3 + (2*c^4 + 16*c^3
*d - 60*c^2*d^2 + 112*c*d^3 - 43*d^4)*cos(f*x + e)^2 + 6*(4*c*d^3 - d^4)*co
s(f*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^4 + 2*a^2*f*cos(f*x + e)^3 +
a^2*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\int \frac{c^4\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int \frac{d^4\sec^5(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int \frac{4cd^3\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx+\int \frac{6c^2d^2\sec^3(e-}{\sec^2(e+fx)+2\sec(}}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**4*sec(e + f*x)**5/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(4*c*d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(6*c**2*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) +
1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x
) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(184) = 368$.

Time = 0.23 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.78

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = -\frac{d^4\left(\frac{6\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{5\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2-\frac{2a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^2\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{\frac{21\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-4}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(d^4*(6*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3)/(a^2 - 2*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4) + (21*sin(f*x + e)/(cos(f*x + e) + 1) + sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 21*log(sin(f*x + e)/(cos(f*x + e) +
1) + 1)/a^2 + 21*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 4*c*d^3*((
15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a
^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + 6*c^2*d^2*((9*sin(f*x + e)/(cos(
f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(sin(f*x +
e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
/a^2) - 4*c^3*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*
x + e) + 1)^3)/a^2 - c^4*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{3(12c^2d^2-16cd^3+7d^4)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{a^2}-\frac{3(12c^2d^2-16cd^3+7d^4)\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{a^2}-\frac{6\left(8cd^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-5d^4\tan\left(\frac{1}{2}\right.\right.}{\left(\mathrm{ta}\right.}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a
^2 - 3*(12*c^2*d^2 - 16*c*d^3 + 7*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a
^2 - 6*(8*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 5*d^4*tan(1/2*f*x + 1/2*e)^3 - 8*c
*d^3*tan(1/2*f*x + 1/2*e) + 3*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2
*e)^2 - 1)^2*a^2) - (a^4*c^4*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(1/2*f
*x + 1/2*e)^3 + 6*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*tan(1/2*
f*x + 1/2*e)^3 + a^4*d^4*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*f*x + 1
/2*e) - 12*a^4*c^3*d*tan(1/2*f*x + 1/2*e) + 54*a^4*c^2*d^2*tan(1/2*f*x + 1/
2*e) - 60*a^4*c*d^3*tan(1/2*f*x + 1/2*e) + 21*a^4*d^4*tan(1/2*f*x + 1/2*e))
/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^2}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(8\,c\,d^3-3\,d^4\right)-{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3\,\left(8\,c\,d^3-5\,d^4\right)}{f\,\left(a^2\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^4-2\,a^2\,{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^2+a^2\right)} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(\frac{3\,{\left(c-d\right)}^4}{2\,a^2}-\frac{2\,\left(c+d\right)\,{\left(c-d\right)}^3}{a^2}\right)}{f} - \frac{{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}^3\,{\left(c-d\right)}^4}{6\,a^2\,f} + \frac{d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)\,\left(12\,c^2-16\,c\,d+7\,d^2\right)}{a^2\,f}$$

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(8*c*d^3 - 3*d^4) - tan(e/2 + (f*x)/2)^3*(8*c*d^3 - 5*d
^4))/(f*(a^2*tan(e/2 + (f*x)/2)^4 - 2*a^2*tan(e/2 + (f*x)/2)^2 + a^2)) - (t
an(e/2 + (f*x)/2)*((3*(c - d)^4)/(2*a^2) - (2*(c + d)*(c - d)^3)/a^2))/f -
(tan(e/2 + (f*x)/2)^3*(c - d)^4)/(6*a^2*f) + (d^2*atanh(tan(e/2 + (f*x)/2))
*(12*c^2 - 16*c*d + 7*d^2))/(a^2*f)
```

3.219 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 133

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx &= \frac{(3c-2d)d^2\operatorname{arctanh}(\sin(e+fx))}{a^2f}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2}\\ &+\frac{(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\sec(e+fx))\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}\end{aligned}$$

```
[Out] (3*c-2*d)*d^2*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)*(c+d*sec(f*x+e))^2*tan(f*
x+e)/f/(a+a*sec(f*x+e))^2+1/3*(c^3+4*c^2*d-12*c*d^2+10*d^3-(c-4*d)*d^2*sec(
f*x+e))*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 100, 148, 65, 223, 209}

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx &= \frac{\tan(e+fx)\left(c^3+4c^2d-d^2(c-4d)\sec(e+fx)-12cd^2+10d^3\right)}{3f\left(a^2\sec(e+fx)+a^2\right)}\\ &+\frac{2d^2(3c-2d)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\ &+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^2}{3f(a\sec(e+fx)+a)^2}\end{aligned}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*(3*c - 2*d)*d^2*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(a*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) +
 ((c - d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2)
 + ((c^3 + 4*c^2*d - 12*c*d^2 + 10*d^3 - (c - 4*d)*d^2*Sec[e + f*x])*Tan[e
+ f*x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)(-a^2(c^2+4cd-2d^2)+a^2(c-4d)dx)}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{3af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\sec(e+fx))\tan(e+fx)}{3f(a^2+a^2\sec(e+fx))} - \frac{((3c-2d)d^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\sec(e+fx))\tan(e+fx)}{3f(a^2+a^2\sec(e+fx))} + \frac{(2(3c-2d)d^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\sec(e+fx))\tan(e+fx)}{3f(a^2+a^2\sec(e+fx))} + \frac{(2(3c-2d)d^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2(3c-2d)d^2\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c^3+4c^2d-12cd^2+10d^3-(c-4d)d^2\sec(e+fx))\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 294 vs. $2(133) = 266$.

Time = 3.65 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.21

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$

$$= \frac{2\cos^6\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\left(6d^2(-3c+2d)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+\right.\right.\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] (2*Cos[(e + f*x)/2]^6*Sec[e + f*x]*(6*d^2*(-3*c + 2*d)*(Log[Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 8*(c -
d)^3*Csc[e + f*x]^3*Sin[(e + f*x)/2]^4 + 32*(c - d)^3*Csc[e + f*x]^5*Sin[(e
 + f*x)/2]^8 + 2*(2*c^3 + 3*c^2*d - 12*c*d^2 + 13*d^3)*Tan[(e + f*x)/2] + 6
*(3*c - 2*d)*d^2*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]])*Tan[(e + f*x)/2]^2 - 2*(c - d)^2*(2*c + 7*d)*Ta
n[(e + f*x)/2]^3))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.32

method	result
parallelrisch	$\frac{-18\left(c-\frac{2d}{3}\right)\cos(fx+e)d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+18\left(c-\frac{2d}{3}\right)\cos(fx+e)d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\sec\left(\frac{fx}{2}+\frac{e}{2}\right)}{6fa^2\cos(fx+e)}$
derivativedivides	$-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2d-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3cd^2+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3d^3}{3}+c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-9\tan\left(\frac{fx}{2}\right.$
default	$-\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^2d-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3cd^2+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3d^3}{3}+c^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-9\tan\left(\frac{fx}{2}\right.$
norman	$\frac{\frac{(c^3-3cd^2+2d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{af}-\frac{(c^3-3c^2d+3cd^2-d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{6af}-\frac{(c^3+3c^2d-9cd^2+9d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}-\frac{(2c^3+3c^2d-12cd^2+}{af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3a}$
risch	$\frac{2i(3c^3e^{4i(fx+e)}-9cd^2e^{4i(fx+e)}+6d^3e^{4i(fx+e)}+3c^3e^{3i(fx+e)}+9c^2de^{3i(fx+e)}-27cd^2e^{3i(fx+e)}+18d^3e^{3i(fx+e)}+5c^3e^{2i(f}}{3fa^2(1}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/6*(-18*(c-2/3*d)*cos(f*x+e)*d^2*ln(tan(1/2*f*x+1/2*e)-1)+18*(c-2/3*d)*cos
(f*x+e)*d^2*ln(tan(1/2*f*x+1/2*e)+1)+tan(1/2*f*x+1/2*e)*sec(1/2*f*x+1/2*e)^
2*((c^3+3/2*c^2*d-6*c*d^2+5*d^3)*cos(2*f*x+2*e)+(c^3+6*c^2*d-15*c*d^2+14*d^
3)*cos(f*x+e)+c^3+3/2*c^2*d-6*c*d^2+8*d^3))/f/a^2/cos(f*x+e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(130) = 260.

Time = 0.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.02

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{3\left((3\,cd^2-2\,d^3)\cos\left(fx+e\right)^3+2\,(3\,cd^2-2\,d^3)\cos\left(fx+e\right)^2+(3\,cd^2-2\,d^3)\cos\left(fx+e\right)\right)\log\left(\sin\left(fx+e\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*((3*c*d^2 - 2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)
^2 + (3*c*d^2 - 2*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1) - 3*((3*c*d^2 -
2*d^3)*cos(f*x + e)^3 + 2*(3*c*d^2 - 2*d^3)*cos(f*x + e)^2 + (3*c*d^2 - 2*d
^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(3*d^3 + (2*c^3 + 3*c^2*d - 12
*c*d^2 + 10*d^3)*cos(f*x + e)^2 + (c^3 + 6*c^2*d - 15*c*d^2 + 14*d^3)*cos(f
```

```
*x + e))*sin(f*x + e))/(a^2*f*cos(f*x + e)^3 + 2*a^2*f*cos(f*x + e)^2 + a^2
*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{\int \frac{c^3\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d^3\sec^4(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{3cd^2\sec^3(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{3c^2d\sec^2(e+}{\sec^2(e+fx)+2\sec(}}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**3*sec(e + f*x)**4/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(3*c*d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x)
+ Integral(3*c**2*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1),
 x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(130) = 260$.

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.57

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx$$
$$= \frac{d^3\left(\frac{\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2} - \frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2} + \frac{12\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2} + \frac{12\sin(fx+e)}{\left(a^2-\frac{a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)}\right) - 3}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(d^3*((15*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 12*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 12*log(si
n(f*x + e)/(cos(f*x + e) + 1) - 1)/a^2 + 12*sin(f*x + e)/((a^2 - a^2*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - 3*c*d^2*((9*sin(f*x +
 e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - 6*log(s
in(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(f*x + e)/(cos(f*x + e)
+ 1) - 1)/a^2) + 3*c^2*d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)/a^2 + c^3*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(
f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = -\frac{\frac{12\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^2} - \frac{6\,(3\,cd^2-2\,d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2} + \frac{6\,(3\,cd^2-2\,d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2} + \frac{a^4c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^2) - 6*(3
*c*d^2 - 2*d^3)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 + 6*(3*c*d^2 - 2*d^3
)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^2 + (a^4*c^3*tan(1/2*f*x + 1/2*e)^3
- 3*a^4*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^4*c*d^2*tan(1/2*f*x + 1/2*e)^3 -
 a^4*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^3*tan(1/2*f*x + 1/2*e) - 9*a^4*c^
2*d*tan(1/2*f*x + 1/2*e) + 27*a^4*c*d^2*tan(1/2*f*x + 1/2*e) - 15*a^4*d^3*t
an(1/2*f*x + 1/2*e))/a^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(3\,c-2\,d)}{a^2\,f} - \frac{2\,d^3\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{f\,\left(a^2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^2\right)} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(c-d)^3}{6\,a^2\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{(c-d)^3}{a^2}-\frac{3\,(c+d)\,(c-d)^2}{2\,a^2}\right)}{f}$$

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*atanh(tan(e/2 + (f*x)/2))*(3*c - 2*d))/(a^2*f) - (2*d^3*tan(e/2 + (f
*x)/2))/(f*(a^2*tan(e/2 + (f*x)/2)^2 - a^2)) - (tan(e/2 + (f*x)/2)^3*(c - d
)^3)/(6*a^2*f) - (tan(e/2 + (f*x)/2)*((c - d)^3/a^2 - (3*(c + d)*(c - d)^2)
/(2*a^2)))/f
```

3.220 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 89

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2} dx = \frac{d^2 \text{arctanh}(\sin(e+fx))}{a^2 f} + \frac{(c-d)^2 \tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\,(a^2+a^2\sec(e+fx))}$$

```
[Out] d^2*arctanh(sin(f*x+e))/a^2/f+1/3*(c-d)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^2+1
/3*(c-d)*(c+5*d)*tan(f*x+e)/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.67, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 91, 79, 65, 223, 209}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2} dx = \frac{(c+5d)(c-d)\tan(e+fx)}{3f\,(a^2\sec(e+fx)+a^2)} + \frac{2d^2\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} + \frac{(c-d)^2\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - d)^2*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + (2*d^2*ArcTan[Sqrt[
a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a*f*Sqrt[a -
```

```
 a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c - d)*(c + 5*d)*Tan[e + f*x
])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
```

```
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c+dx)^2}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} - \frac{\tan(e+fx)\text{Subst}\left(\int\frac{a^3(c^2+4cd-2d^2)+3a^3d^2x}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx,x,\sec(e+fx)\right)}{3a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} - \frac{\left(d^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{\left(2d^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)} + \frac{\left(2d^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{2d^2\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(c+5d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(89) = 178.

Time = 2.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.03

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx =$$
$$-\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(6d^2\cos^3\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^2,x]
[Out] (-2*Cos[(e + f*x)/2]*(6*d^2*Cos[(e + f*x)/2]^3*(Log[Cos[(e + f*x)/2] - Sin[
(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)^2*Sec[e
/2]*Sin[(f*x)/2] - 4*(c^2 + c*d - 2*d^2)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f
*x)/2] + (c - d)^2*Cos[(e + f*x)/2]*Tan[e/2]))/(3*a^2*f*(1 + Cos[e + f*x])^
2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{-6\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d^2+6\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d^2-(c-d)\left((c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-3c-9d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{6a^2f}$
derivativedivides	$\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cd+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3cd}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2-2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d^2-\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{\tan(}{}}{2fa^2}$
default	$\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cd+\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3cd}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2-3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)d^2-2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d^2-\frac{c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{\tan(}{}}{2fa^2}$
risch	$\frac{2i(3c^2e^{2i(fx+e)}-3d^2e^{2i(fx+e)}+3c^2e^{i(fx+e)}+6d\,e^{i(fx+e)}c-9d^2e^{i(fx+e)}+2c^2+2cd-4d^2)}{3fa^2(e^{i(fx+e)}+1)^3}+\frac{d^2\ln(e^{i(fx+e)}+i)}{a^2f}-\frac{d^2\ln}{}$
norman	$\frac{-\frac{(c^2-2cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{6af}+\frac{(c^2+2cd-3d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{(5c^2+2cd-7d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{6af}-\frac{(7c^2+10cd-17d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{6af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2a}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
[Out] 1/6*(-6*ln(tan(1/2*f*x+1/2*e)-1)*d^2+6*ln(tan(1/2*f*x+1/2*e)+1)*d^2-(c-d)*(
(c-d)*tan(1/2*f*x+1/2*e)^2-3*c-9*d)*tan(1/2*f*x+1/2*e))/a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.74

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right)\log\left(\sin\left(fx+e\right)+1\right)-3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)\right.}{6\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\right.}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(sin(f*x + e) + 1
) - 3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*log(-sin(f*x + e) + 1
) + 2*(c^2 + 4*c*d - 5*d^2 + 2*(c^2 + c*d - 2*d^2)*cos(f*x + e))*sin(f*x +
e))/(a^2*f*cos(f*x + e)^2 + 2*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{c^2\sec\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{d^2\sec^3\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx+\int \frac{2cd\sec^2\left(e+fx\right)}{\sec^2\left(e+fx\right)+2\sec\left(e+fx\right)+1}\,dx}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(d**2*sec(e + f*x)**3/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + In
tegral(2*c*d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a*
*2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(85) = 170$.

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = -\frac{d^2\left(\frac{\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^2}\right)-\frac{2cd\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}-\frac{c}{}}{6f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(d^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e
) + 1)^3)/a^2 - 6*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^2 + 6*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/a^2) - 2*c*d*(3*sin(f*x + e)/(cos(f*x + e)
 + 1) + sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2 - c^2*(3*sin(f*x + e)/(cos
(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.78

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{6\,d^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^2}-\frac{6\,d^2\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^2}-\frac{a^4c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-2\,a^4cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+a^4d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,a^4c^2}{a^6}}{6\,f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*d^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^2 - 6*d^2*log(abs(tan(1/2*f
*x + 1/2*e) - 1))/a^2 - (a^4*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^4*c*d*tan(1/2
*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^2*tan(1/2*f*x +
1/2*e) - 6*a^4*c*d*tan(1/2*f*x + 1/2*e) + 9*a^4*d^2*tan(1/2*f*x + 1/2*e))/a
^6)/f
```

Mupad [B] (verification not implemented)

Time = 13.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^2}\,dx = \frac{2\,d^2\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)}{a^2\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{{(c-d)}^2}{2\,a^2}-\frac{c^2-d^2}{a^2}\right)}{f} - \frac{{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^3\,{(c-d)}^2}{6\,a^2\,f}$$

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*atanh(tan(e/2 + (f*x)/2)))/(a^2*f) - (tan(e/2 + (f*x)/2)*((c - d)^2/
(2*a^2) - (c^2 - d^2)/a^2))/f - (tan(e/2 + (f*x)/2)^3*(c - d)^2)/(6*a^2*f)
```

3.221 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 65

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2} + \frac{(c+2d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}$$

```
[Out] 1/3*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^2+1/3*(c+2*d)*tan(f*x+e)/f/(a^2+a^2
*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {4085, 3879}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{(c+2d)\tan(e+fx)}{3f\left(a^2\sec(e+fx)+a^2\right)} + \frac{(c-d)\tan(e+fx)}{3f(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
```

```
[Out] ((c - d)*Tan[e + f*x])/(3*f*(a + a*Sec[e + f*x])^2) + ((c + 2*d)*Tan[e + f*
x])/(3*f*(a^2 + a^2*Sec[e + f*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c+2d)\int\frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{3a}\\ &= \frac{(c-d)\tan(e+fx)}{3f(a+a\sec(e+fx))^2}+\frac{(c+2d)\tan(e+fx)}{3f\left(a^2+a^2\sec(e+fx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int\frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(3(c+d)\sin\left(\frac{fx}{2}\right)-3c\sin\left(e+\frac{fx}{2}\right)+(2c+d)\sin\left(e+\frac{3fx}{2}\right)\right)}{3a^2f(1+\cos(e+fx))^2}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^2,x]
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(3*(c + d)*Sin[(f*x)/2] - 3*c*Sin[e + (f*x)/2] +
 (2*c + d)*Sin[e + (3*f*x)/2]))/(3*a^2*f*(1 + Cos[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{\left(-3c-3d+(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{6a^{2}f}$	42
derivativedivides	$\frac{-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\,a^{2}}$	60
default	$\frac{-\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f\,a^{2}}$	60
risch	$\frac{2i\left(3\,{\rm e}^{2i(fx+e)}c+3\,{\rm e}^{i(fx+e)}c+3d\,{\rm e}^{i(fx+e)}+2c+d\right)}{3f\,a^{2}\left({\rm e}^{i(fx+e)}+1\right)^{3}}$	64
norman	$\frac{-\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{6af}-\frac{(c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{(2c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3af}}{a\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}-1\right)}$	89

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] -1/6*(-3*c-3*d+(c-d)*tan(1/2*f*x+1/2*e)^2)*tan(1/2*f*x+1/2*e)/a^2/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{((2\,c+d)\cos\left(fx+e\right)+c+2\,d)\sin\left(fx+e\right)}{3\left(a^2f\cos\left(fx+e\right)^2+2\,a^2f\cos\left(fx+e\right)+a^2f\right)}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="fric
as")
[Out] 1/3*((2*c + d)*cos(f*x + e) + c + 2*d)*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 +
 2*a^2*f*cos(f*x + e) + a^2*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{\int \frac{c\sec(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx + \int \frac{d\sec^2(e+fx)}{\sec^2(e+fx)+2\sec(e+fx)+1}\,dx}{a^2}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**2,x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x) + Integ
ral(d*sec(e + f*x)**2/(sec(e + f*x)**2 + 2*sec(e + f*x) + 1), x))/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = \frac{\frac{d\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}+\frac{c\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^2}}{6\,f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/6*(d*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e) +
1)^3)/a^2 + c*(3*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3)/a^2)/f
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx = -\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{6\,a^2f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -1/6*(c*tan(1/2*f*x + 1/2*e)^3 - d*tan(1/2*f*x + 1/2*e)^3 - 3*c*tan(1/2*f*x
 + 1/2*e) - 3*d*tan(1/2*f*x + 1/2*e))/(a^2*f)
```

Mupad [B] (verification not implemented)

Time = 13.58 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^2}\,dx=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(c+d)}{2\,a^2\,f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\,(c-d)}{6\,a^2\,f}$$

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^2),x)
[Out] (tan(e/2 + (f*x)/2)*(c + d))/(2*a^2*f) - (tan(e/2 + (f*x)/2)^3*(c - d))/(6*
a^2*f)
```

3.222 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 129

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \frac{2d^2\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{5/2}\sqrt{c+d}f} + \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\left(a^2+a^2\sec(e+fx)\right)}$$

```
[Out] 2*d^2*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^2/(c-d)^(5/2)/f
/(c+d)^(1/2)+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2+1/3*(c-4*d)*tan(f*x+
e)/(c-d)^2/f/(a^2+a^2*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 106, 157, 12, 95, 211}

$$\begin{aligned}&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx\\ &= \frac{(c-4d)\tan(e+fx)}{3f(c-d)^2\left(a^2\sec(e+fx)+a^2\right)}\\ &\quad - \frac{2d^2\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{5/2}\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\ &\quad + \frac{\tan(e+fx)}{3f(c-d)(a\sec(e+fx)+a)^2}\end{aligned}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(3*(c - d)*f*(a + a*Sec[e + f*x])^2) - (2*d^2*ArcTan[(Sqrt[c +
 d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e
 + f*x])/(a*(c - d)^(5/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a
*Sec[e + f*x]]) + ((c - 4*d)*Tan[e + f*x])/(3*(c - d)^2*f*(a^2 + a^2*Sec[e
+ f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{\tan(e+fx)\text{Subst}\left(\int\frac{-a^2(c-3d)-a^2dx}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{3a(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\,(a^2+a^2\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int\frac{3a^4d^2}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{3a^4(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\,(a^2+a^2\sec(e+fx))} - \frac{(d^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\,(a^2+a^2\sec(e+fx))} - \frac{(2d^2\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2} - \frac{2d^2\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{a(c-d)^{5/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-4d)\tan(e+fx)}{3(c-d)^2f\,(a^2+a^2\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.62

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(-\frac{24id^2\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(\cos(e)-i\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}+\sec\left(\frac{e}{2}\right)(3(c\right.}{3a^2(c-d)^2f(1+\cos(e+fx))^2}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]*(((-24*I)*d^2*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d
 + c*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*
Cos[(e + f*x)/2]^3*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*S
in[e])^2]) + Sec[e/2]*(3*(c - 3*d)*Sin[(f*x)/2] - 3*(c - 2*d)*Sin[e + (f*x)
/2] + (2*c - 5*d)*Sin[e + (3*f*x)/2])))/(3*a^2*(c - d)^2*f*(1 + Cos[e + f*x
])^2)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c-d)^2}+\frac{4d^2\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)^2\sqrt{(c+d)(c-d)}}}{2f\,a^2}$
default	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c-d)^2}+\frac{4d^2\,\mathrm{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c-d)^2\sqrt{(c+d)(c-d)}}}{2f\,a^2}$
risch	$\frac{2i\left(3\,\mathrm{e}^{2i(fx+e)}c-6d\,\mathrm{e}^{2i(fx+e)}+3\,\mathrm{e}^{i(fx+e)}c-9d\,\mathrm{e}^{i(fx+e)}+2c-5d\right)}{3f\,a^2(c-d)^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3}+\frac{d^2\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c-d)^2f\,a^2}-\frac{d^2\ln\left(\mathrm{e}^{i}\right.}{}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/a^2*(-1/(c-d)^2*(1/3*c*tan(1/2*f*x+1/2*e)^3-1/3*d*tan(1/2*f*x+1/2*e)^
3-c*tan(1/2*f*x+1/2*e)+3*d*tan(1/2*f*x+1/2*e))+4*d^2/(c-d)^2/((c+d)*(c-d))^
(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(116) = 232$.

Time = 0.29 (sec) , antiderivative size = 598, normalized size of antiderivative = 4.64

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$= \left[\frac{3\left(d^2\cos\left(fx+e\right)^2+2\,d^2\cos\left(fx+e\right)+d^2\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(d\cos(fx+e)}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right.}{6\left(\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)f\cos\left(fx+e\right)^2+2\left(a^2c^4-2\,a^2c^3d\right.\right.}\right.$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/6*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(c^2 - d^2)*log
((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*
cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*c
os(f*x + e) + d^2)) + 2*(c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d -
 2*c*d^2 + 5*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a
^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d
^3 - a^2*d^4)*f*cos(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d
^4)*f), 1/3*(3*(d^2*cos(f*x + e)^2 + 2*d^2*cos(f*x + e) + d^2)*sqrt(-c^2 +
d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e
))) + (c^3 - 4*c^2*d - c*d^2 + 4*d^3 + (2*c^3 - 5*c^2*d - 2*c*d^2 + 5*d^3)*
cos(f*x + e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4
)*f*cos(f*x + e)^2 + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*co
s(f*x + e) + (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$= \frac{\int \frac{\sec\left(e+fx\right)}{c\sec^2\left(e+fx\right)+2c\sec\left(e+fx\right)+c+d\sec^3\left(e+fx\right)+2d\sec^2\left(e+fx\right)+d\sec\left(e+fx\right)}\,dx}{a^2}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x)**2 + 2*c*sec(e + f*x) + c + d*sec(e +
 f*x)**3 + 2*d*sec(e + f*x)**2 + d*sec(e + f*x)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(116) = 232$.

Time = 0.33 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.93

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = -\frac{\frac{12\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{(a^2c^2-2a^2cd+a^2d^2)\sqrt{-c^2+d^2}} + \frac{a^4c^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2a^4cd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+a^4d^2\tan}{}}{6f}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1
/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((a^2*c^2
- 2*a^2*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + (a^4*c^2*tan(1/2*f*x + 1/2*e)^3
- 2*a^4*c*d*tan(1/2*f*x + 1/2*e)^3 + a^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 3*a^4
*c^2*tan(1/2*f*x + 1/2*e) + 12*a^4*c*d*tan(1/2*f*x + 1/2*e) - 9*a^4*d^2*tan
(1/2*f*x + 1/2*e))/(a^6*c^3 - 3*a^6*c^2*d + 3*a^6*c*d^2 - a^6*d^3))/f
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.30

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{1}{a^2(c-d)}-\frac{c+d}{2a^2(c-d)^2}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{6a^2f(c-d)}$$
$$-\frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^3-3\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2d+3\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)cd^2-1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^3}{\sqrt{c+d}\,(c-d)^{5/2}}\right)2\mathrm{i}}{a^2f\sqrt{c+d}\,(c-d)^{5/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(1/(a^2*(c - d)) - (c + d)/(2*a^2*(c - d)^2)))/f - tan(
e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)) - (d^2*atan((c^3*tan(e/2 + (f*x)/2)*1i -
 d^3*tan(e/2 + (f*x)/2)*1i + c*d^2*tan(e/2 + (f*x)/2)*3i - c^2*d*tan(e/2 +
(f*x)/2)*3i)/((c + d)^(1/2)*(c - d)^(5/2)))*2i)/(a^2*f*(c + d)^(1/2)*(c - d
)^(5/2))
```

3.223 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 211

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = \frac{2d^2(3c+2d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{7/2}(c+d)^{3/2}f} + \frac{d(c^2-6cd-10d^2)\tan(e+fx)}{3a^2(c-d)^3(c+d)f(c+d\sec(e+fx))} + \frac{(c-6d)\tan(e+fx)}{3a^2(c-d)^2f(1+\sec(e+fx))(c+d\sec(e+fx))} + \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))}$$

```
[Out] 2*d^2*(3*c+2*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^2/(c-
d)^(7/2)/(c+d)^(3/2)/f+1/3*d*(c^2-6*c*d-10*d^2)*tan(f*x+e)/a^2/(c-d)^3/(c+d
)/f/(c+d*sec(f*x+e))+1/3*(c-6*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1+sec(f*x+e))/(c
+d*sec(f*x+e))+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

$= \{4072, 105, 157, 12, 95, 211\}$

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{(c^2-6cd-10d^2)\tan(e+fx)}{3f(c-d)^3(c+d)\left(a^2\sec(e+fx)+a^2\right)}$$
$$- \frac{2d^2(3c+2d)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{7/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$- \frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))}$$
$$+ \frac{(c+4d)\tan(e+fx)}{3f(c-d)^2(c+d)(a\sec(e+fx)+a)^2}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 4*d)*Tan[e + f*x])/(3*(c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])^2) - (
2*d^2*(3*c + 2*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d
]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a*(c - d)^(7/2)*(c + d)^(3/2)*f
*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c^2 - 6*c*d - 10*d^
2)*Tan[e + f*x])/(3*(c - d)^3*(c + d)*f*(a^2 + a^2*Sec[e + f*x])) - (d*Tan[
e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^2(c+2d)-2a^2dx}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{-a^4(c-6d)(c+d)-a^4d(c+4d)x}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{3a^3(c-d)\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{(c^2-6cd-10d^2)\tan(e+fx)}{3(c-d)^3(c+d)f\,(a^2+a^2\sec(e+fx))} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{3a^6d^2(3c+2d)}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{3a^6(c-d)^2\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{(c^2-6cd-10d^2)\tan(e+fx)}{3(c-d)^3(c+d)f\,(a^2+a^2\sec(e+fx))} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{(d^2(3c+2d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{(c-d)^2\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} + \frac{(c^2-6cd-10d^2)\tan(e+fx)}{3(c-d)^3(c+d)f\,(a^2+a^2\sec(e+fx))} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{(2d^2(3c+2d)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx,x,\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{(c-d)^2\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+4d)\tan(e+fx)}{3(c-d)^2(c+d)f(a+a\sec(e+fx))^2} - \frac{2d^2(3c+2d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{a(c-d)^{7/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c^2-6cd-10d^2)\tan(e+fx)}{3(c-d)^3(c+d)f\,(a^2+a^2\sec(e+fx))} - \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.96 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)(d+c\cos(e+fx))\sec^4(e+fx)\left(\frac{12d^2(3c+2d)\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos^3 \cdots}{(c+d)\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e \cdots}\right.}{\cdots}$$

[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2),x]

[Out] (2*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*Sec[e + f*x]^4*((12*d^2*(3*c + 2*d)*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c*Cos[e])*Tan[(f*x)/2]))/(S qrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[(e + f*x)/2]^3*(d + c*Cos[e + f*x])*(I*Cos[e] + Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Si n[e])^2]) + (c - d)*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] - 4*(c - 4*d)*Cos[(e + f*x)/2]^2*(d + c*Cos[e + f*x])*Sec[e/2]*Sin[(f*x)/2] + (6*d^3*Co s[(e + f*x)/2]^3*(-(d*Sin[e]) + c*Sin[f*x]))/(c*(c + d)*(Cos[e/2] - Sin[e/2])*(Cos[e/2] + Sin[e/2])) + (c - d)*Cos[(e + f*x)/2]*(d + c*Cos[e + f*x])*T an[e/2]))/(3*a^2*(-c + d)^3*f*(1 + Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2)

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+5d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-2cd+d^2\right)(c-d)}-\frac{4d^2\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(3c}{}\right.}{(c-d)^3}}{2fa^2}$
default	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+5d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-2cd+d^2\right)(c-d)}-\frac{4d^2\left(-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c+d)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)}-\frac{(3c}{}\right.}{(c-d)^3}}{2fa^2}$
risch	$\frac{2i\left(-3c^4e^{4i(fx+e)}+6c^3d\,e^{4i(fx+e)}+9c^2d^2e^{4i(fx+e)}+3d^4e^{4i(fx+e)}-3c^4e^{3i(fx+e)}+6c^3d\,e^{3i(fx+e)}+27c^2d^2e^{3i(fx+e)}+21\right.}{}$

[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS E)

[Out] 1/2/f/a^2*(-1/(c^2-2*c*d+d^2)/(c-d)*(1/3*c*tan(1/2*f*x+1/2*e)^3-1/3*d*tan(1 /2*f*x+1/2*e)^3-c*tan(1/2*f*x+1/2*e)+5*d*tan(1/2*f*x+1/2*e))-4*d^2/(c-d)^3* (-d/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d -c-d)-(3*c+2*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/ ((c+d)*(c-d))^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(196) = 392.

Time = 0.33 (sec) , antiderivative size = 1242, normalized size of antiderivative = 5.89

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/6*(3*(3*c*d^3 + 2*d^4 + (3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d
^2 + 7*c*d^3 + 2*d^4)*cos(f*x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*
x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e
)^2 - 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c
^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(c^4*d - 6*c^3*d^2 - 11*
c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^3*d^2 + 3*c^2*d^3 + 8*
c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c^3*d^2 - 10*c^2*d^3 +
13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*c^7 - 2*a^2*c^6*d - a^
2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d^5 + a^2*c*d^6)*f*cos(
f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2 + 7*a^2*c^4*d^3 + 2*a
^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^2 + (a^2*c^7 - 5*a^2*c
^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^
2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^
3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f), 1/3*(3*(3*c*d^3 + 2*d^4 +
(3*c^2*d^2 + 2*c*d^3)*cos(f*x + e)^3 + (6*c^2*d^2 + 7*c*d^3 + 2*d^4)*cos(f*
x + e)^2 + (3*c^2*d^2 + 8*c*d^3 + 4*d^4)*cos(f*x + e))*sqrt(-c^2 + d^2)*arc
tan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (c
^4*d - 6*c^3*d^2 - 11*c^2*d^3 + 6*c*d^4 + 10*d^5 + (2*c^5 - 6*c^4*d - 10*c^
3*d^2 + 3*c^2*d^3 + 8*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (c^5 - 4*c^4*d - 14*c
^3*d^2 - 10*c^2*d^3 + 13*c*d^4 + 14*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*
c^7 - 2*a^2*c^6*d - a^2*c^5*d^2 + 4*a^2*c^4*d^3 - a^2*c^3*d^4 - 2*a^2*c^2*d
^5 + a^2*c*d^6)*f*cos(f*x + e)^3 + (2*a^2*c^7 - 3*a^2*c^6*d - 4*a^2*c^5*d^2
 + 7*a^2*c^4*d^3 + 2*a^2*c^3*d^4 - 5*a^2*c^2*d^5 + a^2*d^7)*f*cos(f*x + e)^
2 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^
5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x + e) + (a^2*c^6*d - 2*a^2*c^5*d^2 -
a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{\int \frac{\sec(e+fx)}{c^2\sec^2(e+fx)+2c^2\sec(e+fx)+c^2+2cd\sec^3(e+fx)+4cd\sec^2(e+fx)+2cd\sec(e+fx)+d^2\sec^4(e+fx)+2d^2\sec^3(e+fx)+d^2\sec^2(e+fx)}\,dx}{a^2}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x)**2 + 2*c**2*sec(e + f*x) + c**2 +
2*c*d*sec(e + f*x)**3 + 4*c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2*s
ec(e + f*x)**4 + 2*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(196) = 392.

Time = 0.33 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.25

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{\frac{12\,d^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\left(c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-c-d\right)} + \frac{12\,\left(3\,cd^2+2\,d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan(}{}\right.\right.}{\left(a^2c^4-2\,a^2c^3d+2\,a^2cd^3-a^2d^4\right)\sqrt{-c}}}{}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(12*d^3*tan(1/2*f*x + 1/2*e)/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^
2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)) + 12*
(3*c*d^2 + 2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arcta
n(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a
^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)) - (a^4*c^4*
tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c^3*d*tan(1/2*f*x + 1/2*e)^3 + 6*a^4*c^2*d^2
*tan(1/2*f*x + 1/2*e)^3 - 4*a^4*c*d^3*tan(1/2*f*x + 1/2*e)^3 + a^4*d^4*tan(
1/2*f*x + 1/2*e)^3 - 3*a^4*c^4*tan(1/2*f*x + 1/2*e) + 24*a^4*c^3*d*tan(1/2*
f*x + 1/2*e) - 54*a^4*c^2*d^2*tan(1/2*f*x + 1/2*e) + 48*a^4*c*d^3*tan(1/2*f
*x + 1/2*e) - 15*a^4*d^4*tan(1/2*f*x + 1/2*e))/(a^6*c^6 - 6*a^6*c^5*d + 15*
a^6*c^4*d^2 - 20*a^6*c^3*d^3 + 15*a^6*c^2*d^4 - 6*a^6*c*d^5 + a^6*d^6))/f
```

Mupad [B] (verification not implemented)

Time = 14.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.49

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3}{2a^2(c-d)^2}-\frac{c^2-d^2}{a^2(c-d)^4}\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{6a^2f(c-d)^2}$$
$$+ \frac{2d^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,(c+d)\left(a^2d^4-a^2c^4+\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(a^2c^4-4a^2c^3d+6a^2c^2d^2-4a^2cd^3+a^2d^4\right)-2a^2cd^3+2c\right.}$$
$$- \frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^4-4\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^3d+6\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2d^2-4\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)cd^3+1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^4}{\sqrt{c+d}\,(c-d)^{7/2}}\right)(3c+2d)\,2\mathrm{i}}{a^2f\,(c+d)^{3/2}\,(c-d)^{7/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3/(2*a^2*(c - d)^2) - (c^2 - d^2)/(a^2*(c - d)^4)))/f
- tan(e/2 + (f*x)/2)^3/(6*a^2*f*(c - d)^2) + (2*d^3*tan(e/2 + (f*x)/2))/(f*
(c + d)*(a^2*d^4 - a^2*c^4 + tan(e/2 + (f*x)/2)^2*(a^2*c^4 + a^2*d^4 - 4*a^
2*c*d^3 - 4*a^2*c^3*d + 6*a^2*c^2*d^2) - 2*a^2*c*d^3 + 2*a^2*c^3*d)) - (d^2
*atan((c^4*tan(e/2 + (f*x)/2)*1i + d^4*tan(e/2 + (f*x)/2)*1i - c*d^3*tan(e/
2 + (f*x)/2)*4i - c^3*d*tan(e/2 + (f*x)/2)*4i + c^2*d^2*tan(e/2 + (f*x)/2)*
6i)/((c + d)^(1/2)*(c - d)^(7/2)))*(3*c + 2*d)*2i)/(a^2*f*(c + d)^(3/2)*(c
- d)^(7/2))
```

3.224 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 284

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx &= \frac{d^2(12c^2+16cd+7d^2)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^2(c-d)^{9/2}(c+d)^{5/2}f} \\ &+ \frac{d(2c^2-16cd-21d^2)\tan(e+fx)}{6a^2(c-d)^3(c+d)f(c+d\sec(e+fx))^2} \\ &+ \frac{(c-8d)\tan(e+fx)}{3a^2(c-d)^2f(1+\sec(e+fx))(c+d\sec(e+fx))^2} \\ &+ \frac{\tan(e+fx)}{3(c-d)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} \\ &+ \frac{d(2c^3-16c^2d-59cd^2-32d^3)\tan(e+fx)}{6a^2(c-d)^4(c+d)^2f(c+d\sec(e+fx))}\end{aligned}$$

```
[Out] d^2*(12*c^2+16*c*d+7*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2
))/a^2/(c-d)^(9/2)/(c+d)^(5/2)/f+1/6*d*(2*c^2-16*c*d-21*d^2)*tan(f*x+e)/a^2
/(c-d)^3/(c+d)/f/(c+d*sec(f*x+e))^2+1/3*(c-8*d)*tan(f*x+e)/a^2/(c-d)^2/f/(1
+sec(f*x+e))/(c+d*sec(f*x+e))^2+1/3*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^2/(
c+d*sec(f*x+e))^2+1/6*d*(2*c^3-16*c^2*d-59*c*d^2-32*d^3)*tan(f*x+e)/a^2/(c-
d)^4/(c+d)^2/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 105, 156, 157, 12, 95, 211}

$$\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx \\
&= \frac{(2c^3-16c^2d-59cd^2-32d^3)\tan(e+fx)}{6f(c-d)^4(c+d)^2\left(a^2\sec(e+fx)+a^2\right)} \\
&\quad - \frac{d^2(12c^2+16cd+7d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{af(c-d)^{9/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\
&\quad - \frac{d(5c+2d)\tan(e+fx)}{2f\left(c^2-d^2\right)^2(a\sec(e+fx)+a)^2(c+d\sec(e+fx))} \\
&\quad - \frac{d\tan(e+fx)}{2f\left(c^2-d^2\right)(a\sec(e+fx)+a)^2(c+d\sec(e+fx))^2} \\
&\quad + \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6f(c-d)^3(c+d)^2(a\sec(e+fx)+a)^2}
\end{aligned}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 22*c*d + 11*d^2)*Tan[e + f*x])/(6*(c - d)^3*(c + d)^2*f*(a + a*Se
c[e + f*x])^2) - (d^2*(12*c^2 + 16*c*d + 7*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a
+ a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a
*(c - d)^(9/2)*(c + d)^(5/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e +
f*x]]) + ((2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3)*Tan[e + f*x])/(6*(c - d)^4
*(c + d)^2*f*(a^2 + a^2*Sec[e + f*x])) - (d*Tan[e + f*x])/(2*(c^2 - d^2)*f*
(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^2) - (d*(5*c + 2*d)*Tan[e + f*x
])/(2*(c^2 - d^2)^2*f*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2(c^2-d^2)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{2a^2(c+d)-3a^2dx}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{2(c^2-d^2)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2(c^2-d^2)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{d(5c+2d)\tan(e+fx)}{2(c^2-d^2)^2f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^4(2c^2+12cd+7d^2)-2a^4d(5c+2d)x}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2a^2(c^2-d^2)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} - \frac{d\tan(e+fx)}{2(c^2-d^2)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{d(5c+2d)\tan(e+fx)}{2(c^2-d^2)^2f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{-a^6(c+d)(2c^2-16cd-21d^2)-a^6d(2c^2+22cd+11d^2)x}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{6a^5(c-d)(c^2-d^2)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^3-16c^2d-59cd^2-32d^3)\tan(e+fx)}{6(c-d)^4(c+d)^2f(a^2+a^2\sec(e+fx))} - \frac{d\tan(e+fx)}{2(c^2-d^2)f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{d(5c+2d)\tan(e+fx)}{2(c^2-d^2)^2f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{3a^8d^2(12c^2+16cd+7d^2)}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{6a^8(c-d)^2(c^2-d^2)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^3-16c^2d-59cd^2-32d^3)\tan(e+fx)}{6(c-d)^4(c+d)^2f\,(a^2+a^2\sec(e+fx))} - \frac{d\tan(e+fx)}{2\,(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{d(5c+2d)\tan(e+fx)}{2\,(c^2-d^2)^2\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{(d^2(12c^2+16cd+7d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{2(c-d)^2\,(c^2-d^2)^2\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^3-16c^2d-59cd^2-32d^3)\tan(e+fx)}{6(c-d)^4(c+d)^2f\,(a^2+a^2\sec(e+fx))} - \frac{d\tan(e+fx)}{2\,(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{d(5c+2d)\tan(e+fx)}{2\,(c^2-d^2)^2\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} - \frac{(d^2(12c^2+16cd+7d^2)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{ac-ad-(-ac-ad)x^2}\,dx,x,\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{(c-d)^2\,(c^2-d^2)^2\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c^2+22cd+11d^2)\tan(e+fx)}{6(c-d)^3(c+d)^2f(a+a\sec(e+fx))^2} - \frac{d^2(12c^2+16cd+7d^2)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{a(c-d)^{9/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(2c^3-16c^2d-59cd^2-32d^3)\tan(e+fx)}{6(c-d)^4(c+d)^2f\,(a^2+a^2\sec(e+fx))} - \frac{d\tan(e+fx)}{2\,(c^2-d^2)\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} - \frac{d(5c+2d)\tan(e+fx)}{2\,(c^2-d^2)^2\,f(a+a\sec(e+fx))^2(c+d\sec(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.18 (sec) , antiderivative size = 2220, normalized size of antiderivative = 7.82

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((12*c^2 + 16*c*d + 7*d^2)*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[
e + f*x]^5*(((-4*I)*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[C
```

```
os[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[
2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d
^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (4*d^2*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sq
rt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sq
rt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*S
in[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d
)^2*(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3) + (Cos[e/2 + (f*x)/2]*(d
 + c*Cos[e + f*x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-16*c^7*Sin[(f*x)/2] + 1
4*c^6*d*Sin[(f*x)/2] + 220*c^5*d^2*Sin[(f*x)/2] + 334*c^4*d^3*Sin[(f*x)/2]
+ 54*c^3*d^4*Sin[(f*x)/2] - 156*c^2*d^5*Sin[(f*x)/2] - 48*c*d^6*Sin[(f*x)/2
] + 18*d^7*Sin[(f*x)/2] + 14*c^7*Sin[(3*f*x)/2] - 16*c^6*d*Sin[(3*f*x)/2] -
 226*c^5*d^2*Sin[(3*f*x)/2] - 532*c^4*d^3*Sin[(3*f*x)/2] - 583*c^3*d^4*Sin[
(3*f*x)/2] - 232*c^2*d^5*Sin[(3*f*x)/2] - 6*c*d^6*Sin[(3*f*x)/2] + 6*d^7*Si
n[(3*f*x)/2] - 12*c^7*Sin[e - (f*x)/2] + 20*c^6*d*Sin[e - (f*x)/2] + 236*c^
5*d^2*Sin[e - (f*x)/2] + 628*c^4*d^3*Sin[e - (f*x)/2] + 778*c^3*d^4*Sin[e -
 (f*x)/2] + 420*c^2*d^5*Sin[e - (f*x)/2] + 48*c*d^6*Sin[e - (f*x)/2] - 18*d
^7*Sin[e - (f*x)/2] + 12*c^7*Sin[e + (f*x)/2] - 20*c^6*d*Sin[e + (f*x)/2] -
 236*c^5*d^2*Sin[e + (f*x)/2] - 460*c^4*d^3*Sin[e + (f*x)/2] - 310*c^3*d^4*
Sin[e + (f*x)/2] + 39*c^2*d^5*Sin[e + (f*x)/2] + 48*c*d^6*Sin[e + (f*x)/2]
- 18*d^7*Sin[e + (f*x)/2] - 16*c^7*Sin[2*e + (f*x)/2] + 14*c^6*d*Sin[2*e +
(f*x)/2] + 220*c^5*d^2*Sin[2*e + (f*x)/2] + 502*c^4*d^3*Sin[2*e + (f*x)/2]
+ 522*c^3*d^4*Sin[2*e + (f*x)/2] + 303*c^2*d^5*Sin[2*e + (f*x)/2] + 48*c*d^
6*Sin[2*e + (f*x)/2] - 18*d^7*Sin[2*e + (f*x)/2] - 6*c^7*Sin[e + (3*f*x)/2]
 + 6*c^6*d*Sin[e + (3*f*x)/2] + 126*c^5*d^2*Sin[e + (3*f*x)/2] + 114*c^4*d^
3*Sin[e + (3*f*x)/2] - 159*c^3*d^4*Sin[e + (3*f*x)/2] - 144*c^2*d^5*Sin[e +
 (3*f*x)/2] - 6*c*d^6*Sin[e + (3*f*x)/2] + 6*d^7*Sin[e + (3*f*x)/2] + 14*c^
7*Sin[2*e + (3*f*x)/2] - 16*c^6*d*Sin[2*e + (3*f*x)/2] - 226*c^5*d^2*Sin[2*
e + (3*f*x)/2] - 412*c^4*d^3*Sin[2*e + (3*f*x)/2] - 235*c^3*d^4*Sin[2*e + (
3*f*x)/2] - 7*c^2*d^5*Sin[2*e + (3*f*x)/2] + 6*c*d^6*Sin[2*e + (3*f*x)/2] -
 6*d^7*Sin[2*e + (3*f*x)/2] - 6*c^7*Sin[3*e + (3*f*x)/2] + 6*c^6*d*Sin[3*e
+ (3*f*x)/2] + 126*c^5*d^2*Sin[3*e + (3*f*x)/2] + 234*c^4*d^3*Sin[3*e + (3*
f*x)/2] + 189*c^3*d^4*Sin[3*e + (3*f*x)/2] + 81*c^2*d^5*Sin[3*e + (3*f*x)/2
] + 6*c*d^6*Sin[3*e + (3*f*x)/2] - 6*d^7*Sin[3*e + (3*f*x)/2] + 6*c^7*Sin[e
 + (5*f*x)/2] - 14*c^6*d*Sin[e + (5*f*x)/2] - 134*c^5*d^2*Sin[e + (5*f*x)/2
] - 274*c^4*d^3*Sin[e + (5*f*x)/2] - 193*c^3*d^4*Sin[e + (5*f*x)/2] - 27*c^
2*d^5*Sin[e + (5*f*x)/2] + 6*c*d^6*Sin[e + (5*f*x)/2] - 6*c^7*Sin[2*e + (5*
f*x)/2] + 12*c^6*d*Sin[2*e + (5*f*x)/2] + 42*c^5*d^2*Sin[2*e + (5*f*x)/2] -
 48*c^4*d^3*Sin[2*e + (5*f*x)/2] - 105*c^3*d^4*Sin[2*e + (5*f*x)/2] - 27*c^
2*d^5*Sin[2*e + (5*f*x)/2] + 6*c*d^6*Sin[2*e + (5*f*x)/2] + 6*c^7*Sin[3*e +
 (5*f*x)/2] - 14*c^6*d*Sin[3*e + (5*f*x)/2] - 134*c^5*d^2*Sin[3*e + (5*f*x)
/2] - 202*c^4*d^3*Sin[3*e + (5*f*x)/2] - 61*c^3*d^4*Sin[3*e + (5*f*x)/2] +
12*c^2*d^5*Sin[3*e + (5*f*x)/2] - 6*c*d^6*Sin[3*e + (5*f*x)/2] - 6*c^7*Sin[
4*e + (5*f*x)/2] + 12*c^6*d*Sin[4*e + (5*f*x)/2] + 42*c^5*d^2*Sin[4*e + (5*
f*x)/2] + 24*c^4*d^3*Sin[4*e + (5*f*x)/2] + 27*c^3*d^4*Sin[4*e + (5*f*x)/2]
 + 12*c^2*d^5*Sin[4*e + (5*f*x)/2] - 6*c*d^6*Sin[4*e + (5*f*x)/2] + 4*c^7*S
```

```
in[2*e + (7*f*x)/2] - 14*c^6*d*Sin[2*e + (7*f*x)/2] - 40*c^5*d^2*Sin[2*e +
(7*f*x)/2] - 46*c^4*d^3*Sin[2*e + (7*f*x)/2] - 12*c^3*d^4*Sin[2*e + (7*f*x)
/2] + 3*c^2*d^5*Sin[2*e + (7*f*x)/2] - 24*c^4*d^3*Sin[3*e + (7*f*x)/2] - 12
*c^3*d^4*Sin[3*e + (7*f*x)/2] + 3*c^2*d^5*Sin[3*e + (7*f*x)/2] + 4*c^7*Sin[
4*e + (7*f*x)/2] - 14*c^6*d*Sin[4*e + (7*f*x)/2] - 40*c^5*d^2*Sin[4*e + (7*
f*x)/2] - 22*c^4*d^3*Sin[4*e + (7*f*x)/2]))/(48*c^2*(-c + d)^4*(c + d)^2*f*
(a + a*Sec[e + f*x])^2*(c + d*Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+7d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)(c-d)}-\frac{8d^{2}\left(\frac{-\frac{d\left(8c^{2}-3cd-5d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{4\left(c^{2}+2cd+d^{2}\right)}+\frac{d(8c+3d)\tan\left(\frac{fx}{2}\right.}{4c+4d}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}\right.}{(c-}}{2f\,a^{2}}$
default	$\frac{-\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-\frac{d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}-c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+7d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)(c-d)}-\frac{8d^{2}\left(\frac{-\frac{d\left(8c^{2}-3cd-5d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{4\left(c^{2}+2cd+d^{2}\right)}+\frac{d(8c+3d)\tan\left(\frac{fx}{2}\right.}{4c+4d}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}\right.}{(c-}}{2f\,a^{2}}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/f/a^2*(-1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c-d)*(1/3*c*tan(1/2*f*x+1/2*e)^3-1
/3*d*tan(1/2*f*x+1/2*e)^3-c*tan(1/2*f*x+1/2*e)+7*d*tan(1/2*f*x+1/2*e))-8*d^
2/(c-d)^4*((-1/4*d*(8*c^2-3*c*d-5*d^2)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3
+1/4*d*(8*c+3*d)/(c+d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*
f*x+1/2*e)^2*d-c-d)^2-1/4*(12*c^2+16*c*d+7*d^2)/(c^2+2*c*d+d^2)/((c+d)*(c-d
))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(267) = 534$.

Time = 0.36 (sec) , antiderivative size = 2030, normalized size of antiderivative = 7.15

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2*
d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*co
s(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)*co
s(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e))
*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2
*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(
f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d^2 - 16*c^4*d^3 - 61*c^
3*d^4 - 16*c^2*d^5 + 59*c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5*d^2 - 3
2*c^4*d^3 + 28*c^3*d^4 + 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x + e)^3 + (2
*c^7 - 8*c^6*d - 68*c^5*d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*d^5 + 89*c
*d^6 + 6*d^7)*cos(f*x + e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^
3*d^4 + 71*c^2*d^5 + 134*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*
c^10 - 2*a^2*c^9*d - 2*a^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5*d^5 + 2*a^2*
c^4*d^6 + 2*a^2*c^3*d^7 - a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(a^2*c^10 - a^2
*c^9*d - 4*a^2*c^8*d^2 + 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a^2*c^5*d^5 - 4*
a^2*c^4*d^6 + 4*a^2*c^3*d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*cos(f*x + e)^3 + (
a^2*c^10 + 2*a^2*c^9*d - 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 22*a^2*c^6*d^4 - 2
2*a^2*c^4*d^6 + 4*a^2*c^3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d^9 - a^2*d^10)*f*c
os(f*x + e)^2 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4
+ 6*a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9
 - a^2*d^10)*f*cos(f*x + e) + (a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4
+ 6*a^2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f
), 1/6*(3*(12*c^2*d^4 + 16*c*d^5 + 7*d^6 + (12*c^4*d^2 + 16*c^3*d^3 + 7*c^2
*d^4)*cos(f*x + e)^4 + 2*(12*c^4*d^2 + 28*c^3*d^3 + 23*c^2*d^4 + 7*c*d^5)*c
os(f*x + e)^3 + (12*c^4*d^2 + 64*c^3*d^3 + 83*c^2*d^4 + 44*c*d^5 + 7*d^6)*c
os(f*x + e)^2 + 2*(12*c^3*d^3 + 28*c^2*d^4 + 23*c*d^5 + 7*d^6)*cos(f*x + e)
)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^
2)*sin(f*x + e))) + (2*c^5*d^2 - 16*c^4*d^3 - 61*c^3*d^4 - 16*c^2*d^5 + 59*
c*d^6 + 32*d^7 + (4*c^7 - 14*c^6*d - 44*c^5*d^2 - 32*c^4*d^3 + 28*c^3*d^4 +
 49*c^2*d^5 + 12*c*d^6 - 3*d^7)*cos(f*x + e)^3 + (2*c^7 - 8*c^6*d - 68*c^5*
d^2 - 140*c^4*d^3 - 23*c^3*d^4 + 142*c^2*d^5 + 89*c*d^6 + 6*d^7)*cos(f*x +
e)^2 + (4*c^6*d - 28*c^5*d^2 - 118*c^4*d^3 - 106*c^3*d^4 + 71*c^2*d^5 + 134
*c*d^6 + 43*d^7)*cos(f*x + e))*sin(f*x + e))/((a^2*c^10 - 2*a^2*c^9*d - 2*a
^2*c^8*d^2 + 6*a^2*c^7*d^3 - 6*a^2*c^5*d^5 + 2*a^2*c^4*d^6 + 2*a^2*c^3*d^7
- a^2*c^2*d^8)*f*cos(f*x + e)^4 + 2*(a^2*c^10 - a^2*c^9*d - 4*a^2*c^8*d^2 +
 4*a^2*c^7*d^3 + 6*a^2*c^6*d^4 - 6*a^2*c^5*d^5 - 4*a^2*c^4*d^6 + 4*a^2*c^3*
d^7 + a^2*c^2*d^8 - a^2*c*d^9)*f*cos(f*x + e)^3 + (a^2*c^10 + 2*a^2*c^9*d -
 9*a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 22*a^2*c^6*d^4 - 22*a^2*c^4*d^6 + 4*a^2*c^
3*d^7 + 9*a^2*c^2*d^8 - 2*a^2*c*d^9 - a^2*d^10)*f*cos(f*x + e)^2 + 2*(a^2*c
^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f*cos(f*x +
 e) + (a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2*
c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{\sec(e+fx)}{c^3\sec^2(e+fx)+2c^3\sec(e+fx)+c^3+3c^2d\sec^3(e+fx)+6c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^4(e+fx)+6cd^2\sec^3(e+fx)+3cd^2\sec^2(e\cdots}}{a^2}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**2/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x)**2 + 2*c**3*sec(e + f*x) + c**3 +
3*c**2*d*sec(e + f*x)**3 + 6*c**2*d*sec(e + f*x)**2 + 3*c**2*d*sec(e + f*x)
 + 3*c*d**2*sec(e + f*x)**4 + 6*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e + f
*x)**2 + d**3*sec(e + f*x)**5 + 2*d**3*sec(e + f*x)**4 + d**3*sec(e + f*x)*
*3), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(267) = 534$.

Time = 0.40 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.64

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{6\,(12\,c^2d^2+16\,cd^3+7\,d^4)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c+2\,d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(a^2c^6-2\,a^2c^5d-a^2c^4d^2+4\,a^2c^3d^3-a^2c^2d^4-2\,a^2cd^5+a^2d^6)\sqrt{-c^2+d^2}} - \frac{a^4c^6\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-6\,a^4c^5d\tan\left(\frac{1}{2}\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

[Out] 1/6*(6*(12*c^2*d^2 + 16*c*d^3 + 7*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*sqrt(-c^2 + d^2)) - (a^4*c^6*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c^5*d*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 20*a^4*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^4*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 6*a^4*c*d^5*tan(1/2*f*x + 1/2*e)^3 + a^4*d^6*tan(1/2*f*x + 1/2*e)^3 - 3*a^4*c^6*tan(1/2*f*x + 1/2*e) + 36*a^4*c^5*d*tan(1/2*f*x + 1/2*e) - 135*a^4*c^4*d^2*tan(1/2*f*x + 1/2*e) + 240*a^4*c^3*d^3*tan(1/2*f*x + 1/2*e) - 225*a^4*c^2*d^4*tan(1/2*f*x + 1/2*e) + 108*a^4*c*d^5*tan(1/2*f*x + 1/2*e) - 21*a^4*d^6*tan(1/2*f*x + 1/2*e))/(a^6*c^9 - 9*a^6*c^8*d + 36*a^6*c^7*d^2 - 84*a^6*c^6*d^3 + 126*a^6*c^5*d^4 - 126*a^6*c^4*d^5 + 84*a^6*c^3*d^6 - 36*a^6*c^2*d^7 + 9*a^6*c*d^8 - a^6*d^9) + 6*(8*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 3*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 5*d^5*tan(1/2*f*x + 1/2*e)^3 - 8*c^2*d^3*tan(1/2*f*x + 1/2*e) - 11*c*d^4*tan(1/2*f*x + 1/2*e) - 3*d^5*tan(1/2*f*x + 1/2*e))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f

Mupad [B] (verification not implemented)

Time = 14.20 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^2(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\left(2a^2c^6-8a^2c^5d+10a^2c^4d^2-10a^2c^2d^4+8a^2cd^5-2a^2d^6\right)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(a^2c^6\right.\right.}$$
$$+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{2}{a^2(c-d)^3}-\frac{3(c+d)}{2a^2(c-d)^4}\right)}{f}-\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3}{6a^2f(c-d)^3}$$
$$-\frac{d^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^5-5\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^4d+10\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^3d^2-10\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2d^3+5\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)cd^4-1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^5}{\sqrt{c+d}\,(c-d)^{9/2}}\right)}{a^2f(c+d)^{5/2}(c-d)^{9/2}}$$

[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^2*(c + d/cos(e + f*x))^3),x)

[Out] ((tan(e/2 + (f*x)/2)^3*(3*c*d^4 + 5*d^5 - 8*c^2*d^3))/(c + d)^2 + (tan(e/2 + (f*x)/2)*(8*c*d^3 + 3*d^4))/(c + d))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^2*c^6 - 2*a^2*d^6 + 8*a^2*c*d^5 - 8*a^2*c^5*d - 10*a^2*c^2*d^4 + 10*a^2*c^4*d^2) - tan(e/2 + (f*x)/2)^4*(a^2*c^6 + a^2*d^6 - 6*a^2*c*d^5 - 6*a^2*c^5*d + 15*a^2*c^2*d^4 - 20*a^2*c^3*d^3 + 15*a^2*c^4*d^2) - a^2*c^6 - a^2*d^6 + 2*a^2*c*d^5 + 2*a^2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)) + (tan(e/2 + (f*x)/2)*(2/(a^2*(c - d)^3) - (3*(c + d))/(2*a^2*(c - d)^4)))/f - tan(e

```
/2 + (f*x)/2)^3/(6*a^2*f*(c - d)^3) - (d^2*atan((c^5*tan(e/2 + (f*x)/2)*1i
- d^5*tan(e/2 + (f*x)/2)*1i + c*d^4*tan(e/2 + (f*x)/2)*5i - c^4*d*tan(e/2 +
 (f*x)/2)*5i - c^2*d^3*tan(e/2 + (f*x)/2)*10i + c^3*d^2*tan(e/2 + (f*x)/2)*
10i)/((c + d)^(1/2)*(c - d)^(9/2)))*(16*c*d + 12*c^2 + 7*d^2)*1i)/(a^2*f*(c
 + d)^(5/2)*(c - d)^(9/2))
```

3.225 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 363

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx \\
&= \frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\operatorname{arctanh}(\sin(e+fx))}{2a^3f} \\
&\quad - \frac{2d(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)\tan(e+fx)}{15a^3f} \\
&\quad - \frac{d^2(4c^4+36c^3d+216c^2d^2-626cd^3+345d^4)\sec(e+fx)\tan(e+fx)}{30a^3f} \\
&\quad - \frac{d(2c^3+18c^2d+111cd^2-136d^3)\,(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} \\
&\quad + \frac{(c-d)\,(2c^2+18cd+115d^2)\,(c+d\sec(e+fx))^3\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))} \\
&\quad + \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}
\end{aligned}$$

```
[Out] 1/2*d^3*(40*c^3-90*c^2*d+78*c*d^2-23*d^3)*arctanh(sin(f*x+e))/a^3/f-2/15*d*
(2*c^5+18*c^4*d+107*c^3*d^2-472*c^2*d^3+456*c*d^4-136*d^5)*tan(f*x+e)/a^3/f
-1/30*d^2*(4*c^4+36*c^3*d+216*c^2*d^2-626*c*d^3+345*d^4)*sec(f*x+e)*tan(f*x
+e)/a^3/f-1/15*d*(2*c^3+18*c^2*d+111*c*d^2-136*d^3)*(c+d*sec(f*x+e))^2*tan(
f*x+e)/a^3/f+1/15*(c-d)*(2*c^2+18*c*d+115*d^2)*(c+d*sec(f*x+e))^3*tan(f*x+e
)/f/(a^3+a^3*sec(f*x+e))+1/15*(c-d)*(2*c+13*d)*(c+d*sec(f*x+e))^4*tan(f*x+e
)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*sec(f*x+e))^5*tan(f*x+e)/f/(a+a*sec
(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4072, 100, 155, 158, 152, 65, 223, 209}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{(c-d)\left(2c^2+18cd+115d^2\right)\tan(e+fx)(c+d\sec(e+fx))^3}{15f\left(a^3\sec(e+fx)+a^3\right)}$$
$$- \frac{d(2c^3+18c^2d+111cd^2-136d^3)\tan(e+fx)(c+d\sec(e+fx))^2}{15a^3f}$$
$$- \frac{d\tan(e+fx)\left(d(4c^4+36c^3d+216c^2d^2-626cd^3+345d^4)\sec(e+fx)+4(2c^5+18c^4d+107c^3d^2-47\right.}{30a^3f}$$
$$+ \frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+ \frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^5}{5f(a\sec(e+fx)+a)^3}$$
$$+ \frac{(c-d)(2c+13d)\tan(e+fx)(c+d\sec(e+fx))^4}{15af(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (d^3*(40*c^3 - 90*c^2*d + 78*c*d^2 - 23*d^3)*ArcTan[Sqrt[a - a*Sec[e + f*x]
]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]
*Sqrt[a + a*Sec[e + f*x]]) - (d*(2*c^3 + 18*c^2*d + 111*c*d^2 - 136*d^3)*(c
 + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a^3*f) + ((c - d)*(2*c^2 + 18*c*d +
115*d^2)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]
)) + ((c - d)*(2*c + 13*d)*(c + d*Sec[e + f*x])^4*Tan[e + f*x])/(15*a*f*(a
+ a*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f*x])^5*Tan[e + f*x])/(5*f*(
a + a*Sec[e + f*x])^3) - (d*(4*(2*c^5 + 18*c^4*d + 107*c^3*d^2 - 472*c^2*d^
3 + 456*c*d^4 - 136*d^5) + d*(4*c^4 + 36*c^3*d + 216*c^2*d^2 - 626*c*d^3 +
345*d^4)*Sec[e + f*x])*Tan[e + f*x])/(30*a^3*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 158

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
```

```
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c+dx)^6}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)^4\left(-a^2\left(2c^2+8cd-5d^2\right)+a^2(3c-8d)dx\right)}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{5af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)^3\left(-a^4\left(2c^3+10c^2d+55cd^2-52d^3\right)+3a^4d\left(2c^2+14cd-21d^2\right)x\right)}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx,x,\sec(e+fx)\right)}{15a^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c-d)\left(2c^2+18cd+115d^2\right)(c+d\sec(e+fx))^3\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}+\frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)^2\left(-3a^6d^2\left(2c^2+118cd-115d^2\right)+3a^6d\left(2c^3+18c^2d+111cd^2-136d^3\right)x\right)}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx\right.}{15a^7f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(2c^3+18c^2d+111cd^2-136d^3)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)(2c^2+18cd+115d^2)(c+d\sec(e+fx))^3\tan(e+fx)}{15f(a^3+a^3\sec(e+fx))} + \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)(3a^8d^2(2c^3+318c^2d-567cd^2+272d^3)-3a^8d(4c^4+36c^3d+216c^2d^2-626cd^3+345d^4)x)}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \text{s}}{45a^9f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(2c^3+18c^2d+111cd^2-136d^3)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)(2c^2+18cd+115d^2)(c+d\sec(e+fx))^3\tan(e+fx)}{15f(a^3+a^3\sec(e+fx))} + \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{d(4(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)+d(4c^4+36c^3d+216c^2d^2-626cd^3+}{30a^3f} - \frac{(d^3(40c^3-90c^2d+78cd^2-23d^3)\tan(e+fx))\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{2af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(2c^3+18c^2d+111cd^2-136d^3)(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)(2c^2+18cd+115d^2)(c+d\sec(e+fx))^3\tan(e+fx)}{15f(a^3+a^3\sec(e+fx))} + \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{d(4(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)+d(4c^4+36c^3d+216c^2d^2-626cd^3+}{30a^3f} + \frac{(d^3(40c^3-90c^2d+78cd^2-23d^3)\tan(e+fx))\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d(2c^3+18c^2d+111cd^2-136d^3)\,(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)\,(2c^2+18cd+115d^2)\,(c+d\sec(e+fx))^3\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))} + \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{d(4(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)+d(4c^4+36c^3d+216c^2d^2-626cd^3+}{30a^3f} + \frac{(d^3(40c^3-90c^2d+78cd^2-23d^3)\tan(e+fx))\,\mathrm{Subst}\left(\int\frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^3(40c^3-90c^2d+78cd^2-23d^3)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} - \frac{d(2c^3+18c^2d+111cd^2-136d^3)\,(c+d\sec(e+fx))^2\tan(e+fx)}{15a^3f} + \frac{(c-d)\,(2c^2+18cd+115d^2)\,(c+d\sec(e+fx))^3\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))} + \frac{(c-d)(2c+13d)(c+d\sec(e+fx))^4\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^5\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{d(4(2c^5+18c^4d+107c^3d^2-472c^2d^3+456cd^4-136d^5)+d(4c^4+36c^3d+216c^2d^2-626cd^3+}{30a^3f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1338 vs. 2(363) = 726.

Time = 9.81 (sec) , antiderivative size = 1338, normalized size of antiderivative = 3.69

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{4(-40c^3d^3+90c^2d^4-78cd^5+23d^6)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(c+d\sec}{f(d+c\cos(e+fx))^6(a+a\sec(e+fx))^3}$$
$$-\frac{4(-40c^3d^3+90c^2d^4-78cd^5+23d^6)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(c+d}{f(d+c\cos(e+fx))^6(a+a\sec(e+fx))^3}$$
$$+\frac{2\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\sec\left(\frac{e}{2}\right)(c+d\sec(e+fx))^6\left(c^6\sin\left(\frac{e}{2}\right)-6c^5d\sin\left(\frac{e}{2}\right)+15c^4d^2\sin\left(\frac{e}{2}\right)-20c\right.}{5f(d+c\cos(e+fx))^6(a+a\sec(e+fx))^3}$$
$$+\frac{8\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\sec\left(\frac{e}{2}\right)(c+d\sec(e+fx))^6\left(-4c^6\sin\left(\frac{e}{2}\right)+9c^5d\sin\left(\frac{e}{2}\right)+15c^4d^2\sin\left(\frac{e}{2}\right)-\right.}{15f(d+c\cos(e+fx))^6(a+a\sec(e+fx))}$$
$$+\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\sec\left(\frac{e}{2}\right)(c+d\sec(e+fx))^6\left(c^6\sin\left(\frac{fx}{2}\right)-6c^5d\sin\left(\frac{fx}{2}\right)+15c^4d^2\sin\left(\frac{fx}{2}\right)-2\right.}{5f(d+c\cos(e+fx))^6(a+a\sec(e+fx))^3}$$
$$+\frac{8\cos^3\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\sec\left(\frac{e}{2}\right)(c+d\sec(e+fx))^6\left(-4c^6\sin\left(\frac{fx}{2}\right)+9c^5d\sin\left(\frac{fx}{2}\right)+15c^4d^2\sin\left(\frac{fx}{2}\right)\right.}{15f(d+c\cos(e+fx))^6(a+a\sec(e+f}$$
$$+\frac{8\cos^5\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^3(e+fx)\sec\left(\frac{e}{2}\right)(c+d\sec(e+fx))^6\left(7c^6\sin\left(\frac{fx}{2}\right)+18c^5d\sin\left(\frac{fx}{2}\right)+30c^4d^2\sin\left(\frac{fx}{2}\right)\right.}{15f(d+c\cos(e+fx))^6(a+a\sec(e+}$$
$$+\frac{8d^6\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\sec(e)(c+d\sec(e+fx))^6\sin(fx)}{3f(d+c\cos(e+fx))^6(a+a\sec(e+fx))^3}$$
$$-\frac{4\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\cos^2(e+fx)\sec(e)(c+d\sec(e+fx))^6\left(-18cd^5\sin(e)+9d^6\sin(e)-90c^2d^4\sin(fx)+10\right.}{3f(d+c\cos(e+fx))^6(a+a\sec(e+fx))^3}$$
$$+\frac{4\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)\cos(e+fx)\sec(e)(c+d\sec(e+fx))^6\left(2d^6\sin(e)+18cd^5\sin(fx)-9d^6\sin(fx)\right)}{3f(d+c\cos(e+fx))^6(a+a\sec(e+fx))^3}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^6)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (4*(-40*c^3*d^3 + 90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[
e + f*x]^3*Log[Cos[e/2 + (f*x)/2] - Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x]
)^6)/(f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*(-40*c^3*d^3 +
90*c^2*d^4 - 78*c*d^5 + 23*d^6)*Cos[e/2 + (f*x)/2]^6*Cos[e + f*x]^3*Log[Cos
[e/2 + (f*x)/2] + Sin[e/2 + (f*x)/2]]*(c + d*Sec[e + f*x])^6)/(f*(d + c*Cos
[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]^2*Cos[e + f*x]
^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[e/2] - 6*c^5*d*Sin[e/2] + 15*c^
4*d^2*Sin[e/2] - 20*c^3*d^3*Sin[e/2] + 15*c^2*d^4*Sin[e/2] - 6*c*d^5*Sin[e/
2] + d^6*Sin[e/2]))/(5*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (
8*Cos[e/2 + (f*x)/2]^4*Cos[e + f*x]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(-4*c
^6*Sin[e/2] + 9*c^5*d*Sin[e/2] + 15*c^4*d^2*Sin[e/2] - 70*c^3*d^3*Sin[e/2]
```

```
+ 90*c^2*d^4*Sin[e/2] - 51*c*d^5*Sin[e/2] + 11*d^6*Sin[e/2]))/(15*f*(d + c*
Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (2*Cos[e/2 + (f*x)/2]*Cos[e + f*x
]^3*Sec[e/2]*(c + d*Sec[e + f*x])^6*(c^6*Sin[(f*x)/2] - 6*c^5*d*Sin[(f*x)/2
] + 15*c^4*d^2*Sin[(f*x)/2] - 20*c^3*d^3*Sin[(f*x)/2] + 15*c^2*d^4*Sin[(f*x
)/2] - 6*c*d^5*Sin[(f*x)/2] + d^6*Sin[(f*x)/2]))/(5*f*(d + c*Cos[e + f*x])^
6*(a + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^3*Cos[e + f*x]^3*Sec[e/2]
*(c + d*Sec[e + f*x])^6*(-4*c^6*Sin[(f*x)/2] + 9*c^5*d*Sin[(f*x)/2] + 15*c^
4*d^2*Sin[(f*x)/2] - 70*c^3*d^3*Sin[(f*x)/2] + 90*c^2*d^4*Sin[(f*x)/2] - 51
*c*d^5*Sin[(f*x)/2] + 11*d^6*Sin[(f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^6*(a
 + a*Sec[e + f*x])^3) + (8*Cos[e/2 + (f*x)/2]^5*Cos[e + f*x]^3*Sec[e/2]*(c
+ d*Sec[e + f*x])^6*(7*c^6*Sin[(f*x)/2] + 18*c^5*d*Sin[(f*x)/2] + 30*c^4*d^
2*Sin[(f*x)/2] - 440*c^3*d^3*Sin[(f*x)/2] + 855*c^2*d^4*Sin[(f*x)/2] - 642*
c*d^5*Sin[(f*x)/2] + 172*d^6*Sin[(f*x)/2]))/(15*f*(d + c*Cos[e + f*x])^6*(a
 + a*Sec[e + f*x])^3) + (8*d^6*Cos[e/2 + (f*x)/2]^6*Sec[e]*(c + d*Sec[e + f
*x])^6*Sin[f*x])/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) - (4*C
os[e/2 + (f*x)/2]^6*Cos[e + f*x]^2*Sec[e]*(c + d*Sec[e + f*x])^6*(-18*c*d^5
*Sin[e] + 9*d^6*Sin[e] - 90*c^2*d^4*Sin[f*x] + 108*c*d^5*Sin[f*x] - 40*d^6*
Sin[f*x]))/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e + f*x])^3) + (4*Cos[e/2
 + (f*x)/2]^6*Cos[e + f*x]*Sec[e]*(c + d*Sec[e + f*x])^6*(2*d^6*Sin[e] + 18
*c*d^5*Sin[f*x] - 9*d^6*Sin[f*x]))/(3*f*(d + c*Cos[e + f*x])^6*(a + a*Sec[e
 + f*x])^3)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.35

method	result
parallelrisch	$\frac{-14400\left(c^{3}-\frac{9}{4}c^{2}d+\frac{39}{20}c\,d^{2}-\frac{23}{40}d^{3}\right)\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)d^{3}\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+14400\left(c^{3}-\frac{9}{4}c^{2}d+\frac{39}{20}c\,d^{2}-\frac{23}{40}d^{3}\right)}{}$
derivativedivides	$\frac{-\frac{4d^{6}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}+c^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+49d^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{4d^{6}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{d^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{2c^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{10}{}}{}$
default	$\frac{-\frac{4d^{6}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{3}}+c^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+49d^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-\frac{4d^{6}}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}+\frac{d^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}{5}-\frac{2c^{6}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{10}{}}{}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/240*(-14400*(c^3-9/4*c^2*d+39/20*c*d^2-23/40*d^3)*(cos(f*x+e)+1/3*cos(3*f
*x+3*e))*d^3*ln(tan(1/2*f*x+1/2*e)-1)+14400*(c^3-9/4*c^2*d+39/20*c*d^2-23/4
0*d^3)*(cos(f*x+e)+1/3*cos(3*f*x+3*e))*d^3*ln(tan(1/2*f*x+1/2*e)+1)+12*tan(
1/2*f*x+1/2*e)*((43/12*c^6+1549/6*d^6-859*c*d^5+95/2*c^4*d^2-1190/3*c^3*d^3
+1035*c^2*d^4+27/2*c^5*d)*cos(3*f*x+3*e)+(36*c^5*d+4*c^6+1382/3*d^6-1524*c*
```

d^5+60*c^4*d^2-680*c^3*d^3+1860*c^2*d^4)*cos(2*f*x+2*e)+(c^6+429/4*d^6+9*c^5*d+15*c^4*d^2-170*c^3*d^3+855/2*c^2*d^4-717/2*c*d^5)*cos(4*f*x+4*e)+(7/12*c^6+68/3*d^6-76*c*d^5+3/2*c^5*d+5/2*c^4*d^2-110/3*c^3*d^3+90*c^2*d^4)*cos(5*f*x+5*e)+(3907/6*d^6+33*c^5*d+47/6*c^6-3020/3*c^3*d^3+2655*c^2*d^4-2137*c*d^5+130*c^4*d^2)*cos(f*x+e)+4321/12*d^6+3*c^6+27*c^5*d+45*c^4*d^2-510*c^3*d^3+2865/2*c^2*d^4-2331/2*c*d^5)*sec(1/2*f*x+1/2*e)^4)/f/a^3/(cos(3*f*x+3*e)+3*cos(f*x+e))

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.71

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \frac{15\left(\left(40\,c^3d^3-90\,c^2d^4+78\,cd^5-23\,d^6\right)\cos\left(fx+e\right)^6+3\left(40\,c^3d^3-90\,c^2d^4+78\,cd^5-23\,d^6\right)\cos\left(fx+e\right)^5\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(15*((40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^3)*log(sin(f*x + e) + 1) - 15*((40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^6 + 3*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^5 + 3*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^4 + (40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*cos(f*x + e)^3)*log(-sin(f*x + e) + 1) + 2*(10*d^6 + 2*(7*c^6 + 18*c^5*d + 30*c^4*d^2 - 440*c^3*d^3 + 1080*c^2*d^4 - 912*c*d^5 + 272*d^6)*cos(f*x + e)^5 + 3*(4*c^6 + 36*c^5*d + 60*c^4*d^2 - 680*c^3*d^3 + 1710*c^2*d^4 - 1434*c*d^5 + 429*d^6)*cos(f*x + e)^4 + (4*c^6 + 36*c^5*d + 210*c^4*d^2 - 1280*c^3*d^3 + 3510*c^2*d^4 - 2874*c*d^5 + 869*d^6)*cos(f*x + e)^3 + 5*(90*c^2*d^4 - 54*c*d^5 + 19*d^6)*cos(f*x + e)^2 + 15*(6*c*d^5 - d^6)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^6 + 3*a^3*f*cos(f*x + e)^5 + 3*a^3*f*cos(f*x + e)^4 + a^3*f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^6 \sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^6 \sec^7(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{6cd^5 \sec^6(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+\dots}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**6/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**6*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**6*sec(e + f*x)**7/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*d**5*sec(e + f*x)**6/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c
**2*d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(20*c**3*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*s
ec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(15*c**4*d**2*sec(e + f*
x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Inte
gral(6*c**5*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(
e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(349) = 698$.

Time = 0.25 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.61

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(d^6*(20*(33*sin(f*x + e)/(cos(f*x + e) + 1) - 76*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 51*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^3 - 3*a^3*sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
 - a^3*sin(f*x + e)^6/(cos(f*x + e) + 1)^6) + (735*sin(f*x + e)/(cos(f*x +
e) + 1) + 50*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3 - 690*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 690
*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 6*c*d^5*(60*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^3 - 2*a^3*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 390*log(sin(f*x + e
)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1
```

```
)/a^3) + 45*c^2*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a
^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)/a^3) - 20*c^3*d^3*((105*sin(f*x + e)/(cos(f*x + e)
+ 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
 e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(
sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 15*c^4*d^2*(15*sin(f*x + e)/(co
s(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5
/(cos(f*x + e) + 1)^5)/a^3 + c^6*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3 + 18*c^5*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*
x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.85

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx$$

$$= \frac{\frac{30\left(40\,c^3d^3-90\,c^2d^4+78\,cd^5-23\,d^6\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3}-\frac{30\left(40\,c^3d^3-90\,c^2d^4+78\,cd^5-23\,d^6\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3}-\frac{20\left(90\,c^2d^4\,\mathrm{ta}\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^6/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*log(abs(tan(1/2*f*x
+ 1/2*e) + 1))/a^3 - 30*(40*c^3*d^3 - 90*c^2*d^4 + 78*c*d^5 - 23*d^6)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - 20*(90*c^2*d^4*tan(1/2*f*x + 1/2*e)^5 -
 126*c*d^5*tan(1/2*f*x + 1/2*e)^5 + 51*d^6*tan(1/2*f*x + 1/2*e)^5 - 180*c^2
*d^4*tan(1/2*f*x + 1/2*e)^3 + 216*c*d^5*tan(1/2*f*x + 1/2*e)^3 - 76*d^6*tan
(1/2*f*x + 1/2*e)^3 + 90*c^2*d^4*tan(1/2*f*x + 1/2*e) - 90*c*d^5*tan(1/2*f*
x + 1/2*e) + 33*d^6*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a
^3) + (3*a^12*c^6*tan(1/2*f*x + 1/2*e)^5 - 18*a^12*c^5*d*tan(1/2*f*x + 1/2*
e)^5 + 45*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e)^5 - 60*a^12*c^3*d^3*tan(1/2*f*x
 + 1/2*e)^5 + 45*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 18*a^12*c*d^5*tan(1/
2*f*x + 1/2*e)^5 + 3*a^12*d^6*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^6*tan(1/2*
f*x + 1/2*e)^3 + 150*a^12*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 400*a^12*c^3*d^3
*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 - 240*a^1
2*c*d^5*tan(1/2*f*x + 1/2*e)^3 + 50*a^12*d^6*tan(1/2*f*x + 1/2*e)^3 + 15*a^
12*c^6*tan(1/2*f*x + 1/2*e) + 90*a^12*c^5*d*tan(1/2*f*x + 1/2*e) + 225*a^12
*c^4*d^2*tan(1/2*f*x + 1/2*e) - 2100*a^12*c^3*d^3*tan(1/2*f*x + 1/2*e) + 38
```

```
25*a^12*c^2*d^4*tan(1/2*f*x + 1/2*e) - 2790*a^12*c*d^5*tan(1/2*f*x + 1/2*e)
 + 735*a^12*d^6*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 13.62 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.90

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^6}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(\frac{5\,(c-d)^6}{2\,a^3}-\frac{6\,(c+d)\,(c-d)^5}{a^3}+\frac{15\,(c+d)^2\,(c-d)^4}{4\,a^3}\right)}{f} - \frac{(30\,c^2\,d^4-42\,c\,d^5+17\,d^6)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5+\left(-60\,c^2\,d^4+72\,c\,d^5-\frac{76\,d^6}{3}\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3+(30\,c^2\,d^4-30}{f\,\left(a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^6-3\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4+3\,a^3\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-a^3\right)} + \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(\frac{(c-d)^6}{3\,a^3}-\frac{(c+d)\,(c-d)^5}{2\,a^3}\right)}{f} + \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(c-d)^6}{20\,a^3\,f} + \frac{d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,(40\,c^3-90\,c^2\,d+78\,c\,d^2-23\,d^3)}{a^3\,f}$$

```
[In] int((c + d/cos(e + f*x))^6/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((5*(c - d)^6)/(2*a^3) - (6*(c + d)*(c - d)^5)/a^3 + (1
5*(c + d)^2*(c - d)^4)/(4*a^3)))/f - (tan(e/2 + (f*x)/2)*(11*d^6 - 30*c*d^5
 + 30*c^2*d^4) + tan(e/2 + (f*x)/2)^5*(17*d^6 - 42*c*d^5 + 30*c^2*d^4) - ta
n(e/2 + (f*x)/2)^3*((76*d^6)/3 - 72*c*d^5 + 60*c^2*d^4))/(f*(3*a^3*tan(e/2
+ (f*x)/2)^2 - 3*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^6 - a^3)
) + (tan(e/2 + (f*x)/2)^3*((c - d)^6/(3*a^3) - ((c + d)*(c - d)^5)/(2*a^3))
)/f + (tan(e/2 + (f*x)/2)^5*(c - d)^6)/(20*a^3*f) + (d^3*atanh(tan(e/2 + (f
*x)/2))*(78*c*d^2 - 90*c^2*d + 40*c^3 - 23*d^3))/(a^3*f)
```

3.226 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 287

$$\begin{aligned}
\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx
&= \frac{d^3(20c^2-30cd+13d^2)\,\text{arctanh}(\sin(e+fx))}{2a^3f} \\
&\quad - \frac{2d(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)\tan(e+fx)}{15a^3f} \\
&\quad - \frac{d^2(4c^3+30c^2d+146cd^2-195d^3)\sec(e+fx)\tan(e+fx)}{30a^3f} \\
&\quad + \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} \\
&\quad + \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}
\end{aligned}$$

```
[Out] 1/2*d^3*(20*c^2-30*c*d+13*d^2)*arctanh(sin(f*x+e))/a^3/f-2/15*d*(2*c^4+15*c
^3*d+72*c^2*d^2-180*c*d^3+76*d^4)*tan(f*x+e)/a^3/f-1/30*d^2*(4*c^3+30*c^2*d
+146*c*d^2-195*d^3)*sec(f*x+e)*tan(f*x+e)/a^3/f+1/15*(c-d)*(2*c^2+15*c*d+76
*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))+1/15*(c-d)*(2*c+
11*d)*(c+d*sec(f*x+e))^3*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*s
ec(f*x+e))^4*tan(f*x+e)/f/(a+a*sec(f*x+e))^3
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 100, 155, 152, 65, 223, 209}

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx \\
&= \frac{(c-d)\left(2c^2+15cd+76d^2\right)\tan(e+fx)(c+d\sec(e+fx))^2}{15f\left(a^3\sec(e+fx)+a^3\right)} \\
&\quad -\frac{d\tan(e+fx)\left(d\left(4c^3+30c^2d+146cd^2-195d^3\right)\sec(e+fx)+4\left(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4\right)\right)}{30a^3f} \\
&\quad +\frac{d^3\left(20c^2-30cd+13d^2\right)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} \\
&\quad +\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^4}{5f(a\sec(e+fx)+a)^3} \\
&\quad +\frac{(c-d)(2c+11d)\tan(e+fx)(c+d\sec(e+fx))^3}{15af(a\sec(e+fx)+a)^2}
\end{aligned}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (d^3*(20*c^2 - 30*c*d + 13*d^2)*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 +
 Sec[e + f*x])]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) + ((c - d)*(2*c^2 + 15*c*d + 76*d^2)*(c + d*Sec[e + f*x])^2*T
an[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x])) + ((c - d)*(2*c + 11*d)*(c + d
*Sec[e + f*x])^3*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((c - d)*(
c + d*Sec[e + f*x])^4*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) - (d*(4*(2
*c^4 + 15*c^3*d + 72*c^2*d^2 - 180*c*d^3 + 76*d^4) + d*(4*c^3 + 30*c^2*d +
146*c*d^2 - 195*d^3)*Sec[e + f*x])*Tan[e + f*x])/(30*a^3*f)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
```

```
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
```

```
egerQ[m - 1/2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int\frac{(c+dx)^5}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)^3(-a^2(2c-d)(c+4d)+a^2(2c-7d)dx)}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{5af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\\
&\quad+\frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)^2\left(-a^4\left(2c^3+9c^2d+37cd^2-33d^3\right)+a^4d\left(4c^2+24cd-43d^2\right)x\right)}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx,x,\sec(e+fx)\right)}{15a^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}\\
&\quad+\frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\\
&\quad+\frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{(c+dx)\left(-a^6d^2\left(2c^2+165cd-152d^2\right)+a^6d\left(4c^3+30c^2d+146cd^2-195d^3\right)x\right)}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{15a^7f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}\\
&\quad+\frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\\
&\quad+\frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad-\frac{d\left(4\left(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4\right)+d\left(4c^3+30c^2d+146cd^2-195d^3\right)\sec(e+fx)\right)\text{ta}}{30a^3f}\\
&\quad-\frac{\left(d^3\left(20c^2-30cd+13d^2\right)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{2af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{d(4(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)+d(4c^3+30c^2d+146cd^2-195d^3)\sec(e+fx))\tan}{30a^3f} + \frac{\left(d^3(20c^2-30cd+13d^2)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{d(4(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)+d(4c^3+30c^2d+146cd^2-195d^3)\sec(e+fx))\tan}{30a^3f} + \frac{\left(d^3(20c^2-30cd+13d^2)\tan(e+fx)\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{d^3(20c^2-30cd+13d^2)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)\left(2c^2+15cd+76d^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)} + \frac{(c-d)(2c+11d)(c+d\sec(e+fx))^3\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^4\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{d(4(2c^4+15c^3d+72c^2d^2-180cd^3+76d^4)+d(4c^3+30c^2d+146cd^2-195d^3)\sec(e+fx))\tan}{30a^3f}$$

Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.53

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{-480d^3(20c^2-30cd+13d^2)\cos^6\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-480*d^3*(20*c^2 - 30*c*d + 13*d^2)*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/
2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Cos[
(e + f*x)/2]*(29*c^5 + 105*c^4*d + 340*c^3*d^2 - 1940*c^2*d^3 + 3420*c*d^4
- 1354*d^5 + 3*(12*c^5 + 90*c^4*d + 120*c^3*d^2 - 1020*c^2*d^3 + 1910*c*d^4
 - 777*d^5)*Cos[e + f*x] + 6*(6*c^5 + 20*c^4*d + 60*c^3*d^2 - 360*c^2*d^3 +
 630*c*d^4 - 261*d^5)*Cos[2*(e + f*x)] + 12*c^5*Cos[3*(e + f*x)] + 90*c^4*d
*Cos[3*(e + f*x)] + 120*c^3*d^2*Cos[3*(e + f*x)] - 1020*c^2*d^3*Cos[3*(e +
f*x)] + 1710*c*d^4*Cos[3*(e + f*x)] - 717*d^5*Cos[3*(e + f*x)] + 7*c^5*Cos[
4*(e + f*x)] + 15*c^4*d*Cos[4*(e + f*x)] + 20*c^3*d^2*Cos[4*(e + f*x)] - 22
0*c^2*d^3*Cos[4*(e + f*x)] + 360*c*d^4*Cos[4*(e + f*x)] - 152*d^5*Cos[4*(e
+ f*x)])*Sec[e + f*x]^2*Sin[(e + f*x)/2])/(120*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.25

method	result
parallelrisch	$\frac{-2400\left(c^2-\frac{3}{2}cd+\frac{13}{20}d^2\right)(1+\cos(2fx+2e))d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+2400\left(c^2-\frac{3}{2}cd+\frac{13}{20}d^2\right)(1+\cos(2fx+2e))d^3\ln\left(\tan\left(\frac{f}{2}\right.\right.}{}$
derivativedivides	$\frac{\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-c^4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+2c^3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-2c^2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+c\,d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-\frac{d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-}{}$
default	$\frac{\frac{c^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-c^4d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+2c^3d^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-2c^2d^3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5+c\,d^4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5-\frac{d^5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-}{}$
norman	$\frac{\frac{\left(c^5-5c^4d+10c^3d^2-10c^2d^3+5c\,d^4-d^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{15}}{20af}-\frac{5\left(c^5-3c^4d+2c^3d^2+2c^2d^3-3c\,d^4+d^5\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{12af}-\frac{\left(c^5+5c^4d+10c^3\right.}{}}{}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/240*(-2400*(c^2-3/2*c*d+13/20*d^2)*(1+cos(2*f*x+2*e))*d^3*ln(tan(1/2*f*x+
1/2*e)-1)+2400*(c^2-3/2*c*d+13/20*d^2)*(1+cos(2*f*x+2*e))*d^3*ln(tan(1/2*f*
x+1/2*e)+1)+7*sec(1/2*f*x+1/2*e)^4*tan(1/2*f*x+1/2*e)*(6*(20/7*c^4*d+6/7*c^
5-261/7*d^5+60/7*c^3*d^2-360/7*c^2*d^3+90*c*d^4)*cos(2*f*x+2*e)+3/7*(4*c^5+
30*c^4*d+40*c^3*d^2-340*c^2*d^3+570*c*d^4-239*d^5)*cos(3*f*x+3*e)+(c^5-152/
7*d^5+15/7*c^4*d+20/7*c^3*d^2-220/7*c^2*d^3+360/7*c*d^4)*cos(4*f*x+4*e)+3*(
-111*d^5+120/7*c^3*d^2+12/7*c^5+90/7*c^4*d-1020/7*c^2*d^3+1910/7*c*d^4)*cos
(f*x+e)-1354/7*d^5+29/7*c^5+15*c^4*d+340/7*c^3*d^2-1940/7*c^2*d^3+3420/7*c*
d^4))/f/a^3/(1+cos(2*f*x+2*e))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.75

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx = \frac{15\left((20\,c^2d^3-30\,cd^4+13\,d^5)\cos\left(fx+e\right)^5+3\left(20\,c^2d^3-30\,cd^4+13\,d^5\right)\cos\left(fx+e\right)^4+3\left(20\,c^2d^3-30\,cd\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2*d^3 -
 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos
(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(sin(f*x
+ e) + 1) - 15*((20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^5 + 3*(20*c^2
*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^4 + 3*(20*c^2*d^3 - 30*c*d^4 + 13*d^
5)*cos(f*x + e)^3 + (20*c^2*d^3 - 30*c*d^4 + 13*d^5)*cos(f*x + e)^2)*log(-s
in(f*x + e) + 1) + 2*(15*d^5 + 2*(7*c^5 + 15*c^4*d + 20*c^3*d^2 - 220*c^2*d
^3 + 360*c*d^4 - 152*d^5)*cos(f*x + e)^4 + 3*(4*c^5 + 30*c^4*d + 40*c^3*d^2
 - 340*c^2*d^3 + 570*c*d^4 - 239*d^5)*cos(f*x + e)^3 + (4*c^5 + 30*c^4*d +
140*c^3*d^2 - 640*c^2*d^3 + 1170*c*d^4 - 479*d^5)*cos(f*x + e)^2 + 15*(10*c
*d^4 - 3*d^5)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^5 + 3*a^3*f*c
os(f*x + e)^4 + 3*a^3*f*cos(f*x + e)^3 + a^3*f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^5\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^5\sec^6(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{5cd^4\sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**5*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**5*sec(e + f*x)**6/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c*d**4*sec(e + f*x)**5/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(10*c
**2*d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(10*c**3*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*s
ec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(5*c**4*d*sec(e + f*x)**
2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(275) = 550$.

Time = 0.24 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx =$$
$$-\frac{d^5\left(\frac{60\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{a^3-\frac{2a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}} + \frac{\frac{465\sin(fx+e)}{\cos(fx+e)+1}+\frac{40\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{390\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3} + \frac{390\log\left(\frac{\sin}{\cos}\right.}{}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(d^5*(60*(5*sin(f*x + e)/(cos(f*x + e) + 1) - 7*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3)/(a^3 - 2*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4) + (465*sin(f*x + e)/(cos(f*x + e) + 1) + 4
0*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5)/a^3 - 390*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 390*log(sin(f*
x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 15*c*d^4*(40*sin(f*x + e)/((a^3 - a^3
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e
)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e
)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)
/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) + 10*c^2*d^3*((105*
```

```
sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*x + e)/(cos(f*x +
 e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/a^3) - 10*c
^3*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^5*(15*sin(f*x + e
)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x +
 e)^5/(cos(f*x + e) + 1)^5)/a^3 - 15*c^4*d*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.76

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$

$$= \frac{\frac{30\,(20\,c^2d^3-30\,cd^4+13\,d^5)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} - \frac{30\,(20\,c^2d^3-30\,cd^4+13\,d^5)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} - \frac{60\left(10\,cd^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3-7\,d}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*log(abs(tan(1/2*f*x + 1/2*e) + 1)
)/a^3 - 30*(20*c^2*d^3 - 30*c*d^4 + 13*d^5)*log(abs(tan(1/2*f*x + 1/2*e) -
1))/a^3 - 60*(10*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 7*d^5*tan(1/2*f*x + 1/2*e)^
3 - 10*c*d^4*tan(1/2*f*x + 1/2*e) + 5*d^5*tan(1/2*f*x + 1/2*e))/((tan(1/2*f
*x + 1/2*e)^2 - 1)^2*a^3) + (3*a^12*c^5*tan(1/2*f*x + 1/2*e)^5 - 15*a^12*c^
4*d*tan(1/2*f*x + 1/2*e)^5 + 30*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 30*a^
12*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 15*a^12*c*d^4*tan(1/2*f*x + 1/2*e)^5 -
3*a^12*d^5*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^5*tan(1/2*f*x + 1/2*e)^3 + 10
0*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 200*a^12*c^2*d^3*tan(1/2*f*x + 1/2*
e)^3 + 150*a^12*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 40*a^12*d^5*tan(1/2*f*x + 1/
2*e)^3 + 15*a^12*c^5*tan(1/2*f*x + 1/2*e) + 75*a^12*c^4*d*tan(1/2*f*x + 1/2
*e) + 150*a^12*c^3*d^2*tan(1/2*f*x + 1/2*e) - 1050*a^12*c^2*d^3*tan(1/2*f*x
 + 1/2*e) + 1275*a^12*c*d^4*tan(1/2*f*x + 1/2*e) - 465*a^12*d^5*tan(1/2*f*x
 + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 13.86 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3(c-d)^5}{2a^3}-\frac{15(c+d)(c-d)^4}{4a^3}+\frac{5(c+d)^2(c-d)^3}{2a^3}\right)}{f}$$
$$+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(10cd^4-5d^5)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(10cd^4-7d^5)}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-2a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+a^3\right)}$$
$$+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{(c-d)^5}{4a^3}-\frac{5(c+d)(c-d)^4}{12a^3}\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(c-d)^5}{20a^3f}$$
$$+\frac{d^3\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(20c^2-30cd+13d^2)}{a^3f}$$

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*(c - d)^5)/(2*a^3) - (15*(c + d)*(c - d)^4)/(4*a^3)
 + (5*(c + d)^2*(c - d)^3)/(2*a^3)))/f + (tan(e/2 + (f*x)/2)*(10*c*d^4 - 5*
d^5) - tan(e/2 + (f*x)/2)^3*(10*c*d^4 - 7*d^5))/(f*(a^3*tan(e/2 + (f*x)/2)^
4 - 2*a^3*tan(e/2 + (f*x)/2)^2 + a^3)) + (tan(e/2 + (f*x)/2)^3*((c - d)^5/(
4*a^3) - (5*(c + d)*(c - d)^4)/(12*a^3)))/f + (tan(e/2 + (f*x)/2)^5*(c - d)
^5)/(20*a^3*f) + (d^3*atanh(tan(e/2 + (f*x)/2))*(20*c^2 - 30*c*d + 13*d^2))
/(a^3*f)
```

3.227 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 205

$$
\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx \\
&= \frac{(4c-3d)d^3\operatorname{arctanh}(\sin(e+fx))}{a^3f} + \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} \\
&\quad + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} \\
&\quad + \frac{(2c^4+8c^3d+21c^2d^2-88cd^3+72d^4-d^2(2c^2+10cd-27d^2)\sec(e+fx))\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))}
\end{aligned}
$$

```
[Out] (4*c-3*d)*d^3*arctanh(sin(f*x+e))/a^3/f+1/15*(c-d)*(2*c+9*d)*(c+d*sec(f*x+e
))^2*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2+1/5*(c-d)*(c+d*sec(f*x+e))^3*tan(f*x
+e)/f/(a+a*sec(f*x+e))^3+1/15*(2*c^4+8*c^3*d+21*c^2*d^2-88*c*d^3+72*d^4-d^2
*(2*c^2+10*c*d-27*d^2)*sec(f*x+e))*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used

= {4072, 100, 155, 148, 65, 223, 209}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\tan(e+fx)\left(2c^4+8c^3d-d^2(2c^2+10cd-27d^2)\sec(e+fx)+21c^2d^2-88cd^3+72d^4\right)}{15f\left(a^3\sec(e+fx)+a^3\right)}$$
$$+ \frac{2d^3(4c-3d)\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+ \frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^3}{5f(a\sec(e+fx)+a)^3}$$
$$+ \frac{(c-d)(2c+9d)\tan(e+fx)(c+d\sec(e+fx))^2}{15af(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*(4*c - 3*d)*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x]
)]]*Tan[e + f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])
 + ((c - d)*(2*c + 9*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(15*a*f*(a + a
*Sec[e + f*x])^2) + ((c - d)*(c + d*Sec[e + f*x])^3*Tan[e + f*x])/(5*f*(a +
 a*Sec[e + f*x])^3) + ((2*c^4 + 8*c^3*d + 21*c^2*d^2 - 88*c*d^3 + 72*d^4 -
d^2*(2*c^2 + 10*c*d - 27*d^2)*Sec[e + f*x])*Tan[e + f*x])/(15*f*(a^3 + a^3*
Sec[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b^2*d*e*g - a^2*d*f*h*m - a*b*(d
```

```
*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d*(b*c - a*d)*(m + 1))), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2
*p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{(c+dx)^4}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)^2(-a^2(2c^2+6cd-3d^2)+a^2(c-6d)dx)}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)(-a^4(2c^3+8c^2d+23cd^2-18d^3)+a^4d(2c^2+10cd-27d^2)x)}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{15a^4f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c^4+8c^3d+21c^2d^2-88cd^3+72d^4-d^2(2c^2+10cd-27d^2)\sec(e+fx))\tan(e+fx)}{15f(a^3+a^3\sec(e+fx))} - \frac{((4c-3d)d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx, x, \sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c^4+8c^3d+21c^2d^2-88cd^3+72d^4-d^2(2c^2+10c}{15f(a^3+a^3\sec(e+} + \frac{(2(4c-3d)d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx, x, \sqrt{a-a\sec(e+fx)}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c^4+8c^3d+21c^2d^2-88cd^3+72d^4-d^2(2c^2+10cd-27d^2)\sec(e+fx))\tan(e+fx)}{15f(a^3+a^3\sec(e+fx))} + \frac{(2(4c-3d)d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{2(4c-3d)d^3\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(c-d)(2c+9d)(c+d\sec(e+fx))^2\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(c-d)(c+d\sec(e+fx))^3\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c^4+8c^3d+21c^2d^2-88cd^3+72d^4-d^2(2c^2+10cd-27d^2)\sec(e+fx))\tan(e+fx)}{15f(a^3+a^3\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 3.86 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.42

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(3(c-d)^4\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)-8(c-d)^3(2c+3d)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+4(c-\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*Cos[(e + f*x)/2]*(3*(c - d)^4*Sec[e/2]*Sin[(f*x)/2] - 8*(c - d)^3*(2*c +
 3*d)*Cos[(e + f*x)/2]^2*Sec[e/2]*Sin[(f*x)/2] + 4*(c - d)^2*(7*c^2 + 26*c*
d + 57*d^2)*Cos[(e + f*x)/2]^4*Sec[e/2]*Sin[(f*x)/2] - 60*d^3*Cos[(e + f*x)
/2]^5*((4*c - 3*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]) - d*Sec[e]*Sec[e + f*x]*Sin[f*x]) + 3*(c - d)
^4*Cos[(e + f*x)/2]*Tan[e/2] - 8*(c - d)^3*(2*c + 3*d)*Cos[(e + f*x)/2]^3*T
an[e/2]))/(15*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{-960\left(c-\frac{3d}{4}\right)\cos(fx+e)d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+960\left(c-\frac{3d}{4}\right)\cos(fx+e)d^3\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+29\sec\left(\frac{fx}{2}+\frac{e}{2}\right)^4\tan\left(\right.}{}$
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^4}{5}-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^3d}{5}+\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^2d^2}{5}-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c\,d^3}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5d^4}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^4}{3}+4\tan}{}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^4}{5}-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^3d}{5}+\frac{6\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c^2d^2}{5}-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5c\,d^3}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5d^4}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^4}{3}+4\tan}{}$
norman	$\frac{\frac{\left(c^4-4c^3d+6c^2d^2-4c\,d^3+d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{13}}{20af}+\frac{\left(c^4+4c^3d+6c^2d^2-28c\,d^3+25d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4af}-\frac{\left(7c^4+24c^3d+30c^2d^2-160c\,d^3+13\right.}{6af}}{}$
risch	$\frac{2i\left(120c^2d^2e^{4i(fx+e)}-668c\,d^3e^{2i(fx+e)}+72c^3d\,e^{2i(fx+e)}-300c\,d^3e^{5i(fx+e)}-640c\,d^3e^{4i(fx+e)}+60c^2d^2e^{i(fx+e)}-380c\,d\right.}{}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/240*(-960*(c-3/4*d)*cos(f*x+e)*d^3*ln(tan(1/2*f*x+1/2*e)-1)+960*(c-3/4*d)
*cos(f*x+e)*d^3*ln(tan(1/2*f*x+1/2*e)+1)+29*sec(1/2*f*x+1/2*e)^4*tan(1/2*f*
x+1/2*e)*(6/29*(2*c^4+12*c^3*d+12*c^2*d^2-68*c*d^3+57*d^4)*cos(2*f*x+2*e)+1
/29*(7*c^4+12*c^3*d+12*c^2*d^2-88*c*d^3+72*d^4)*cos(3*f*x+3*e)+(c^4+84/29*c
^3*d+204/29*c^2*d^2-776/29*c*d^3+684/29*d^4)*cos(f*x+e)+12/29*c^4+72/29*c^3
*d+72/29*c^2*d^2-408/29*c*d^3+402/29*d^4))/f/a^3/cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{15\left(\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^4+3\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^3+3\left(4\,cd^3-3\,d^4\right)\cos\left(fx+e\right)^2+\left(4\,cd^3-\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3 - 3*d^4)*cos(f*x +
e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3*d^4)*cos(f*x + e))
```

```
*log(sin(f*x + e) + 1) - 15*((4*c*d^3 - 3*d^4)*cos(f*x + e)^4 + 3*(4*c*d^3
- 3*d^4)*cos(f*x + e)^3 + 3*(4*c*d^3 - 3*d^4)*cos(f*x + e)^2 + (4*c*d^3 - 3
*d^4)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*(15*d^4 + (7*c^4 + 12*c^3*d
+ 12*c^2*d^2 - 88*c*d^3 + 72*d^4)*cos(f*x + e)^3 + 3*(2*c^4 + 12*c^3*d + 12
*c^2*d^2 - 68*c*d^3 + 57*d^4)*cos(f*x + e)^2 + (2*c^4 + 12*c^3*d + 42*c^2*d
^2 - 128*c*d^3 + 117*d^4)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos(f*x + e)^4
 + 3*a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + a^3*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^4\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^4\sec^5(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{4cd^3\sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec}}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**4*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**4*sec(e + f*x)**5/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(4*c*d**3*sec(e + f*x)**4/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(6*c*
*2*d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*
x) + 1), x) + Integral(4*c**3*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(199) = 398$.

Time = 0.23 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.32

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{3\,d^4\left(\frac{40\,\sin(fx+e)}{\left(a^3-\frac{a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)} + \frac{\frac{85\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\,\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3} - \frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3} + \frac{60\,\log\left(\frac{\sin(fx+e)}{\cos(fx+e)}\right.}{a^3}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*d^4*(40*sin(f*x + e)/((a^3 - a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^
2)*(cos(f*x + e) + 1)) + (85*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x +
 e)^3/(cos(f*x + e) + 1)^3 + sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*
```

```
log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x
 + e) + 1) - 1)/a^3) - 4*c*d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)
/a^3 - 60*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e
)/(cos(f*x + e) + 1) - 1)/a^3) + 6*c^2*d^2*(15*sin(f*x + e)/(cos(f*x + e) +
 1) + 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)/a^3 + c^4*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 12*c^
3*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^
5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.82

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = -\frac{\frac{120\,d^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)a^3} - \frac{60\,(4\,cd^3-3\,d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} + \frac{60\,(4\,cd^3-3\,d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} - \frac{3\,a^{12}c^4\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a^3) - 60
*(4*c*d^3 - 3*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 + 60*(4*c*d^3 - 3
*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/a^3 - (3*a^12*c^4*tan(1/2*f*x + 1/
2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12*c^2*d^2*tan(1/2*f*x
 + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^4*tan(1/2*f*x
 + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 60*a^12*c^2*d^2*tan(1/2*
f*x + 1/2*e)^3 - 80*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*d^4*tan(1/2
*f*x + 1/2*e)^3 + 15*a^12*c^4*tan(1/2*f*x + 1/2*e) + 60*a^12*c^3*d*tan(1/2*
f*x + 1/2*e) + 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 420*a^12*c*d^3*tan(1/
2*f*x + 1/2*e) + 255*a^12*d^4*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 13.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3(c-d)^4}{4a^3}+\frac{3(c^2-d^2)^2}{2a^3}-\frac{2(c+d)(c-d)^3}{a^3}\right)}{f} + \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{(c-d)^4}{6a^3}-\frac{(c+d)(c-d)^3}{3a^3}\right)}{f} - \frac{2d^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\left(a^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-a^3\right)} + \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(c-d)^4}{20a^3f} + \frac{2d^3\operatorname{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)(4c-3d)}{a^3f}$$

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((3*(c - d)^4)/(4*a^3) + (3*(c^2 - d^2)^2)/(2*a^3) - (2
*(c + d)*(c - d)^3)/a^3))/f + (tan(e/2 + (f*x)/2)^3*((c - d)^4/(6*a^3) - ((
c + d)*(c - d)^3)/(3*a^3)))/f - (2*d^4*tan(e/2 + (f*x)/2))/(f*(a^3*tan(e/2
+ (f*x)/2)^2 - a^3)) + (tan(e/2 + (f*x)/2)^5*(c - d)^4)/(20*a^3*f) + (2*d^3
*atanh(tan(e/2 + (f*x)/2))*(4*c - 3*d))/(a^3*f)
```

3.228 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 133

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx &= \frac{d^3\text{arctanh}(\sin(e+fx))}{a^3f}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\ &\quad+\frac{(c-d)\left(2(2c^2+8cd+11d^2)+(2c^2+11cd+29d^2)\sec(e+fx)\right)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\end{aligned}$$

```
[Out] d^3*arctanh(sin(f*x+e))/a^3/f+1/5*(c-d)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f/(a+
a*sec(f*x+e))^3+1/15*(c-d)*(4*c^2+16*c*d+22*d^2+(2*c^2+11*c*d+29*d^2)*sec(f
*x+e))*tan(f*x+e)/a/f/(a+a*sec(f*x+e))^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 100, 150, 65, 223, 209}

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx &= \frac{2d^3\tan(e+fx)\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a(\sec(e+fx)+1)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}\\ &\quad+\frac{(c-d)\tan(e+fx)\left((2c^2+11cd+29d^2)\sec(e+fx)+2(2c^2+8cd+11d^2)\right)}{15af(a\sec(e+fx)+a)^2}\\ &\quad+\frac{(c-d)\tan(e+fx)(c+d\sec(e+fx))^2}{5f(a\sec(e+fx)+a)^3}\end{aligned}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (2*d^3*ArcTan[Sqrt[a - a*Sec[e + f*x]]/Sqrt[a*(1 + Sec[e + f*x])]]*Tan[e +
f*x])/(a^2*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((c - d)*
(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((c - d
)*(2*(2*c^2 + 8*c*d + 11*d^2) + (2*c^2 + 11*c*d + 29*d^2)*Sec[e + f*x])*Tan
[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\left(a^2\tan(e+fx)\right)\text{Subst}\left(\int \frac{(c+dx)^3}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad+\frac{\tan(e+fx)\text{Subst}\left(\int \frac{(c+dx)\left(-a^2\left(2c^2+5cd-2d^2\right)-5a^2d^2x\right)}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx,x,\sec(e+fx)\right)}{5af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad+\frac{(c-d)\left(2\left(2c^2+8cd+11d^2\right)+\left(2c^2+11cd+29d^2\right)\sec(e+fx)\right)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\\
&\quad-\frac{\left(d^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}}\,dx,x,\sec(e+fx)\right)}{af\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\\
&= \frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}\\
&\quad+\frac{(c-d)\left(2\left(2c^2+8cd+11d^2\right)+\left(2c^2+11cd+29d^2\right)\sec(e+fx)\right)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}\\
&\quad+\frac{\left(2d^3\tan(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{2a-x^2}}\,dx,x,\sqrt{a-a\sec(e+fx)}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}
\end{aligned}
$$

$$=\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(c-d)\left(2(2c^2+8cd+11d^2)+(2c^2+11cd+29d^2)\sec(e+fx)\right)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{(2d^3\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{2d^3\arctan\left(\frac{\sqrt{a-a\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\right)\tan(e+fx)}{a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}+\frac{(c-d)(c+d\sec(e+fx))^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(c-d)\left(2(2c^2+8cd+11d^2)+(2c^2+11cd+29d^2)\sec(e+fx)\right)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 295 vs. 2(133) = 266.

Time = 2.59 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.22

$$\int\frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx=\frac{-240d^3\cos^6\left(\frac{1}{2}(e+fx)\right)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right.\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (-240*d^3*Cos[(e + f*x)/2]^6*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Lo
g[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (c - d)*Cos[(e + f*x)/2]*Sec[e/2]
*(5*(8*c^2 + 17*c*d + 29*d^2)*Sin[(f*x)/2] - 15*(2*c^2 + 5*c*d + 5*d^2)*Sin
[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] + 65*c*d*Sin[e + (3*f*x)/2] + 95*
d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] - 15*c*d*Sin[2*e + (3*
f*x)/2] - 15*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (5*f*x)/2] + 16*c*d
*Sin[2*e + (5*f*x)/2] + 22*d^2*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(1 + Cos[e
+ f*x])^3)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{-60\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)d^3+60\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)d^3+3(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left((c-d)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\frac{10(c+2d)(c-d)\tan}{3}\right.}{60a^3f}$
derivativedivides	$\frac{-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3cd^2+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cd^2-\frac{3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{3cd^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}}{}$
default	$\frac{-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3c^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3cd^2+3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)cd^2-\frac{3c^2d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{3cd^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}}{}$
risch	$\frac{2i(15c^3e^{4i(fx+e)}-15d^3e^{4i(fx+e)}+30c^3e^{3i(fx+e)}+45c^2d\,e^{3i(fx+e)}-75d^3e^{3i(fx+e)}+40c^3e^{2i(fx+e)}+45c^2d\,e^{2i(fx+e)}+60}{15f\,a^3(e^{i(}}$
norman	$\frac{\frac{(c^3-3c^2d+3c\,d^2-d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}{20af}-\frac{(c^3+3c^2d+3c\,d^2-7d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4af}+\frac{3(3c^3+c^2d-c\,d^2-3d^3)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{10af}+\frac{(11c^3+27c}{}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2\right.}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/60*(-60*ln(tan(1/2*f*x+1/2*e)-1)*d^3+60*ln(tan(1/2*f*x+1/2*e)+1)*d^3+3*(c
-d)*tan(1/2*f*x+1/2*e)*((c-d)^2*tan(1/2*f*x+1/2*e)^4-10/3*(c+2*d)*(c-d)*tan
(1/2*f*x+1/2*e)^2+5*c^2+20*c*d+35*d^2))/a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{15\left(d^3\cos(fx+e)^3+3\,d^3\cos(fx+e)^2+3\,d^3\cos(fx+e)+d^3\right)\log(\sin(fx+e)+1)-15\left(d^3\cos(fx+\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e) +
d^3)*log(sin(f*x + e) + 1) - 15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2
+ 3*d^3*cos(f*x + e) + d^3)*log(-sin(f*x + e) + 1) + 2*(2*c^3 + 9*c^2*d + 2
1*c*d^2 - 32*d^3 + (7*c^3 + 9*c^2*d + 6*c*d^2 - 22*d^3)*cos(f*x + e)^2 + 3*
(2*c^3 + 9*c^2*d + 6*c*d^2 - 17*d^3)*cos(f*x + e))*sin(f*x + e))/(a^3*f*cos
(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^3\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d^3\sec^4(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{3cd^2\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\ldots}}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**3*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**3*sec(e + f*x)**4/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*d**2*sec(e + f*x)**3/(se
c(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(3*c*
*2*d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x)
+ 1), x))/a**3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(126) = 252$.

Time = 0.23 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.31

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx =$$
$$-\frac{d^3\left(\frac{\frac{105\sin(fx+e)}{\cos(fx+e)+1}+\frac{20\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{a^3}\right)-\frac{3cd^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\ldots}{\ldots}\right.}{\ldots}\ldots}{60f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(d^3*((105*sin(f*x + e)/(cos(f*x + e) + 1) + 20*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 60*log(sin(f*
x + e)/(cos(f*x + e) + 1) + 1)/a^3 + 60*log(sin(f*x + e)/(cos(f*x + e) + 1)
 - 1)/a^3) - 3*c*d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - c^3*(
15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 - 9*c^2*d*(5*sin(f*x + e)/(co
s(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(126) = 252$.

Time = 0.37 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.95

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{60\,d^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{a^3} - \frac{60\,d^3\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{a^3} + \frac{3\,a^{12}c^3\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 9\,a^{12}c^2d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 + 9\,a^{12}cd^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*d^3*log(abs(tan(1/2*f*x + 1/2*e) + 1))/a^3 - 60*d^3*log(abs(tan(1/
2*f*x + 1/2*e) - 1))/a^3 + (3*a^12*c^3*tan(1/2*f*x + 1/2*e)^5 - 9*a^12*c^2*
d*tan(1/2*f*x + 1/2*e)^5 + 9*a^12*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 3*a^12*d^3
*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^3*tan(1/2*f*x + 1/2*e)^3 + 30*a^12*c*d^
2*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^3*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^3
*tan(1/2*f*x + 1/2*e) + 45*a^12*c^2*d*tan(1/2*f*x + 1/2*e) + 45*a^12*c*d^2*
tan(1/2*f*x + 1/2*e) - 105*a^12*d^3*tan(1/2*f*x + 1/2*e))/a^15)/f
```

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{(c-d)^3}{4\,a^3}-\frac{3\,(c+d)\,(c-d)^2}{4\,a^3}+\frac{3\,(c+d)^2\,(c-d)}{4\,a^3}\right)}{f} + \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{(c-d)^3}{12\,a^3}-\frac{(c+d)\,(c-d)^2}{4\,a^3}\right)}{f} + \frac{2\,d^3\,\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^3\,f} + \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5\,(c-d)^3}{20\,a^3\,f}$$

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*((c - d)^3/(4*a^3) - (3*(c + d)*(c - d)^2)/(4*a^3) + (3
*(c + d)^2*(c - d))/(4*a^3)))/f + (tan(e/2 + (f*x)/2)^3*((c - d)^3/(12*a^3)
 - ((c + d)*(c - d)^2)/(4*a^3)))/f + (2*d^3*atanh(tan(e/2 + (f*x)/2)))/(a^3
*f) + (tan(e/2 + (f*x)/2)^5*(c - d)^3)/(20*a^3*f)
```

3.229 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3} dx$

Optimal result

Integrand size = 31, antiderivative size = 115

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3} dx = \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c^2+6cd+7d^2)\tan(e+fx)}{15f(a^3+a^3\sec(e+fx))}$$

```
[Out] 1/5*(c-d)^2*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+2/15*(c-d)*(c+4*d)*tan(f*x+e)/a
/f/(a+a*sec(f*x+e))^2+1/15*(2*c^2+6*c*d+7*d^2)*tan(f*x+e)/f/(a^3+a^3*sec(f*
x+e))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4072, 91, 79, 37}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3} dx = \frac{(2c^2+6cd+7d^2)\tan(e+fx)}{15f(a^3\sec(e+fx)+a^3)} + \frac{(c-d)^2\tan(e+fx)}{5f(a\sec(e+fx)+a)^3} + \frac{2(c+4d)(c-d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - d)^2*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + (2*(c - d)*(c + 4*d
)*Tan[e + f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c^2 + 6*c*d + 7*d^2)*
Tan[e + f*x])/(15*f*(a^3 + a^3*Sec[e + f*x]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\Big(\int\frac{(c+dx)^2}{\sqrt{a-ax}(a+ax)^{7/2}}\,dx, x, \sec(e+fx)\Big)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^3(2c^2+6cd-3d^2)+5a^3d^2x}{\sqrt{a-ax}(a+ax)^{5/2}}\,dx, x, \sec(e+fx)\right)}{5a^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} - \frac{((2c^2+6cd+7d^2)\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{3/2}}\,dx, x, \sec(e+fx)\right)}{15f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c-d)^2\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{2(c-d)(c+4d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c^2+6cd+7d^2)\tan(e+fx)}{15f\,(a^3+a^3\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.73

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{(2c^2+6cd+7d^2+6(c^2+3cd+d^2)\cos(e+fx)+(7c^2+6cd+2d^2)\cos^2(e+fx))\sin(e+fx)}{15a^3f(1+\cos(e+fx))^3}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((2*c^2 + 6*c*d + 7*d^2 + 6*(c^2 + 3*c*d + d^2)*Cos[e + f*x] + (7*c^2 + 6*c
*d + 2*d^2)*Cos[e + f*x]^2)*Sin[e + f*x])/(15*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

method	result
parallelrisch	$\frac{\left((c-d)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4+\frac{10(-c^2+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2}{3}+5(c+d)^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{20a^3f}$
derivativedivides	$\frac{\frac{(c-d)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{2(-c-d)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(-c-d)^2}{4f\,a^3}$
default	$\frac{\frac{(c-d)^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}+\frac{2(-c-d)(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)(-c-d)^2}{4f\,a^3}$
risch	$\frac{2i\left(15c^2e^{4i(fx+e)}+30c^2e^{3i(fx+e)}+30cd\,e^{3i(fx+e)}+40c^2e^{2i(fx+e)}+30cd\,e^{2i(fx+e)}+20d^2e^{2i(fx+e)}+20c^2e^{i(fx+e)}+30d\,e^{i(f}\cdots\right.}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}$
norman	$\frac{\frac{(c^2-2cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}{20af}+\frac{(c^2+2cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4af}-\frac{(2c^2+3cd+d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3af}-\frac{(4c^2-3cd-d^2)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{15af}+\frac{(19c^2+\cdots}{\cdots}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2a^2}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/20*((c-d)^2*tan(1/2*f*x+1/2*e)^4+10/3*(-c^2+d^2)*tan(1/2*f*x+1/2*e)^2+5*(
c+d)^2)*tan(1/2*f*x+1/2*e)/a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\left((7c^2+6cd+2d^2)\cos(fx+e)^2+2c^2+6cd+7d^2+6(c^2+3cd+d^2)\cos(fx+e)\right)\sin(fx+e)}{15\left(a^3f\cos(fx+e)^3+3a^3f\cos(fx+e)^2+3a^3f\cos(fx+e)+a^3f\right)}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((7*c^2 + 6*c*d + 2*d^2)*cos(f*x + e)^2 + 2*c^2 + 6*c*d + 7*d^2 + 6*(c
^2 + 3*c*d + d^2)*cos(f*x + e))*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*
f*cos(f*x + e)^2 + 3*a^3*f*cos(f*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \frac{c^2\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int \frac{d^2\sec^3(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx+\int \frac{2cd\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+\ldots}}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c**2*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e
+ f*x) + 1), x) + Integral(d**2*sec(e + f*x)**3/(sec(e + f*x)**3 + 3*sec(e
+ f*x)**2 + 3*sec(e + f*x) + 1), x) + Integral(2*c*d*sec(e + f*x)**2/(sec(e
 + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.60

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{d^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}+\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{c^2\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1}-\frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}+\frac{6\,cd\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sin(fx+e)^5}{(\cos(fx+e)+1}\right.}{a^3}}{60\,f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(d^2*(15*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + c^2*(15*sin(f*x
+ e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 6*c*d*(5*sin(f*x + e)/(cos(f*x + e) +
1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{3\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-6\,cd\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5+3\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-10\,c^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3+10\,d^2\tan}{60\,a^3f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+a*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(3*c^2*tan(1/2*f*x + 1/2*e)^5 - 6*c*d*tan(1/2*f*x + 1/2*e)^5 + 3*d^2*t
an(1/2*f*x + 1/2*e)^5 - 10*c^2*tan(1/2*f*x + 1/2*e)^3 + 10*d^2*tan(1/2*f*x
+ 1/2*e)^3 + 15*c^2*tan(1/2*f*x + 1/2*e) + 30*c*d*tan(1/2*f*x + 1/2*e) + 15
*d^2*tan(1/2*f*x + 1/2*e))/(a^3*f)
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(c+d)^2}{4\,a^3\,f} - \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,(2\,c^2-2\,d^2)}{12\,a^3\,f} + \frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^5\,(c-d)^2}{20\,a^3\,f}$$

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
[Out] (tan(e/2 + (f*x)/2)*(c + d)^2)/(4*a^3*f) - (tan(e/2 + (f*x)/2)^3*(2*c^2 - 2
*d^2))/(12*a^3*f) + (tan(e/2 + (f*x)/2)^5*(c - d)^2)/(20*a^3*f)
```

3.230 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 102

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3} + \frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2} + \frac{(2c+3d)\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}$$

```
[Out] 1/5*(c-d)*tan(f*x+e)/f/(a+a*sec(f*x+e))^3+1/15*(2*c+3*d)*tan(f*x+e)/a/f/(a+
a*sec(f*x+e))^2+1/15*(2*c+3*d)*tan(f*x+e)/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4085, 3881, 3879}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{(2c+3d)\tan(e+fx)}{15f\left(a^3\sec(e+fx)+a^3\right)} + \frac{(2c+3d)\tan(e+fx)}{15af(a\sec(e+fx)+a)^2} + \frac{(c-d)\tan(e+fx)}{5f(a\sec(e+fx)+a)^3}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] ((c - d)*Tan[e + f*x])/(5*f*(a + a*Sec[e + f*x])^3) + ((2*c + 3*d)*Tan[e +
f*x])/(15*a*f*(a + a*Sec[e + f*x])^2) + ((2*c + 3*d)*Tan[e + f*x])/(15*f*(a
^3 + a^3*Sec[e + f*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(2c+3d)\int\frac{\sec(e+fx)}{(a+a\sec(e+fx))^2}\,dx}{5a}\\
&= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{(2c+3d)\int\frac{\sec(e+fx)}{a+a\sec(e+fx)}\,dx}{15a^2}\\
&= \frac{(c-d)\tan(e+fx)}{5f(a+a\sec(e+fx))^3}+\frac{(2c+3d)\tan(e+fx)}{15af(a+a\sec(e+fx))^2}+\frac{(2c+3d)\tan(e+fx)}{15f\left(a^3+a^3\sec(e+fx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.32

$$\int\frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx$$
$$=\frac{\cos\left(\frac{1}{2}(e+fx)\right)\sec\left(\frac{e}{2}\right)\left(5(8c+3d)\sin\left(\frac{fx}{2}\right)-15(2c+d)\sin\left(e+\frac{fx}{2}\right)+20c\sin\left(e+\frac{3fx}{2}\right)+15d\sin\left(e+\right.\right.}{30a^3f(1+\cos(e+fx))^3}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + a*Sec[e + f*x])^3,x]
```

```
[Out] (Cos[(e + f*x)/2]*Sec[e/2]*(5*(8*c + 3*d)*Sin[(f*x)/2] - 15*(2*c + d)*Sin[e
 + (f*x)/2] + 20*c*Sin[e + (3*f*x)/2] + 15*d*Sin[e + (3*f*x)/2] - 15*c*Sin[
2*e + (3*f*x)/2] + 7*c*Sin[2*e + (5*f*x)/2] + 3*d*Sin[2*e + (5*f*x)/2]))/(3
0*a^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result	siz
parallelrisch	$\frac{\left((c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^4-\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c}{3}+5c+5d\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{20a^3f}$	56
derivativedivides	$\frac{\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f\,a^3}$	64
default	$\frac{\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{5}-\frac{2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4f\,a^3}$	64
risch	$\frac{2i\left(15c\,e^{4i(fx+e)}+30c\,e^{3i(fx+e)}+15d\,e^{3i(fx+e)}+40\,e^{2i(fx+e)}c+15d\,e^{2i(fx+e)}+20\,e^{i(fx+e)}c+15d\,e^{i(fx+e)}+7c+3d\right)}{15f\,a^3\left(e^{i(fx+e)}+1\right)^5}$	11
norman	$\frac{\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{20af}-\frac{(c+d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{4af}+\frac{(5c+3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{12af}-\frac{(13c-3d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{60af}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)a^2}$	11

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*((c-d)*tan(1/2*f*x+1/2*e)^4-10/3*tan(1/2*f*x+1/2*e)^2*c+5*c+5*d)*tan(1
/2*f*x+1/2*e)/a^3/f
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{((7\,c+3\,d)\cos(fx+e)^2+3\,(2\,c+3\,d)\cos(fx+e)+2\,c+3\,d)\sin(fx+e)}{15\,(a^3f\cos(fx+e)^3+3\,a^3f\cos(fx+e)^2+3\,a^3f\cos(fx+e)+a^3f)}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((7*c + 3*d)*cos(f*x + e)^2 + 3*(2*c + 3*d)*cos(f*x + e) + 2*c + 3*d)*
sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 + 3*a^3*f*cos(f
*x + e) + a^3*f)
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\int \frac{c\sec(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx + \int \frac{d\sec^2(e+fx)}{\sec^3(e+fx)+3\sec^2(e+fx)+3\sec(e+fx)+1}\,dx}{a^3}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**3,x)
```

```
[Out] (Integral(c*sec(e + f*x)/(sec(e + f*x)**3 + 3*sec(e + f*x)**2 + 3*sec(e + f
*x) + 1), x) + Integral(d*sec(e + f*x)**2/(sec(e + f*x)**3 + 3*sec(e + f*x)
**2 + 3*sec(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\frac{c\left(\frac{15\sin(fx+e)}{\cos(fx+e)+1} - \frac{10\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3} + \frac{3\,d\left(\frac{5\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^3}}{60\,f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(c*(15*sin(f*x + e)/(cos(f*x + e) + 1) - 10*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3 + 3*d*(5*sin(f*x + e
)/(cos(f*x + e) + 1) - sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/a^3)/f
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{3\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 3\,d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5 - 10\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^3 + 15\,c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right) + 15\,d\tan\left(\frac{1}{2}\right.}{60\,a^3f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(3*c*tan(1/2*f*x + 1/2*e)^5 - 3*d*tan(1/2*f*x + 1/2*e)^5 - 10*c*tan(1/
2*f*x + 1/2*e)^3 + 15*c*tan(1/2*f*x + 1/2*e) + 15*d*tan(1/2*f*x + 1/2*e))/(
a^3*f)
```

Mupad [B] (verification not implemented)

Time = 13.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+a\sec(e+fx))^3}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(15\,c+15\,d-10\,c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+3\,c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4-3\,d\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4\right)}{60\,a^3\,f}$$

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + a/cos(e + f*x))^3),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(15*c + 15*d - 10*c*tan(e/2 + (f*x)/2)^2 + 3*c*tan(e/2
+ (f*x)/2)^4 - 3*d*tan(e/2 + (f*x)/2)^4))/(60*a^3*f)
```

3.231 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 181

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = -\frac{2d^3\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{7/2}\sqrt{c+d}f} + \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15(c-d)^3f\left(a^3+a^3\sec(e+fx)\right)}$$

```
[Out] -2*d^3*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^3/(c-d)^(7/2)/
f/(c+d)^(1/2)+1/5*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e))^3+1/15*(2*c-7*d)*tan(
f*x+e)/a/(c-d)^2/f/(a+a*sec(f*x+e))^2+1/15*(2*c^2-9*c*d+22*d^2)*tan(f*x+e)/
(c-d)^3/f/(a^3+a^3*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used

$= \{4072, 106, 157, 12, 95, 211\}$

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$$
$$= \frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15f(c-d)^3\left(a^3\sec(e+fx)+a^3\right)}$$
$$+ \frac{2d^3\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{7/2}\sqrt{c+d}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$+ \frac{(2c-7d)\tan(e+fx)}{15af(c-d)^2(a\sec(e+fx)+a)^2} + \frac{\tan(e+fx)}{5f(c-d)(a\sec(e+fx)+a)^3}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

```
[Out] Tan[e + f*x]/(5*(c - d)*f*(a + a*Sec[e + f*x])^3) + ((2*c - 7*d)*Tan[e + f*
x])/(15*a*(c - d)^2*f*(a + a*Sec[e + f*x])^2) + (2*d^3*ArcTan[(Sqrt[c + d]*
Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f
*x])/(a^2*(c - d)^(7/2)*Sqrt[c + d]*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*S
ec[e + f*x]]) + ((2*c^2 - 9*c*d + 22*d^2)*Tan[e + f*x])/(15*(c - d)^3*f*(a^
3 + a^3*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 106

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)}\,dx,x,\sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3}+\frac{\tan(e+fx)\text{Subst}\left(\int \frac{-a^2(2c-5d)-2a^2dx}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)}\,dx,x,\sec(e+fx)\right)}{5a(c-d)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3}+\frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^4(2c^2-7cd+15d^2)+a^4(2c-7d)dx}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx,x,\sec(e+fx)\right)}{15a^4(c-d)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3}+\frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15(c-d)^3f\left(a^3+a^3\sec(e+fx)\right)} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{15a^6d^3}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{15a^7(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15(c-d)^3f(a^3+a^3\sec(e+fx))} + \frac{(d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15(c-d)^3f(a^3+a^3\sec(e+fx))} + \frac{(2d^3\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{a(c-d)^3f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3} + \frac{(2c-7d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2} + \frac{2d^3\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{a^2(c-d)^{7/2}\sqrt{c+d}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} + \frac{(2c^2-9cd+22d^2)\tan(e+fx)}{15(c-d)^3f(a^3+a^3\sec(e+fx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.91

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = \frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(\frac{480d^3\arctan\left(\frac{(i\cos(e)+\sin(e))\left(c\sin(e)+(-d+c\cos(e))\tan\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}}\right)\cos^5\left(\frac{1}{2}(e+fx)\right)(i\cos(e)+\sin(e))}{\sqrt{c^2-d^2}\sqrt{(\cos(e)-i\sin(e))^2}} + \sec\left(\frac{e}{2}\right)\left(5(8c^2 -\right.\right.}{}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Cos[(e + f*x)/2]*((480*d^3*ArcTan[((I*Cos[e] + Sin[e])*(c*Sin[e] + (-d + c
*Cos[e])*Tan[(f*x)/2]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])]*Cos[
(e + f*x)/2]^5*(I*Cos[e] + Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e
])^2]) + Sec[e/2]*(5*(8*c^2 - 27*c*d + 37*d^2)*Sin[(f*x)/2] - 15*(2*c^2 - 7
*c*d + 9*d^2)*Sin[e + (f*x)/2] + 20*c^2*Sin[e + (3*f*x)/2] - 75*c*d*Sin[e +
 (3*f*x)/2] + 115*d^2*Sin[e + (3*f*x)/2] - 15*c^2*Sin[2*e + (3*f*x)/2] + 45
```

```
*c*d*Sin[2*e + (3*f*x)/2] - 45*d^2*Sin[2*e + (3*f*x)/2] + 7*c^2*Sin[2*e + (
5*f*x)/2] - 24*c*d*Sin[2*e + (5*f*x)/2] + 32*d^2*Sin[2*e + (5*f*x)/2])))/(3
0*a^3*(c - d)^3*f*(1 + Cos[e + f*x])^3)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 c^2}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 d^2}{5}-\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 cd-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 d^2}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c}{(c-d)^3}}{4f a^3}$
default	$\frac{\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 c^2}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 d^2}{5}-\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 cd-\frac{4\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 d^2}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c}{(c-d)^3}}{4f a^3}$
risch	$\frac{2i\left(15c^2 e^{4i(fx+e)}-45cd\, e^{4i(fx+e)}+45d^2 e^{4i(fx+e)}+30c^2 e^{3i(fx+e)}-105cd\, e^{3i(fx+e)}+135d^2 e^{3i(fx+e)}+40c^2 e^{2i(fx+e)}-13\right.}{15f a^3(c-d)^3\left(e^{i(fx+e)}+1\right.}$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/a^3*(1/(c-d)^3*(1/5*tan(1/2*f*x+1/2*e)^5*c^2-2/5*tan(1/2*f*x+1/2*e)^5
*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*c^2*tan(1/2*f*x+1/2*e)^3+2*tan(1/2*f*
x+1/2*e)^3*c*d-4/3*tan(1/2*f*x+1/2*e)^3*d^2+tan(1/2*f*x+1/2*e)*c^2-4*tan(1/
2*f*x+1/2*e)*c*d+7*tan(1/2*f*x+1/2*e)*d^2)-8*d^3/(c-d)^3/((c+d)*(c-d))^(1/2
)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(166) = 332$.

Time = 0.30 (sec) , antiderivative size = 1001, normalized size of antiderivative = 5.53

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx$$
$$= \left[-\frac{15\left(d^3\cos\left(fx+e\right)^3+3\,d^3\cos\left(fx+e\right)^2+3\,d^3\cos\left(fx+e\right)+d^3\right)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos}{c^2\,c}\right.}{30\left(\left(a^3c^5-3\,a^3c^4d+2\,a^3c^3d^2+2\,a^3c^2d^3-3\,a^3cd^4+a^3d^5\right)f\cos\left(fx+e\right)^3+3\left(a^3c^5-\right.\right.}\right.$$
$$-\frac{15\left(d^3\cos\left(fx+e\right)^3+3\,d^3\cos\left(fx+e\right)^2+3\,d^3\cos\left(fx+e\right)+d^3\right)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(}{(c^2-d^2)\sin(}\right.}{15\left(\left(a^3c^5-3\,a^3c^4d+2\,a^3c^3d^2+2\,a^3c^2d^3-3\,a^3cd^4+a^3d^5\right)f\cos\left(fx+e\right)^3+3\left(a^3c^5-3\,a^3c^4d+2\,a^3c^3\right.\right.}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/30*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos(f*x + e)^2 + 3*d^3*cos(f*x + e)
+ d^3)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)
^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^
2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(2*c^4 - 9*c^3*d + 20*c^2
*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3 - 32*d^
4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*d^4)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^
2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d
+ 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 +
 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a
^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2
*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f), -1/15*(15*(d^3*cos(f*x + e)^3 + 3*d^3*cos
(f*x + e)^2 + 3*d^3*cos(f*x + e) + d^3)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2
+ d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^4 - 9*c^3*d
+ 20*c^2*d^2 + 9*c*d^3 - 22*d^4 + (7*c^4 - 24*c^3*d + 25*c^2*d^2 + 24*c*d^3
 - 32*d^4)*cos(f*x + e)^2 + 3*(2*c^4 - 9*c^3*d + 15*c^2*d^2 + 9*c*d^3 - 17*
d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 +
2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^
3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x
+ e)^2 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c
*d^4 + a^3*d^5)*f*cos(f*x + e) + (a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2
*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{c\sec^3(e+fx)+3c\sec^2(e+fx)+3c\sec(e+fx)+c+d\sec^4(e+fx)+3d\sec^3(e+fx)+3d\sec^2(e+fx)+d\sec(e+fx)}\,dx}{a^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/(c*sec(e + f*x)**3 + 3*c*sec(e + f*x)**2 + 3*c*sec(e
+ f*x) + c + d*sec(e + f*x)**4 + 3*d*sec(e + f*x)**3 + 3*d*sec(e + f*x)**2
+ d*sec(e + f*x)), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(166) = 332$.

Time = 0.36 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.60

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = -\frac{\frac{120\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^3}{(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)\sqrt{-c^2+d^2}} - \frac{3a^{12}c^4\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-12a^{12}c^3d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5}{}}{}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] -1/60*(120*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*t
an(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^3/((a^3*
c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*sqrt(-c^2 + d^2)) - (3*a^12*c^4*
tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 18*a^12*c^2
*d^2*tan(1/2*f*x + 1/2*e)^5 - 12*a^12*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a^12
*d^4*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^4*tan(1/2*f*x + 1/2*e)^3 + 50*a^12*
c^3*d*tan(1/2*f*x + 1/2*e)^3 - 90*a^12*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 + 70*
a^12*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 20*a^12*d^4*tan(1/2*f*x + 1/2*e)^3 + 15
*a^12*c^4*tan(1/2*f*x + 1/2*e) - 90*a^12*c^3*d*tan(1/2*f*x + 1/2*e) + 240*a
^12*c^2*d^2*tan(1/2*f*x + 1/2*e) - 270*a^12*c*d^3*tan(1/2*f*x + 1/2*e) + 10
5*a^12*d^4*tan(1/2*f*x + 1/2*e))/(a^15*c^5 - 5*a^15*c^4*d + 10*a^15*c^3*d^2
 - 10*a^15*c^2*d^3 + 5*a^15*c*d^4 - a^15*d^5))/f
```

Mupad [B] (verification not implemented)

Time = 13.80 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.26

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))}\,dx = \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(\frac{3}{4a^3(c-d)}-\frac{(c+d)\left(\frac{3}{4a^3(c-d)}-\frac{c+d}{4a^3(c-d)^2}\right)}{c-d}\right)}{f} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\frac{1}{4a^3(c-d)}-\frac{c+d}{12a^3(c-d)^2}\right)}{f} + \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{20a^3f\,(c-d)} - \frac{2d^3\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2c-2d)\,(a^3c^3-3a^3c^2d+3a^3cd^2-a^3d^3)}{2a^3\sqrt{c+d}\,(c-d)^{7/2}}\right)}{a^3f\sqrt{c+d}\,(c-d)^{7/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))),x)
```

```
[Out] (tan(e/2 + (f*x)/2)*(3/(4*a^3*(c - d)) - ((c + d)*(3/(4*a^3*(c - d)) - (c +
 d)/(4*a^3*(c - d)^2)))/(c - d)))/f - (tan(e/2 + (f*x)/2)^3*(1/(4*a^3*(c -
d)) - (c + d)/(12*a^3*(c - d)^2)))/f + tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c -
d)) - (2*d^3*atanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(a^3*c^3 - a^3*d^3 + 3*a
^3*c*d^2 - 3*a^3*c^2*d))/(2*a^3*(c + d)^(1/2)*(c - d)^(7/2))))/(a^3*f*(c +
d)^(1/2)*(c - d)^(7/2))
```

3.232 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 288

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = -\frac{2d^3(4c+3d)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{9/2}(c+d)^{3/2}f} + \frac{d(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15a^3(c-d)^4(c+d)f(c+d\sec(e+fx))} + \frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} + \frac{(2c-9d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2(c+d\sec(e+fx))} + \frac{(2c^2-12cd+45d^2)\tan(e+fx)}{15(c-d)^3f\,(a^3+a^3\sec(e+fx))\,(c+d\sec(e+fx))}$$

```
[Out] -2*d^3*(4*c+3*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/a^3/(c
-d)^(9/2)/(c+d)^(3/2)/f+1/15*d*(2*c^3-12*c^2*d+43*c*d^2+72*d^3)*tan(f*x+e)/
a^3/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))+1/5*tan(f*x+e)/(c-d)/f/(a+a*sec(f*x+e)
)^3/(c+d*sec(f*x+e))+1/15*(2*c-9*d)*tan(f*x+e)/a/(c-d)^2/f/(a+a*sec(f*x+e))
^2/(c+d*sec(f*x+e))+1/15*(2*c^2-12*c*d+45*d^2)*tan(f*x+e)/(c-d)^3/f/(a^3+a^
3*sec(f*x+e))/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4072, 105, 157, 12, 95, 211}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15f(c-d)^4(c+d)\left(a^3\sec(e+fx)+a^3\right)} + \frac{2d^3(4c+3d)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{9/2}(c+d)^{3/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}} - \frac{d\tan(e+fx)}{f\left(c^2-d^2\right)(a\sec(e+fx)+a)^3(c+d\sec(e+fx))} + \frac{(2c^2-10cd-27d^2)\tan(e+fx)}{15af(c-d)^3(c+d)(a\sec(e+fx)+a)^2} + \frac{(c+6d)\tan(e+fx)}{5f(c-d)^2(c+d)(a\sec(e+fx)+a)^3}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((c + 6*d)*Tan[e + f*x])/(5*(c - d)^2*(c + d)*f*(a + a*Sec[e + f*x])^3) + (
(2*c^2 - 10*c*d - 27*d^2)*Tan[e + f*x])/(15*a*(c - d)^3*(c + d)*f*(a + a*Se
c[e + f*x])^2) + (2*d^3*(4*c + 3*d)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e +
f*x]])/(Sqrt[c - d]*Sqrt[a - a*Sec[e + f*x]])]*Tan[e + f*x])/(a^2*(c - d)^(
9/2)*(c + d)^(3/2)*f*Sqrt[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + (
(2*c^3 - 12*c^2*d + 43*c*d^2 + 72*d^3)*Tan[e + f*x])/(15*(c - d)^4*(c + d)*
f*(a^3 + a^3*Sec[e + f*x])) - (d*Tan[e + f*x])/((c^2 - d^2)*f*(a + a*Sec[e
+ f*x])^3*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
```

```
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^2(c+3d)-3a^2dx}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3}-\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))}+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{-a^4(2c^2-8cd-15d^2)-2a^4d(c+6d)x}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)}\,dx,x,\sec(e+fx)\right)}{5a^3(c-d)\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3}+\frac{(2c^2-10cd-27d^2)\tan(e+fx)}{15a(c-d)^3(c+d)f(a+a\sec(e+fx))^2}-\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))}-\frac{\tan(e+fx)\text{Subst}\left(\int\frac{a^6(c+d)(2c^2-12cd+45d^2)+a^6d(2c^2-10cd-27d^2)x}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx,x,\sec(e+fx)\right)}{15a^6(c-d)^2\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3}+\frac{(2c^2-10cd-27d^2)\tan(e+fx)}{15a(c-d)^3(c+d)f(a+a\sec(e+fx))^2}+\frac{(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15(c-d)^4(c+d)f\,(a^3+a^3\sec(e+fx))}-\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))}+\frac{\tan(e+fx)\text{Subst}\left(\int\frac{15a^8d^3(4c+3d)}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{15a^9(c-d)^3\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3}+\frac{(2c^2-10cd-27d^2)\tan(e+fx)}{15a(c-d)^3(c+d)f(a+a\sec(e+fx))^2}+\frac{(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15(c-d)^4(c+d)f\,(a^3+a^3\sec(e+fx))}-\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))}+\frac{(d^3(4c+3d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx,x,\sec(e+fx)\right)}{a(c-d)^3\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$=\frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3}+\frac{(2c^2-10cd-27d^2)\tan(e+fx)}{15a(c-d)^3(c+d)f(a+a\sec(e+fx))^2}+\frac{(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15(c-d)^4(c+d)f\,(a^3+a^3\sec(e+fx))}-\frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))}+\frac{(2d^3(4c+3d)\tan(e+fx))\,\text{Subst}\left(\int\frac{1}{ac-ad-(-ac-ad)x^2}\,dx,x,\frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{a(c-d)^3\,(c^2-d^2)\,f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(c+6d)\tan(e+fx)}{5(c-d)^2(c+d)f(a+a\sec(e+fx))^3} + \frac{(2c^2-10cd-27d^2)\tan(e+fx)}{15a(c-d)^3(c+d)f(a+a\sec(e+fx))^2}$$
$$+ \frac{2d^3(4c+3d)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{a^2(c-d)^{9/2}(c+d)^{3/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$
$$+ \frac{(2c^3-12c^2d+43cd^2+72d^3)\tan(e+fx)}{15(c-d)^4(c+d)f\,(a^3+a^3\sec(e+fx))}$$
$$- \frac{d\tan(e+fx)}{(c^2-d^2)\,f(a+a\sec(e+fx))^3(c+d\sec(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.96 (sec) , antiderivative size = 1772, normalized size of antiderivative = 6.15

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{(4c+3d)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^2\sec^5(e+fx)\left(\frac{16id^3\arctan\left(\sec\left(\frac{fx}{2}\right)\left(\frac{\cos(e)}{\sqrt{c^2-d^2}\sqrt{\cos(2e)-i\sin(2e)}}-\frac{i s}{\sqrt{c^2-d^2}\sqrt{c}}\right.\right.}{\sqrt{c^2-d^2}f\sqrt{\cos(2e}}\right.}{(-c+d)^4(c+}$$
$$+\frac{\cos\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))\sec\left(\frac{e}{2}\right)\sec(e)\sec^5(e+fx)\left(-55c^5\sin\left(\frac{fx}{2}\right)+135c^4d\sin\left(\frac{fx}{2}\right)-20c^3d^2\right.}{}$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] ((4*c + 3*d)*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*Sec[e + f*x]^5*(((
16*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Si
n[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*
d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos
[2*e] - I*Sin[2*e]]) + (16*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]
*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] -
 I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[
c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])))/((-c + d)^4*(c + d)*(a + a*Sec[
e + f*x])^3*(c + d*Sec[e + f*x])^2) + (Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*
x])*Sec[e/2]*Sec[e]*Sec[e + f*x]^5*(-55*c^5*Sin[(f*x)/2] + 135*c^4*d*Sin[(f
*x)/2] - 20*c^3*d^2*Sin[(f*x)/2] - 810*c^2*d^3*Sin[(f*x)/2] - 450*c*d^4*Sin
[(f*x)/2] + 150*d^5*Sin[(f*x)/2] + 47*c^5*Sin[(3*f*x)/2] - 137*c^4*d*Sin[(3
*f*x)/2] + 88*c^3*d^2*Sin[(3*f*x)/2] + 812*c^2*d^3*Sin[(3*f*x)/2] + 690*c*d
^4*Sin[(3*f*x)/2] + 75*d^5*Sin[(3*f*x)/2] - 50*c^5*Sin[e - (f*x)/2] + 130*c
^4*d*Sin[e - (f*x)/2] - 10*c^3*d^2*Sin[e - (f*x)/2] - 1030*c^2*d^3*Sin[e -
(f*x)/2] - 990*c*d^4*Sin[e - (f*x)/2] - 150*d^5*Sin[e - (f*x)/2] + 50*c^5*S
in[e + (f*x)/2] - 130*c^4*d*Sin[e + (f*x)/2] + 10*c^3*d^2*Sin[e + (f*x)/2]
+ 1030*c^2*d^3*Sin[e + (f*x)/2] + 765*c*d^4*Sin[e + (f*x)/2] - 150*d^5*Sin[
```

```
e + (f*x)/2] - 55*c^5*Sin[2*e + (f*x)/2] + 135*c^4*d*Sin[2*e + (f*x)/2] - 2
0*c^3*d^2*Sin[2*e + (f*x)/2] - 810*c^2*d^3*Sin[2*e + (f*x)/2] - 675*c*d^4*S
in[2*e + (f*x)/2] - 150*d^5*Sin[2*e + (f*x)/2] - 30*c^5*Sin[e + (3*f*x)/2]
+ 90*c^4*d*Sin[e + (3*f*x)/2] - 60*c^3*d^2*Sin[e + (3*f*x)/2] - 360*c^2*d^3
*Sin[e + (3*f*x)/2] - 30*c*d^4*Sin[e + (3*f*x)/2] + 75*d^5*Sin[e + (3*f*x)/
2] + 47*c^5*Sin[2*e + (3*f*x)/2] - 137*c^4*d*Sin[2*e + (3*f*x)/2] + 88*c^3*
d^2*Sin[2*e + (3*f*x)/2] + 812*c^2*d^3*Sin[2*e + (3*f*x)/2] + 525*c*d^4*Sin
[2*e + (3*f*x)/2] - 75*d^5*Sin[2*e + (3*f*x)/2] - 30*c^5*Sin[3*e + (3*f*x)/
2] + 90*c^4*d*Sin[3*e + (3*f*x)/2] - 60*c^3*d^2*Sin[3*e + (3*f*x)/2] - 360*
c^2*d^3*Sin[3*e + (3*f*x)/2] - 195*c*d^4*Sin[3*e + (3*f*x)/2] - 75*d^5*Sin[
3*e + (3*f*x)/2] + 20*c^5*Sin[e + (5*f*x)/2] - 76*c^4*d*Sin[e + (5*f*x)/2]
+ 106*c^3*d^2*Sin[e + (5*f*x)/2] + 346*c^2*d^3*Sin[e + (5*f*x)/2] + 219*c*d
^4*Sin[e + (5*f*x)/2] + 15*d^5*Sin[e + (5*f*x)/2] - 15*c^5*Sin[2*e + (5*f*x
)/2] + 45*c^4*d*Sin[2*e + (5*f*x)/2] - 30*c^3*d^2*Sin[2*e + (5*f*x)/2] - 90
*c^2*d^3*Sin[2*e + (5*f*x)/2] + 75*c*d^4*Sin[2*e + (5*f*x)/2] + 15*d^5*Sin[
2*e + (5*f*x)/2] + 20*c^5*Sin[3*e + (5*f*x)/2] - 76*c^4*d*Sin[3*e + (5*f*x)
/2] + 106*c^3*d^2*Sin[3*e + (5*f*x)/2] + 346*c^2*d^3*Sin[3*e + (5*f*x)/2] +
 144*c*d^4*Sin[3*e + (5*f*x)/2] - 15*d^5*Sin[3*e + (5*f*x)/2] - 15*c^5*Sin[
4*e + (5*f*x)/2] + 45*c^4*d*Sin[4*e + (5*f*x)/2] - 30*c^3*d^2*Sin[4*e + (5*
f*x)/2] - 90*c^2*d^3*Sin[4*e + (5*f*x)/2] - 15*d^5*Sin[4*e + (5*f*x)/2] + 7
*c^5*Sin[2*e + (7*f*x)/2] - 27*c^4*d*Sin[2*e + (7*f*x)/2] + 38*c^3*d^2*Sin[
2*e + (7*f*x)/2] + 72*c^2*d^3*Sin[2*e + (7*f*x)/2] + 15*c*d^4*Sin[2*e + (7*
f*x)/2] + 15*c*d^4*Sin[3*e + (7*f*x)/2] + 7*c^5*Sin[4*e + (7*f*x)/2] - 27*c
^4*d*Sin[4*e + (7*f*x)/2] + 38*c^3*d^2*Sin[4*e + (7*f*x)/2] + 72*c^2*d^3*Si
n[4*e + (7*f*x)/2]))/(120*c*(-c + d)^4*(c + d)*f*(a + a*Sec[e + f*x])^3*(c
+ d*Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 c^2}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 d^2}{5}-\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 cd}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 d^2+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2}{\left(c^2-2cd+d^2\right)(c-d)^2}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 c^2}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5 d^2}{5}-\frac{2c^2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3}+\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 cd}{3}-2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3 d^2+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)c^2}{\left(c^2-2cd+d^2\right)(c-d)^2}$
risch	$2i\left(7c^5+10c^3d^2e^{3i(fx+e)}-137c^4d\,e^{2i(fx+e)}+106c^3d^2e^{i(fx+e)}-76c^4d\,e^{i(fx+e)}+195c\,d^4e^{5i(fx+e)}+990c\,d^4e^{3i(fx+e)}+60c^3\right.$

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f/a^3*(1/(c^2-2*c*d+d^2)/(c-d)^2*(1/5*tan(1/2*f*x+1/2*e)^5*c^2-2/5*tan(
1/2*f*x+1/2*e)^5*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*c^2*tan(1/2*f*x+1/2*e
)^3+8/3*tan(1/2*f*x+1/2*e)^3*c*d-2*tan(1/2*f*x+1/2*e)^3*d^2+tan(1/2*f*x+1/2
*e)*c^2-6*tan(1/2*f*x+1/2*e)*c*d+17*tan(1/2*f*x+1/2*e)*d^2)+16*d^3/(c-d)^4*
(-1/2*d/(c+d)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)
^2*d-c-d)-1/2*(4*c+3*d)/(c+d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x
+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(271) = 542$.

Time = 0.34 (sec) , antiderivative size = 1693, normalized size of antiderivative = 5.88

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/30*(15*(4*c*d^4 + 3*d^5 + (4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2
*d^3 + 13*c*d^4 + 3*d^5)*cos(f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*c
os(f*x + e)^2 + (4*c^2*d^3 + 15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(c^2 - d^2
)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2 - d^2
)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*
c*d*cos(f*x + e) + d^2)) + 2*(2*c^5*d - 12*c^4*d^2 + 41*c^3*d^3 + 84*c^2*d^
4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2 + 99*c^3*d^3 - 23*c^
2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 29*c^5*d + 51*c^4*d^2
+ 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(f*x + e)^2 + (2*c^6 -
 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141*c*d^5 - 171*d^6)*cos
(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3*c^6*d^2 + 5*a^3*c^5*
d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^3*c*d^7)*f*cos(f*x +
e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*a^3*c^4*d^4 - 8*a^3*c
^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3*(a^3*c^8 - 2*a^3*c^7
*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*
c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^
3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*cos(f*x +
e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d
^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f), -1/15*(15*(4*c*d^4 + 3*d^5 +
(4*c^2*d^3 + 3*c*d^4)*cos(f*x + e)^4 + (12*c^2*d^3 + 13*c*d^4 + 3*d^5)*cos(
f*x + e)^3 + 3*(4*c^2*d^3 + 7*c*d^4 + 3*d^5)*cos(f*x + e)^2 + (4*c^2*d^3 +
15*c*d^4 + 9*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(
d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (2*c^5*d - 12*c^4*d^2 + 4
```

```
1*c^3*d^3 + 84*c^2*d^4 - 43*c*d^5 - 72*d^6 + (7*c^6 - 27*c^5*d + 31*c^4*d^2
 + 99*c^3*d^3 - 23*c^2*d^4 - 72*c*d^5 - 15*d^6)*cos(f*x + e)^3 + (6*c^6 - 2
9*c^5*d + 51*c^4*d^2 + 193*c^3*d^3 + 60*c^2*d^4 - 164*c*d^5 - 117*d^6)*cos(
f*x + e)^2 + (2*c^6 - 6*c^5*d + 5*c^4*d^2 + 147*c^3*d^3 + 164*c^2*d^4 - 141
*c*d^5 - 171*d^6)*cos(f*x + e))*sin(f*x + e))/((a^3*c^8 - 3*a^3*c^7*d + a^3
*c^6*d^2 + 5*a^3*c^5*d^3 - 5*a^3*c^4*d^4 - a^3*c^3*d^5 + 3*a^3*c^2*d^6 - a^
3*c*d^7)*f*cos(f*x + e)^4 + (3*a^3*c^8 - 8*a^3*c^7*d + 16*a^3*c^5*d^3 - 10*
a^3*c^4*d^4 - 8*a^3*c^3*d^5 + 8*a^3*c^2*d^6 - a^3*d^8)*f*cos(f*x + e)^3 + 3
*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2
*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*cos(f*x + e)^2 + (a^3*c^8 - 8*a^3*c
^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*
a^3*d^8)*f*cos(f*x + e) + (a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*
c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$$
$$= \frac{\int \frac{\sec(e+fx)}{c^2\sec^3(e+fx)+3c^2\sec^2(e+fx)+3c^2\sec(e+fx)+c^2+2cd\sec^4(e+fx)+6cd\sec^3(e+fx)+6cd\sec^2(e+fx)+2cd\sec(e+fx)+d^2\sec^5(e+fx)+3d^2}}{a^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/(c**2*sec(e + f*x)**3 + 3*c**2*sec(e + f*x)**2 + 3*c*
*2*sec(e + f*x) + c**2 + 2*c*d*sec(e + f*x)**4 + 6*c*d*sec(e + f*x)**3 + 6*
c*d*sec(e + f*x)**2 + 2*c*d*sec(e + f*x) + d**2*sec(e + f*x)**5 + 3*d**2*se
c(e + f*x)**4 + 3*d**2*sec(e + f*x)**3 + d**2*sec(e + f*x)**2), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(271) = 542$.

Time = 0.39 (sec) , antiderivative size = 918, normalized size of antiderivative = 3.19

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/60*(120*d^4*tan(1/2*f*x + 1/2*e)/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2
 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan
(1/2*f*x + 1/2*e)^2 - c - d)) + 120*(4*c*d^3 + 3*d^4)*(pi*floor(1/2*(f*x +
e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*
f*x + 1/2*e))/sqrt(-c^2 + d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 +
2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(-c^2 + d^2)) - (3*a^12*c^8*tan(
1/2*f*x + 1/2*e)^5 - 24*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^5 + 84*a^12*c^6*d^2
*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^5 + 210*a^1
2*c^4*d^4*tan(1/2*f*x + 1/2*e)^5 - 168*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^5
+ 84*a^12*c^2*d^6*tan(1/2*f*x + 1/2*e)^5 - 24*a^12*c*d^7*tan(1/2*f*x + 1/2*
e)^5 + 3*a^12*d^8*tan(1/2*f*x + 1/2*e)^5 - 10*a^12*c^8*tan(1/2*f*x + 1/2*e)
^3 + 100*a^12*c^7*d*tan(1/2*f*x + 1/2*e)^3 - 420*a^12*c^6*d^2*tan(1/2*f*x +
 1/2*e)^3 + 980*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e)^3 - 1400*a^12*c^4*d^4*tan
(1/2*f*x + 1/2*e)^3 + 1260*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e)^3 - 700*a^12*c
^2*d^6*tan(1/2*f*x + 1/2*e)^3 + 220*a^12*c*d^7*tan(1/2*f*x + 1/2*e)^3 - 30*
a^12*d^8*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^8*tan(1/2*f*x + 1/2*e) - 180*a^
12*c^7*d*tan(1/2*f*x + 1/2*e) + 1020*a^12*c^6*d^2*tan(1/2*f*x + 1/2*e) - 31
80*a^12*c^5*d^3*tan(1/2*f*x + 1/2*e) + 5850*a^12*c^4*d^4*tan(1/2*f*x + 1/2*
e) - 6540*a^12*c^3*d^5*tan(1/2*f*x + 1/2*e) + 4380*a^12*c^2*d^6*tan(1/2*f*x
 + 1/2*e) - 1620*a^12*c*d^7*tan(1/2*f*x + 1/2*e) + 255*a^12*d^8*tan(1/2*f*x
 + 1/2*e))/(a^15*c^10 - 10*a^15*c^9*d + 45*a^15*c^8*d^2 - 120*a^15*c^7*d^3
+ 210*a^15*c^6*d^4 - 252*a^15*c^5*d^5 + 210*a^15*c^4*d^6 - 120*a^15*c^3*d^7
 + 45*a^15*c^2*d^8 - 10*a^15*c*d^9 + a^15*d^10))/f
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.61

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{20\,a^3\,f\,(c-d)^2} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(\frac{2\,(c^2-d^2)\,\left(\frac{1}{a^3\,(c-d)^2}-\frac{c^2-d^2}{2\,a^3\,(c-d)^4}\right)}{(c-d)^2}-\frac{3}{2\,a^3\,(c-d)^2}+\frac{(c+d)^2}{4\,a^3\,(c-d)^4}\right)}{f}$$

$$- \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\,\left(\frac{1}{3\,a^3\,(c-d)^2}-\frac{c^2-d^2}{6\,a^3\,(c-d)^4}\right)}{f}$$

$$+ \frac{2\,d^4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f\,(c+d)\,\left(a^3\,c^5-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,(a^3\,c^5-5\,a^3\,c^4\,d+10\,a^3\,c^3\,d^2-10\,a^3\,c^2\,d^3+5\,a^3\,c\,d^4-a^3\,d^5)+a^3\,d^5-\right.}$$

$$+ \frac{d^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^5-5\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^4\,d+10\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^3\,d^2-10\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2\,d^3+5\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c\,d^4-1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)d^5}{\sqrt{c+d}\,(c-d)^{9/2}}\right)}{a^3\,f\,(c+d)^{3/2}\,(c-d)^{9/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^2),x)
```

```
[Out] tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c - d)^2) - (tan(e/2 + (f*x)/2)*((2*(c^2 -
d^2)*(1/(a^3*(c - d)^2) - (c^2 - d^2)/(2*a^3*(c - d)^4)))/(c - d)^2 - 3/(2*
a^3*(c - d)^2) + (c + d)^2/(4*a^3*(c - d)^4)))/f - (tan(e/2 + (f*x)/2)^3*(1
/(3*a^3*(c - d)^2) - (c^2 - d^2)/(6*a^3*(c - d)^4)))/f + (2*d^4*tan(e/2 + (
f*x)/2))/(f*(c + d)*(a^3*c^5 - tan(e/2 + (f*x)/2)^2*(a^3*c^5 - a^3*d^5 + 5*
a^3*c*d^4 - 5*a^3*c^4*d - 10*a^3*c^2*d^3 + 10*a^3*c^3*d^2) + a^3*d^5 - 3*a^
3*c*d^4 - 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2)) + (d^3*atan((c^5*ta
n(e/2 + (f*x)/2)*1i - d^5*tan(e/2 + (f*x)/2)*1i + c*d^4*tan(e/2 + (f*x)/2)*
5i - c^4*d*tan(e/2 + (f*x)/2)*5i - c^2*d^3*tan(e/2 + (f*x)/2)*10i + c^3*d^2
*tan(e/2 + (f*x)/2)*10i)/((c + d)^(1/2)*(c - d)^(9/2)))*(4*c + 3*d)*2i)/(a^
3*f*(c + d)^(3/2)*(c - d)^(9/2))
```

3.233 $\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 368

$$\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx \\
&= -\frac{d^3(20c^2+30cd+13d^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{a^3(c-d)^{11/2}(c+d)^{5/2}f} \\
&\quad+\frac{d(4c^3-30c^2d+146cd^2+195d^3)\tan(e+fx)}{30a^3(c-d)^4(c+d)f(c+d\sec(e+fx))^2} \\
&\quad+\frac{\tan(e+fx)}{5(c-d)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} \\
&\quad+\frac{(2c-11d)\tan(e+fx)}{15a(c-d)^2f(a+a\sec(e+fx))^2(c+d\sec(e+fx))^2} \\
&\quad+\frac{(2c^2-15cd+76d^2)\tan(e+fx)}{15(c-d)^3f\,(a^3+a^3\sec(e+fx))\,(c+d\sec(e+fx))^2} \\
&\quad+\frac{d(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\tan(e+fx)}{30a^3(c-d)^5(c+d)^2f(c+d\sec(e+fx))}
\end{aligned}$$

```
[Out] -d^3*(20*c^2+30*c*d+13*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1
/2))/a^3/(c-d)^(11/2)/(c+d)^(5/2)/f+1/30*d*(4*c^3-30*c^2*d+146*c*d^2+195*d^
3)*tan(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*sec(f*x+e))^2+1/5*tan(f*x+e)/(c-d)/f
/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^2+1/15*(2*c-11*d)*tan(f*x+e)/a/(c-d)^2
/f/(a+a*sec(f*x+e))^2/(c+d*sec(f*x+e))^2+1/15*(2*c^2-15*c*d+76*d^2)*tan(f*x
+e)/(c-d)^3/f/(a^3+a^3*sec(f*x+e))/(c+d*sec(f*x+e))^2+1/30*d*(4*c^4-30*c^3*
d+142*c^2*d^2+525*c*d^3+304*d^4)*tan(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*sec(
f*x+e))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4072, 105, 156, 157, 12, 95, 211}

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\tan(e+fx)}{30f(c-d)^5(c+d)^2\,(a^3\sec(e+fx)+a^3)}$$
$$+ \frac{d^3(20c^2+30cd+13d^2)\tan(e+fx)\arctan\left(\frac{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)}{a^2f(c-d)^{11/2}(c+d)^{5/2}\sqrt{a-a\sec(e+fx)}\sqrt{a\sec(e+fx)+a}}$$
$$- \frac{3d(2c+d)\tan(e+fx)}{2f\,(c^2-d^2)^2\,(a\sec(e+fx)+a)^3(c+d\sec(e+fx))}$$
$$- \frac{d\tan(e+fx)}{2f\,(c^2-d^2)\,(a\sec(e+fx)+a)^3(c+d\sec(e+fx))^2}$$
$$+ \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10f(c-d)^3(c+d)^2(a\sec(e+fx)+a)^3}$$
$$+ \frac{(4c^3-26c^2d-184cd^2-109d^3)\tan(e+fx)}{30af(c-d)^4(c+d)^2(a\sec(e+fx)+a)^2}$$

```
[In] Int[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] ((2*c^2 + 39*c*d + 22*d^2)*Tan[e + f*x])/(10*(c - d)^3*(c + d)^2*f*(a + a*S
ec[e + f*x])^3) + ((4*c^3 - 26*c^2*d - 184*c*d^2 - 109*d^3)*Tan[e + f*x])/(
30*a*(c - d)^4*(c + d)^2*f*(a + a*Sec[e + f*x])^2) + (d^3*(20*c^2 + 30*c*d
+ 13*d^2)*ArcTan[(Sqrt[c + d]*Sqrt[a + a*Sec[e + f*x]])/(Sqrt[c - d]*Sqrt[a
 - a*Sec[e + f*x]])]*Tan[e + f*x])/(a^2*(c - d)^(11/2)*(c + d)^(5/2)*f*Sqrt
[a - a*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]]) + ((4*c^4 - 30*c^3*d + 142*c
^2*d^2 + 525*c*d^3 + 304*d^4)*Tan[e + f*x])/(30*(c - d)^5*(c + d)^2*f*(a^3
+ a^3*Sec[e + f*x])) - (d*Tan[e + f*x])/(2*(c^2 - d^2)*f*(a + a*Sec[e + f*x
])^3*(c + d*Sec[e + f*x])^2) - (3*d*(2*c + d)*Tan[e + f*x])/(2*(c^2 - d^2)^
2*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 156

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4072

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[a
^2*g*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])),
Subst[Int[(g*x)^(p - 1)*(a + b*x)^(m - 1/2)*((c + d*x)^n/Sqrt[a - b*x]), x]
, x, Csc[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (EqQ[p, 1] || Int
```

```
egerQ[m - 1/2])
```

Rubi steps

$$\text{integral} = -\frac{(a^2\tan(e+fx))\,\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)^3}\,dx, x, \sec(e+fx)\right)}{f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^2(2c+3d)-4a^2dx}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)^2}\,dx, x, \sec(e+fx)\right)}{2\left(c^2-d^2\right)f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= -\frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{3d(2c+d)\tan(e+fx)}{2\left(c^2-d^2\right)^2f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^4(2c^2+21cd+13d^2)-9a^4d(2c+d)x}{\sqrt{a-ax}(a+ax)^{7/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2a^2\left(c^2-d^2\right)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} - \frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{3d(2c+d)\tan(e+fx)}{2\left(c^2-d^2\right)^2f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{-a^6(4c^3-22c^2d-106cd^2-65d^3)-2a^6d(2c^2+39cd+22d^2)x}{\sqrt{a-ax}(a+ax)^{5/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{10a^5(c-d)\left(c^2-d^2\right)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{\left(2c^2+39cd+22d^2\right)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{\left(4c^3-26c^2d-184cd^2-109d^3\right)\tan(e+fx)}{30a(c-d)^4(c+d)^2f(a+a\sec(e+fx))^2} - \frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{3d(2c+d)\tan(e+fx)}{2\left(c^2-d^2\right)^2f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} - \frac{\tan(e+fx)\text{Subst}\left(\int \frac{a^8(c+d)(4c^3-30c^2d+146cd^2+195d^3)+a^8d(4c^3-26c^2d-184cd^2-109d^3)x}{\sqrt{a-ax}(a+ax)^{3/2}(c+dx)}\,dx, x, \sec(e+fx)\right)}{30a^8(c-d)^2\left(c^2-d^2\right)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{(4c^3-26c^2d-184cd^2-109d^3)\tan(e+fx)}{30a(c-d)^4(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\tan(e+fx)}{30(c-d)^5(c+d)^2f(a^3+a^3\sec(e+fx))} - \frac{d\tan(e+fx)}{2(c^2-d^2)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{3d(2c+d)\tan(e+fx)}{2(c^2-d^2)^2f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} + \frac{\tan(e+fx)\text{Subst}\left(\int \frac{15a^{10}d^3(20c^2+30cd+13d^2)}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{30a^{11}(c-d)^3(c^2-d^2)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{(4c^3-26c^2d-184cd^2-109d^3)\tan(e+fx)}{30a(c-d)^4(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\tan(e+fx)}{30(c-d)^5(c+d)^2f(a^3+a^3\sec(e+fx))} - \frac{d\tan(e+fx)}{2(c^2-d^2)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{3d(2c+d)\tan(e+fx)}{2(c^2-d^2)^2f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} + \frac{(d^3(20c^2+30cd+13d^2)\tan(e+fx))\text{Subst}\left(\int \frac{1}{\sqrt{a-ax}\sqrt{a+ax}(c+dx)}\,dx, x, \sec(e+fx)\right)}{2a(c-d)^3(c^2-d^2)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} + \frac{(4c^3-26c^2d-184cd^2-109d^3)\tan(e+fx)}{30a(c-d)^4(c+d)^2f(a+a\sec(e+fx))^2} + \frac{(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\tan(e+fx)}{30(c-d)^5(c+d)^2f(a^3+a^3\sec(e+fx))} - \frac{d\tan(e+fx)}{2(c^2-d^2)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} - \frac{3d(2c+d)\tan(e+fx)}{2(c^2-d^2)^2f(a+a\sec(e+fx))^3(c+d\sec(e+fx))} + \frac{(d^3(20c^2+30cd+13d^2)\tan(e+fx))\text{Subst}\left(\int \frac{1}{ac-ad-(-ac-ad)x^2}\,dx, x, \frac{\sqrt{a+a\sec(e+fx)}}{\sqrt{a-a\sec(e+fx)}}\right)}{a(c-d)^3(c^2-d^2)^2f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}$$

$$
\begin{aligned}
&= \frac{(2c^2+39cd+22d^2)\tan(e+fx)}{10(c-d)^3(c+d)^2f(a+a\sec(e+fx))^3} \\
&\quad + \frac{(4c^3-26c^2d-184cd^2-109d^3)\tan(e+fx)}{30a(c-d)^4(c+d)^2f(a+a\sec(e+fx))^2} \\
&\quad + \frac{d^3(20c^2+30cd+13d^2)\arctan\left(\frac{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}{\sqrt{c-d}\sqrt{a-a\sec(e+fx)}}\right)\tan(e+fx)}{a^2(c-d)^{11/2}(c+d)^{5/2}f\sqrt{a-a\sec(e+fx)}\sqrt{a+a\sec(e+fx)}} \\
&\quad + \frac{(4c^4-30c^3d+142c^2d^2+525cd^3+304d^4)\tan(e+fx)}{30(c-d)^5(c+d)^2f\left(a^3+a^3\sec(e+fx)\right)} \\
&\quad - \frac{d\tan(e+fx)}{2\left(c^2-d^2\right)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^2} \\
&\quad - \frac{3d(2c+d)\tan(e+fx)}{2\left(c^2-d^2\right)^2f(a+a\sec(e+fx))^3(c+d\sec(e+fx))}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 9.05 (sec) , antiderivative size = 1096, normalized size of antiderivative = 2.98

$$
\begin{aligned}
&\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx \\
&= \frac{4\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec\left(\frac{e}{2}\right)\sec^6(e+fx)\left(-8c\sin\left(\frac{e}{2}\right)+23d\sin\left(\frac{e}{2}\right)\right)}{15(-c+d)^4f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3} \\
&\quad + \frac{(20c^2+30cd+13d^2)\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec^6(e+fx)\left(-\frac{8id^3\arctan\left(\sec\left(\frac{fx}{2}\right)\left(\frac{\cos(e)}{\sqrt{c^2-d^2}\sqrt{\cos(2e)-i\sin(2e)}}\right.\right.}{}\right.}{(-c+d)} \\
&\quad - \frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec\left(\frac{e}{2}\right)\sec^6(e+fx)\sin\left(\frac{fx}{2}\right)}{5(-c+d)^3f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3} \\
&\quad + \frac{4\cos^3\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec\left(\frac{e}{2}\right)\sec^6(e+fx)\left(-8c\sin\left(\frac{fx}{2}\right)+23d\sin\left(\frac{fx}{2}\right)\right)}{15(-c+d)^4f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3} \\
&\quad - \frac{8\cos^5\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec\left(\frac{e}{2}\right)\sec^6(e+fx)\left(7c^2\sin\left(\frac{fx}{2}\right)-44cd\sin\left(\frac{fx}{2}\right)+127d^2\sin\left(\frac{fx}{2}\right)\right)}{15(-c+d)^5f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3} \\
&\quad + \frac{4\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))\sec(e)\sec^6(e+fx)\left(d^6\sin(e)-cd^5\sin(fx)\right)}{c^2(-c+d)^4(c+d)f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3} \\
&\quad - \frac{4\cos^6\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^2\sec(e)\sec^6(e+fx)\left(-11c^2d^5\sin(e)-6cd^6\sin(e)+2d^7\sin(e)+10c^3\right.}{c^2(-c+d)^5(c+d)^2f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3} \\
&\quad - \frac{2\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(d+c\cos(e+fx))^3\sec^6(e+fx)\tan\left(\frac{e}{2}\right)}{5(-c+d)^3f(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]/((a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3),x]
```

```
[Out] (4*Cos[e/2 + (f*x)/2]^4*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8*
c*Sin[e/2] + 23*d*Sin[e/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(c + d
*Sec[e + f*x])^3) + ((20*c^2 + 30*c*d + 13*d^2)*Cos[e/2 + (f*x)/2]^6*(d + c
*Cos[e + f*x])^3*Sec[e + f*x]^6*(((-8*I)*d^3*ArcTan[Sec[(f*x)/2]*(Cos[e]/(S
qrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e])/(Sqrt[c^2 - d^2]*S
qrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c*Sin[e + (f*x)/2])]*
Cos[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (8*d^3*ArcTan[Sec
[(f*x)/2]*(Cos[e]/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]) - (I*Sin[e]
)/(Sqrt[c^2 - d^2]*Sqrt[Cos[2*e] - I*Sin[2*e]]))*((-I)*d*Sin[(f*x)/2] + I*c
*Sin[e + (f*x)/2])]*Sin[e])/(Sqrt[c^2 - d^2]*f*Sqrt[Cos[2*e] - I*Sin[2*e]])
))/((-c + d)^5*(c + d)^2*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (
2*Cos[e/2 + (f*x)/2]*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*Sin[(f*
x)/2])/(5*(-c + d)^3*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) + (4*
Cos[e/2 + (f*x)/2]^3*(d + c*Cos[e + f*x])^3*Sec[e/2]*Sec[e + f*x]^6*(-8*c*S
in[(f*x)/2] + 23*d*Sin[(f*x)/2]))/(15*(-c + d)^4*f*(a + a*Sec[e + f*x])^3*(
c + d*Sec[e + f*x])^3) - (8*Cos[e/2 + (f*x)/2]^5*(d + c*Cos[e + f*x])^3*Sec
[e/2]*Sec[e + f*x]^6*(7*c^2*Sin[(f*x)/2] - 44*c*d*Sin[(f*x)/2] + 127*d^2*Si
n[(f*x)/2]))/(15*(-c + d)^5*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3
) + (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])*Sec[e]*Sec[e + f*x]^6*(d^6
*Sin[e] - c*d^5*Sin[f*x]))/(c^2*(-c + d)^4*(c + d)*f*(a + a*Sec[e + f*x])^3
*(c + d*Sec[e + f*x])^3) - (4*Cos[e/2 + (f*x)/2]^6*(d + c*Cos[e + f*x])^2*S
ec[e]*Sec[e + f*x]^6*(-11*c^2*d^5*Sin[e] - 6*c*d^6*Sin[e] + 2*d^7*Sin[e] +
10*c^3*d^4*Sin[f*x] + 6*c^2*d^5*Sin[f*x] - c*d^6*Sin[f*x]))/(c^2*(-c + d)^5
*(c + d)^2*f*(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3) - (2*Cos[e/2 +
(f*x)/2]^2*(d + c*Cos[e + f*x])^3*Sec[e + f*x]^6*Tan[e/2])/(5*(-c + d)^3*f*
(a + a*Sec[e + f*x])^3*(c + d*Sec[e + f*x])^3)
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} c^{2}}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} d^{2}}{5}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3} cd}{3}-\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3} d^{2}}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)\left(c^{2}-2cd+d^{2}\right)}$
default	$\frac{\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} c^{2}}{5}-\frac{2\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} cd}{5}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5} d^{2}}{5}-\frac{2c^{2}\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{3}+\frac{10\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3} cd}{3}-\frac{8\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3} d^{2}}{3}+\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{\left(c^{3}-3c^{2}d+3c\,d^{2}-d^{3}\right)\left(c^{2}-2cd+d^{2}\right)}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f/a^3*(1/(c^3-3*c^2*d+3*c*d^2-d^3)/(c^2-2*c*d+d^2)*(1/5*tan(1/2*f*x+1/2
*e)^5*c^2-2/5*tan(1/2*f*x+1/2*e)^5*c*d+1/5*tan(1/2*f*x+1/2*e)^5*d^2-2/3*c^2
*tan(1/2*f*x+1/2*e)^3+10/3*tan(1/2*f*x+1/2*e)^3*c*d-8/3*tan(1/2*f*x+1/2*e)^
3*d^2+tan(1/2*f*x+1/2*e)*c^2-8*tan(1/2*f*x+1/2*e)*c*d+31*tan(1/2*f*x+1/2*e)
*d^2)+16*d^3/(c-d)^5*((-1/4*d*(10*c^2-3*c*d-7*d^2)/(c^2+2*c*d+d^2)*tan(1/2*
f*x+1/2*e)^3+5/4*d*(2*c+d)/(c+d)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*
c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2-1/4*(20*c^2+30*c*d+13*d^2)/(c^2+2*c*d+d^2)/
((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(349) = 698$.

Time = 0.39 (sec) , antiderivative size = 2677, normalized size of antiderivative = 7.27

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/60*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13*
c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d^6
)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 + 13
*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*d^6
+ 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)*
cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f
*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d
^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(4*c^6*d^2 - 30*c^
5*d^3 + 138*c^4*d^4 + 555*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8 + (14
*c^8 - 60*c^7*d + 78*c^6*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^3*d^5 - 41
9*c^2*d^6 - 90*c*d^7 + 15*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7*d + 114*c^
6*d^2 + 1056*c^5*d^3 + 1626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6 - 913*c*d^7
 - 45*d^8)*cos(f*x + e)^3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 828*c^5*d^3 + 2
400*c^4*d^4 + 1197*c^3*d^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*d^8)*cos(f*x +
 e)^2 + (8*c^7*d - 48*c^6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*d^5 -
 459*c^2*d^6 - 1733*c*d^7 - 717*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^11
 - 3*a^3*c^10*d + 8*a^3*c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^6*d^5 + 8*a^3*c^5
*d^6 - 3*a^3*c^3*d^8 + a^3*c^2*d^9)*f*cos(f*x + e)^5 + (3*a^3*c^11 - 7*a^3*
c^10*d - 6*a^3*c^9*d^2 + 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4 - 30*a^3*c^6*d^5 +
12*a^3*c^5*d^6 + 16*a^3*c^4*d^7 - 9*a^3*c^3*d^8 - 3*a^3*c^2*d^9 + 2*a^3*c*d
^10)*f*cos(f*x + e)^4 + (3*a^3*c^11 - 3*a^3*c^10*d - 17*a^3*c^9*d^2 + 21*a^
3*c^8*d^3 + 30*a^3*c^7*d^4 - 46*a^3*c^6*d^5 - 18*a^3*c^5*d^6 + 42*a^3*c^4*d
```

```
^7 - a^3*c^3*d^8 - 15*a^3*c^2*d^9 + 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)
^3 + (a^3*c^11 + 3*a^3*c^10*d - 15*a^3*c^9*d^2 - a^3*c^8*d^3 + 42*a^3*c^7*d
^4 - 18*a^3*c^6*d^5 - 46*a^3*c^5*d^6 + 30*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 17
*a^3*c^2*d^9 - 3*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e)^2 + (2*a^3*c^10*d
- 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*
c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3
*a^3*d^11)*f*cos(f*x + e) + (a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 -
6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f)
, -1/30*(15*(20*c^2*d^5 + 30*c*d^6 + 13*d^7 + (20*c^4*d^3 + 30*c^3*d^4 + 13
*c^2*d^5)*cos(f*x + e)^5 + (60*c^4*d^3 + 130*c^3*d^4 + 99*c^2*d^5 + 26*c*d^
6)*cos(f*x + e)^4 + (60*c^4*d^3 + 210*c^3*d^4 + 239*c^2*d^5 + 108*c*d^6 + 1
3*d^7)*cos(f*x + e)^3 + (20*c^4*d^3 + 150*c^3*d^4 + 253*c^2*d^5 + 168*c*d^6
 + 39*d^7)*cos(f*x + e)^2 + (40*c^3*d^4 + 120*c^2*d^5 + 116*c*d^6 + 39*d^7)
*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) +
c)/((c^2 - d^2)*sin(f*x + e))) - (4*c^6*d^2 - 30*c^5*d^3 + 138*c^4*d^4 + 55
5*c^3*d^5 + 162*c^2*d^6 - 525*c*d^7 - 304*d^8 + (14*c^8 - 60*c^7*d + 78*c^6
*d^2 + 480*c^5*d^3 + 312*c^4*d^4 - 330*c^3*d^5 - 419*c^2*d^6 - 90*c*d^7 + 1
5*d^8)*cos(f*x + e)^4 + (12*c^8 - 62*c^7*d + 114*c^6*d^2 + 1056*c^5*d^3 + 1
626*c^4*d^4 - 81*c^3*d^5 - 1707*c^2*d^6 - 913*c*d^7 - 45*d^8)*cos(f*x + e)^
3 + (4*c^8 - 6*c^7*d - 28*c^6*d^2 + 828*c^5*d^3 + 2400*c^4*d^4 + 1197*c^3*d
^5 - 1897*c^2*d^6 - 2019*c*d^7 - 479*d^8)*cos(f*x + e)^2 + (8*c^7*d - 48*c^
6*d^2 + 186*c^5*d^3 + 1224*c^4*d^4 + 1539*c^3*d^5 - 459*c^2*d^6 - 1733*c*d^
7 - 717*d^8)*cos(f*x + e))*sin(f*x + e))/((a^3*c^11 - 3*a^3*c^10*d + 8*a^3*
c^8*d^3 - 6*a^3*c^7*d^4 - 6*a^3*c^6*d^5 + 8*a^3*c^5*d^6 - 3*a^3*c^3*d^8 + a
^3*c^2*d^9)*f*cos(f*x + e)^5 + (3*a^3*c^11 - 7*a^3*c^10*d - 6*a^3*c^9*d^2 +
 24*a^3*c^8*d^3 - 2*a^3*c^7*d^4 - 30*a^3*c^6*d^5 + 12*a^3*c^5*d^6 + 16*a^3*
c^4*d^7 - 9*a^3*c^3*d^8 - 3*a^3*c^2*d^9 + 2*a^3*c*d^10)*f*cos(f*x + e)^4 +
(3*a^3*c^11 - 3*a^3*c^10*d - 17*a^3*c^9*d^2 + 21*a^3*c^8*d^3 + 30*a^3*c^7*d
^4 - 46*a^3*c^6*d^5 - 18*a^3*c^5*d^6 + 42*a^3*c^4*d^7 - a^3*c^3*d^8 - 15*a^
3*c^2*d^9 + 3*a^3*c*d^10 + a^3*d^11)*f*cos(f*x + e)^3 + (a^3*c^11 + 3*a^3*c
^10*d - 15*a^3*c^9*d^2 - a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 18*a^3*c^6*d^5 - 46
*a^3*c^5*d^6 + 30*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 3*a^3*c*d
^10 + 3*a^3*d^11)*f*cos(f*x + e)^2 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*
c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7
+ 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*cos(f*x + e
) + (a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^
4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{\int \frac{\sec(e+fx)}{c^3\sec^3(e+fx)+3c^3\sec^2(e+fx)+3c^3\sec(e+fx)+c^3+3c^2d\sec^4(e+fx)+9c^2d\sec^3(e+fx)+9c^2d\sec^2(e+fx)+3c^2d\sec(e+fx)+3cd^2\sec^5(e+f}}{a^3}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**3/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral(sec(e + f*x)/(c**3*sec(e + f*x)**3 + 3*c**3*sec(e + f*x)**2 + 3*c*
*3*sec(e + f*x) + c**3 + 3*c**2*d*sec(e + f*x)**4 + 9*c**2*d*sec(e + f*x)**
3 + 9*c**2*d*sec(e + f*x)**2 + 3*c**2*d*sec(e + f*x) + 3*c*d**2*sec(e + f*x
)**5 + 9*c*d**2*sec(e + f*x)**4 + 9*c*d**2*sec(e + f*x)**3 + 3*c*d**2*sec(e
 + f*x)**2 + d**3*sec(e + f*x)**6 + 3*d**3*sec(e + f*x)**5 + 3*d**3*sec(e +
 f*x)**4 + d**3*sec(e + f*x)**3), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. $2(349) = 698$.

Time = 0.47 (sec) , antiderivative size = 1369, normalized size of antiderivative = 3.72

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^3/(c+d*sec(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(20*c^2*d^3 + 30*c*d^4 + 13*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2
)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e
```

```
))/sqrt(-c^2 + d^2)))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4*d^3
 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*sqrt(-c^2 + d^2)) -
 (3*a^12*c^12*tan(1/2*f*x + 1/2*e)^5 - 36*a^12*c^11*d*tan(1/2*f*x + 1/2*e)^
5 + 198*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^5 - 660*a^12*c^9*d^3*tan(1/2*f*x
 + 1/2*e)^5 + 1485*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^5 - 2376*a^12*c^7*d^5*
tan(1/2*f*x + 1/2*e)^5 + 2772*a^12*c^6*d^6*tan(1/2*f*x + 1/2*e)^5 - 2376*a^
12*c^5*d^7*tan(1/2*f*x + 1/2*e)^5 + 1485*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e)^
5 - 660*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^5 + 198*a^12*c^2*d^10*tan(1/2*f*x
 + 1/2*e)^5 - 36*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^5 + 3*a^12*d^12*tan(1/2*f
*x + 1/2*e)^5 - 10*a^12*c^12*tan(1/2*f*x + 1/2*e)^3 + 150*a^12*c^11*d*tan(1
/2*f*x + 1/2*e)^3 - 990*a^12*c^10*d^2*tan(1/2*f*x + 1/2*e)^3 + 3850*a^12*c^
9*d^3*tan(1/2*f*x + 1/2*e)^3 - 9900*a^12*c^8*d^4*tan(1/2*f*x + 1/2*e)^3 + 1
7820*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e)^3 - 23100*a^12*c^6*d^6*tan(1/2*f*x +
 1/2*e)^3 + 21780*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e)^3 - 14850*a^12*c^4*d^8*
tan(1/2*f*x + 1/2*e)^3 + 7150*a^12*c^3*d^9*tan(1/2*f*x + 1/2*e)^3 - 2310*a^
12*c^2*d^10*tan(1/2*f*x + 1/2*e)^3 + 450*a^12*c*d^11*tan(1/2*f*x + 1/2*e)^3
 - 40*a^12*d^12*tan(1/2*f*x + 1/2*e)^3 + 15*a^12*c^12*tan(1/2*f*x + 1/2*e)
- 270*a^12*c^11*d*tan(1/2*f*x + 1/2*e) + 2340*a^12*c^10*d^2*tan(1/2*f*x + 1
/2*e) - 11850*a^12*c^9*d^3*tan(1/2*f*x + 1/2*e) + 38475*a^12*c^8*d^4*tan(1/
2*f*x + 1/2*e) - 84780*a^12*c^7*d^5*tan(1/2*f*x + 1/2*e) + 131040*a^12*c^6*
d^6*tan(1/2*f*x + 1/2*e) - 144180*a^12*c^5*d^7*tan(1/2*f*x + 1/2*e) + 11272
5*a^12*c^4*d^8*tan(1/2*f*x + 1/2*e) - 61350*a^12*c^3*d^9*tan(1/2*f*x + 1/2*
e) + 22140*a^12*c^2*d^10*tan(1/2*f*x + 1/2*e) - 4770*a^12*c*d^11*tan(1/2*f*
x + 1/2*e) + 465*a^12*d^12*tan(1/2*f*x + 1/2*e))/(a^15*c^15 - 15*a^15*c^14*
d + 105*a^15*c^13*d^2 - 455*a^15*c^12*d^3 + 1365*a^15*c^11*d^4 - 3003*a^15*
c^10*d^5 + 5005*a^15*c^9*d^6 - 6435*a^15*c^8*d^7 + 6435*a^15*c^7*d^8 - 5005
*a^15*c^6*d^9 + 3003*a^15*c^5*d^10 - 1365*a^15*c^4*d^11 + 455*a^15*c^3*d^12
 - 105*a^15*c^2*d^13 + 15*a^15*c*d^14 - a^15*d^15) + 60*(10*c^2*d^4*tan(1/2
*f*x + 1/2*e)^3 - 3*c*d^5*tan(1/2*f*x + 1/2*e)^3 - 7*d^6*tan(1/2*f*x + 1/2*
e)^3 - 10*c^2*d^4*tan(1/2*f*x + 1/2*e) - 15*c*d^5*tan(1/2*f*x + 1/2*e) - 5*
d^6*tan(1/2*f*x + 1/2*e))/((a^3*c^7 - 3*a^3*c^6*d + a^3*c^5*d^2 + 5*a^3*c^4
*d^3 - 5*a^3*c^3*d^4 - a^3*c^2*d^5 + 3*a^3*c*d^6 - a^3*d^7)*(c*tan(1/2*f*x
+ 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 14.09 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.78

$$\int \frac{\sec(e+fx)}{(a+a\sec(e+fx))^3(c+d\sec(e+fx))^3}\,dx$$

$$= \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5}{20\,a^3\,f\,(c-d)^3} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,\left(\frac{3\,(c+d)^2}{4\,a^3\,(c-d)^5}-\frac{5}{2\,a^3\,(c-d)^3}+\frac{3\,(c+d)\,\left(\frac{5}{4\,a^3\,(c-d)^3}-\frac{3\,(c+d)}{4\,a^3\,(c-d)^4}\right)}{c-d}\right)}{f}$$
$$- \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\,\left(\frac{5}{12\,a^3\,(c-d)^3}-\frac{c+d}{4\,a^3\,(c-d)^4}\right)}{f}$$
$$- \frac{\quad}{f\,\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2\,(2\,a^3\,c^7-10\,a^3\,c^6\,d+18\,a^3\,c^5\,d^2-10\,a^3\,c^4\,d^3-10\,a^3\,c^3\,d^4+18\,a^3\,c^2\,d^5-10\,a^3\,c\,d^6+2\cdots}$$
$$+ \frac{d^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^6-6\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^5\,d+15\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^4\,d^2-20\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^3\,d^3+15\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c^2\,d^4-6\mathrm{i}\tan\left(\frac{e}{2}+\frac{fx}{2}\right)c\,d^5\cdots}{\sqrt{c+d}\,(c-d)^{11/2}}\cdots}{a^3\,f\,(c+d)^{5/2}\,(c-d)^{11/2}}$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^3*(c + d/cos(e + f*x))^3),x)
```

```
[Out] tan(e/2 + (f*x)/2)^5/(20*a^3*f*(c - d)^3) - (tan(e/2 + (f*x)/2)*((3*(c + d)
^2)/(4*a^3*(c - d)^5) - 5/(2*a^3*(c - d)^3) + (3*(c + d)*(5/(4*a^3*(c - d)^
3) - (3*(c + d))/(4*a^3*(c - d)^4)))/(c - d)))/f - (tan(e/2 + (f*x)/2)^3*(5
/(12*a^3*(c - d)^3) - (c + d)/(4*a^3*(c - d)^4)))/f - ((tan(e/2 + (f*x)/2)^
3*(3*c*d^5 + 7*d^6 - 10*c^2*d^4))/(c + d)^2 + (5*tan(e/2 + (f*x)/2)*(2*c*d^
4 + d^5))/(c + d))/(f*(tan(e/2 + (f*x)/2)^2*(2*a^3*c^7 + 2*a^3*d^7 - 10*a^3
*c*d^6 - 10*a^3*c^6*d + 18*a^3*c^2*d^5 - 10*a^3*c^3*d^4 - 10*a^3*c^4*d^3 +
18*a^3*c^5*d^2) - tan(e/2 + (f*x)/2)^4*(a^3*c^7 - a^3*d^7 + 7*a^3*c*d^6 - 7
*a^3*c^6*d - 21*a^3*c^2*d^5 + 35*a^3*c^3*d^4 - 35*a^3*c^4*d^3 + 21*a^3*c^5*
d^2) - a^3*c^7 + a^3*d^7 - 3*a^3*c*d^6 + 3*a^3*c^6*d + a^3*c^2*d^5 + 5*a^3*
c^3*d^4 - 5*a^3*c^4*d^3 - a^3*c^5*d^2)) + (d^3*atan((c^6*tan(e/2 + (f*x)/2)
*1i + d^6*tan(e/2 + (f*x)/2)*1i - c*d^5*tan(e/2 + (f*x)/2)*6i - c^5*d*tan(e
/2 + (f*x)/2)*6i + c^2*d^4*tan(e/2 + (f*x)/2)*15i - c^3*d^3*tan(e/2 + (f*x)
/2)*20i + c^4*d^2*tan(e/2 + (f*x)/2)*15i)/((c + d)^(1/2)*(c - d)^(11/2)))*(
30*c*d + 20*c^2 + 13*d^2)*1i)/(a^3*f*(c + d)^(5/2)*(c - d)^(11/2))
```

3.234 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

```
[Out] 2*arctanh(a^(1/2)*d^(1/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)
)^(1/2))*a^(1/2)/f/d^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {4065, 212}

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]
*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4065

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-2*(b/f), Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2a)\text{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$= \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\sqrt{d+c\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{d}f\sqrt{c+d\sec(e+fx)}}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],
x]
[Out] (Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]*Sqrt[d + c*Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(S
qrt[d]*f*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(49) = 98$.

Time = 5.51 (sec) , antiderivative size = 278, normalized size of antiderivative = 4.56

method	result
default	$\frac{\sqrt{2}\sqrt{a(\sec(fx+e)+1)}\sqrt{c+d\sec(fx+e)}\left(\ln\left(\frac{2\sqrt{2}\sqrt{-d}\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\sin(fx+e)-2\sin(fx+e)c-2\sin(fx+e)d+2c\cos(fx+e)-2d\cos(fx}{-\cos(fx+e)+1+\sin(fx+e)}\right.\right.}{f\sqrt{-d}(\cos(fx+e)+1)}$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/f*2^(1/2)/(-d)^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(ln(
2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)
-sin(f*x+e)*c-sin(f*x+e)*d+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(-cos(f*x+e)+1+si
n(f*x+e)))-ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(
1/2)*sin(f*x+e)-sin(f*x+e)*c-sin(f*x+e)*d-c*cos(f*x+e)+d*cos(f*x+e)+c-d)/(c
os(f*x+e)-1+sin(f*x+e))))*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(c
os(f*x+e)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(49) = 98$.

Time = 0.41 (sec) , antiderivative size = 307, normalized size of antiderivative = 5.03

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \left[\frac{\sqrt{\frac{a}{d}}\log\left(-\frac{8\,acd\cos(fx+e)+\left(ac^2-6\,acd+ad^2\right)\cos(fx+e)^3+4\left(2\,d^2\cos(fx+e)+\left(cd-d^2\right)\cos(fx+e)^2\right)\sqrt{\frac{a}{d}}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+}{\cos(fx+e)}}}{\cos(fx+e)^3+\cos(fx+e)^2}\right.}{2\,f}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(a/d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f
*x + e)^3 + 4*(2*d^2*cos(f*x + e) + (c*d - d^2)*cos(f*x + e)^2)*sqrt(a/d)*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x +
e))*sin(f*x + e) + 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(c
os(f*x + e)^3 + cos(f*x + e)^2))/f, sqrt(-a/d)*arctan(-2*d*sqrt(-a/d)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*
cos(f*x + e)*sin(f*x + e)/((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)
*cos(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)),
x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),x)
[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)), x
)
```

3.235 $\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 140

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[Out] arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2))*2^(1/2)*(c-d)^(1/2)/f/a^(1/2)+2*arctanh(a^(1/2)*d^(1/
2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))*d^(1/2)/f/a^(1
/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4066, 4068, 209, 4065, 212}

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}+\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqr
t[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*f) + (2*Sqrt[d]*
ArcTanh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d
*Sec[e + f*x]])])/(Sqrt[a]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4065

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-2*(b/f), Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4066

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-(b*c - a*d)/d,
 Int[Csc[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x],
x] + Dist[b/d, Int[Csc[e + f*x]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && Ne
Q[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = (c-d)\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx + \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a}$$

$$= -\frac{(2(c-d))\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} - \frac{(2d)\text{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$= \frac{\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f} + \frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [A] (verified)

Time = 14.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.34

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \frac{\sqrt{c}\left(-\sqrt{2}\sqrt{c-d}\arctan\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{c-d}\sqrt{c-c\cos(e+fx)}}\right) + 2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+c\cos(e+fx)}}{\sqrt{d}\sqrt{c-c\cos(e+fx)}}\right)\right)\sqrt{c+d\sec(e+fx)}\sin(e}{f\sqrt{c-c\cos(e+fx)}\sqrt{d+c\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + a*Sec[e + f*x]],
x]
```

```
[Out] (Sqrt[c]*(-(Sqrt[2]*Sqrt[c - d]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + c*Cos[e +
f*x]])/(Sqrt[c - d]*Sqrt[c - c*Cos[e + f*x]])]) + 2*Sqrt[d]*ArcTanh[(Sqrt[c
]*Sqrt[d + c*Cos[e + f*x]])/(Sqrt[d]*Sqrt[c - c*Cos[e + f*x]])])*Sqrt[c + d
*Sec[e + f*x]]*Sin[e + f*x])/(f*Sqrt[c - c*Cos[e + f*x]]*Sqrt[d + c*Cos[e +
 f*x]]*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(113) = 226$.

Time = 5.30 (sec) , antiderivative size = 478, normalized size of antiderivative = 3.41

method	result
default	$-\frac{\sqrt{2}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{a(\sec(fx+e)+1)}\left(\ln\left(\frac{\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}\,\sqrt{c-d}-c\cot(fx+e)+d\cot(fx+e)+c\csc(fx+e)-d\csc(fx+e)}{\sqrt{c-d}}\right)\sqrt{2}\,\sqrt{-}\right.}{}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/f/a/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a*(sec(f*x+e)
+1))^(1/2)*(ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c
```

```
-d)^(1/2)-c*cot(f*x+e)+d*cot(f*x+e)+c*csc(f*x+e)-d*csc(f*x+e)))*2^(1/2)*(-d
)^(1/2)*c-ln(1/(c-d)^(1/2)*((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(c-d
)^(1/2)-c*cot(f*x+e)+d*cot(f*x+e)+c*csc(f*x+e)-d*csc(f*x+e)))*2^(1/2)*(-d)^
(1/2)*d+d*ln(2*(-2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1
/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)+sin(f*x+e)*c+sin(f*x+e)*d-c+d)/(co
s(f*x+e)-1+sin(f*x+e)))*(c-d)^(1/2)-d*ln(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*co
s(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)-sin(f*x+e)*c-sin(f*x+e)*d+c*cos(
f*x+e)-d*cos(f*x+e)-c+d)/(cos(f*x+e)-1-sin(f*x+e)))*(c-d)^(1/2))*cos(f*x+e)
/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 1048, normalized size of antiderivative = 7.49

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqrt(-(c - d)/a)*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*
x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*cos(f*x + e) + c
 - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(d/a)*log(-((c^2 - 6*c
*d + d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sq
rt(d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/c
os(f*x + e))*sin(f*x + e) + 8*c*d*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(
f*x + e)^2 + 8*d^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/f, 1/2*(2*sqrt(2)*s
qrt((c - d)/a)*arctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/co
s(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*s
in(f*x + e))) + sqrt(d/a)*log(-((c^2 - 6*c*d + d^2)*cos(f*x + e)^3 + 4*((c
- d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(d/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8*c*d
*cos(f*x + e) + (c^2 + 2*c*d - 7*d^2)*cos(f*x + e)^2 + 8*d^2)/(cos(f*x + e)
^3 + cos(f*x + e)^2)))/f, 1/2*(sqrt(2)*sqrt(-(c - d)/a)*log(-(2*sqrt(2)*sqr
t(-(c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2
*(c + d)*cos(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2
*sqrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sq
rt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((c - d)*co
s(f*x + e)^2 + (c + d)*cos(f*x + e) + 2*d))/f, (sqrt(2)*sqrt((c - d)/a)*ar
ctan(-sqrt(2)*sqrt((c - d)/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(
(c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)/((c - d)*sin(f*x + e))) + s
```

```
qrt(-d/a)*arctan(-2*sqrt(-d/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt
((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((c - d)*cos(
f*x + e)^2 + (c + d)*cos(f*x + e) + 2*d)))/f]
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{c+d\sec\left(e+fx\right)}\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**(1/2)/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(c + d*sec(e + f*x))*sec(e + f*x)/sqrt(a*(sec(e + f*x) + 1)),
x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)+c}\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/sqrt(a*sec(f*x + e) + a), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{c+d\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx=\int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((c + d/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)),x)
[Out] int((c + d/cos(e + f*x))^(1/2)/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)), x
)
```

3.236 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 78

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

```
[Out] arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(c
+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {4068, 209}

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}f\sqrt{c-d}}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[
e + f*x]]*Sqrt[c + d*Sec[e + f*x]])])/(Sqrt[a]*Sqrt[c - d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$
$$= \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{2\arctan\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+fx)}\sec(e+fx)}{\sqrt{c-d}f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),
x]
[Out] (2*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]]]*Cos[(e +
 f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x])/(Sqrt[c - d]*f*Sqrt[a*(1 +
Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(63) = 126.

Time = 2.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.73

method	result	size
default	$-\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a(\sec(fx+e)+1)}\,\ln\left(\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}-\sqrt{c-d}\,\cot(fx+e)+\sqrt{c-d}\,\csc(fx+e)\right)\cos(fx+e)}{fa\sqrt{c-d}\,(\cos(fx+e)+1)\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}}$	135

```
[In] int(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/f/a/(c-d)^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a*(sec(f*x+e)+1))^(1/2)*ln((-2*(
d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*x+e)+(c-d)^(1/2)*cs
c(f*x+e))*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.15

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{-\frac{1}{ac-ad}}\log\left(-\frac{2\sqrt{2}(c-d)\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\sqrt{-\frac{1}{ac-ad}}\cos(fx+e)\sin(fx+e)-(3c-d)\cos(fx+e)^2-2(c+d)\cos(fx+}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{2f}\right.$$
$$\left. -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(fx+e)+a}{\cos(fx+e)}}\sqrt{\frac{c\cos(fx+e)+d}{\cos(fx+e)}}\cos(fx+e)}{\sqrt{ac-ad}\sin(fx+e)}\right)}{\sqrt{ac-ad}f}\right]$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/(a*c - a*d))*log(-(2*sqrt(2)*(c - d)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/(a*c
 - a*d))*cos(f*x + e)*sin(f*x + e) - (3*c - d)*cos(f*x + e)^2 - 2*(c + d)*c
os(f*x + e) + c - 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))/f, -sqrt(2)*a
rctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e)
+ d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/(sqrt(a*c -
 a*d)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x)))
, x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
 x)
```

3.237 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 141

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}\,f} + \frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}\,f}$$

```
[Out] -arctan(1/2*a^(1/2)*(c-d)^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2)/(
c+d*sec(f*x+e))^(1/2))*2^(1/2)/f/a^(1/2)/(c-d)^(1/2)+2*arctanh(a^(1/2)*d^(1
/2)*tan(f*x+e)/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2))/f/a^(1/2)/d^(
1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {4070, 4068, 209, 4065, 212}

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}\,f} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\,f\sqrt{c-d}}$$

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[c - d]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Se
c[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])]))/(Sqrt[a]*Sqrt[c - d]*f)) + (2*ArcTa
nh[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[
e + f*x]])])/(Sqrt[a]*Sqrt[d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4065

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Dist[-2*(b/f), Subst
[Int[1/(1 - b*d*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c
+ d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0
] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4068

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-2*(a/(b*f)), S
ubst[Int[1/(2 + (a*c - b*d)*x^2), x], x, Cot[e + f*x]/(Sqrt[a + b*Csc[e + f
*x]]*Sqrt[c + d*Csc[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4070

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-a/b, Int[Csc
[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dis
t[1/b, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{a} - \int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{1-adx^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f} + \frac{2\text{Subst}\left(\int \frac{1}{2+(ac-ad)x^2}\,dx, x, -\frac{\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{f}$$

$$= -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sqrt{c-d}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{c-d}f} + \frac{2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\right)}{\sqrt{a}\sqrt{d}f}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\left(-\sqrt{d}\arctan\left(\frac{\sqrt{c-d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right) + \sqrt{2}\sqrt{c-d}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{d+c\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)\sqrt{d+c\cos(e+}}{\sqrt{c-d}\sqrt{d}f\sqrt{a(1+\sec(e+fx))}\sqrt{c+d\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]
),x]

[Out] (2*(-(Sqrt[d]*ArcTan[(Sqrt[c - d]*Sin[(e + f*x)/2])/Sqrt[d + c*Cos[e + f*x]
]]) + Sqrt[2]*Sqrt[c - d]*ArcTanh[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/Sqrt[d
 + c*Cos[e + f*x]]])*Cos[(e + f*x)/2]*Sqrt[d + c*Cos[e + f*x]]*Sec[e + f*x]
)/(Sqrt[c - d]*Sqrt[d]*f*Sqrt[a*(1 + Sec[e + f*x])]*Sqrt[c + d*Sec[e + f*x]
])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(114) = 228.

Time = 5.16 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.60

method	result
default	$\frac{\sqrt{2}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{a(\sec(fx+e)+1)}\left(\ln\left(\sqrt{-\frac{2(d+c\cos(fx+e))}{\cos(fx+e)+1}}-\sqrt{c-d}\,\cot(fx+e)+\sqrt{c-d}\,\csc(fx+e)\right)\sqrt{2}\,\sqrt{-d}-\ln\left(\frac{-2\sqrt{2}\,\sqrt{-d}\,\sqrt{}}{}\right.\right.}{}$

```
[In] int(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)

[Out] 1/f/a/(c-d)^(1/2)*2^(1/2)/(-d)^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a*(sec(f*x+e)+
1))^(1/2)*(ln((-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)-(c-d)^(1/2)*cot(f*
```

```
x+e)+(c-d)^(1/2)*csc(f*x+e))*2^(1/2)*(-d)^(1/2)-ln(2*(-2^(1/2)*(-d)^(1/2)*(
-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x
+e)+sin(f*x+e)*c+sin(f*x+e)*d-c+d)/(cos(f*x+e)-1+sin(f*x+e)))*(c-d)^(1/2)+l
n(-2*(2^(1/2)*(-d)^(1/2)*(-2*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*sin(f*x
+e)-sin(f*x+e)*c-sin(f*x+e)*d+c*cos(f*x+e)-d*cos(f*x+e)-c+d)/(cos(f*x+e)-1-
sin(f*x+e)))*(c-d)^(1/2))*cos(f*x+e)/(cos(f*x+e)+1)/(-2*(d+c*cos(f*x+e))/(c
os(f*x+e)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 1100, normalized size of antiderivative = 7.80

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt((a*cos(f
*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sqrt(-1/
(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 + 2*(c +
d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(a*
d)*log(-(8*a*c*d*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 +
4*((c - d)*cos(f*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e
) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) +
 8*a*d^2 + (a*c^2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + co
s(f*x + e)^2)))/(a*d*f), 1/2*(2*sqrt(2)*a*d*arctan(sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a*c - a*d) + sqrt(a*d)*log(-(8*a*c*d
*cos(f*x + e) + (a*c^2 - 6*a*c*d + a*d^2)*cos(f*x + e)^3 + 4*((c - d)*cos(f
*x + e)^2 + 2*d*cos(f*x + e))*sqrt(a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x +
 e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*sin(f*x + e) + 8*a*d^2 + (a*c^
2 + 2*a*c*d - 7*a*d^2)*cos(f*x + e)^2)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/
(a*d*f), 1/2*(sqrt(2)*a*d*sqrt(-1/(a*c - a*d))*log((2*sqrt(2)*(c - d)*sqrt(
(a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*
sqrt(-1/(a*c - a*d))*cos(f*x + e)*sin(f*x + e) + (3*c - d)*cos(f*x + e)^2 +
 2*(c + d)*cos(f*x + e) - c + 3*d)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) +
 2*sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c - a
*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))))/(a*d*f), (sqrt(2)*
a*d*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt((c*cos(f*x
+ e) + d)/cos(f*x + e))*cos(f*x + e)/(sqrt(a*c - a*d)*sin(f*x + e)))/sqrt(a
*c - a*d) + sqrt(-a*d)*arctan(-2*sqrt(-a*d)*sqrt((a*cos(f*x + e) + a)/cos(f
```

```
*x + e))*sqrt((c*cos(f*x + e) + d)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/
((a*c - a*d)*cos(f*x + e)^2 + 2*a*d + (a*c + a*d)*cos(f*x + e))))/(a*d*f)]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec^2\left(e+fx\right)}{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*sqrt(c + d*sec(e + f*x
))), x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{a\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="giac")
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$=\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
), x)
```

3.238 $\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 61

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}f}$$

```
[Out] 2*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(
1/2)/f/d^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {4052, 211}

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{d}f\sqrt{c+d}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Se
c[e + f*x]])])/(Sqrt[d]*Sqrt[c + d]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(2a)\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{d}\sqrt{c+d}f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)\sqrt{\cos(e+fx)}\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sec(e+fx))}}{\sqrt{d}\sqrt{c+d}f}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
[Out] (Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e
+ f*x]])]*Sqrt[Cos[e + f*x]]*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(
Sqrt[d]*Sqrt[c + d]*f)
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(47) = 94$.

Time = 19.38 (sec) , antiderivative size = 424, normalized size of antiderivative = 6.95

method	result
default	$\sqrt{2}\left(\ln\left(-\frac{2\left(\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,c-\sqrt{2}\,\sqrt{\frac{d}{c-d}}\,\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\,d+\sqrt{(c+d)(c-d)}\,(-\cot(fx+e)+\csc(fx+\right.}{-c(-\cot(fx+e)+\csc(fx+e))+(-\cot(fx+e)+\csc(fx+e))d+\sqrt{(c+d)(c-d)}}\right.\right.$

```
[In] int(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/f*2^(1/2)/(d/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)*(ln(-2*(((1-cos(f*x+e))^2
*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((
1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+cs
c(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+
d)*(c-d))^(1/2)))-ln(2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/
(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1
/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(c*(-cot(f*x+e)+csc
(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2))))*((1-cos(f*x+e))^
2*csc(f*x+e)^2-1)^(1/2)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(47) = 94$.

Time = 0.44 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.62

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \left[\frac{\sqrt{-\frac{a}{cd+d^2}}\log\left(-\frac{(ac^2+8\,acd+8\,ad^2)\cos(fx+e)^3+ad^2+(ac^2+2\,acd)\cos(fx+e)^2-4\left((c^2d+3\,cd^2+2\,d^3)\cos(fx+e)^2-(cd^2+d^3)\cos(fx+e\right.}{c^2\cos(fx+e)^3+(c^2+2\,cd)\cos(fx+e)^2+d^2+(2\,cd+d^2)\cos(f}\right.}{2\,f}\right.$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] [1/2*sqrt(-a/(c*d + d^2))*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3
+ a*d^2 + (a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*((c^2*d + 3*c*d^2 + 2*d^3)*c
os(f*x + e)^2 - (c*d^2 + d^3)*cos(f*x + e))*sqrt(-a/(c*d + d^2))*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*c*d + 7*a*d^2)*cos(f*x +
e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2
)*cos(f*x + e)))/f, sqrt(a/(c*d + d^2))*arctan(2*(c*d + d^2)*sqrt(a/(c*d +
d^2))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a
*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e)))/f]
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\sec\left(e+fx\right)}{c+d\sec\left(e+fx\right)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")

[Out] integrate(sqrt(a*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

[In] integrate(sec(f*x+e)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

[In] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)

[Out] int((a + a/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)

3.239 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 149

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\frac{2\sqrt{a}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df}-\frac{2\sqrt{a}\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{d\sqrt{c+d}f}$$

```
[Out] 2*g^(3/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2))*a^(1/2)/d/f-2*g^(3/2)*arctanh(a^(1/2)*c^(1/2)*g^(1/2)*tan(f*
x+e)/(c+d)^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))*a^(1/2)*c^(1/
2)/d/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4055, 3887, 214, 4050}

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\frac{2\sqrt{a}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df}-\frac{2\sqrt{a}\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{df\sqrt{c+d}}$$

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e + f*x]),
x]
```

```
[Out] (2*Sqrt[a]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f
*x]]*Sqrt[a + a*Sec[e + f*x]])])/(d*f) - (2*Sqrt[a]*Sqrt[c]*g^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*S
qrt[a + a*Sec[e + f*x]])])/(d*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3887

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/f), Subst[Int[1/(b - d*x^2), x], x, b*(C
ot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[a*(d/b), 0]
```

Rule 4050

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*b*(g
/f), Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc[
e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4055

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[g/d,
Int[Sqrt[g*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c*(g/d), I
nt[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x]
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{g\int\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx}{d}-\frac{(cg)\int\frac{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{d}\\
&=-\frac{(2ag^2)\operatorname{Subst}\left(\int\frac{1}{a-gx^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df}\\
&\quad+\frac{(2acg^2)\operatorname{Subst}\left(\int\frac{1}{ac+ad-cgx^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df}
\end{aligned}$$

$$= \frac{2\sqrt{a}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df} - \frac{2\sqrt{a}\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{d\sqrt{c+d}f}$$

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.26

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = -\frac{g^2\left(\sqrt{c+d}\log\left(\sqrt{2}-2\sin\left(\frac{1}{2}(e+fx)\right)\right)-\sqrt{c+d}\log\left(\sqrt{2}+2\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{c}\left(-\log\left(\sqrt{2}\sqrt{c+d}-2\right.\right.}{\sqrt{2}d\sqrt{c+d}f\sqrt{g}}$$

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + a*Sec[e + f*x]])/(c + d*Sec[e +
f*x]),x]
```

```
[Out] -((g^2*(Sqrt[c + d]*Log[Sqrt[2] - 2*Sin[(e + f*x)/2]] - Sqrt[c + d]*Log[Sqr
t[2] + 2*Sin[(e + f*x)/2]] + Sqrt[c]*(-Log[Sqrt[2]*Sqrt[c + d] - 2*Sqrt[c]*
Sin[(e + f*x)/2]] + Log[Sqrt[2]*Sqrt[c + d] + 2*Sqrt[c]*Sin[(e + f*x)/2]]))
*Sec[(e + f*x)/2]*Sqrt[a*(1 + Sec[e + f*x])])/(Sqrt[2]*d*Sqrt[c + d]*f*Sqrt
[g*Sec[e + f*x]]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. 2(117) = 234.

Time = 22.71 (sec) , antiderivative size = 685, normalized size of antiderivative = 4.60

method	result
default	$-\frac{g\sqrt{2}\,(c-d)\sqrt{-\frac{g\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(\sqrt{(c+d)(c-d)}\right.}{}$

```
[In] int((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] -g/f*2^(1/2)*(c-d)/((c+d)*(c-d))^(1/2)/(c-d+((c+d)*(c-d))^(1/2))/(-c+d+((c+
d)*(c-d))^(1/2))/(c/(c-d))^(1/2)*(-g*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)/((1-
cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)*(-2*
a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(((c+d)*(c-d))^(1/2)*arctanh(1/2
*(-cot(f*x+e)+csc(f*x+e)+1)*2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(1/2)
)*(c/(c-d))^(1/2)+((c+d)*(c-d))^(1/2)*arctanh(1/2*(-cot(f*x+e)+csc(f*x+e)-1
```

```
)*2^(1/2)/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(1/2))*(c/(c-d))^(1/2)-c*ln(-2*
(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(1/2)*(c/(c-d))^(1/2)*c-2^(1/2)*
((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(1/2)*(c/(c-d))^(1/2)*d+((c+d)*(c-d))^(1/
2)*(-cot(f*x+e)+csc(f*x+e))+c-d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+
csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))+c*ln(2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f
*x+e)^2+1)^(1/2)*(c/(c-d))^(1/2)*c-2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2+1
)^(1/2)*(c/(c-d))^(1/2)*d-((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+c-d)
/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)
)))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 4.81 (sec) , antiderivative size = 1126, normalized size of antiderivative = 7.56

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*c*g/(c + d))*g*log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*
d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)
*cos(f*x + e))*sqrt(a*c*g/(c + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*
sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) + (8*a
*c^2 + 8*a*c*d + a*d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)
^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + sqrt(a*g)*g*log((a*g*cos(f*x + e)
^3 - 7*a*g*cos(f*x + e)^2 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*s
qrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) +
8*a*g)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/(d*f), -1/2*(2*sqrt(-a*c*g/(c +
d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(a*c*g*
sin(f*x + e))) - sqrt(a*g)*g*log((a*g*cos(f*x + e)^3 - 7*a*g*cos(f*x + e)^2
 - 4*sqrt(a*g)*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/
cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + 8*a*g)/(cos(f*x + e)^3 +
cos(f*x + e)^2)))/(d*f), 1/2*(2*sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*co
s(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e
)/(a*g*cos(f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g)) + sqrt(a*c*g/(c + d))*g*
log((a*c^2*g*cos(f*x + e)^3 - (7*a*c^2 + 6*a*c*d)*g*cos(f*x + e)^2 + 4*((c^
2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(a*c*g/(c
 + d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x
 + e) + (2*a*c*d + a*d^2)*g*cos(f*x + e) + (8*a*c^2 + 8*a*c*d + a*d^2)*g)/(
c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos
(f*x + e))))/(d*f), (sqrt(-a*g)*g*arctan(2*sqrt(-a*g)*sqrt((a*cos(f*x + e)
```

```
+ a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e)/(a*g*cos(
f*x + e)^2 - a*g*cos(f*x + e) - 2*a*g)) - sqrt(-a*c*g/(c + d))*g*arctan(1/2
*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-a*c*g/(c + d))*sqrt((a*c
os(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(a*c*g*sin(f*x + e))))/
(d*f)]
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a\left(\sec\left(e+fx\right)+1\right)}\left(g\sec\left(e+fx\right)\right)^{\frac{3}{2}}}{c+d\sec\left(e+fx\right)}\,dx$$

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+a*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
[Out] Integral(sqrt(a*(sec(e + f*x) + 1))*(g*sec(e + f*x))**(3/2)/(c + d*sec(e +
f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagin
ary; found %i
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a\sec\left(fx+e\right)+a}\left(g\sec\left(fx+e\right)\right)^{\frac{3}{2}}}{d\sec\left(fx+e\right)+c}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+a*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx=\int \frac{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

```
[In] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
),x)
```

```
[Out] int(((a + a/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)
), x)
```

3.240 $\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 122

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

```
[Out] arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)
/f/a^(1/2)-2*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+e))
^(1/2))*d^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {4057, 3880, 209, 4052, 211}

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)} - \frac{2\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])])
/(Sqrt[a]*(c - d)*f) - (2*Sqrt[d]*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sq
rt[c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 4057

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[
Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[Csc
[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^
2 - d^2, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} - \frac{d\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)} \\
&= -\frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} + \frac{(2d)\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} \\
&= \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2\sqrt{d}\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$= \frac{2\left(\sqrt{-c-d}\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{2}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{-c-d}\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}\right)\right)}{\sqrt{-c-d}(c-d)f\sqrt{\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*(Sqrt[-c - d]*ArcSin[Tan[(e + f*x)/2]] + Sqrt[2]*Sqrt[d]*ArcTanh[(Sqrt[d
]*Tan[(e + f*x)/2])/(Sqrt[-c - d]*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])]))
/(Sqrt[-c - d]*(c - d)*f*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^2]*Sqrt[a*(1 +
Sec[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(99) = 198.

Time = 19.78 (sec) , antiderivative size = 504, normalized size of antiderivative = 4.13

method	result
default	$\frac{\left(2\sqrt{(c+d)(c-d)}\,\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\right)\sqrt{\frac{d}{c-d}}+d\sqrt{2}\,\ln\left(-\frac{2\left(-\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-}\right.}{}\right.\right.}{}$

```
[In] int(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/f/(d/(c-d))^(1/2)/(c-d)/((c+d)*(c-d))^(1/2)/a*(2*((c+d)*(c-d))^(1/2)*ln
(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2))*(d/(c-d))^(
1/2)+d*2^(1/2)*ln(-2*(-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(
c-d))^(1/2)*c+2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/
2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+c-d)/(c*(-cot(f*x+e)+csc(
f*x+e))-(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))-d*2^(1/2)*ln(-2*((
(1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d
/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)
*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+cs
c(f*x+e))*d+((c+d)*(c-d))^(1/2))))*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 963, normalized size of antiderivative = 7.89

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x
+ e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x +
 e) - 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + sqrt(-d/(a*c + a*d))*log(
-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 - 4*(
(c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d^2)*cos(f*x + e))*sqrt(-d/(a
*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) + d^2 - (6*
c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)
^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*(sqrt(2)*sqrt(-1
/a)*log((2*sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f
*x + e)*sin(f*x + e) + 3*cos(f*x + e)^2 + 2*cos(f*x + e) - 1)/(cos(f*x + e)
^2 + 2*cos(f*x + e) + 1)) + 2*sqrt(d/(a*c + a*d))*arctan(2*(c + d)*sqrt(d/(
a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*sin(f*x +
e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)))/((c - d)*f), -1/
2*(sqrt(-d/(a*c + a*d))*log(-((c^2 + 8*c*d + 8*d^2)*cos(f*x + e)^3 + (c^2 +
 2*c*d)*cos(f*x + e)^2 - 4*((c^2 + 3*c*d + 2*d^2)*cos(f*x + e)^2 - (c*d + d
^2)*cos(f*x + e))*sqrt(-d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x +
e))*sin(f*x + e) + d^2 - (6*c*d + 7*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3
+ (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqr
t(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(s
qrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f), -(sqrt(d/(a*c + a*d))*arctan(2*
(c + d)*sqrt(d/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x
 + e)*sin(f*x + e)/((c + 2*d)*cos(f*x + e)^2 + (c + d)*cos(f*x + e) - d)) +
 sqrt(2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e
)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((c - d)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,(c+d\sec(e+fx))}\,dx$$

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
maxima")
[Out] integrate(sec(f*x + e)/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm="
giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/(cos(e + f*x)*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.241 $\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 124

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2c\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

```
[Out] -arctan(1/2*a^(1/2)*tan(f*x+e)*2^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d
)/f/a^(1/2)+2*c*arctan(a^(1/2)*d^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(a+a*sec(f*x+
e))^(1/2))/(c-d)/f/a^(1/2)/d^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4061, 3880, 209, 4052, 211}

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2c\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}\sqrt{d}f(c-d)\sqrt{c+d}} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}}\right)}{\sqrt{a}f(c-d)}$$

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Tan[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sec[e + f*x]])
])/(Sqrt[a]*(c - d)*f)) + (2*c*ArcTan[(Sqrt[a]*Sqrt[d]*Tan[e + f*x])/(Sqrt[
c + d]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[d]*Sqrt[c + d]*f)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4052

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*(b/f), Subst[Int
[1/(b*c + a*d + d*x^2), x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
]
```

Rule 4061

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[-a/(b*c - a*d), I
nt[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[c/(b*c - a*d), Int[
Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ
[c^2 - d^2, 0])
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\sec(e+fx)}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{c\int \frac{\sec(e+fx)\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} - \frac{(2c)\text{Subst}\left(\int \frac{1}{ac+ad+dx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f}$$

$$= -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\tan(e+fx)}{\sqrt{2}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2c\arctan\left(\frac{\sqrt{a}\sqrt{d}\tan(e+fx)}{\sqrt{c+d}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{d}\sqrt{c+d}f}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$= -\frac{2\left(\sqrt{d}\sqrt{c+d}\arctan\left(\frac{\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\cos(e+fx)}}\right)-\sqrt{2}c\arctan\left(\frac{\sqrt{2}\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}\sqrt{\cos(e+fx)}}\right)\right)\cos\left(\frac{1}{2}(e+fx)\right)}{(c-d)\sqrt{d}\sqrt{c+d}f\sqrt{\cos(e+fx)}\sqrt{a(1+\sec(e+fx))}}$$

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-2*(Sqrt[d]*Sqrt[c + d]*ArcTan[Sin[(e + f*x)/2]/Sqrt[Cos[e + f*x]]] - Sqrt
[2]*c*ArcTan[(Sqrt[2]*Sqrt[d]*Sin[(e + f*x)/2])/(Sqrt[c + d]*Sqrt[Cos[e + f
*x]])])*Cos[(e + f*x)/2])/((c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[Cos[e + f*x]]
*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(101) = 202$.

Time = 18.95 (sec) , antiderivative size = 504, normalized size of antiderivative = 4.06

method	result
default	$-\frac{\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\sqrt{(1-\cos(fx+e))^2\csc(fx+e)^2-1}\left(2\sqrt{(c+d)(c-d)}\ln\left(\csc(fx+e)-\cot(fx+e)+\sqrt{(1-\cos(fx+e}\right.\right.}{}$

```
[In] int(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2/f/a/((c+d)*(c-d))^(1/2)/(c-d)/(d/(c-d))^(1/2)*(-2*a/((1-cos(f*x+e))^2*
csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*(2*((c+d)*(c
-d))^(1/2)*ln(csc(f*x+e)-cot(f*x+e)+((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)
)*(d/(c-d))^(1/2)-c*2^(1/2)*ln(-2*(((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*
2^(1/2)*(d/(c-d))^(1/2)*c-2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x
+e)^2-1)^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))-c+d)/(-c*(-co
t(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))+c*2^(
1/2)*ln(-2*(-((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*2^(1/2)*(d/(c-d))^(1/2
)*c+2^(1/2)*(d/(c-d))^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2-1)^(1/2)*d+((c+d
)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f*x+e))+c-d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-
cot(f*x+e)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 1041, normalized size of antiderivative = 8.40

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x +
 e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x + e
)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - sqrt(-a*
c*d - a*d^2)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (
a*c^2 + 2*a*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*cos(f*x
 + e)^2 - d*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x +
 e) - (6*a*c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)
*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f)
, -1/2*(sqrt(2)*(a*c*d + a*d^2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(f*x
+ e) + a)/cos(f*x + e))*sqrt(-1/a)*cos(f*x + e)*sin(f*x + e) - 3*cos(f*x +
e)^2 - 2*cos(f*x + e) + 1)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt(
a*c*d + a*d^2)*c*arctan(2*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*cos(f*x + e)*sin(f*x + e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d +
(a*c + a*d)*cos(f*x + e))))/((a*c^2*d - a*d^3)*f), 1/2*(sqrt(-a*c*d - a*d^2
)*c*log(-((a*c^2 + 8*a*c*d + 8*a*d^2)*cos(f*x + e)^3 + a*d^2 + (a*c^2 + 2*a
*c*d)*cos(f*x + e)^2 - 4*sqrt(-a*c*d - a*d^2)*((c + 2*d)*cos(f*x + e)^2 - d
*cos(f*x + e))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sin(f*x + e) - (6*a*
c*d + 7*a*d^2)*cos(f*x + e))/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x +
e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))) + 2*sqrt(2)*(a*c*d + a*d^2)*arcta
n(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)/(sqrt(a)*sin
(f*x + e)))/sqrt(a))/((a*c^2*d - a*d^3)*f), (sqrt(a*c*d + a*d^2)*c*arctan(2
*sqrt(a*c*d + a*d^2)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*cos(f*x + e)*s
in(f*x + e)/((a*c + 2*a*d)*cos(f*x + e)^2 - a*d + (a*c + a*d)*cos(f*x + e))
) + sqrt(2)*(a*c*d + a*d^2)*arctan(sqrt(2)*sqrt((a*cos(f*x + e) + a)/cos(f*
x + e))*cos(f*x + e)/(sqrt(a)*sin(f*x + e)))/sqrt(a))/((a*c^2*d - a*d^3)*f)
]
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a(\sec(e+fx)+1)}\,(c+d\sec(e+fx))}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral(sec(e + f*x)**2/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e + f*x))),
 x)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="maxima")
[Out] integrate(sec(f*x + e)^2/(sqrt(a*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, algorithm
="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$=\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
[Out] int(1/(cos(e + f*x)^2*(a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.242 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 167

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{\sqrt{2}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

```
[Out] -g^(3/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2^(1/2)/(g*sec(f*x+e))^(1/2
)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)+2*g^(3/2)*arctanh(a^(1/2)
*c^(1/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e
))^(1/2))*c^(1/2)/(c-d)/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {4059, 3893, 214, 4050}

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)\sqrt{c+d}} - \frac{\sqrt{2}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)}$$

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] -((Sqrt[2]*g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*S
ec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*f)) + (2*Sqrt[c]*
g^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*
Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]])])/(Sqrt[a]*(c - d)*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4050

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*b*(g
/f), Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc[
e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4059

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(-a)*
(g/(b*c - a*d)), Int[Sqrt[g*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x]
+ Dist[c*(g/(b*c - a*d)), Int[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*x]
]/(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b
*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{g\int\frac{\sqrt{g\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d}+\frac{(cg)\int\frac{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)}\\
&= \frac{(2g^2)\,\mathrm{Subst}\Big(\int\frac{1}{2a-gx^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\Big)}{(c-d)f}\\
&\quad-\frac{(2cg^2)\,\mathrm{Subst}\Big(\int\frac{1}{ac+ad-cgx^2}\,dx,x,-\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\Big)}{(c-d)f}
\end{aligned}$$

$$= -\frac{\sqrt{2}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} + \frac{2\sqrt{c}g^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)\sqrt{c+d}f}$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.19

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{g\cos\left(\frac{1}{2}(e+fx)\right)\left(2\sqrt{c+d}\log\left(\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e\right.\right.\right.}{}$$

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (g*Cos[(e + f*x)/2]*(2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] - Sin[(e + f*x)/4]]
 - 2*Sqrt[c + d]*Log[Cos[(e + f*x)/4] + Sin[(e + f*x)/4]] + Sqrt[2]*Sqrt[c]
*(-Log[Sqrt[2]*Sqrt[c + d] - 2*Sqrt[c]*Sin[(e + f*x)/2]] + Log[Sqrt[2]*Sqrt
[c + d] + 2*Sqrt[c]*Sin[(e + f*x)/2]]))*Sqrt[g*Sec[e + f*x]])/((c - d)*Sqrt
[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(132) = 264$.

Time = 20.66 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.34

method	result
default	$\frac{g\sqrt{-\frac{g\left((1-\cos(fx+e))^2\csc(fx+e)^2+1\right)}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left((1-\cos(fx+e))^2\csc(fx+e)^2-1\right)\sqrt{-\frac{2a}{(1-\cos(fx+e))^2\csc(fx+e)^2-1}}\left(c\sqrt{2}\ln\left(\frac{2\sqrt{2}\sqrt{(1-\cos(fx}}{}\right.\right.}{}$

```
[In] int((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/2*g/f/a/(c/(c-d))^(1/2)/(c-d)/((c+d)*(c-d))^(1/2)*(-g*((1-cos(f*x+e))^2*c
sc(f*x+e)^2+1)/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*((1-cos(f*x+e))^2*c
sc(f*x+e)^2-1)*(-2*a/((1-cos(f*x+e))^2*csc(f*x+e)^2-1))^(1/2)*(c*2^(1/2)*ln
(2*(2^(1/2)*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(1/2)*(c/(c-d))^(1/2)*c-2^(1/
2)*((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(1/2)*(c/(c-d))^(1/2)*d-((c+d)*(c-d))^
(1/2)*(-cot(f*x+e)+csc(f*x+e))+c-d)/(c*(-cot(f*x+e)+csc(f*x+e))-(-cot(f*x+e
)+csc(f*x+e))*d+((c+d)*(c-d))^(1/2)))-c*2^(1/2)*ln(-2*(2^(1/2)*((1-cos(f*x+
e))^2*csc(f*x+e)^2+1)^(1/2)*(c/(c-d))^(1/2)*c-2^(1/2)*((1-cos(f*x+e))^2*csc
```

```
(f*x+e)^2+1)^(1/2)*(c/(c-d))^(1/2)*d+((c+d)*(c-d))^(1/2)*(-cot(f*x+e)+csc(f
*x+e))+c-d)/(-c*(-cot(f*x+e)+csc(f*x+e))+(-cot(f*x+e)+csc(f*x+e))*d+((c+d)*
(c-d))^(1/2)))-2*((c+d)*(c-d))^(1/2)*arcsinh(cot(f*x+e)-csc(f*x+e))*(c/(c-d
))^(1/2))/((1-cos(f*x+e))^2*csc(f*x+e)^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 1103, normalized size of antiderivative = 6.60

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) +
a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f*x
 + e)^2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) +
sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f
*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*cos(f*x +
 e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/c
os(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 + 8*c*d +
 d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 + (2*c*d
+ d^2)*cos(f*x + e))))/((c - d)*f), 1/2*(2*sqrt(2)*g*sqrt(-g/a)*arctan(sqrt
(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))
*cos(f*x + e)/(g*sin(f*x + e))) - sqrt(c*g/(a*c + a*d))*g*log((c^2*g*cos(f*
x + e)^3 - (7*c^2 + 6*c*d)*g*cos(f*x + e)^2 + 4*((c^2 + c*d)*cos(f*x + e)^2
 - (2*c^2 + 3*c*d + d^2)*cos(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*
x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)
*g*cos(f*x + e) + (8*c^2 + 8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c
*d)*cos(f*x + e)^2 + d^2 + (2*c*d + d^2)*cos(f*x + e))))/((c - d)*f), -1/2*
(sqrt(2)*g*sqrt(g/a)*log((2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos
(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) - g*cos(f*x + e)^
2 + 2*g*cos(f*x + e) + 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 2*sqrt
(-c*g/(a*c + a*d))*g*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))
*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(
f*x + e))/(c*g*sin(f*x + e))))/((c - d)*f), (sqrt(2)*g*sqrt(-g/a)*arctan(sq
rt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e
))*cos(f*x + e)/(g*sin(f*x + e))) + sqrt(-c*g/(a*c + a*d))*g*arctan(1/2*(c*
cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos
(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))/(c*g*sin(f*x + e))))/((c
- d)*f)]
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a(\sec(e+fx)+1)}\,(c+d\sec(e+fx))}\,dx$$

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a*(sec(e + f*x) + 1))*(c + d*sec(e +
 f*x))), x)
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
[Out] 1/2*(sqrt(2)*c*f*g*integrate(((c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x + 2*e
)^2 - 2*(c*d - 2*d^2)*cos(f*x + e)^2 - (c^2 - 4*c*d)*sin(2*f*x + 2*e)*sin(f
*x + e) - 2*(c*d - 2*d^2)*sin(f*x + e)^2 + (c^2 - (c^2 - 4*c*d)*cos(f*x + e
))*cos(2*f*x + 2*e) - (c^2 - 2*c*d)*cos(f*x + e))*cos(1/2*arctan2(sin(f*x +
 e), cos(f*x + e))) - (c^2*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2*cos(f*x + e
) + c^2)*sin(2*f*x + 2*e) + (c^2 - 2*c*d)*sin(f*x + e))*sin(1/2*arctan2(sin
(f*x + e), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2
+ c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin(
f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2*
f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2*cos(2*f*x
 + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x
 + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*
(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x + e),
cos(f*x + e)))^2), x) + sqrt(2)*c*f*g*integrate(((2*c*d*cos(f*x + e)^2 + 2*
c*d*sin(f*x + e)^2 - (c^2 - 2*c*d)*cos(2*f*x + 2*e)^2 + c^2*cos(f*x + e) -
(c^2 - 2*c*d)*sin(2*f*x + 2*e)^2 + (c^2 - 2*c*d + 4*d^2)*sin(2*f*x + 2*e)*s
in(f*x + e) - (c^2 - 2*c*d - (c^2 - 2*c*d + 4*d^2)*cos(f*x + e))*cos(2*f*x
+ 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + (c^2*sin(f*x + e) +
(c^2 + 2*c*d - 4*d^2)*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2 - 2*c*d + (c^2 +
 2*c*d - 4*d^2)*cos(f*x + e))*sin(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x + e
), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*si
n(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)
^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*
e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2*cos(2*f*x + 2*e)^
```

```
2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*
sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*c
os(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x + e), cos(f*x
+ e)))^2), x) - sqrt(2)*d*g*log(cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))
)^2 + sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(1/2*arctan2(si
n(f*x + e), cos(f*x + e))) + 1) + sqrt(2)*d*g*log(cos(1/2*arctan2(sin(f*x +
 e), cos(f*x + e)))^2 + sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*
sin(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + 1))*sqrt(g)/((c*d - d^2)*sqr
t(a)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

3.243 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 231

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}df} + \frac{\sqrt{2}g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2c^{3/2}g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)d\sqrt{c+d}f}$$

```
[Out] 2*g^(5/2)*arctanh(a^(1/2)*g^(1/2)*tan(f*x+e)/(g*sec(f*x+e))^(1/2)/(a+a*sec(
f*x+e))^(1/2))/d/f/a^(1/2)+g^(5/2)*arctanh(1/2*a^(1/2)*g^(1/2)*tan(f*x+e)*2
^(1/2)/(g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))*2^(1/2)/(c-d)/f/a^(1/2)
-2*c^(3/2)*g^(5/2)*arctanh(a^(1/2)*c^(1/2)*g^(1/2)*tan(f*x+e)/(c+d)^(1/2)/(
g*sec(f*x+e))^(1/2)/(a+a*sec(f*x+e))^(1/2))/(c-d)/d/f/a^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

$= \{4063, 4050, 214, 4108, 3893, 3887\}$

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df(c-d)\sqrt{c+d}} + \frac{\sqrt{2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}f(c-d)} + \frac{2g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{a\sec(e+fx)+a}\sqrt{g\sec(e+fx)}}\right)}{\sqrt{a}df}$$

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[g]*Tan[e + f*x])/(Sqrt[g*Sec[e + f*x]]*Sqr
t[a + a*Sec[e + f*x]])])/(Sqrt[a]*d*f) + (Sqrt[2]*g^(5/2)*ArcTanh[(Sqrt[a]*
Sqrt[g]*Tan[e + f*x])/(Sqrt[2]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*f) - (2*c^(3/2)*g^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c]*Sqrt
[g]*Tan[e + f*x])/(Sqrt[c + d]*Sqrt[g*Sec[e + f*x]]*Sqrt[a + a*Sec[e + f*x]
])])/(Sqrt[a]*(c - d)*d*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3887

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/f), Subst[Int[1/(b - d*x^2), x], x, b*(C
ot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] &&  !GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4050

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(g_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[-2*b*(g
```

```
/f), Subst[Int[1/(b*c + a*d - c*g*x^2), x], x, b*(Cot[e + f*x]/(Sqrt[g*Csc[
e + f*x]]*Sqrt[a + b*Csc[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f, g}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4063

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[(-c^2
)*(g^2/(d*(b*c - a*d))), Int[Sqrt[g*Csc[e + f*x]]*(Sqrt[a + b*Csc[e + f*x]]
/(c + d*Csc[e + f*x])), x], x] + Dist[g^2/(d*(b*c - a*d)), Int[Sqrt[g*Csc[e
 + f*x]]*((a*c + (b*c - a*d)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x
] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2,
 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{g^2 \int \frac{\sqrt{g\sec(e+fx)}(ac+(ac-ad)\sec(e+fx))}{\sqrt{a+a\sec(e+fx)}}\,dx}{a(c-d)d} - \frac{(c^2g^2)\int \frac{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}{c+d\sec(e+fx)}\,dx}{a(c-d)d}$$

$$= \frac{g^2 \int \frac{\sqrt{g\sec(e+fx)}}{\sqrt{a+a\sec(e+fx)}}\,dx}{c-d} + \frac{g^2 \int \sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}\,dx}{ad} + \frac{(2c^2g^3)\,\text{Subst}\left(\int \frac{1}{ac+ad-cgx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)df}$$

$$= -\frac{2c^{3/2}g^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)d\sqrt{c+d}f} - \frac{(2g^3)\,\text{Subst}\left(\int \frac{1}{2a-gx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{(c-d)f} - \frac{(2g^3)\,\text{Subst}\left(\int \frac{1}{a-gx^2}\,dx, x, -\frac{a\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{df}$$

$$= \frac{2g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}df} + \frac{\sqrt{2}g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{g}\tan(e+fx)}{\sqrt{2}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)f} - \frac{2c^{3/2}g^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c}\sqrt{g}\tan(e+fx)}{\sqrt{c+d}\sqrt{g\sec(e+fx)}\sqrt{a+a\sec(e+fx)}}\right)}{\sqrt{a}(c-d)d\sqrt{c+d}f}$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.67

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g^2\left(d\sqrt{c+d}\operatorname{arctanh}\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{2}\left((c-d)\sqrt{c+d}a\right.\right.}{(}$$

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + a*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (2*g^2*(d*Sqrt[c + d]*ArcTanh[Sin[(e + f*x)/2]] + Sqrt[2]*((c - d)*Sqrt[c +
 d]*ArcTanh[Sqrt[2]*Sin[(e + f*x)/2]] - c^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[c]*Si
n[(e + f*x)/2])/Sqrt[c + d]]))*Cos[(e + f*x)/2]*Sqrt[g*Sec[e + f*x]])/((c -
 d)*d*Sqrt[c + d]*f*Sqrt[a*(1 + Sec[e + f*x])])
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. 2(184) = 368.

Time = 22.87 (sec) , antiderivative size = 796, normalized size of antiderivative = 3.45

method	result
default	$-\frac{2\left(\sqrt{(c+d)(c-d)}\,\operatorname{arcsinh}(\cot(fx+e)-\csc(fx+e))\sqrt{\frac{c}{c-d}}\,d\sqrt{2}-\operatorname{arctanh}\left(\frac{\cos(fx+e)+\sin(fx+e)+1}{2(\cos(fx+e)+1)\sqrt{\frac{1}{\cos(fx+e)+1}}}\right)\sqrt{\frac{c}{c-d}}\,\sqrt{(c+d)(c-d)}\,c+\mathrm{ar}}{}$

```
[In] int((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -2/f/a/(c/(c-d))^(1/2)/((c+d)*(c-d))^(1/2)/(-c+d+((c+d)*(c-d))^(1/2))/(c-d+
((c+d)*(c-d))^(1/2))*(((c+d)*(c-d))^(1/2)*arcsinh(cot(f*x+e)-csc(f*x+e))*(c
/(c-d))^(1/2)*d*2^(1/2)-arctanh(1/2*(cos(f*x+e)+sin(f*x+e)+1)/(cos(f*x+e)+1
)/(1/(cos(f*x+e)+1))^(1/2))*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c+arctanh(1
/2*(cos(f*x+e)+sin(f*x+e)+1)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2))*(c/(c
-d))^(1/2)*((c+d)*(c-d))^(1/2)*d-arctanh(1/2*(-cos(f*x+e)+sin(f*x+e)-1)/(co
```

```
s(f*x+e)+1)/(1/(cos(f*x+e)+1))^(1/2))*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*c
+arctanh(1/2*(-cos(f*x+e)+sin(f*x+e)-1)/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^(
1/2))*(c/(c-d))^(1/2)*((c+d)*(c-d))^(1/2)*d+ln(-2*(2*(1/(cos(f*x+e)+1))^(1/
2)*(c/(c-d))^(1/2)*c*sin(f*x+e)-2*(1/(cos(f*x+e)+1))^(1/2)*(c/(c-d))^(1/2)*
d*sin(f*x+e)+sin(f*x+e)*c-sin(f*x+e)*d-((c+d)*(c-d))^(1/2)*cos(f*x+e)+((c+d
)*(c-d))^(1/2))/(((c+d)*(c-d))^(1/2)*sin(f*x+e)+c*cos(f*x+e)-d*cos(f*x+e)-c
+d))*c^2-ln(-2*(-2*(c/(c-d))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*c*cos(f*x+e)+2*
(c/(c-d))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*d-2*(1/(cos(f*x+e)+1))^
(1/2)*(c/(c-d))^(1/2)*c+2*(1/(cos(f*x+e)+1))^(1/2)*(c/(c-d))^(1/2)*d+((c+d)
*(c-d))^(1/2)*sin(f*x+e)-c*cos(f*x+e)+d*cos(f*x+e)-c+d)/(((c+d)*(c-d))^(1/2
)*cos(f*x+e)+sin(f*x+e)*c-sin(f*x+e)*d+((c+d)*(c-d))^(1/2)))*c^2)*(a*(sec(f
*x+e)+1))^(1/2)*(g*sec(f*x+e))^(1/2)*g^2/(cos(f*x+e)+1)/(1/(cos(f*x+e)+1))^
(1/2)*cos(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 52.75 (sec) , antiderivative size = 1597, normalized size of antiderivative = 6.91

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*d*g^2*sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x +
e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*co
s(f*x + e)^2 - 2*g*cos(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1
)) + c*sqrt(c*g/(a*c + a*d))*g^2*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d
)*g*cos(f*x + e)^2 - 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*
cos(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2
+ 8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2
+ (2*c*d + d^2)*cos(f*x + e))) - (c - d)*g^2*sqrt(g/a)*log((g*cos(f*x + e)^
3 - 4*(cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)
/cos(f*x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g
)/(cos(f*x + e)^3 + cos(f*x + e)^2)))/((c*d - d^2)*f), -1/2*(sqrt(2)*d*g^2*
sqrt(g/a)*log(-(2*sqrt(2)*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))
*sqrt(g/cos(f*x + e))*cos(f*x + e)*sin(f*x + e) + g*cos(f*x + e)^2 - 2*g*co
s(f*x + e) - 3*g)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 2*c*sqrt(-c*g/(a
*c + a*d))*g^2*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e))*sqrt(
-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x +
e))/(c*g*sin(f*x + e))) - (c - d)*g^2*sqrt(g/a)*log((g*cos(f*x + e)^3 - 4*(
cos(f*x + e)^2 - 2*cos(f*x + e))*sqrt(g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*
```

```
x + e))*sqrt(g/cos(f*x + e))*sin(f*x + e) - 7*g*cos(f*x + e)^2 + 8*g)/(cos(
f*x + e)^3 + cos(f*x + e)^2)))/((c*d - d^2)*f), -1/2*(2*sqrt(2)*d*g^2*sqrt(
-g/a)*arctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqr
t(g/cos(f*x + e))*cos(f*x + e)/(g*sin(f*x + e))) - 2*(c - d)*g^2*sqrt(-g/a)
*arctan(2*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x
 + e))*cos(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(f*x + e) - 2*g))
 + c*sqrt(c*g/(a*c + a*d))*g^2*log((c^2*g*cos(f*x + e)^3 - (7*c^2 + 6*c*d)*
g*cos(f*x + e)^2 - 4*((c^2 + c*d)*cos(f*x + e)^2 - (2*c^2 + 3*c*d + d^2)*co
s(f*x + e))*sqrt(c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*s
qrt(g/cos(f*x + e))*sin(f*x + e) + (2*c*d + d^2)*g*cos(f*x + e) + (8*c^2 +
8*c*d + d^2)*g)/(c^2*cos(f*x + e)^3 + (c^2 + 2*c*d)*cos(f*x + e)^2 + d^2 +
(2*c*d + d^2)*cos(f*x + e))))/((c*d - d^2)*f), -(sqrt(2)*d*g^2*sqrt(-g/a)*a
rctan(sqrt(2)*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos
(f*x + e))*cos(f*x + e)/(g*sin(f*x + e))) - (c - d)*g^2*sqrt(-g/a)*arctan(2
*sqrt(-g/a)*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e)/(g*cos(f*x + e)^2 - g*cos(f*x + e) - 2*g)) + c*sqrt
(-c*g/(a*c + a*d))*g^2*arctan(1/2*(c*cos(f*x + e)^2 - (2*c + d)*cos(f*x + e
))*sqrt(-c*g/(a*c + a*d))*sqrt((a*cos(f*x + e) + a)/cos(f*x + e))*sqrt(g/co
s(f*x + e))/(c*g*sin(f*x + e))))/((c*d - d^2)*f)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{a\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
[Out] -1/2*(sqrt(2)*c^2*f*g^2*integrate((((c^2*cos(2*f*x + 2*e)^2 + c^2*sin(2*f*x
+ 2*e)^2 - 2*(c*d - 2*d^2)*cos(f*x + e)^2 - (c^2 - 4*c*d)*sin(2*f*x + 2*e)*
sin(f*x + e) - 2*(c*d - 2*d^2)*sin(f*x + e)^2 + (c^2 - (c^2 - 4*c*d)*cos(f*
x + e))*cos(2*f*x + 2*e) - (c^2 - 2*c*d)*cos(f*x + e))*cos(1/2*arctan2(sin(
f*x + e), cos(f*x + e))) - (c^2*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2*cos(f*
```

```
x + e) + c^2)*sin(2*f*x + 2*e) + (c^2 - 2*c*d)*sin(f*x + e))*sin(1/2*arctan
2(sin(f*x + e), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x +
e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2
*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*c
os(2*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2*cos(
2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(
2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2
 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x +
 e), cos(f*x + e)))^2), x) + sqrt(2)*c^2*f*g^2*integrate(((2*c*d*cos(f*x +
e)^2 + 2*c*d*sin(f*x + e)^2 - (c^2 - 2*c*d)*cos(2*f*x + 2*e)^2 + c^2*cos(f*
x + e) - (c^2 - 2*c*d)*sin(2*f*x + 2*e)^2 + (c^2 - 2*c*d + 4*d^2)*sin(2*f*x
 + 2*e)*sin(f*x + e) - (c^2 - 2*c*d - (c^2 - 2*c*d + 4*d^2)*cos(f*x + e))*c
os(2*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + (c^2*sin(f*
x + e) + (c^2 + 2*c*d - 4*d^2)*cos(2*f*x + 2*e)*sin(f*x + e) - (c^2 - 2*c*d
 + (c^2 + 2*c*d - 4*d^2)*cos(f*x + e))*sin(2*f*x + 2*e))*sin(1/2*arctan2(si
n(f*x + e), cos(f*x + e))))/((c^2*cos(2*f*x + 2*e)^2 + 4*d^2*cos(f*x + e)^2
 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*x + 2*e)*sin(f*x + e) + 4*d^2*sin
(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2*(2*c*d*cos(f*x + e) + c^2)*cos(2
*f*x + 2*e))*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + (c^2*cos(2*f*
x + 2*e)^2 + 4*d^2*cos(f*x + e)^2 + c^2*sin(2*f*x + 2*e)^2 + 4*c*d*sin(2*f*
x + 2*e)*sin(f*x + e) + 4*d^2*sin(f*x + e)^2 + 4*c*d*cos(f*x + e) + c^2 + 2
*(2*c*d*cos(f*x + e) + c^2)*cos(2*f*x + 2*e))*sin(1/2*arctan2(sin(f*x + e),
 cos(f*x + e)))^2), x) - sqrt(2)*d^2*g^2*log(cos(1/2*arctan2(sin(f*x + e),
cos(f*x + e)))^2 + sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(1
/2*arctan2(sin(f*x + e), cos(f*x + e))) + 1) + sqrt(2)*d^2*g^2*log(cos(1/2*
arctan2(sin(f*x + e), cos(f*x + e)))^2 + sin(1/2*arctan2(sin(f*x + e), cos(
f*x + e)))^2 - 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + 1) - (c*d -
 d^2)*g^2*log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(1/2*
arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(f*x
+ e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)
)) + 2) + (c*d - d^2)*g^2*log(2*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))
)^2 + 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sqrt(2)*cos(1/2*
arctan2(sin(f*x + e), cos(f*x + e))) - 2*sqrt(2)*sin(1/2*arctan2(sin(f*x +
e), cos(f*x + e))) + 2) - (c*d - d^2)*g^2*log(2*cos(1/2*arctan2(sin(f*x + e
), cos(f*x + e)))^2 + 2*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e)))^2 - 2*
sqrt(2)*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + 2*sqrt(2)*sin(1/2*ar
ctan2(sin(f*x + e), cos(f*x + e))) + 2) + (c*d - d^2)*g^2*log(2*cos(1/2*arc
tan2(sin(f*x + e), cos(f*x + e)))^2 + 2*sin(1/2*arctan2(sin(f*x + e), cos(f
*x + e)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(f*x + e), cos(f*x + e))) - 2*sq
rt(2)*sin(1/2*arctan2(sin(f*x + e), cos(f*x + e))) + 2))*sqrt(g)/((c*d^2 -
d^3)*sqrt(a)*f)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+a*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Val
ue
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+a\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{a}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(5/2)/((a + a/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

3.244 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$

Optimal result

Integrand size = 29, antiderivative size = 250

$$\begin{aligned}
&\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx \\
&= \frac{(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4)\operatorname{arctanh}(\sin(e+fx))}{8f} \\
&\quad+\frac{(12bc^4+95ac^3d+112bc^2d^2+80acd^3+16bd^4)\tan(e+fx)}{30f} \\
&\quad+\frac{d(24bc^3+130ac^2d+116bcd^2+45ad^3)\sec(e+fx)\tan(e+fx)}{120f} \\
&\quad+\frac{(12bc^2+35acd+16bd^2)\,(c+d\sec(e+fx))^2\tan(e+fx)}{60f} \\
&\quad+\frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f}+\frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f}
\end{aligned}$$

```
[Out] 1/8*(8*a*c^4+24*a*c^2*d^2+3*a*d^4+16*b*c^3*d+12*b*c*d^3)*arctanh(sin(f*x+e)
)/f+1/30*(95*a*c^3*d+80*a*c*d^3+12*b*c^4+112*b*c^2*d^2+16*b*d^4)*tan(f*x+e)
/f+1/120*d*(130*a*c^2*d+45*a*d^3+24*b*c^3+116*b*c*d^2)*sec(f*x+e)*tan(f*x+e
)/f+1/60*(35*a*c*d+12*b*c^2+16*b*d^2)*(c+d*sec(f*x+e))^2*tan(f*x+e)/f+1/20*
(5*a*d+4*b*c)*(c+d*sec(f*x+e))^3*tan(f*x+e)/f+1/5*b*(c+d*sec(f*x+e))^4*tan(
f*x+e)/f
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$= \frac{(8ac^4+24ac^2d^2+3ad^4+16bc^3d+12bcd^3)\,\text{arctanh}(\sin(e+fx))}{8f} + \frac{(35acd+12bc^2+16bd^2)\tan(e+fx)(c+d\sec(e+fx))^2}{60f} + \frac{d(130ac^2d+45ad^3+24bc^3+116bcd^2)\tan(e+fx)\sec(e+fx)}{120f} + \frac{(95ac^3d+80acd^3+12bc^4+112bc^2d^2+16bd^4)\tan(e+fx)}{30f} + \frac{(5ad+4bc)\tan(e+fx)(c+d\sec(e+fx))^3}{20f} + \frac{b\tan(e+fx)(c+d\sec(e+fx))^4}{5f}$$

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*ArcTanh[Sin[e
 + f*x]])/(8*f) + ((12*b*c^4 + 95*a*c^3*d + 112*b*c^2*d^2 + 80*a*c*d^3 + 16
*b*d^4)*Tan[e + f*x])/(30*f) + (d*(24*b*c^3 + 130*a*c^2*d + 116*b*c*d^2 + 4
5*a*d^3)*Sec[e + f*x]*Tan[e + f*x])/(120*f) + ((12*b*c^2 + 35*a*c*d + 16*b*
d^2)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(60*f) + ((4*b*c + 5*a*d)*(c + d*
Sec[e + f*x])^3*Tan[e + f*x])/(20*f) + (b*(c + d*Sec[e + f*x])^4*Tan[e + f*
x])/(5*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f} \\
&\quad + \frac{1}{5}\int \sec(e+fx)(c+d\sec(e+fx))^3(5ac+4bd+(4bc+5ad)\sec(e+fx))\,dx \\
&= \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f} \\
&\quad + \frac{1}{20}\int \sec(e+fx)(c+d\sec(e+fx))^2\left(20ac^2+28bcd+15ad^2\right. \\
&\qquad \left.+\left(12bc^2+35acd+16bd^2\right)\sec(e+fx)\right)\,dx \\
&= \frac{\left(12bc^2+35acd+16bd^2\right)(c+d\sec(e+fx))^2\tan(e+fx)}{60f} \\
&\quad + \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f} \\
&\quad + \frac{1}{60}\int \sec(e+fx)(c+d\sec(e+fx))\left(60ac^3+108bc^2d+115acd^2+32bd^3\right. \\
&\qquad \left.+\left(24bc^3+130ac^2d+116bcd^2+45ad^3\right)\sec(e+fx)\right)\,dx
\end{aligned}$$

$$= \frac{d(24bc^3+130ac^2d+116bcd^2+45ad^3)\sec(e+fx)\tan(e+fx)}{120f} + \frac{(12bc^2+35acd+16bd^2)(c+d\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f} + \frac{1}{120}\int \sec(e+fx)\left(15\left(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4\right) + 4\left(12bc^4+95ac^3d+112bc^2d^2+80acd^3+16bd^4\right)\sec(e+fx)\right)\,dx$$

$$= \frac{d(24bc^3+130ac^2d+116bcd^2+45ad^3)\sec(e+fx)\tan(e+fx)}{120f} + \frac{(12bc^2+35acd+16bd^2)(c+d\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f} + \frac{1}{8}\left(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4\right)\int \sec(e+fx)\,dx + \frac{1}{30}\left(12bc^4+95ac^3d+112bc^2d^2+80acd^3+16bd^4\right)\int \sec^2(e+fx)\,dx$$

$$= \frac{(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4)\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{d(24bc^3+130ac^2d+116bcd^2+45ad^3)\sec(e+fx)\tan(e+fx)}{120f} + \frac{(12bc^2+35acd+16bd^2)(c+d\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f} - \frac{(12bc^4+95ac^3d+112bc^2d^2+80acd^3+16bd^4)\,\mathrm{Subst}(\int 1\,dx, x, -\tan(e+fx))}{30f}$$

$$= \frac{(8ac^4+16bc^3d+24ac^2d^2+12bcd^3+3ad^4)\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{(12bc^4+95ac^3d+112bc^2d^2+80acd^3+16bd^4)\tan(e+fx)}{30f} + \frac{d(24bc^3+130ac^2d+116bcd^2+45ad^3)\sec(e+fx)\tan(e+fx)}{120f} + \frac{(12bc^2+35acd+16bd^2)(c+d\sec(e+fx))^2\tan(e+fx)}{60f} + \frac{(4bc+5ad)(c+d\sec(e+fx))^3\tan(e+fx)}{20f} + \frac{b(c+d\sec(e+fx))^4\tan(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 2.84 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.80

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$= \frac{15(4bcd(4c^2+3d^2)+a(8c^4+24c^2d^2+3d^4))\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(15d(3ad(8c^2+d^2)+4\right.}{}$$

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^4,x]
```

```
[Out] (15*(4*b*c*d*(4*c^2 + 3*d^2) + a*(8*c^4 + 24*c^2*d^2 + 3*d^4))*ArcTanh[Sin[
e + f*x]] + Tan[e + f*x]*(15*d*(3*a*d*(8*c^2 + d^2) + 4*b*(4*c^3 + 3*c*d^2)
)*Sec[e + f*x] + 30*d^3*(4*b*c + a*d)*Sec[e + f*x]^3 + 8*(15*(4*a*c*d*(c^2
+ d^2) + b*(c^4 + 6*c^2*d^2 + d^4)) + 10*d^2*(2*a*c*d + b*(3*c^2 + d^2))*Ta
n[e + f*x]^2 + 3*b*d^4*Tan[e + f*x]^4)))/(120*f)
```

Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.95

method	result
parts	$\frac{(a\,d^4+4bc\,d^3)\left(-\left(-\frac{\sec(fx+e)^3}{4}-\frac{3\sec(fx+e)}{8}\right)\tan(fx+e)+\frac{3\ln(\sec(fx+e)+\tan(fx+e))}{8}\right)}{f}-\frac{(4ac\,d^3+6b\,c^2d^2)\left(-\frac{2}{3}-\frac{\sec}{}\right.}{f}$
derivativedivides	$\frac{a\,c^4\ln(\sec(fx+e)+\tan(fx+e))+4a\,c^3d\tan(fx+e)+6a\,c^2d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4ac\,d^3\left(-\right.}{}$
default	$\frac{a\,c^4\ln(\sec(fx+e)+\tan(fx+e))+4a\,c^3d\tan(fx+e)+6a\,c^2d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-4ac\,d^3\left(-\right.}{}$
parallelrisch	$\frac{-120(\cos(5fx+5e)+5\cos(3fx+3e)+10\cos(fx+e))\left(a\,c^4+3a\,c^2d^2+\frac{3}{8}a\,d^4+2b\,c^3d+\frac{3}{2}bc\,d^3\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+120(c}{}$
norman	$\frac{-\frac{4\left(180a\,c^3d+100ac\,d^3+45b\,c^4+150b\,c^2d^2+29b\,d^4\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{15f}-\frac{\left(32a\,c^3d-24a\,c^2d^2+32ac\,d^3-5a\,d^4+8b\,c^4-16b\,c^3d+48b\,c^2d^2-\right.}{4f}}{}$
risch	$\frac{i\left(480b\,c^2d^2+320ac\,d^3+480a\,c^3d+64b\,d^4+120b\,c^4-210a\,d^4\mathrm{e}^{7i(fx+e)}+480b\,c^4\mathrm{e}^{6i(fx+e)}+320b\,d^4\mathrm{e}^{2i(fx+e)}+720b\,c^4\mathrm{e}^{4i(f}\right.}{}$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] (a*d^4+4*b*c*d^3)/f*(-(-1/4*sec(f*x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(
sec(f*x+e)+tan(f*x+e)))-(4*a*c*d^3+6*b*c^2*d^2)/f*(-2/3-1/3*sec(f*x+e)^2)*t
an(f*x+e)+(6*a*c^2*d^2+4*b*c^3*d)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f
*x+e)+tan(f*x+e)))+(4*a*c^3*d+b*c^4)/f*tan(f*x+e)+a*c^4/f*ln(sec(f*x+e)+tan
(f*x+e))-b*d^4/f*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.12

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4 \, dx = \frac{15\left(8\,ac^4+16\,bc^3d+24\,ac^2d^2+12\,bcd^3+3\,ad^4\right)\cos\left(fx+e\right)^5\log\left(\sin\left(fx+e\right)+1\right)-15\left(8\,ac^4+16\,bc^3d+\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(
f*x + e)^5*log(sin(f*x + e) + 1) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2
+ 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)^5*log(-sin(f*x + e) + 1) + 2*(24*b*d^4
 + 8*(15*b*c^4 + 60*a*c^3*d + 60*b*c^2*d^2 + 40*a*c*d^3 + 8*b*d^4)*cos(f*x
+ e)^4 + 15*(16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*cos(f*x + e)
^3 + 16*(15*b*c^2*d^2 + 10*a*c*d^3 + 2*b*d^4)*cos(f*x + e)^2 + 30*(4*b*c*d^
3 + a*d^4)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^5)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4 \, dx = \int \left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^4\sec\left(e+fx\right)\, dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**4*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.52

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4 \, dx = \frac{480\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)bc^2d^2+320\left(\tan\left(fx+e\right)^3+3\,\tan\left(fx+e\right)\right)acd^3+16\left(3\,\tan\left(fx+e\right.\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c^2*d^2 + 320*(tan(f*x + e)^
3 + 3*tan(f*x + e))*a*c*d^3 + 16*(3*tan(f*x + e)^5 + 10*tan(f*x + e)^3 + 15
*tan(f*x + e))*b*d^4 - 60*b*c*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(s
in(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin
(f*x + e) - 1)) - 15*a*d^4*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x
+ e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x +
e) - 1)) - 240*b*c^3*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x +
 e) + 1) + log(sin(f*x + e) - 1)) - 360*a*c^2*d^2*(2*sin(f*x + e)/(sin(f*x
+ e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 240*a*c^4*lo
g(sec(f*x + e) + tan(f*x + e)) + 240*b*c^4*tan(f*x + e) + 960*a*c^3*d*tan(f
*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(238) = 476$.

Time = 0.37 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.40

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 + 12*b*c*d^3 + 3*a*d^4)*log(
abs(tan(1/2*f*x + 1/2*e) + 1)) - 15*(8*a*c^4 + 16*b*c^3*d + 24*a*c^2*d^2 +
12*b*c*d^3 + 3*a*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(120*b*c^4*tan
(1/2*f*x + 1/2*e)^9 + 480*a*c^3*d*tan(1/2*f*x + 1/2*e)^9 - 240*b*c^3*d*tan(
1/2*f*x + 1/2*e)^9 - 360*a*c^2*d^2*tan(1/2*f*x + 1/2*e)^9 + 720*b*c^2*d^2*t
an(1/2*f*x + 1/2*e)^9 + 480*a*c*d^3*tan(1/2*f*x + 1/2*e)^9 - 300*b*c*d^3*ta
n(1/2*f*x + 1/2*e)^9 - 75*a*d^4*tan(1/2*f*x + 1/2*e)^9 + 120*b*d^4*tan(1/2*
f*x + 1/2*e)^9 - 480*b*c^4*tan(1/2*f*x + 1/2*e)^7 - 1920*a*c^3*d*tan(1/2*f*
x + 1/2*e)^7 + 480*b*c^3*d*tan(1/2*f*x + 1/2*e)^7 + 720*a*c^2*d^2*tan(1/2*f
*x + 1/2*e)^7 - 1920*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^7 - 1280*a*c*d^3*tan(1/
2*f*x + 1/2*e)^7 + 120*b*c*d^3*tan(1/2*f*x + 1/2*e)^7 + 30*a*d^4*tan(1/2*f*
x + 1/2*e)^7 - 160*b*d^4*tan(1/2*f*x + 1/2*e)^7 + 720*b*c^4*tan(1/2*f*x + 1
/2*e)^5 + 2880*a*c^3*d*tan(1/2*f*x + 1/2*e)^5 + 2400*b*c^2*d^2*tan(1/2*f*x
+ 1/2*e)^5 + 1600*a*c*d^3*tan(1/2*f*x + 1/2*e)^5 + 464*b*d^4*tan(1/2*f*x +
1/2*e)^5 - 480*b*c^4*tan(1/2*f*x + 1/2*e)^3 - 1920*a*c^3*d*tan(1/2*f*x + 1/
2*e)^3 - 480*b*c^3*d*tan(1/2*f*x + 1/2*e)^3 - 720*a*c^2*d^2*tan(1/2*f*x + 1
/2*e)^3 - 1920*b*c^2*d^2*tan(1/2*f*x + 1/2*e)^3 - 1280*a*c*d^3*tan(1/2*f*x
+ 1/2*e)^3 - 120*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 30*a*d^4*tan(1/2*f*x + 1/
2*e)^3 - 160*b*d^4*tan(1/2*f*x + 1/2*e)^3 + 120*b*c^4*tan(1/2*f*x + 1/2*e)
```

```
+ 480*a*c^3*d*tan(1/2*f*x + 1/2*e) + 240*b*c^3*d*tan(1/2*f*x + 1/2*e) + 360
*a*c^2*d^2*tan(1/2*f*x + 1/2*e) + 720*b*c^2*d^2*tan(1/2*f*x + 1/2*e) + 480*
a*c*d^3*tan(1/2*f*x + 1/2*e) + 300*b*c*d^3*tan(1/2*f*x + 1/2*e) + 75*a*d^4*
tan(1/2*f*x + 1/2*e) + 120*b*d^4*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e
)^2 - 1)^5)/f
```

Mupad [B] (verification not implemented)

Time = 17.35 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.22

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^4\,dx$$
$$=\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a\,c^4+2\,b\,c^3\,d+3\,a\,c^2\,d^2+\frac{3\,b\,c\,d^3}{2}+\frac{3\,a\,d^4}{8}\right)}{4\,a\,c^4+8\,b\,c^3\,d+12\,a\,c^2\,d^2+6\,b\,c\,d^3+\frac{3\,a\,d^4}{2}}\right)\left(2\,a\,c^4+4\,b\,c^3\,d+6\,a\,c^2\,d^2+3\,b\,c\,d^3+\frac{3\,a\,d^4}{4}\right)}{f}$$
$$-\frac{\left(2\,b\,c^4-\frac{5\,a\,d^4}{4}+2\,b\,d^4-6\,a\,c^2\,d^2+12\,b\,c^2\,d^2+8\,a\,c\,d^3+8\,a\,c^3\,d-5\,b\,c\,d^3-4\,b\,c^3\,d\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^9+\left(a\right.}{}$$

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^4)/cos(e + f*x),x)
```

```
[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^4 + (3*a*d^4)/8 + 3*a*c^2*d^2 + (3*b*c*d^
3)/2 + 2*b*c^3*d))/(4*a*c^4 + (3*a*d^4)/2 + 12*a*c^2*d^2 + 6*b*c*d^3 + 8*b*
c^3*d))*(2*a*c^4 + (3*a*d^4)/4 + 6*a*c^2*d^2 + 3*b*c*d^3 + 4*b*c^3*d))/f -
(tan(e/2 + (f*x)/2)^5*(12*b*c^4 + (116*b*d^4)/15 + 40*b*c^2*d^2 + (80*a*c*d
^3)/3 + 48*a*c^3*d) + tan(e/2 + (f*x)/2)*((5*a*d^4)/4 + 2*b*c^4 + 2*b*d^4 +
 6*a*c^2*d^2 + 12*b*c^2*d^2 + 8*a*c*d^3 + 8*a*c^3*d + 5*b*c*d^3 + 4*b*c^3*d
) + tan(e/2 + (f*x)/2)^9*(2*b*c^4 - (5*a*d^4)/4 + 2*b*d^4 - 6*a*c^2*d^2 + 1
2*b*c^2*d^2 + 8*a*c*d^3 + 8*a*c^3*d - 5*b*c*d^3 - 4*b*c^3*d) - tan(e/2 + (f
*x)/2)^3*((a*d^4)/2 + 8*b*c^4 + (8*b*d^4)/3 + 12*a*c^2*d^2 + 32*b*c^2*d^2 +
 (64*a*c*d^3)/3 + 32*a*c^3*d + 2*b*c*d^3 + 8*b*c^3*d) - tan(e/2 + (f*x)/2)^
7*(8*b*c^4 - (a*d^4)/2 + (8*b*d^4)/3 - 12*a*c^2*d^2 + 32*b*c^2*d^2 + (64*a*
c*d^3)/3 + 32*a*c^3*d - 2*b*c*d^3 - 8*b*c^3*d))/(f*(5*tan(e/2 + (f*x)/2)^2
- 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 - 5*tan(e/2 + (f*x)/2)^
8 + tan(e/2 + (f*x)/2)^10 - 1))
```

3.245 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 180

$$\begin{aligned}\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx &= \frac{(8ac^3+12bc^2d+12acd^2+3bd^3)\operatorname{arctanh}(\sin(e+fx))}{8f} \\ &+ \frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\tan(e+fx)}{6f} \\ &+ \frac{d(6bc^2+20acd+9bd^2)\sec(e+fx)\tan(e+fx)}{24f} \\ &+ \frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f}\end{aligned}$$

```
[Out] 1/8*(8*a*c^3+12*a*c*d^2+12*b*c^2*d+3*b*d^3)*arctanh(sin(f*x+e))/f+1/6*(4*a*
d*(4*c^2+d^2)+3*b*(c^3+4*c*d^2))*tan(f*x+e)/f+1/24*d*(20*a*c*d+6*b*c^2+9*b*
d^2)*sec(f*x+e)*tan(f*x+e)/f+1/12*(4*a*d+3*b*c)*(c+d*sec(f*x+e))^2*tan(f*x+
e)/f+1/4*b*(c+d*sec(f*x+e))^3*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\begin{aligned}&\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx\\&=\frac{(8ac^3+12acd^2+12bc^2d+3bd^3)\operatorname{arctanh}(\sin(e+fx))}{8f}\\&+\frac{d(20acd+6bc^2+9bd^2)\tan(e+fx)\sec(e+fx)}{24f}\\&+\frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\tan(e+fx)}{6f}\\&+\frac{(4ad+3bc)\tan(e+fx)(c+d\sec(e+fx))^2}{12f}+\frac{b\tan(e+fx)(c+d\sec(e+fx))^3}{4f}\end{aligned}$$

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*ArcTanh[Sin[e + f*x]])/(8*f)
 + ((4*a*d*(4*c^2 + d^2) + 3*b*(c^3 + 4*c*d^2))*Tan[e + f*x])/(6*f) + (d*(6
*b*c^2 + 20*a*c*d + 9*b*d^2)*Sec[e + f*x]*Tan[e + f*x])/(24*f) + ((3*b*c +
4*a*d)*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(12*f) + (b*(c + d*Sec[e + f*x]
)^3*Tan[e + f*x])/(4*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad+\frac{1}{4}\int \sec(e+fx)(c+d\sec(e+fx))^2(4ac+3bd+(3bc+4ad)\sec(e+fx))\,dx \\
&= \frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad+\frac{1}{12}\int \sec(e+fx)(c+d\sec(e+fx))\left(12ac^2+15bcd+8ad^2\right. \\
&\qquad\left.+\left(6bc^2+20acd+9bd^2\right)\sec(e+fx)\right)\,dx \\
&= \frac{d(6bc^2+20acd+9bd^2)\sec(e+fx)\tan(e+fx)}{24f} \\
&\quad+\frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad+\frac{1}{24}\int \sec(e+fx)\left(3\left(3bd\left(4c^2+d^2\right)+4a\left(2c^3+3cd^2\right)\right)\right. \\
&\qquad\left.+4\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\sec(e+fx)\right)\,dx \\
&= \frac{d(6bc^2+20acd+9bd^2)\sec(e+fx)\tan(e+fx)}{24f} \\
&\quad+\frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f}+\frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f} \\
&\quad+\frac{1}{8}\left(8ac^3+12bc^2d+12acd^2+3bd^3\right)\int \sec(e+fx)\,dx \\
&\quad+\frac{1}{6}\left(4ad\left(4c^2+d^2\right)+3b\left(c^3+4cd^2\right)\right)\int \sec^2(e+fx)\,dx
\end{aligned}$$

$$= \frac{(8ac^3+12bc^2d+12acd^2+3bd^3)\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{d(6bc^2+20acd+9bd^2)\sec(e+fx)\tan(e+fx)}{24f} + \frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f} - \frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\operatorname{Subst}(\int 1\,dx, x, -\tan(e+fx))}{6f}$$

$$= \frac{(8ac^3+12bc^2d+12acd^2+3bd^3)\operatorname{arctanh}(\sin(e+fx))}{8f} + \frac{(4ad(4c^2+d^2)+3b(c^3+4cd^2))\tan(e+fx)}{6f} + \frac{d(6bc^2+20acd+9bd^2)\sec(e+fx)\tan(e+fx)}{24f} + \frac{(3bc+4ad)(c+d\sec(e+fx))^2\tan(e+fx)}{12f} + \frac{b(c+d\sec(e+fx))^3\tan(e+fx)}{4f}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.79

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx = \frac{3(3bd(4c^2+d^2)+4a(2c^3+3cd^2))\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(9d(4acd+b(4c^2+d^2))\sec(e+fx\right.}{24f}$$

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^3,x]
```

```
[Out] (3*(3*b*d*(4*c^2 + d^2) + 4*a*(2*c^3 + 3*c*d^2))*ArcTanh[Sin[e + f*x]] + Ta
n[e + f*x]*(9*d*(4*a*c*d + b*(4*c^2 + d^2))*Sec[e + f*x] + 6*b*d^3*Sec[e +
f*x]^3 + 8*(3*a*d*(3*c^2 + d^2) + 3*b*(c^3 + 3*c*d^2) + d^2*(3*b*c + a*d)*T
an[e + f*x]^2)))/(24*f)
```

Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03

method	result
parts	$-\frac{(a\,d^3+3bc\,d^2)\left(-\frac{2}{3}-\frac{\sec(fx+e)^2}{3}\right)\tan(fx+e)}{f}+\frac{(3ac\,d^2+3b\,c^2d)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
derivativedivides	$\frac{a\,c^3\ln(\sec(fx+e)+\tan(fx+e))+3a\,c^2d\tan(fx+e)+3ac\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,d^3\left(-\frac{2}{3}-\right.}{}$
default	$\frac{a\,c^3\ln(\sec(fx+e)+\tan(fx+e))+3a\,c^2d\tan(fx+e)+3ac\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)-a\,d^3\left(-\frac{2}{3}-\right.}{}$
parallelrisch	$\frac{-96\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx+2e)\right)\left(a\,c^3+\frac{3}{2}ac\,d^2+\frac{3}{2}b\,c^2d+\frac{3}{8}b\,d^3\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+96\left(\frac{3}{4}+\frac{\cos(4fx+4e)}{4}+\cos(2fx\right.}{}$
norman	$\frac{-\frac{\left(24a\,c^2d-12ac\,d^2+8a\,d^3+8b\,c^3-12b\,c^2d+24bc\,d^2-5b\,d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}{4f}+\frac{\left(24a\,c^2d+12ac\,d^2+8a\,d^3+8b\,c^3+12b\,c^2d+24bc\,d^2+5b\,d^3\right.}{4f}}{}$
risch	$-\frac{i\left(-48bc\,d^2-72a\,c^2d-16a\,d^3-24b\,c^3-33b\,d^3\mathrm{e}^{3i(fx+e)}-9d^3b\,\mathrm{e}^{i(fx+e)}-72b\,c^3\mathrm{e}^{4i(fx+e)}+33b\,d^3\mathrm{e}^{5i(fx+e)}-48a\,d^3\mathrm{e}^{4i(f\right.}{}$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(a*d^3+3*b*c*d^2)/f*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+(3*a*c*d^2+3*b*c^2*
d)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))+(3*a*c^2*d+b
*c^3)/f*tan(f*x+e)+a*c^3/f*ln(sec(f*x+e)+tan(f*x+e))+b*d^3/f*(-(-1/4*sec(f*
x+e)^3-3/8*sec(f*x+e))*tan(f*x+e)+3/8*ln(sec(f*x+e)+tan(f*x+e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.17

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$=\frac{3\left(8\,ac^3+12\,bc^2d+12\,acd^2+3\,bd^3\right)\cos\left(fx+e\right)^4\log\left(\sin\left(fx+e\right)+1\right)-3\left(8\,ac^3+12\,bc^2d+12\,acd^2+3\right.}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x + e)^4*log(si
n(f*x + e) + 1) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*cos(f*x +
 e)^4*log(-sin(f*x + e) + 1) + 2*(6*b*d^3 + 8*(3*b*c^3 + 9*a*c^2*d + 6*b*c*
d^2 + 2*a*d^3)*cos(f*x + e)^3 + 9*(4*b*c^2*d + 4*a*c*d^2 + b*d^3)*cos(f*x +
 e)^2 + 8*(3*b*c*d^2 + a*d^3)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^4
)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$=\int (a+b\sec(e+fx))\,(c+d\sec(e+fx))^3\sec(e+fx)\,dx$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**3,x)

[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**3*sec(e + f*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.48

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$=\frac{48\left(\tan(fx+e)^3+3\tan(fx+e)\right)bcd^2+16\left(\tan(fx+e)^3+3\tan(fx+e)\right)ad^3-3\,bd^3\left(\frac{2\left(3\sin(fx+e)^3-5\sin\right.}{\sin(fx+e)^4-2\sin(}\right.}{}$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="maxima")

[Out] 1/48*(48*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*c*d^2 + 16*(tan(f*x + e)^3 + 3*tan(f*x + e))*a*d^3 - 3*b*d^3*(2*(3*sin(f*x + e)^3 - 5*sin(f*x + e))/(sin(f*x + e)^4 - 2*sin(f*x + e)^2 + 1) - 3*log(sin(f*x + e) + 1) + 3*log(sin(f*x + e) - 1)) - 36*b*c^2*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 36*a*c*d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) + 48*a*c^3*log(sec(f*x + e) + tan(f*x + e)) + 48*b*c^3*tan(f*x + e) + 144*a*c^2*d*tan(f*x + e))/f

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(170) = 340.

Time = 0.36 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.26

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$=\frac{3\left(8\,ac^3+12\,bc^2d+12\,acd^2+3\,bd^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(8\,ac^3+12\,bc^2d+12\,acd^2+3\,bd^3\right)\log}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] 1/24*(3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*log(abs(tan(1/2*f*x +
 1/2*e) + 1)) - 3*(8*a*c^3 + 12*b*c^2*d + 12*a*c*d^2 + 3*b*d^3)*log(abs(tan
(1/2*f*x + 1/2*e) - 1)) - 2*(24*b*c^3*tan(1/2*f*x + 1/2*e)^7 + 72*a*c^2*d*t
an(1/2*f*x + 1/2*e)^7 - 36*b*c^2*d*tan(1/2*f*x + 1/2*e)^7 - 36*a*c*d^2*tan(
1/2*f*x + 1/2*e)^7 + 72*b*c*d^2*tan(1/2*f*x + 1/2*e)^7 + 24*a*d^3*tan(1/2*f
*x + 1/2*e)^7 - 15*b*d^3*tan(1/2*f*x + 1/2*e)^7 - 72*b*c^3*tan(1/2*f*x + 1/
2*e)^5 - 216*a*c^2*d*tan(1/2*f*x + 1/2*e)^5 + 36*b*c^2*d*tan(1/2*f*x + 1/2*
e)^5 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^5 - 120*b*c*d^2*tan(1/2*f*x + 1/2*e)
^5 - 40*a*d^3*tan(1/2*f*x + 1/2*e)^5 - 9*b*d^3*tan(1/2*f*x + 1/2*e)^5 + 72*
b*c^3*tan(1/2*f*x + 1/2*e)^3 + 216*a*c^2*d*tan(1/2*f*x + 1/2*e)^3 + 36*b*c^
2*d*tan(1/2*f*x + 1/2*e)^3 + 36*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + 120*b*c*d^
2*tan(1/2*f*x + 1/2*e)^3 + 40*a*d^3*tan(1/2*f*x + 1/2*e)^3 - 9*b*d^3*tan(1/
2*f*x + 1/2*e)^3 - 24*b*c^3*tan(1/2*f*x + 1/2*e) - 72*a*c^2*d*tan(1/2*f*x +
 1/2*e) - 36*b*c^2*d*tan(1/2*f*x + 1/2*e) - 36*a*c*d^2*tan(1/2*f*x + 1/2*e)
 - 72*b*c*d^2*tan(1/2*f*x + 1/2*e) - 24*a*d^3*tan(1/2*f*x + 1/2*e) - 15*b*d
^3*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^4)/f
```

Mupad [B] (verification not implemented)

Time = 17.20 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.19

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^3\,dx$$
$$=\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(a\,c^3+\frac{3\,b\,c^2\,d}{2}+\frac{3\,a\,c\,d^2}{2}+\frac{3\,b\,d^3}{8}\right)}{4\,a\,c^3+6\,b\,c^2\,d+6\,a\,c\,d^2+\frac{3\,b\,d^3}{2}}\right)\,\left(2\,a\,c^3+3\,b\,c^2\,d+3\,a\,c\,d^2+\frac{3\,b\,d^3}{4}\right)}{f}$$
$$-\frac{\left(2\,a\,d^3+2\,b\,c^3-\frac{5\,b\,d^3}{4}-3\,a\,c\,d^2+6\,a\,c^2\,d+6\,b\,c\,d^2-3\,b\,c^2\,d\right)\,\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+\left(3\,a\,c\,d^2-6\,b\,c^3-\frac{3\,b\,d^3}{4}\right.}{}$$

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^3)/cos(e + f*x),x)
```

```
[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^3 + (3*b*d^3)/8 + (3*a*c*d^2)/2 + (3*b*c^
2*d)/2))/(4*a*c^3 + (3*b*d^3)/2 + 6*a*c*d^2 + 6*b*c^2*d))*(2*a*c^3 + (3*b*d
^3)/4 + 3*a*c*d^2 + 3*b*c^2*d))/f - (tan(e/2 + (f*x)/2)^7*(2*a*d^3 + 2*b*c^
3 - (5*b*d^3)/4 - 3*a*c*d^2 + 6*a*c^2*d + 6*b*c*d^2 - 3*b*c^2*d) + tan(e/2
+ (f*x)/2)^3*((10*a*d^3)/3 + 6*b*c^3 - (3*b*d^3)/4 + 3*a*c*d^2 + 18*a*c^2*d
 + 10*b*c*d^2 + 3*b*c^2*d) - tan(e/2 + (f*x)/2)^5*((10*a*d^3)/3 + 6*b*c^3 +
 (3*b*d^3)/4 - 3*a*c*d^2 + 18*a*c^2*d + 10*b*c*d^2 - 3*b*c^2*d) - tan(e/2 +
 (f*x)/2)*(2*a*d^3 + 2*b*c^3 + (5*b*d^3)/4 + 3*a*c*d^2 + 6*a*c^2*d + 6*b*c*
d^2 + 3*b*c^2*d))/(f*(6*tan(e/2 + (f*x)/2)^4 - 4*tan(e/2 + (f*x)/2)^2 - 4*t
an(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))
```

3.246 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 115

$$\begin{aligned}\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx &= \frac{(2bcd+a(2c^2+d^2))\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{2(3acd+b(c^2+d^2))\tan(e+fx)}{3f}\\ &+\frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f}\end{aligned}$$

```
[Out] 1/2*(2*b*c*d+a*(2*c^2+d^2))*arctanh(sin(f*x+e))/f+2/3*(3*a*c*d+b*(c^2+d^2))
*tan(f*x+e)/f+1/6*d*(3*a*d+2*b*c)*sec(f*x+e)*tan(f*x+e)/f+1/3*b*(c+d*sec(f*
x+e))^2*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\begin{aligned}\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx &= \frac{(a(2c^2+d^2)+2bcd)\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{2(3acd+b(c^2+d^2))\tan(e+fx)}{3f}\\ &+\frac{d(3ad+2bc)\tan(e+fx)\sec(e+fx)}{6f}+\frac{b\tan(e+fx)(c+d\sec(e+fx))^2}{3f}\end{aligned}$$

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]])/(2*f) + (2*(3*a*c*d + b
*(c^2 + d^2))*Tan[e + f*x])/(3*f) + (d*(2*b*c + 3*a*d)*Sec[e + f*x]*Tan[e +
 f*x])/(6*f) + (b*(c + d*Sec[e + f*x])^2*Tan[e + f*x])/(3*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f} \\
&\quad+\frac{1}{3}\int \sec(e+fx)(c+d\sec(e+fx))(3ac+2bd+(2bc+3ad)\sec(e+fx))\,dx \\
&= \frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f} \\
&\quad+\frac{1}{6}\int \sec(e+fx)\left(3\left(2bcd+a\left(2c^2+d^2\right)\right)+4\left(3acd+b\left(c^2+d^2\right)\right)\sec(e+fx)\right)\,dx \\
&= \frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f} \\
&\quad+\frac{1}{3}\left(2\left(3acd+b\left(c^2+d^2\right)\right)\right)\int \sec^2(e+fx)\,dx+\frac{1}{2}\left(2bcd+a\left(2c^2+d^2\right)\right)\int \sec(e \\
&\quad+fx)\,dx \\
&= \frac{(2bcd+a(2c^2+d^2))\operatorname{arctanh}(\sin(e+fx))}{2f} \\
&\quad+\frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f} \\
&\quad-\frac{(2(3acd+b(c^2+d^2)))\operatorname{Subst}(\int 1\,dx,x,-\tan(e+fx))}{3f} \\
&= \frac{(2bcd+a(2c^2+d^2))\operatorname{arctanh}(\sin(e+fx))}{2f}+\frac{2(3acd+b(c^2+d^2))\tan(e+fx)}{3f} \\
&\quad+\frac{d(2bc+3ad)\sec(e+fx)\tan(e+fx)}{6f}+\frac{b(c+d\sec(e+fx))^2\tan(e+fx)}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$
\begin{aligned}
&\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx \\
&= \frac{3(2bcd+a(2c^2+d^2))\operatorname{arctanh}(\sin(e+fx))+\tan(e+fx)\left(12acd+6b(c^2+d^2)+3d(2bc+ad)\sec(e+fx\right.}{6f}
\end{aligned}
$$

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (3*(2*b*c*d + a*(2*c^2 + d^2))*ArcTanh[Sin[e + f*x]] + Tan[e + f*x]*(12*a*c
*d + 6*b*(c^2 + d^2) + 3*d*(2*b*c + a*d)*Sec[e + f*x] + 2*b*d^2*Tan[e + f*x
]^2))/(6*f)
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

method	result
parts	$\frac{(a\,d^2+2bcd)\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}+\frac{(2acd+b\,c^2)\tan(fx+e)}{f}+\frac{a\,c^2\ln(\sec(fx+e)+\tan(f}{f}$
derivativedivides	$\frac{a\,c^2\ln(\sec(fx+e)+\tan(fx+e))+2acd\tan(fx+e)+a\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\,c^2\tan(fx+e}{f}$
default	$\frac{a\,c^2\ln(\sec(fx+e)+\tan(fx+e))+2acd\tan(fx+e)+a\,d^2\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)+b\,c^2\tan(fx+e}{f}$
parallelrisch	$\frac{-9\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)(a\,c^2+\frac{1}{2}a\,d^2+bcd)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+9\left(\cos(fx+e)+\frac{\cos(3fx+3e)}{3}\right)(a\,c^2+\frac{1}{2}a\,d^2+bcd)}{3f(\cos(3fx+3e)-}$
norman	$\frac{\frac{4\left(6acd+3b\,c^2+b\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3f}-\frac{\left(4acd-a\,d^2+2b\,c^2-2bcd+2b\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{f}-\frac{\left(4acd+a\,d^2+2b\,c^2+2bcd+2b\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right.}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^3}$
risch	$-\frac{i\left(3a\,d^2\mathrm{e}^{5i(fx+e)}+6bcd\,\mathrm{e}^{5i(fx+e)}-12acd\,\mathrm{e}^{4i(fx+e)}-6b\,c^2\mathrm{e}^{4i(fx+e)}-24acd\,\mathrm{e}^{2i(fx+e)}-12b\,c^2\mathrm{e}^{2i(fx+e)}-12b\,d^2\mathrm{e}^{2i(fx+}\right.}{3f\left(1+\mathrm{e}^{2i(fx+e)}\right)^3}$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (a*d^2+2*b*c*d)/f*(1/2*sec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e)))
+(2*a*c*d+b*c^2)/f*tan(f*x+e)+a*c^2/f*ln(sec(f*x+e)+tan(f*x+e))-b*d^2/f*(-2
/3-1/3*sec(f*x+e)^2)*tan(f*x+e)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.30

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{3\,(2\,ac^2+2\,bcd+ad^2)\cos{(fx+e)}^3\log{(\sin{(fx+e)}+1)}-3\,(2\,ac^2+2\,bcd+ad^2)\cos{(fx+e)}^3\log{(-\mathrm{si}}}{12\,f\,\mathrm{co}}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(sin(f*x + e) + 1) -
3*(2*a*c^2 + 2*b*c*d + a*d^2)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*
b*d^2 + 2*(3*b*c^2 + 6*a*c*d + 2*b*d^2)*cos(f*x + e)^2 + 3*(2*b*c*d + a*d^2
)*cos(f*x + e))*sin(f*x + e))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\int (a+b\sec(e+fx))(c+d\sec(e+fx))^2\sec(e+fx)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))**2,x)
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.43

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{4\left(\tan(fx+e)^3+3\tan(fx+e)\right)bd^2-6\,bcd\left(\frac{2\sin(fx+e)}{\sin(fx+e)^2-1}-\log(\sin(fx+e)+1)+\log(\sin(fx+e)-1)\right)}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
[Out] 1/12*(4*(tan(f*x + e)^3 + 3*tan(f*x + e))*b*d^2 - 6*b*c*d*(2*sin(f*x + e)/(
sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(f*x + e) - 1)) - 3*a*
d^2*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + log(sin(
f*x + e) - 1)) + 12*a*c^2*log(sec(f*x + e) + tan(f*x + e)) + 12*b*c^2*tan(f
*x + e) + 24*a*c*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(107) = 214.

Time = 0.34 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.56

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$=\frac{3\left(2\,ac^2+2\,bcd+ad^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-3\left(2\,ac^2+2\,bcd+ad^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*a*c^2 + 2*b*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - 3*(
2*a*c^2 + 2*b*c*d + a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(6*b*c^2*
tan(1/2*f*x + 1/2*e)^5 + 12*a*c*d*tan(1/2*f*x + 1/2*e)^5 - 6*b*c*d*tan(1/2*
f*x + 1/2*e)^5 - 3*a*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*b*d^2*tan(1/2*f*x + 1/2
*e)^5 - 12*b*c^2*tan(1/2*f*x + 1/2*e)^3 - 24*a*c*d*tan(1/2*f*x + 1/2*e)^3 -
 4*b*d^2*tan(1/2*f*x + 1/2*e)^3 + 6*b*c^2*tan(1/2*f*x + 1/2*e) + 12*a*c*d*t
an(1/2*f*x + 1/2*e) + 6*b*c*d*tan(1/2*f*x + 1/2*e) + 3*a*d^2*tan(1/2*f*x +
1/2*e) + 6*b*d^2*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x + 1/2*e)^2 - 1)^3)/f
```

Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.97

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))^2\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\left(ac^2+bcd+\frac{ad^2}{2}\right)}{4ac^2+4bcd+2ad^2}\right)(2ac^2+2bcd+ad^2)}{f} - \frac{(2bc^2-ad^2+2bd^2+4acd-2bcd)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+\left(-4bc^2-8acd-\frac{4bd^2}{3}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+(ac}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6-3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2-\right.}$$

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^2)/cos(e + f*x),x)
```

```
[Out] (atanh((4*tan(e/2 + (f*x)/2)*(a*c^2 + (a*d^2)/2 + b*c*d))/(4*a*c^2 + 2*a*d^
2 + 4*b*c*d))*(2*a*c^2 + a*d^2 + 2*b*c*d))/f - (tan(e/2 + (f*x)/2)*(a*d^2 +
 2*b*c^2 + 2*b*d^2 + 4*a*c*d + 2*b*c*d) - tan(e/2 + (f*x)/2)^3*(4*b*c^2 + (
4*b*d^2)/3 + 8*a*c*d) + tan(e/2 + (f*x)/2)^5*(2*b*c^2 - a*d^2 + 2*b*d^2 + 4
*a*c*d - 2*b*c*d))/(f*(3*tan(e/2 + (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + ta
n(e/2 + (f*x)/2)^6 - 1))
```

3.247 $\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$

Optimal result

Integrand size = 27, antiderivative size = 61

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{(2ac+bd)\text{arctanh}(\sin(e+fx))}{2f}+\frac{(bc+ad)\tan(e+fx)}{f}+\frac{bd\sec(e+fx)\tan(e+fx)}{2f}$$

```
[Out] 1/2*(2*a*c+b*d)*arctanh(sin(f*x+e))/f+(a*d+b*c)*tan(f*x+e)/f+1/2*b*d*sec(f*
x+e)*tan(f*x+e)/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {4082, 3872, 3855, 3852, 8}

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{(2ac+bd)\text{arctanh}(\sin(e+fx))}{2f}+\frac{(ad+bc)\tan(e+fx)}{f}+\frac{bd\tan(e+fx)\sec(e+fx)}{2f}$$

```
[In] Int[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] ((2*a*c + b*d)*ArcTanh[Sin[e + f*x]])/(2*f) + ((b*c + a*d)*Tan[e + f*x])/f
+ (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bd\sec(e+fx)\tan(e+fx)}{2f}+\frac{1}{2}\int\sec(e+fx)(2ac+bd+2(bc+ad)\sec(e+fx))\,dx\\
&= \frac{bd\sec(e+fx)\tan(e+fx)}{2f}+(bc+ad)\int\sec^2(e+fx)\,dx+\frac{1}{2}(2ac+bd)\int\sec(e+fx)\,dx\\
&= \frac{(2ac+bd)\text{arctanh}(\sin(e+fx))}{2f}+\frac{bd\sec(e+fx)\tan(e+fx)}{2f}\\
&\quad-\frac{(bc+ad)\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{f}\\
&= \frac{(2ac+bd)\text{arctanh}(\sin(e+fx))}{2f}+\frac{(bc+ad)\tan(e+fx)}{f}+\frac{bd\sec(e+fx)\tan(e+fx)}{2f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\begin{aligned}&\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx\\&=\frac{ac\operatorname{arctanh}(\sin(e+fx))}{f}+\frac{bd\operatorname{arctanh}(\sin(e+fx))}{2f}\\&\quad+\frac{bc\tan(e+fx)}{f}+\frac{ad\tan(e+fx)}{f}+\frac{bd\sec(e+fx)\tan(e+fx)}{2f}\end{aligned}$$

```
[In] Integrate[Sec[e + f*x]*(a + b*Sec[e + f*x])*(c + d*Sec[e + f*x]),x]
```

```
[Out] (a*c*ArcTanh[Sin[e + f*x]])/f + (b*d*ArcTanh[Sin[e + f*x]])/(2*f) + (b*c*Ta
n[e + f*x])/f + (a*d*Tan[e + f*x])/f + (b*d*Sec[e + f*x]*Tan[e + f*x])/(2*f
)
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)+bc\tan(fx+e)+bd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
default	$\frac{ac\ln(\sec(fx+e)+\tan(fx+e))+ad\tan(fx+e)+bc\tan(fx+e)+bd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parts	$\frac{(ad+bc)\tan(fx+e)}{f}+\frac{ac\ln(\sec(fx+e)+\tan(fx+e))}{f}+\frac{bd\left(\frac{\sec(fx+e)\tan(fx+e)}{2}+\frac{\ln(\sec(fx+e)+\tan(fx+e))}{2}\right)}{f}$
parallelrisch	$\frac{-\left(ac+\frac{bd}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)+\left(ac+\frac{bd}{2}\right)(1+\cos(2fx+2e))\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)+(ad+bc)\sin(2fx+2}{f(1+\cos(2fx+2e))}$
norman	$\frac{\frac{(2ad+2bc+bd)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}-\frac{(2ad+2bc-bd)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{f}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)^2}-\frac{(2ac+bd)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{2f}+\frac{(2ac+bd)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}{2f}$
risch	$-\frac{i\left(bd\,e^{3i(fx+e)}-2ad\,e^{2i(fx+e)}-2bc\,e^{2i(fx+e)}-bd\,e^{i(fx+e)}-2ad-2bc\right)}{f\left(1+e^{2i(fx+e)}\right)^2}-\frac{ac\ln\left(e^{i(fx+e)}-i\right)}{f}-\frac{\ln\left(e^{i(fx+e)}-i\right)bd}{2f}+$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*c*ln(sec(f*x+e)+tan(f*x+e))+a*d*tan(f*x+e)+b*c*tan(f*x+e)+b*d*(1/2*s
ec(f*x+e)*tan(f*x+e)+1/2*ln(sec(f*x+e)+tan(f*x+e))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx = \frac{(2\,ac+bd)\cos\left(fx+e\right)^2\log\left(\sin\left(fx+e\right)+1\right)-(2\,ac+bd)\cos\left(fx+e\right)^2\log\left(-\sin\left(fx+e\right)+1\right)+2\,(bd}{4\,f\cos\left(fx+e\right)^2}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*a*c + b*d)*cos(f*x + e)^2*log(sin(f*x + e) + 1) - (2*a*c + b*d)*cos
(f*x + e)^2*log(-sin(f*x + e) + 1) + 2*(b*d + 2*(b*c + a*d)*cos(f*x + e))*s
in(f*x + e))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx = \int \left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)\sec\left(e+fx\right)\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))*(c + d*sec(e + f*x))*sec(e + f*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx = -\frac{bd\left(\frac{2\,\sin(fx+e)}{\sin(fx+e)^2-1}-\log\left(\sin\left(fx+e\right)+1\right)+\log\left(\sin\left(fx+e\right)-1\right)\right)-4\,ac\log\left(\sec\left(fx+e\right)+\tan\left(fx+e\right.\right.}{4\,f}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] -1/4*(b*d*(2*sin(f*x + e)/(sin(f*x + e)^2 - 1) - log(sin(f*x + e) + 1) + lo
g(sin(f*x + e) - 1)) - 4*a*c*log(sec(f*x + e) + tan(f*x + e)) - 4*b*c*tan(f
*x + e) - 4*a*d*tan(f*x + e))/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(57) = 114$.

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.51

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=\frac{(2\,ac+bd)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)-(2\,ac+bd)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)-\frac{2\left(2\,bc\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{3}+2\,ad\,\mathrm{ta}}{}}{2\,f}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))*(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*((2*a*c + b*d)*log(abs(tan(1/2*f*x + 1/2*e) + 1)) - (2*a*c + b*d)*log(a
bs(tan(1/2*f*x + 1/2*e) - 1)) - 2*(2*b*c*tan(1/2*f*x + 1/2*e)^3 + 2*a*d*tan
(1/2*f*x + 1/2*e)^3 - b*d*tan(1/2*f*x + 1/2*e)^3 - 2*b*c*tan(1/2*f*x + 1/2*
e) - 2*a*d*tan(1/2*f*x + 1/2*e) - b*d*tan(1/2*f*x + 1/2*e))/(tan(1/2*f*x +
1/2*e)^2 - 1)^2)/f
```

Mupad [B] (verification not implemented)

Time = 14.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \sec(e+fx)(a+b\sec(e+fx))(c+d\sec(e+fx))\,dx$$
$$=\frac{\mathrm{atanh}\left(\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\right)\,\left(2\,a\,c+b\,d\right)}{f}+\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\left(2\,a\,d+2\,b\,c+b\,d\right)-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^3\,\left(2\,a\,d+2\,b\,c-b\,d\right)}{f\,\left({\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^4-2\,{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)}^2+1\right)}$$

```
[In] int(((a + b/cos(e + f*x))*(c + d/cos(e + f*x)))/cos(e + f*x),x)
```

```
[Out] (atanh(tan(e/2 + (f*x)/2))*(2*a*c + b*d))/f + (tan(e/2 + (f*x)/2)*(2*a*d +
2*b*c + b*d) - tan(e/2 + (f*x)/2)^3*(2*a*d + 2*b*c - b*d))/(f*(tan(e/2 + (f
*x)/2)^4 - 2*tan(e/2 + (f*x)/2)^2 + 1))
```

3.248 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{b\,\text{arctanh}(\sin(e+fx))}{df} - \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,d\sqrt{c+d}\,f}$$

```
[Out] b*arctanh(sin(f*x+e))/d/f-2*(-a*d+b*c)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*
e)/(c+d)^(1/2))/d/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4083, 3855, 3916, 2738, 214}

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{b\,\text{arctanh}(\sin(e+fx))}{df} - \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{df\sqrt{c-d}\sqrt{c+d}}$$

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] (b*ArcTanh[Sin[e + f*x]])/(d*f) - (2*(b*c - a*d)*ArcTanh[(Sqrt[c - d]*Tan[(
e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*d*Sqrt[c + d]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\int \sec(e+fx)\,dx}{d} + \frac{(-bc+ad)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{d} \\
&= \frac{b\operatorname{arctanh}(\sin(e+fx))}{df} - \frac{(bc-ad)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d^2} \\
&= \frac{b\operatorname{arctanh}(\sin(e+fx))}{df} - \frac{(2(bc-ad))\operatorname{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d^2f} \\
&= \frac{b\operatorname{arctanh}(\sin(e+fx))}{df} - \frac{2(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\,d\sqrt{c+d}\,f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{\frac{2(bc-ad)\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + b\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.\right.}{df}$$

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x]),x]
```

```
[Out] ((2*(b*c - a*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[
c^2 - d^2] + b*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(d*f)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{d}-\frac{2(-ad+bc)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{d\sqrt{(c+d)(c-d)}}}{f}$
default	$\frac{\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{d}-\frac{2(-ad+bc)\operatorname{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{d\sqrt{(c+d)(c-d)}}}{f}$
risch	$\frac{\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)a}{\sqrt{c^2-d^2}\,f}-\frac{\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)bc}{\sqrt{c^2-d^2}\,fd}-\frac{\ln\left(e^{i(fx+e)}-\frac{ic^2-id^2-\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)a}{\sqrt{c^2-d^2}\,f}$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(b/d*ln(tan(1/2*f*x+1/2*e)+1)-b/d*ln(tan(1/2*f*x+1/2*e)-1)-2/d*(-a*d+b*
c)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)
))
```

Fricas [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.16

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$
$$=\left[-\frac{(bc-ad)\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(fx+e)+2\,c^2-d^2}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right)-(bc^2-bd^2)}{2\,(c^2d-d^3)f}\right.$$
$$-\frac{2\,(bc-ad)\sqrt{-c^2+d^2}\arctan\left(-\frac{\sqrt{-c^2+d^2}(d\cos(fx+e)+c)}{(c^2-d^2)\sin(fx+e)}\right)-(bc^2-bd^2)\log\left(\sin\left(fx+e\right)+1\right)+(bc^2-bd^2)}{2\,(c^2d-d^3)f}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*
cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^
2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - (b*c^2 - b*d^2)
*log(sin(f*x + e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d -
d^3)*f), -1/2*(2*(b*c - a*d)*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*c
os(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (b*c^2 - b*d^2)*log(sin(f*x
+ e) + 1) + (b*c^2 - b*d^2)*log(-sin(f*x + e) + 1))/((c^2*d - d^3)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx=\int \frac{(a+b\sec{(e+fx)})\sec{(e+fx)}}{c+d\sec{(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx$$
$$= \frac{\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{d} - \frac{b\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{d} + \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)(bc-a}{\sqrt{-c^2+d^2}d}}{f}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*f*x + 1/2*e) + 1))/d - b*log(abs(tan(1/2*f*x + 1/2*e) -
1))/d + 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(
1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(b*c - a*d)/(
sqrt(-c^2 + d^2)*d))/f
```

Mupad [B] (verification not implemented)

Time = 14.54 (sec) , antiderivative size = 573, normalized size of antiderivative = 7.54

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{c+d\sec(e+fx)}\,dx = \frac{a\,c^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c^2-d^2\right)^{3/2}} - \frac{a\,d^2\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c^2-d^2\right)^{3/2}} - \frac{2\,b\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c^2-d^2\right)} - \frac{a\,\ln\left(\frac{c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{\left(c+d\right)\,\left(c-d\right)}}{f\,\left(c^2-d^2\right)} + \frac{b\,c\,d\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(c^2-d^2\right)^{3/2}} + \frac{2\,b\,c^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d\,f\,\left(c^2-d^2\right)} - \frac{b\,c^3\,\ln\left(\frac{c\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-d\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{d\,f\,\left(c^2-d^2\right)^{3/2}} + \frac{b\,c\,\ln\left(\frac{c\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+d\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)-\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c^2-d^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\,\sqrt{\left(c+d\right)\,\left(c-d\right)}}{d\,f\,\left(c^2-d^2\right)}$$

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] (a*c^2*log((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2
)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (a*d^2*lo
g((c*sin(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 -
d^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) - (2*b*d*atanh(sin(e
/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)) - (a*log((c*cos(e/2 + (f
*x)/2) + d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e
/2 + (f*x)/2))*((c + d)*(c - d))^(1/2))/(f*(c^2 - d^2)) + (b*c*d*log((c*sin
(e/2 + (f*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/
2))/cos(e/2 + (f*x)/2)))/(f*(c^2 - d^2)^(3/2)) + (2*b*c^2*atanh(sin(e/2 + (
f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(c^2 - d^2)) - (b*c^3*log((c*sin(e/2 + (f
*x)/2) - d*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e
/2 + (f*x)/2)))/(d*f*(c^2 - d^2)^(3/2)) + (b*c*log((c*cos(e/2 + (f*x)/2) +
d*cos(e/2 + (f*x)/2) - sin(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2))/cos(e/2 + (f*x
)/2))*((c + d)*(c - d))^(1/2))/(d*f*(c^2 - d^2))
```

3.249 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 99

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2(ac-bd)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(c-d)^(3/2)
/(c+d)^(3/2)/f+(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4088, 12, 3916, 2738, 214}

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2(ac-bd)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{3/2}} + \frac{(bc-ad)\tan(e+fx)}{f\,(c^2-d^2)\,(c+d\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d
)^(3/2)*(c + d)^(3/2)*f) + ((b*c - a*d)*Tan[e + f*x])/((c^2 - d^2)*f*(c + d
*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{\int \frac{(-ac+bd)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{-c^2+d^2}\\
&= \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{(ac-bd)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{c^2-d^2}\\
&= \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{(ac-bd)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{d\,(c^2-d^2)}\\
&= \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}+\frac{(2(ac-bd))\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{d\,(c^2-d^2)\,f}
\end{aligned}$$

$$= \frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}f} + \frac{(bc-ad)\tan(e+fx)}{(c^2-d^2)\,f(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{-\frac{2(ac-bd)\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}} + \frac{(bc-ad)\sin(e+fx)}{(c-d)(c+d)(d+c\cos(e+fx))}}{f}$$

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(a*c - b*d)*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c^2
 - d^2)^(3/2) + ((b*c - a*d)*Sin[e + f*x])/((c - d)*(c + d)*(d + c*Cos[e +
f*x])))/f
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d-c-d\right)}+\frac{2(ac-bd)\,\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{f}$
default	$\frac{\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(c^2-d^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d-c-d\right)}+\frac{2(ac-bd)\,\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}}{f}$
risch	$\frac{2i(-ad+bc)\left(d\,e^{i(fx+e)}+c\right)}{c(c^2-d^2)f\left(e^{2i(fx+e)}c+2d\,e^{i(fx+e)}+c\right)}+\frac{\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)ac}{\sqrt{c^2-d^2}\,(c+d)(c-d)f}-\frac{\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(c+d)(c-d)f}$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*(a*d-b*c)/(c^2-d^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1
/2*f*x+1/2*e)^2*d-c-d)+2*(a*c-b*d)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh(
(c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.93

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \left[\frac{(acd-bd^2+(ac^2-bcd)\cos(fx+e))\sqrt{c^2-d^2}\log\left(\frac{2\,cd\cos(fx+e)-(c^2-2\,d^2)\cos(fx+e)^2+2\,\sqrt{c^2-d^2}(d\cos(fx+e)+c)\sin(\ldots}{c^2\cos(fx+e)^2+2\,cd\cos(fx+e)+d^2}\right.}{2\,((c^5-2\,c^3d^2+cd^4)f\cos(fx+e)+(c^4d-2\,c^2d^3+\ldots}\right.$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*c*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2
*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos
(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(
f*x + e) + d^2)) + 2*(b*c^3 - a*c^2*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^
5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x + e) + (c^4*d - 2*c^2*d^3 + d^5)*f), ((a*c
*d - b*d^2 + (a*c^2 - b*c*d)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c
^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^3 - a*c^2
*d - b*c*d^2 + a*d^3)*sin(f*x + e))/((c^5 - 2*c^3*d^2 + c*d^4)*f*cos(f*x +
e) + (c^4*d - 2*c^2*d^3 + d^5)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \int \frac{(a+b\sec(e+fx))\sec(e+fx)}{(c+d\sec(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c-2d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)(ac-bd)}{(c^2-d^2)\sqrt{-c^2+d^2}}+\frac{bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-c-d\right)(c}$$

$$f$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*
x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*(a*c - b*d)/((c^2 -
 d^2)*sqrt(-c^2 + d^2)) + (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x + 1/2
*e))/((c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)*(c^2 -
d^2)))/f
```

Mupad [B] (verification not implemented)

Time = 13.88 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,\sqrt{c-d}}{\sqrt{c+d}}\right)\,(a\,c-b\,d)}{f\,(c+d)^{3/2}\,(c-d)^{3/2}} - \frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(a\,d-b\,c)}{f\,\left(c+d\right)\,\left(c-d\right)\,\left(\left(d-c\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+c+d\right)}$$

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*atanh((tan(e/2 + (f*x)/2)*(c - d)^(1/2))/(c + d)^(1/2))*(a*c - b*d))/(f*
(c + d)^(3/2)*(c - d)^(3/2)) - (2*tan(e/2 + (f*x)/2)*(a*d - b*c))/(f*(c + d
)*(c - d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)))
```

3.250 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 166

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = -\frac{(3bcd-a(2c^2+d^2))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)\tan(e+fx)}{2\left(c^2-d^2\right)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2\left(c^2-d^2\right)^2 f(c+d\sec(e+fx))}$$

```
[Out] -(3*b*c*d-a*(2*c^2+d^2))*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2)
)/(c-d)^(5/2)/(c+d)^(5/2)/f+1/2*(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/f/(c+d*sec(
f*x+e))^2-1/2*(3*a*c*d-b*(c^2+2*d^2))*tan(f*x+e)/(c^2-d^2)^2/f/(c+d*sec(f*x
+e))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4088, 12, 3916, 2738, 214}

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = -\frac{(3bcd-a(2c^2+d^2))\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{5/2}(c+d)^{5/2}} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2f\left(c^2-d^2\right)^2(c+d\sec(e+fx))} + \frac{(bc-ad)\tan(e+fx)}{2f\left(c^2-d^2\right)(c+d\sec(e+fx))^2}$$

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] -(((3*b*c*d - a*(2*c^2 + d^2))*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[
c + d]])/((c - d)^(5/2)*(c + d)^(5/2)*f)) + ((b*c - a*d)*Tan[e + f*x])/(2*(
c^2 - d^2)*f*(c + d*Sec[e + f*x])^2) - ((3*a*c*d - b*(c^2 + 2*d^2))*Tan[e +
 f*x])/(2*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)\tan(e+fx)}{2\,(c^2-d^2)\,f(c+d\sec(e+fx))^2} - \frac{\int \frac{\sec(e+fx)(-2(ac-bd)-(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^2}\,dx}{2\,(c^2-d^2)}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2(c^2-d^2)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2(c^2-d^2)^2 f(c+d\sec(e+fx))} + \frac{\int \frac{(-3bcd+a(2c^2+d^2))\sec(e+fx)}{c+d\sec(e+fx)}dx}{2(c^2-d^2)^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2(c^2-d^2)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2(c^2-d^2)^2 f(c+d\sec(e+fx))} - \frac{(3bcd-a(2c^2+d^2))\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}dx}{2(c^2-d^2)^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2(c^2-d^2)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2(c^2-d^2)^2 f(c+d\sec(e+fx))} - \frac{(3bcd-a(2c^2+d^2))\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}dx}{2d(c^2-d^2)^2}$$

$$= \frac{(bc-ad)\tan(e+fx)}{2(c^2-d^2)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2(c^2-d^2)^2 f(c+d\sec(e+fx))} - \frac{(3bcd-a(2c^2+d^2))\,\text{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d(c^2-d^2)^2 f}$$

$$= \frac{(2ac^2-3bcd+ad^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{5/2}(c+d)^{5/2}f} + \frac{(bc-ad)\tan(e+fx)}{2(c^2-d^2)f(c+d\sec(e+fx))^2} - \frac{(3acd-b(c^2+2d^2))\tan(e+fx)}{2(c^2-d^2)^2 f(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}dx = \frac{-\frac{2(-3bcd+a(2c^2+d^2))\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{5/2}} + \frac{d(-bc+ad)\sin(e+fx)}{c(c-d)(c+d)(d+c\cos(e+fx))^2} + \frac{(ad(-4c^2+d^2)+bc(2c^2+d^2))\sin(e+fx)}{c(c-d)^2(c+d)^2(d+c\cos(e+fx))}}{2f}$$

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^3,x]
```

```
[Out] ((-2*(-3*b*c*d + a*(2*c^2 + d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[
c^2 - d^2]])/(c^2 - d^2)^(5/2) + (d*(-(b*c) + a*d)*Sin[e + f*x])/(c*(c - d)
*(c + d)*(d + c*Cos[e + f*x])^2) + ((a*d*(-4*c^2 + d^2) + b*c*(2*c^2 + d^2)
)*Sin[e + f*x])/(c*(c - d)^2*(c + d)^2*(d + c*Cos[e + f*x])))/(2*f)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(4acd+a\,d^{2}-2b\,c^{2}-bcd-2b\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2(c-d)\left(c^{2}+2cd+d^{2}\right)}+\frac{\left(4acd-a\,d^{2}-2b\,c^{2}+bcd-2b\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c^{2}-2cd+d^{2}\right)}\right)}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}+\frac{\left(2a\,c^{2}+a\,d^{2}-3bcd\right)\operatorname{arctanh}}{\left(c^{4}-2c^{2}d^{2}+d^{4}\right)}}{f}$
default	$\frac{-\frac{2\left(-\frac{\left(4acd+a\,d^{2}-2b\,c^{2}-bcd-2b\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}}{2(c-d)\left(c^{2}+2cd+d^{2}\right)}+\frac{\left(4acd-a\,d^{2}-2b\,c^{2}+bcd-2b\,d^{2}\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2(c+d)\left(c^{2}-2cd+d^{2}\right)}\right)}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}d-c-d\right)^{2}}+\frac{\left(2a\,c^{2}+a\,d^{2}-3bcd\right)\operatorname{arctanh}}{\left(c^{4}-2c^{2}d^{2}+d^{4}\right)}}{f}$
risch	$\frac{i\left(-5a\,c^{3}d^{2}e^{3i(fx+e)}+2ac\,d^{4}e^{3i(fx+e)}+3b\,c^{4}d\,e^{3i(fx+e)}-4a\,c^{4}d\,e^{2i(fx+e)}-7a\,c^{2}d^{3}e^{2i(fx+e)}+2a\,d^{5}e^{2i(fx+e)}+2b\,c^{5}e^{2i(}\right.}{c^{2}\left(-c^{2}+d^{2}\right.}$

[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(-2*(-1/2*(4*a*c*d+a*d^2-2*b*c^2-b*c*d-2*b*d^2)/(c-d)/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3+1/2*(4*a*c*d-a*d^2-2*b*c^2+b*c*d-2*b*d^2)/(c+d)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)^2+(2*a*c^2+a*d^2-3*b*c*d)/(c^4-2*c^2*d^2+d^4)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(153) = 306.

Time = 0.32 (sec) , antiderivative size = 752, normalized size of antiderivative = 4.53

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$
$$= \left[\frac{\left(2\,ac^2d^2-3\,bcd^3+ad^4+\left(2\,ac^4-3\,bc^3d+ac^2d^2\right)\cos\left(fx+e\right)^2+2\left(2\,ac^3d-3\,bc^2d^2+acd^3\right)\cos\left(fx+e\right.\right.}{4\left(\left(c^8-3\,c^6d^2\right.\right.}\right.$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*((2*a*c^2*d^2 - 3*b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*(b*c^4*d - 3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^3*d^2 + 5*a*c^2*d^3 - b*c*d

```
^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 - 3*c^6*d^2 + 3*c^4*d^4 - c^2
*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 + 3*c^3*d^5 - c*d^7)*f*cos(f*
x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)*f), 1/2*((2*a*c^2*d^2 - 3*
b*c*d^3 + a*d^4 + (2*a*c^4 - 3*b*c^3*d + a*c^2*d^2)*cos(f*x + e)^2 + 2*(2*a
*c^3*d - 3*b*c^2*d^2 + a*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt
(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^4*d -
3*a*c^3*d^2 + b*c^2*d^3 + 3*a*c*d^4 - 2*b*d^5 + (2*b*c^5 - 4*a*c^4*d - b*c^
3*d^2 + 5*a*c^2*d^3 - b*c*d^4 - a*d^5)*cos(f*x + e))*sin(f*x + e))/((c^8 -
3*c^6*d^2 + 3*c^4*d^4 - c^2*d^6)*f*cos(f*x + e)^2 + 2*(c^7*d - 3*c^5*d^3 +
3*c^3*d^5 - c*d^7)*f*cos(f*x + e) + (c^6*d^2 - 3*c^4*d^4 + 3*c^2*d^6 - d^8)
*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \int \frac{(a+b\sec{(e+fx)})\sec{(e+fx)}}{(c+d\sec{(e+fx)})^3}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**3,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(153) = 306.

Time = 0.38 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.40

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{(2ac^2-3bcd+ad^2)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^4-2c^2d^2+d^4)\sqrt{-c^2+d^2}} - \frac{2bc^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-4ac^2d\tan\left(\frac{1}{2}fx+\frac{1}{2}\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^3,x, algorithm="giac
")
```

```
[Out] ((2*a*c^2 - 3*b*c*d + a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2
*d) + arctan(-(c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 +
 d^2)))/((c^4 - 2*c^2*d^2 + d^4)*sqrt(-c^2 + d^2)) - (2*b*c^3*tan(1/2*f*x +
 1/2*e)^3 - 4*a*c^2*d*tan(1/2*f*x + 1/2*e)^3 - b*c^2*d*tan(1/2*f*x + 1/2*e)
^3 + 3*a*c*d^2*tan(1/2*f*x + 1/2*e)^3 + b*c*d^2*tan(1/2*f*x + 1/2*e)^3 + a*
d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*b*c^3*tan(1
/2*f*x + 1/2*e) + 4*a*c^2*d*tan(1/2*f*x + 1/2*e) - b*c^2*d*tan(1/2*f*x + 1/
2*e) + 3*a*c*d^2*tan(1/2*f*x + 1/2*e) - b*c*d^2*tan(1/2*f*x + 1/2*e) - a*d^
3*tan(1/2*f*x + 1/2*e) - 2*b*d^3*tan(1/2*f*x + 1/2*e))/((c^4 - 2*c^2*d^2 +
d^4)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)^2 - c - d)^2))/f
```

Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.51

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx$$
$$= \frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(a\,d^2+2\,b\,c^2+2\,b\,d^2-4\,a\,c\,d-b\,c\,d)}{(c+d)\,(c^2-2\,c\,d+d^2)} - \frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3\,(2\,b\,c^2-a\,d^2+2\,b\,d^2-4\,a\,c\,d+b\,c\,d)}{(c+d)^2\,(c-d)}}{f\,\left(2\,c\,d-\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2\,(2\,c^2-2\,d^2)+\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4\,(c^2-2\,c\,d+d^2)+c^2+d^2\right)}$$
$$+ \frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\,(2\,c-2\,d)\,(c^2-2\,c\,d+d^2)}{2\,\sqrt{c+d}\,(c-d)^{5/2}}\right)\,(2\,a\,c^2-3\,b\,c\,d+a\,d^2)}{f\,(c+d)^{5/2}\,(c-d)^{5/2}}$$

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^3),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)*(a*d^2 + 2*b*c^2 + 2*b*d^2 - 4*a*c*d - b*c*d))/((c + d
)*(c^2 - 2*c*d + d^2)) - (tan(e/2 + (f*x)/2)^3*(2*b*c^2 - a*d^2 + 2*b*d^2 -
```

```
 4*a*c*d + b*c*d))/((c + d)^2*(c - d)))/(f*(2*c*d - tan(e/2 + (f*x)/2)^2*(2
*c^2 - 2*d^2) + tan(e/2 + (f*x)/2)^4*(c^2 - 2*c*d + d^2) + c^2 + d^2)) + (a
tanh((tan(e/2 + (f*x)/2)*(2*c - 2*d)*(c^2 - 2*c*d + d^2))/(2*(c + d)^(1/2)*
(c - d)^(5/2)))*(2*a*c^2 + a*d^2 - 3*b*c*d))/(f*(c + d)^(5/2)*(c - d)^(5/2)
)
```

3.251 $\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 237

$$\begin{aligned}\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx &= \frac{(2ac^3-4bc^2d+3acd^2-bd^3)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{7/2}(c+d)^{7/2}f}\\&\quad+\frac{(bc-ad)\tan(e+fx)}{3\left(c^2-d^2\right)f(c+d\sec(e+fx))^3}+\frac{(2bc^2-5acd+3bd^2)\tan(e+fx)}{6\left(c^2-d^2\right)^2f(c+d\sec(e+fx))^2}\\&\quad+\frac{(2bc^3-11ac^2d+13bcd^2-4ad^3)\tan(e+fx)}{6\left(c^2-d^2\right)^3f(c+d\sec(e+fx))}\end{aligned}$$

```
[Out] (2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/
(c+d)^(1/2))/(c-d)^(7/2)/(c+d)^(7/2)/f+1/3*(-a*d+b*c)*tan(f*x+e)/(c^2-d^2)/
f/(c+d*sec(f*x+e))^3+1/6*(-5*a*c*d+2*b*c^2+3*b*d^2)*tan(f*x+e)/(c^2-d^2)^2/
f/(c+d*sec(f*x+e))^2+1/6*(-11*a*c^2*d-4*a*d^3+2*b*c^3+13*b*c*d^2)*tan(f*x+e
)/(c^2-d^2)^3/f/(c+d*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used

$= \{4088, 12, 3916, 2738, 214\}$

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \frac{(2ac^3+3acd^2-4bc^2d-bd^3)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{7/2}(c+d)^{7/2}} + \frac{(-5acd+2bc^2+3bd^2)\tan(e+fx)}{6f\left(c^2-d^2\right)^2(c+d\sec(e+fx))^2} + \frac{(bc-ad)\tan(e+fx)}{3f\left(c^2-d^2\right)(c+d\sec(e+fx))^3} + \frac{(-11ac^2d-4ad^3+2bc^3+13bcd^2)\tan(e+fx)}{6f\left(c^2-d^2\right)^3(c+d\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*ArcTanh[(Sqrt[c - d]*Tan[(e + f*
x)/2])/Sqrt[c + d]])/((c - d)^(7/2)*(c + d)^(7/2)*f) + ((b*c - a*d)*Tan[e +
 f*x])/(3*(c^2 - d^2)*f*(c + d*Sec[e + f*x])^3) + ((2*b*c^2 - 5*a*c*d + 3*b
*d^2)*Tan[e + f*x])/(6*(c^2 - d^2)^2*f*(c + d*Sec[e + f*x])^2) + ((2*b*c^3
- 11*a*c^2*d + 13*b*c*d^2 - 4*a*d^3)*Tan[e + f*x])/(6*(c^2 - d^2)^3*f*(c +
d*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
```

```
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3}-\frac{\int \frac{\sec(e+fx)(-3(ac-bd)-2(bc-ad)\sec(e+fx))}{(c+d\sec(e+fx))^3}\,dx}{3(c^2-d^2)}\\
&= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3}+\frac{(2bc^2-5acd+3bd^2)\tan(e+fx)}{6(c^2-d^2)^2f(c+d\sec(e+fx))^2}\\
&\quad+\frac{\int \frac{\sec(e+fx)\left(2(3ac^2-5bcd+2ad^2)+(2bc^2-5acd+3bd^2)\sec(e+fx)\right)}{(c+d\sec(e+fx))^2}\,dx}{6(c^2-d^2)^2}\\
&= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3}+\frac{(2bc^2-5acd+3bd^2)\tan(e+fx)}{6(c^2-d^2)^2f(c+d\sec(e+fx))^2}\\
&\quad+\frac{(2bc^3-11ac^2d+13bcd^2-4ad^3)\tan(e+fx)}{6(c^2-d^2)^3f(c+d\sec(e+fx))}\\
&\quad-\frac{\int -\frac{3(2ac^3-4bc^2d+3acd^2-bd^3)\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{6(c^2-d^2)^3}\\
&= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3}+\frac{(2bc^2-5acd+3bd^2)\tan(e+fx)}{6(c^2-d^2)^2f(c+d\sec(e+fx))^2}\\
&\quad+\frac{(2bc^3-11ac^2d+13bcd^2-4ad^3)\tan(e+fx)}{6(c^2-d^2)^3f(c+d\sec(e+fx))}\\
&\quad+\frac{(2ac^3-4bc^2d+3acd^2-bd^3)\int \frac{\sec(e+fx)}{c+d\sec(e+fx)}\,dx}{2(c^2-d^2)^3}\\
&= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3}+\frac{(2bc^2-5acd+3bd^2)\tan(e+fx)}{6(c^2-d^2)^2f(c+d\sec(e+fx))^2}\\
&\quad+\frac{(2bc^3-11ac^2d+13bcd^2-4ad^3)\tan(e+fx)}{6(c^2-d^2)^3f(c+d\sec(e+fx))}\\
&\quad+\frac{(2ac^3-4bc^2d+3acd^2-bd^3)\int \frac{1}{1+\frac{c\cos(e+fx)}{d}}\,dx}{2d(c^2-d^2)^3}
\end{aligned}$$

$$= \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(2bc^2-5acd+3bd^2)\tan(e+fx)}{6(c^2-d^2)^2 f(c+d\sec(e+fx))^2} + \frac{(2bc^3-11ac^2d+13bcd^2-4ad^3)\tan(e+fx)}{6(c^2-d^2)^3 f(c+d\sec(e+fx))} + \frac{(2ac^3-4bc^2d+3acd^2-bd^3)\operatorname{Subst}\left(\int \frac{1}{1+\frac{c}{d}+\left(1-\frac{c}{d}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{d(c^2-d^2)^3 f}$$

$$= \frac{(2ac^3-4bc^2d+3acd^2-bd^3)\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{7/2}(c+d)^{7/2}f} + \frac{(bc-ad)\tan(e+fx)}{3(c^2-d^2)f(c+d\sec(e+fx))^3} + \frac{(2bc^2-5acd+3bd^2)\tan(e+fx)}{6(c^2-d^2)^2 f(c+d\sec(e+fx))^2} + \frac{(2bc^3-11ac^2d+13bcd^2-4ad^3)\tan(e+fx)}{6(c^2-d^2)^3 f(c+d\sec(e+fx))}$$

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.71

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$$

$$= \frac{(d+c\cos(e+fx))\sec^3(e+fx)(a+b\sec(e+fx))\left(\frac{24(-bd(4c^2+d^2)+a(2c^3+3cd^2))\operatorname{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)(d+}{\sqrt{c^2-d^2}}\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(a + b*Sec[e + f*x]))/(c + d*Sec[e + f*x])^4,x]
```

```
[Out] ((d + c*Cos[e + f*x])*Sec[e + f*x]^3*(a + b*Sec[e + f*x])*((24*(-(b*d*(4*c^
2 + d^2)) + a*(2*c^3 + 3*c*d^2))*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c
^2 - d^2]]*(d + c*Cos[e + f*x])^3)/Sqrt[c^2 - d^2] - 6*b*c^5*Sin[e + f*x] +
 18*a*c^4*d*Sin[e + f*x] - 18*b*c^3*d^2*Sin[e + f*x] + 39*a*c^2*d^3*Sin[e +
 f*x] - 51*b*c*d^4*Sin[e + f*x] + 18*a*d^5*Sin[e + f*x] - 12*b*c^4*d*Sin[2*
(e + f*x)] + 54*a*c^3*d^2*Sin[2*(e + f*x)] - 54*b*c^2*d^3*Sin[2*(e + f*x)]
+ 6*a*c*d^4*Sin[2*(e + f*x)] + 6*b*d^5*Sin[2*(e + f*x)] - 6*b*c^5*Sin[3*(e
+ f*x)] + 18*a*c^4*d*Sin[3*(e + f*x)] - 10*b*c^3*d^2*Sin[3*(e + f*x)] - 5*a
*c^2*d^3*Sin[3*(e + f*x)] + b*c*d^4*Sin[3*(e + f*x)] + 2*a*d^5*Sin[3*(e + f
*x)]))/(24*(-c^2 + d^2)^3*f*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4)
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.59

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{\left(6a\,c^2d+3ac\,d^2+2a\,d^3-2b\,c^3-2b\,c^2d-6bc\,d^2-b\,d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{2\left(9a\,c^2d+a\,d^3-3b\,c^3-7bc\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3\left(c^2+2cd+d^2\right)\left(c^2-2cd+d^2\right)}-\frac{(6a\,c^2d-\cdots}{\cdots}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}\cdots}{f}$
default	$-\frac{\frac{2\left(-\frac{\left(6a\,c^2d+3ac\,d^2+2a\,d^3-2b\,c^3-2b\,c^2d-6bc\,d^2-b\,d^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}{2(c-d)\left(c^3+3c^2d+3c\,d^2+d^3\right)}+\frac{2\left(9a\,c^2d+a\,d^3-3b\,c^3-7bc\,d^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^3}{3\left(c^2+2cd+d^2\right)\left(c^2-2cd+d^2\right)}-\frac{(6a\,c^2d-\cdots}{\cdots}\right.}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2d-c-d\right)^3}\cdots}{f}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(-1/2*(6*a*c^2*d+3*a*c*d^2+2*a*d^3-2*b*c^3-2*b*c^2*d-6*b*c*d^2-b*d^
3)/(c-d)/(c^3+3*c^2*d+3*c*d^2+d^3)*tan(1/2*f*x+1/2*e)^5+2/3*(9*a*c^2*d+a*d^
3-3*b*c^3-7*b*c*d^2)/(c^2+2*c*d+d^2)/(c^2-2*c*d+d^2)*tan(1/2*f*x+1/2*e)^3-1
/2*(6*a*c^2*d-3*a*c*d^2+2*a*d^3-2*b*c^3+2*b*c^2*d-6*b*c*d^2+b*d^3)/(c+d)/(c
^3-3*c^2*d+3*c*d^2-d^3)*tan(1/2*f*x+1/2*e))/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2
*f*x+1/2*e)^2*d-c-d)^3+(2*a*c^3+3*a*c*d^2-4*b*c^2*d-b*d^3)/(c^6-3*c^4*d^2+3
*c^2*d^4-d^6)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(
c-d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(222) = 444$.

Time = 0.36 (sec) , antiderivative size = 1238, normalized size of antiderivative = 5.22

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c^
5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^2
+ 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3 +
3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e
) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*s
in(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2))
 + 2*(2*b*c^5*d^2 - 11*a*c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6
+ 4*a*d^7 + (6*b*c^7 - 18*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11*b*c^3*d
```

```
^4 - 7*a*c^2*d^5 + b*c*d^6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*d - 9*a*c
^5*d^2 + 7*b*c^4*d^3 + 8*a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d^7)*cos(f*
x + e))*sin(f*x + e))/((c^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6 + c^3*d^8)
*f*cos(f*x + e)^3 + 3*(c^10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d^7 + c^2*d^9
)*f*cos(f*x + e)^2 + 3*(c^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^1
0)*f*cos(f*x + e) + (c^8*d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f)
, 1/6*(3*(2*a*c^3*d^3 - 4*b*c^2*d^4 + 3*a*c*d^5 - b*d^6 + (2*a*c^6 - 4*b*c^
5*d + 3*a*c^4*d^2 - b*c^3*d^3)*cos(f*x + e)^3 + 3*(2*a*c^5*d - 4*b*c^4*d^2
+ 3*a*c^3*d^3 - b*c^2*d^4)*cos(f*x + e)^2 + 3*(2*a*c^4*d^2 - 4*b*c^3*d^3 +
3*a*c^2*d^4 - b*c*d^5)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d
^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (2*b*c^5*d^2 - 11*a*
c^4*d^3 + 11*b*c^3*d^4 + 7*a*c^2*d^5 - 13*b*c*d^6 + 4*a*d^7 + (6*b*c^7 - 18
*a*c^6*d + 4*b*c^5*d^2 + 23*a*c^4*d^3 - 11*b*c^3*d^4 - 7*a*c^2*d^5 + b*c*d^
6 + 2*a*d^7)*cos(f*x + e)^2 + 3*(2*b*c^6*d - 9*a*c^5*d^2 + 7*b*c^4*d^3 + 8*
a*c^3*d^4 - 10*b*c^2*d^5 + a*c*d^6 + b*d^7)*cos(f*x + e))*sin(f*x + e))/((c
^11 - 4*c^9*d^2 + 6*c^7*d^4 - 4*c^5*d^6 + c^3*d^8)*f*cos(f*x + e)^3 + 3*(c^
10*d - 4*c^8*d^3 + 6*c^6*d^5 - 4*c^4*d^7 + c^2*d^9)*f*cos(f*x + e)^2 + 3*(c
^9*d^2 - 4*c^7*d^4 + 6*c^5*d^6 - 4*c^3*d^8 + c*d^10)*f*cos(f*x + e) + (c^8*
d^3 - 4*c^6*d^5 + 6*c^4*d^7 - 4*c^2*d^9 + d^11)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \int \frac{(a+b\sec{(e+fx)})\sec{(e+fx)}}{(c+d\sec{(e+fx)})^4}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**4,x)
```

```
[Out] Integral((a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x))**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(222) = 444$.

Time = 0.40 (sec) , antiderivative size = 693, normalized size of antiderivative = 2.92

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx = -\frac{\frac{3\,(2\,ac^3-4\,bc^2d+3\,acd^2-bd^3)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,c-2\,d)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-d\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-c^2+d^2}}\right)\right)}{(c^6-3\,c^4d^2+3\,c^2d^4-d^6)\sqrt{-c^2+d^2}}+\frac{6\,bc^5\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-18\,ac^4d\,\mathrm{t}}{}}{}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*a*c^3 - 4*b*c^2*d + 3*a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/p
i + 1/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x +
1/2*e))/sqrt(-c^2 + d^2)))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*sqrt(-c^2 +
 d^2)) + (6*b*c^5*tan(1/2*f*x + 1/2*e)^5 - 18*a*c^4*d*tan(1/2*f*x + 1/2*e)^
5 - 6*b*c^4*d*tan(1/2*f*x + 1/2*e)^5 + 27*a*c^3*d^2*tan(1/2*f*x + 1/2*e)^5
+ 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^5 - 6*a*c^2*d^3*tan(1/2*f*x + 1/2*e)^5
- 27*b*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 + 3*a*c*d^4*tan(1/2*f*x + 1/2*e)^5 +
12*b*c*d^4*tan(1/2*f*x + 1/2*e)^5 - 6*a*d^5*tan(1/2*f*x + 1/2*e)^5 + 3*b*d^
5*tan(1/2*f*x + 1/2*e)^5 - 12*b*c^5*tan(1/2*f*x + 1/2*e)^3 + 36*a*c^4*d*tan
(1/2*f*x + 1/2*e)^3 - 16*b*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 32*a*c^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 + 28*b*c*d^4*tan(1/2*f*x + 1/2*e)^3 - 4*a*d^5*tan(1/2*
f*x + 1/2*e)^3 + 6*b*c^5*tan(1/2*f*x + 1/2*e) - 18*a*c^4*d*tan(1/2*f*x + 1/
2*e) + 6*b*c^4*d*tan(1/2*f*x + 1/2*e) - 27*a*c^3*d^2*tan(1/2*f*x + 1/2*e) +
 12*b*c^3*d^2*tan(1/2*f*x + 1/2*e) - 6*a*c^2*d^3*tan(1/2*f*x + 1/2*e) + 27*
b*c^2*d^3*tan(1/2*f*x + 1/2*e) - 3*a*c*d^4*tan(1/2*f*x + 1/2*e) + 12*b*c*d^
4*tan(1/2*f*x + 1/2*e) - 6*a*d^5*tan(1/2*f*x + 1/2*e) - 3*b*d^5*tan(1/2*f*x
 + 1/2*e))/((c^6 - 3*c^4*d^2 + 3*c^2*d^4 - d^6)*(c*tan(1/2*f*x + 1/2*e)^2 -
 d*tan(1/2*f*x + 1/2*e)^2 - c - d)^3))/f
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.85

$$\int \frac{\sec(e+fx)(a+b\sec(e+fx))}{(c+d\sec(e+fx))^4}\,dx$$
$$= \frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5(2bc^3-2ad^3+bd^3-3acd^2-6ac^2d+6bcd^2+2bc^2d)}{(c+d)^3(c-d)}+\frac{4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3(-3bc^3+9ac^2d-7bcd^2+ad^3)}{3(c+d)^2(c^2-2cd+d^2)}-\frac{\tan}{}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2(-3c^3-3c^2d+3cd^2+3d^3)-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4(-3c^3+3c^2d+3cd^2-3d^3)+3cd^2+\right.}$$
$$+\frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(2c-2d)(c^3-3c^2d+3cd^2-d^3)}{2\sqrt{c+d}(c-d)^{7/2}}\right)(2ac^3-4bc^2d+3acd^2-bd^3)}{f(c+d)^{7/2}(c-d)^{7/2}}$$

```
[In] int((a + b/cos(e + f*x))/(cos(e + f*x)*(c + d/cos(e + f*x))^4),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)^5*(2*b*c^3 - 2*a*d^3 + b*d^3 - 3*a*c*d^2 - 6*a*c^2*d +
 6*b*c*d^2 + 2*b*c^2*d))/((c + d)^3*(c - d)) + (4*tan(e/2 + (f*x)/2)^3*(a*d
^3 - 3*b*c^3 + 9*a*c^2*d - 7*b*c*d^2))/(3*(c + d)^2*(c^2 - 2*c*d + d^2)) -
(tan(e/2 + (f*x)/2)*(2*a*d^3 - 2*b*c^3 + b*d^3 - 3*a*c*d^2 + 6*a*c^2*d - 6*
b*c*d^2 + 2*b*c^2*d))/((c + d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)))/(f*(tan(e/
2 + (f*x)/2)^2*(3*c*d^2 - 3*c^2*d - 3*c^3 + 3*d^3) - tan(e/2 + (f*x)/2)^4*(
3*c*d^2 + 3*c^2*d - 3*c^3 - 3*d^3) + 3*c*d^2 + 3*c^2*d + c^3 + d^3 - tan(e/
2 + (f*x)/2)^6*(3*c*d^2 - 3*c^2*d + c^3 - d^3))) + (atanh((tan(e/2 + (f*x)/
2)*(2*c - 2*d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3))/(2*(c + d)^(1/2)*(c - d)^(7
/2)))*(2*a*c^3 - b*d^3 + 3*a*c*d^2 - 4*b*c^2*d))/(f*(c + d)^(7/2)*(c - d)^(
7/2))
```

3.252 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 247

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx &= \frac{d^3(4bc-ad)\text{arctanh}(\sin(e+fx))}{2b^2f} \\ &\quad+\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\text{arctanh}(\sin(e+fx))}{b^4f} \\ &\quad+\frac{2(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}f} \\ &\quad+\frac{d^4\tan(e+fx)}{bf}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)\tan(e+fx)}{b^3f} \\ &\quad+\frac{d^3(4bc-ad)\sec(e+fx)\tan(e+fx)}{2b^2f}+\frac{d^4\tan^3(e+fx)}{3bf}\end{aligned}$$

```
[Out] 1/2*d^3*(-a*d+4*b*c)*arctanh(sin(f*x+e))/b^2/f+d*(-a*d+2*b*c)*(a^2*d^2-2*a*
b*c*d+2*b^2*c^2)*arctanh(sin(f*x+e))/b^4/f+2*(-a*d+b*c)^4*arctanh((a-b)^(1/
2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/b^4/f/(a-b)^(1/2)/(a+b)^(1/2)+d^4*tan(f*
x+e)/b/f+d^2*(a^2*d^2-4*a*b*c*d+6*b^2*c^2)*tan(f*x+e)/b^3/f+1/2*d^3*(-a*d+4
*b*c)*sec(f*x+e)*tan(f*x+e)/b^2/f+1/3*d^4*tan(f*x+e)^3/b/f
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4073, 3031, 2738, 214, 3855, 3852, 8, 3853}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx$$
$$=\frac{d(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)\text{arctanh}(\sin(e+fx))}{b^4f}$$
$$+\frac{d^2\left(a^2d^2-4abcd+6b^2c^2\right)\tan(e+fx)}{b^3f}$$
$$+\frac{2(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^4f\sqrt{a-b}\sqrt{a+b}}+\frac{d^3(4bc-ad)\text{arctanh}(\sin(e+fx))}{2b^2f}$$
$$+\frac{d^3(4bc-ad)\tan(e+fx)\sec(e+fx)}{2b^2f}+\frac{d^4\tan^3(e+fx)}{3bf}+\frac{d^4\tan(e+fx)}{bf}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d^3*(4*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d*(2*b*c - a*d)*(2*b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*
d)^4*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*
Sqrt[a + b]*f) + (d^4*Tan[e + f*x])/(b*f) + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a
^2*d^2)*Tan[e + f*x])/(b^3*f) + (d^3*(4*b*c - a*d)*Sec[e + f*x]*Tan[e + f*x
])/(2*b^2*f) + (d^4*Tan[e + f*x]^3)/(3*b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
```

```
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d+c\cos(e+fx))^4\sec^4(e+fx)}{b+a\cos(e+fx)}\,dx \\
&= \int \left(\frac{(bc-ad)^4}{b^4(b+a\cos(e+fx))}+\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)\sec(e+fx)}{b^4}\right. \\
&\quad +\frac{d^2(6b^2c^2-4abcd+a^2d^2)\sec^2(e+fx)}{b^3}+\frac{d^3(4bc-ad)\sec^3(e+fx)}{b^2} \\
&\quad \left.+\frac{d^4\sec^4(e+fx)}{b}\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d^4 \int \sec^4(e+fx)\,dx}{b} + \frac{(bc-ad)^4 \int \frac{1}{b+a\cos(e+fx)}\,dx}{b^4} \\
&\quad + \frac{(d^3(4bc-ad)) \int \sec^3(e+fx)\,dx}{b^2} + \frac{(d^2(6b^2c^2-4abcd+a^2d^2)) \int \sec^2(e+fx)\,dx}{b^3} \\
&\quad + \frac{(d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)) \int \sec(e+fx)\,dx}{b^4} \\
&= \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,\text{arctanh}(\sin(e+fx))}{b^4f} \\
&\quad + \frac{d^3(4bc-ad)\sec(e+fx)\tan(e+fx)}{2b^2f} + \frac{(d^3(4bc-ad)) \int \sec(e+fx)\,dx}{2b^2} \\
&\quad - \frac{d^4\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(e+fx)\right)}{bf} \\
&\quad + \frac{(2(bc-ad)^4)\,\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{b^4f} \\
&\quad - \frac{(d^2(6b^2c^2-4abcd+a^2d^2))\,\text{Subst}\left(\int 1\,dx, x, -\tan(e+fx)\right)}{b^3f} \\
&= \frac{d^3(4bc-ad)\text{arctanh}(\sin(e+fx))}{2b^2f} \\
&\quad + \frac{d(2bc-ad)\,(2b^2c^2-2abcd+a^2d^2)\,\text{arctanh}(\sin(e+fx))}{b^4f} \\
&\quad + \frac{2(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}f} \\
&\quad + \frac{d^4\tan(e+fx)}{bf} + \frac{d^2(6b^2c^2-4abcd+a^2d^2)\tan(e+fx)}{b^3f} \\
&\quad + \frac{d^3(4bc-ad)\sec(e+fx)\tan(e+fx)}{2b^2f} + \frac{d^4\tan^3(e+fx)}{3bf}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 580 vs. $2(247) = 494$.

Time = 5.68 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.35

$$
\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx
$$

$$
= \frac{\cos^3(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^4\left(-\frac{24(bc-ad)^4\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 6d(8a^2bcd^2 -\right.}{}
$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^3*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((-24*(b*c - a*
d)^4*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
- 6*d*(8*a^2*b*c*d^2 - 2*a^3*d^3 + 4*b^3*c*(2*c^2 + d^2) - a*b^2*d*(12*c^2
+ d^2))*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 6*d*(-8*a^2*b*c*d^2 + 2*
a^3*d^3 - 4*b^3*c*(2*c^2 + d^2) + a*b^2*d*(12*c^2 + d^2))*Log[Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]] + (b^2*d^3*(-3*a*d + b*(12*c + d)))/(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 + (2*b^3*d^4*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3 + (4*b*d^2*(-12*a*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d
^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (2*b^3*d^4*S
in[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (b^2*d^3*(-3*a*d
 + b*(12*c + d)))/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*b*d^2*(-12*a
*b*c*d + 3*a^2*d^2 + 2*b^2*(9*c^2 + d^2))*Sin[(e + f*x)/2])/(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])))/(12*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e + f*
x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(232) = 464$.

Time = 1.44 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.94

method	result
derivativedivides	$-\frac{2\left(-a^4d^4+4a^3bc\,d^3-6a^2b^2c^2d^2+4a\,b^3c^3d-b^4c^4\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}}-\frac{d^4}{3b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{d\left(2a^3d^3-8a^2bc\,d^2+1\right.}{}$
default	$-\frac{2\left(-a^4d^4+4a^3bc\,d^3-6a^2b^2c^2d^2+4a\,b^3c^3d-b^4c^4\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^4\sqrt{(a-b)(a+b)}}-\frac{d^4}{3b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{d\left(2a^3d^3-8a^2bc\,d^2+1\right.}{}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2/b^4*(-a^4*d^4+4*a^3*b*c*d^3-6*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d-b^4*c^4
)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a-b)*(a+b))^(1/2))
-1/3*d^4/b/(tan(1/2*f*x+1/2*e)+1)^3-1/2*d*(2*a^3*d^3-8*a^2*b*c*d^2+12*a*b^2
*c^2*d+a*b^2*d^3-8*b^3*c^3-4*b^3*c*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)+1)-1/2*d^
2*(2*a^2*d^2-8*a*b*c*d+a*b*d^2+12*b^2*c^2-4*b^2*c*d+2*b^2*d^2)/b^3/(tan(1/2
*f*x+1/2*e)+1)+1/2*d^3*(a*d-4*b*c+b*d)/b^2/(tan(1/2*f*x+1/2*e)+1)^2-1/3*d^4
/b/(tan(1/2*f*x+1/2*e)-1)^3+1/2*d*(2*a^3*d^3-8*a^2*b*c*d^2+12*a*b^2*c^2*d+a
*b^2*d^3-8*b^3*c^3-4*b^3*c*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)-1)-1/2*d^2*(2*a^2
*d^2-8*a*b*c*d+a*b*d^2+12*b^2*c^2-4*b^2*c*d+2*b^2*d^2)/b^3/(tan(1/2*f*x+1/2
*e)-1)-1/2*d^3*(a*d-4*b*c+b*d)/b^2/(tan(1/2*f*x+1/2*e)-1)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(232) = 464.

Time = 102.85 (sec) , antiderivative size = 1093, normalized size of antiderivative = 4.43

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/12*(6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*sqrt(a^2 - b^2)*cos(f*x + e)^3*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2
)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*
a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + 3*(8*(a^2*b^3
 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*
c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)^3*log(sin(f*x + e) + 1)
 - 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*(2*a^4*b -
 a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x + e)^3*log(-
sin(f*x + e) + 1) + 2*(2*(a^2*b^3 - b^5)*d^4 + 2*(18*(a^2*b^3 - b^5)*c^2*d^
2 - 12*(a^3*b^2 - a*b^4)*c*d^3 + (3*a^4*b - a^2*b^3 - 2*b^5)*d^4)*cos(f*x +
 e)^2 + 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a*b^4)*d^4)*cos(f*x + e))*s
in(f*x + e))/((a^2*b^4 - b^6)*f*cos(f*x + e)^3), 1/12*(12*(b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(
f*x + e)^3 + 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b^4)*c^2*d^2 + 4*
(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)*d^4)*cos(f*x +
e)^3*log(sin(f*x + e) + 1) - 3*(8*(a^2*b^3 - b^5)*c^3*d - 12*(a^3*b^2 - a*b
^4)*c^2*d^2 + 4*(2*a^4*b - a^2*b^3 - b^5)*c*d^3 - (2*a^5 - a^3*b^2 - a*b^4)
*d^4)*cos(f*x + e)^3*log(-sin(f*x + e) + 1) + 2*(2*(a^2*b^3 - b^5)*d^4 + 2*
(18*(a^2*b^3 - b^5)*c^2*d^2 - 12*(a^3*b^2 - a*b^4)*c*d^3 + (3*a^4*b - a^2*b
^3 - 2*b^5)*d^4)*cos(f*x + e)^2 + 3*(4*(a^2*b^3 - b^5)*c*d^3 - (a^3*b^2 - a
*b^4)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^2*b^4 - b^6)*f*cos(f*x + e)^3)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \int \frac{(c+d\sec{(e+fx)})^4\sec{(e+fx)}}{a+b\sec{(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(232) = 464$.

Time = 0.40 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.45

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx$$

$$= \frac{\frac{3\,(8\,b^3c^3d-12\,ab^2c^2d^2+8\,a^2bcd^3+4\,b^3cd^3-2\,a^3d^4-ab^2d^4)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^4} - \frac{3\,(8\,b^3c^3d-12\,ab^2c^2d^2+8\,a^2bcd^3+4\,b^3cd^3-2\,a^3d^4-ab^2d}{b^4}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(8*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^
3*d^4 - a*b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^4 - 3*(8*b^3*c^3*d
- 12*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + 4*b^3*c*d^3 - 2*a^3*d^4 - a*b^2*d^4)*l
og(abs(tan(1/2*f*x + 1/2*e) - 1))/b^4 - 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*s
gn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sq
rt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^4) - 2*(36*b^2*c^2*d^2*tan(1/2*f*x + 1
/2*e)^5 - 24*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^5 - 12*b^2*c*d^3*tan(1/2*f*x +
1/2*e)^5 + 6*a^2*d^4*tan(1/2*f*x + 1/2*e)^5 + 3*a*b*d^4*tan(1/2*f*x + 1/2*e
)^5 + 6*b^2*d^4*tan(1/2*f*x + 1/2*e)^5 - 72*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e
)^3 + 48*a*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 12*a^2*d^4*tan(1/2*f*x + 1/2*e)
^3 - 4*b^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 36*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e)
 - 24*a*b*c*d^3*tan(1/2*f*x + 1/2*e) + 12*b^2*c*d^3*tan(1/2*f*x + 1/2*e) +
6*a^2*d^4*tan(1/2*f*x + 1/2*e) - 3*a*b*d^4*tan(1/2*f*x + 1/2*e) + 6*b^2*d^4
*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*b^3))/f
```

Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 9987, normalized size of antiderivative = 40.43

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] (atan(((((((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16
*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*
d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^
3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b
^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2
)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*
(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a
*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 + (8*ta
n(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*
b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 +
 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b
^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 19
2*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 -
104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d
^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*
a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c
^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 -
376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*
b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d
^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*
b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^
2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3)*1i)/b^4 - (((((
8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d
+ 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*
b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3
*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2
- 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 + (8*tan(e/2 + (f*x)/2)*(8*a*b^10
- 16*a^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4
*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) -
b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x
)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*
a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*
d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 +
 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*
d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*
c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2
```

```
*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*
d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880
*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*
c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3
+ 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6
*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 +
32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3
*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3)*1i)/b^4)/((16*(4*a^11*d^12
- 6*a^10*b*d^12 + 16*b^11*c^11*d - a^6*b^5*d^12 + 2*a^7*b^4*d^12 - 5*a^8*b^
3*d^12 + 6*a^9*b^2*d^12 - 16*b^11*c^6*d^6 - 64*b^11*c^8*d^4 + 8*b^11*c^9*d^
3 - 64*b^11*c^10*d^2 + 72*a*b^10*c^5*d^7 + 32*a*b^10*c^6*d^6 + 368*a*b^10*c
^7*d^5 + 62*a*b^10*c^8*d^4 + 440*a*b^10*c^9*d^3 - 24*a*b^10*c^10*d^2 + 12*a
^5*b^6*c*d^11 - 24*a^6*b^5*c*d^11 + 60*a^7*b^4*c*d^11 - 72*a^8*b^3*c*d^11 +
 72*a^9*b^2*c*d^11 - 129*a^2*b^9*c^4*d^8 - 144*a^2*b^9*c^5*d^7 - 936*a^2*b^
9*c^6*d^6 - 496*a^2*b^9*c^7*d^5 - 1422*a^2*b^9*c^8*d^4 - 240*a^2*b^9*c^9*d^
3 + 88*a^2*b^9*c^10*d^2 + 116*a^3*b^8*c^3*d^9 + 258*a^3*b^8*c^4*d^8 + 1384*
a^3*b^8*c^5*d^7 + 1336*a^3*b^8*c^6*d^6 + 2848*a^3*b^8*c^7*d^5 + 1148*a^3*b^
8*c^8*d^4 - 208*a^3*b^8*c^9*d^3 - 54*a^4*b^7*c^2*d^10 - 232*a^4*b^7*c^3*d^9
 - 1301*a^4*b^7*c^4*d^8 - 1952*a^4*b^7*c^5*d^7 - 3888*a^4*b^7*c^6*d^6 - 249
6*a^4*b^7*c^7*d^5 + 276*a^4*b^7*c^8*d^4 + 108*a^5*b^6*c^2*d^10 + 788*a^5*b^
6*c^3*d^9 + 1756*a^5*b^6*c^4*d^8 + 3744*a^5*b^6*c^5*d^7 + 3360*a^5*b^6*c^6*
d^6 - 224*a^5*b^6*c^7*d^5 - 294*a^6*b^5*c^2*d^10 - 1008*a^6*b^5*c^3*d^9 - 2
556*a^6*b^5*c^4*d^8 - 3072*a^6*b^5*c^5*d^7 + 112*a^6*b^5*c^6*d^6 + 360*a^7*
b^4*c^2*d^10 + 1216*a^7*b^4*c^3*d^9 + 1968*a^7*b^4*c^4*d^8 - 32*a^7*b^4*c^5
*d^7 - 384*a^8*b^3*c^2*d^10 - 880*a^8*b^3*c^3*d^9 + 4*a^8*b^3*c^4*d^8 + 264
*a^9*b^2*c^2*d^10 - 16*a*b^10*c^11*d - 48*a^10*b*c*d^11))/b^9 + (((((8*(4*b
^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^
2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^
4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*
c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*
b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2)*(8*a*b^10 - 16*a
^2*b^9 + 8*a^3*b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d
) + a^3*d^4 - 4*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2
*c*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 + (8*tan(e/2 + (f*x)/2)*(
8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^
6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 -
16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a
*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 -
24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7
+ 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c
^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 -
304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b
^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^
6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*
a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c
```

^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b
^8*c^7*d - 64*a^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*
d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 + (((((8*(4*b^13*c^4 - 8*a*b
^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*
a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c
^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*
b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32
*a*b^12*c^3*d))/b^9 + (8*tan(e/2 + (f*x)/2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*
b^8)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d) + a^3*d^4 - 4
*a^2*b*c*d^3))/b^10)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*
d) + a^3*d^4 - 4*a^2*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b
^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b
^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6
- 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 +
192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^
7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c
*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^
2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6
*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 80
0*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5
*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6
 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^
6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a
^8*b*c*d^7))/b^6)*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c*d^3 + 4*c^3*d)
+ a^3*d^4 - 4*a^2*b*c*d^3))/b^4))*(b^2*((a*d^4)/2 + 6*a*c^2*d^2) - b^3*(2*c
*d^3 + 4*c^3*d) + a^3*d^4 - 4*a^2*b*c*d^3)*2i)/(b^4*f) - ((tan(e/2 + (f*x)/
2)*(2*a^2*d^4 + 2*b^2*d^4 + 4*b^2*c*d^3 + 12*b^2*c^2*d^2 - a*b*d^4 - 8*a*b*
c*d^3))/b^3 - (4*tan(e/2 + (f*x)/2)^3*(3*a^2*d^4 + b^2*d^4 + 18*b^2*c^2*d^2
 - 12*a*b*c*d^3))/(3*b^3) + (tan(e/2 + (f*x)/2)^5*(2*a^2*d^4 + 2*b^2*d^4 -
4*b^2*c*d^3 + 12*b^2*c^2*d^2 + a*b*d^4 - 8*a*b*c*d^3))/b^3)/(f*(3*tan(e/2 +
 (f*x)/2)^2 - 3*tan(e/2 + (f*x)/2)^4 + tan(e/2 + (f*x)/2)^6 - 1)) + (atan((
(((a + b)*(a - b))^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*
a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13
*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^
4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c
^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*
b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*
a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d
^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*
a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c
^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 -
1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*
b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d
^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7)
)/b^6 + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(4*b^13*c^4 - 8*a*b^12*c

^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b
^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^
2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c
*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^
12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^
4*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4))))/(b^6 - a^2*b
^4))*(a*d - b*c)^4*1i)/(b^6 - a^2*b^4) + (((a + b)*(a - b))^(1/2)*((8*tan(e
/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7
*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16
*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*
c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a
*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104
*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7
- 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2
*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*
d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376
*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5
*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4
- 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8
*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6 - (((a + b)*(a - b))^(1/2)*(
a*d - b*c)^4*((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 +
16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b
^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11
*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^
3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 + (8*tan(e/2 + (f*x
)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b
^8))/(b^6*(b^6 - a^2*b^4))))/(b^6 - a^2*b^4))*(a*d - b*c)^4*1i)/(b^6 - a^2*
b^4))/((16*(4*a^11*d^12 - 6*a^10*b*d^12 + 16*b^11*c^11*d - a^6*b^5*d^12 + 2
*a^7*b^4*d^12 - 5*a^8*b^3*d^12 + 6*a^9*b^2*d^12 - 16*b^11*c^6*d^6 - 64*b^11
*c^8*d^4 + 8*b^11*c^9*d^3 - 64*b^11*c^10*d^2 + 72*a*b^10*c^5*d^7 + 32*a*b^1
0*c^6*d^6 + 368*a*b^10*c^7*d^5 + 62*a*b^10*c^8*d^4 + 440*a*b^10*c^9*d^3 - 2
4*a*b^10*c^10*d^2 + 12*a^5*b^6*c*d^11 - 24*a^6*b^5*c*d^11 + 60*a^7*b^4*c*d^
11 - 72*a^8*b^3*c*d^11 + 72*a^9*b^2*c*d^11 - 129*a^2*b^9*c^4*d^8 - 144*a^2*
b^9*c^5*d^7 - 936*a^2*b^9*c^6*d^6 - 496*a^2*b^9*c^7*d^5 - 1422*a^2*b^9*c^8*
d^4 - 240*a^2*b^9*c^9*d^3 + 88*a^2*b^9*c^10*d^2 + 116*a^3*b^8*c^3*d^9 + 258
*a^3*b^8*c^4*d^8 + 1384*a^3*b^8*c^5*d^7 + 1336*a^3*b^8*c^6*d^6 + 2848*a^3*b
^8*c^7*d^5 + 1148*a^3*b^8*c^8*d^4 - 208*a^3*b^8*c^9*d^3 - 54*a^4*b^7*c^2*d^
10 - 232*a^4*b^7*c^3*d^9 - 1301*a^4*b^7*c^4*d^8 - 1952*a^4*b^7*c^5*d^7 - 38
88*a^4*b^7*c^6*d^6 - 2496*a^4*b^7*c^7*d^5 + 276*a^4*b^7*c^8*d^4 + 108*a^5*b
^6*c^2*d^10 + 788*a^5*b^6*c^3*d^9 + 1756*a^5*b^6*c^4*d^8 + 3744*a^5*b^6*c^5
*d^7 + 3360*a^5*b^6*c^6*d^6 - 224*a^5*b^6*c^7*d^5 - 294*a^6*b^5*c^2*d^10 -
1008*a^6*b^5*c^3*d^9 - 2556*a^6*b^5*c^4*d^8 - 3072*a^6*b^5*c^5*d^7 + 112*a^
6*b^5*c^6*d^6 + 360*a^7*b^4*c^2*d^10 + 1216*a^7*b^4*c^3*d^9 + 1968*a^7*b^4*
c^4*d^8 - 32*a^7*b^4*c^5*d^7 - 384*a^8*b^3*c^2*d^10 - 880*a^8*b^3*c^3*d^9 +
4*a^8*b^3*c^4*d^8 + 264*a^9*b^2*c^2*d^10 - 16*a*b^10*c^11*d - 48*a^10*b*c*

d^11))/b^9 + (((a + b)*(a - b))^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6 + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 - (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4))))/(b^6 - a^2*b^4))*(a*d - b*c)^4)/(b^6 - a^2*b^4) - (((a + b)*(a - b))^(1/2)*((8*tan(e/2 + (f*x)/2)*(8*a^9*d^8 - 4*b^9*c^8 + 4*a*b^8*c^8 - 16*a^8*b*d^8 - a^2*b^7*d^8 + 3*a^3*b^6*d^8 - 7*a^4*b^5*d^8 + 13*a^5*b^4*d^8 - 16*a^6*b^3*d^8 + 16*a^7*b^2*d^8 - 16*b^9*c^2*d^6 - 64*b^9*c^4*d^4 - 64*b^9*c^6*d^2 + 48*a*b^8*c^2*d^6 + 112*a*b^8*c^3*d^5 + 192*a*b^8*c^4*d^4 + 192*a*b^8*c^5*d^3 + 192*a*b^8*c^6*d^2 - 24*a^2*b^7*c*d^7 - 32*a^2*b^7*c^7*d + 56*a^3*b^6*c*d^7 - 104*a^4*b^5*c*d^7 + 128*a^5*b^4*c*d^7 - 128*a^6*b^3*c*d^7 + 128*a^7*b^2*c*d^7 - 136*a^2*b^7*c^2*d^6 - 336*a^2*b^7*c^3*d^5 - 464*a^2*b^7*c^4*d^4 - 576*a^2*b^7*c^5*d^3 - 304*a^2*b^7*c^6*d^2 + 280*a^3*b^6*c^2*d^6 + 560*a^3*b^6*c^3*d^5 + 880*a^3*b^6*c^4*d^4 + 800*a^3*b^6*c^5*d^3 + 176*a^3*b^6*c^6*d^2 - 376*a^4*b^5*c^2*d^6 - 784*a^4*b^5*c^3*d^5 - 1096*a^4*b^5*c^4*d^4 - 416*a^4*b^5*c^5*d^3 + 424*a^5*b^4*c^2*d^6 + 896*a^5*b^4*c^3*d^5 + 552*a^5*b^4*c^4*d^4 - 448*a^6*b^3*c^2*d^6 - 448*a^6*b^3*c^3*d^5 + 224*a^7*b^2*c^2*d^6 + 8*a*b^8*c*d^7 + 32*a*b^8*c^7*d - 64*a^8*b*c*d^7))/b^6 - (((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*((8*(4*b^13*c^4 - 8*a*b^12*c^4 - 2*a*b^12*d^4 + 8*b^13*c*d^3 + 16*b^13*c^3*d + 4*a^2*b^11*c^4 + 2*a^2*b^11*d^4 - 2*a^3*b^10*d^4 + 6*a^4*b^9*d^4 - 4*a^5*b^8*d^4 - 24*a*b^12*c^2*d^2 + 8*a^2*b^11*c*d^3 + 16*a^2*b^11*c^3*d - 24*a^3*b^10*c*d^3 + 16*a^4*b^9*c*d^3 + 48*a^2*b^11*c^2*d^2 - 24*a^3*b^10*c^2*d^2 - 8*a*b^12*c*d^3 - 32*a*b^12*c^3*d))/b^9 + (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8))/(b^6*(b^6 - a^2*b^4))))/(b^6 - a^2*b^4))*(a*d - b*c)^4)/(b^6 - a^2*b^4)))*((a + b)*(a - b))^(1/2)*(a*d - b*c)^4*2i)/(f*(b^6 - a^2*b^4))

3.253 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 170

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \frac{d^3\text{arctanh}(\sin(e+fx))}{2bf} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,\text{arctanh}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}f} + \frac{d^2(3bc-ad)\tan(e+fx)}{b^2f} + \frac{d^3\sec(e+fx)\tan(e+fx)}{2bf}$$

```
[Out] 1/2*d^3*arctanh(sin(f*x+e))/b/f+d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*arctanh(sin
(f*x+e))/b^3/f+2*(-a*d+b*c)^3*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^
(1/2))/b^3/f/(a-b)^(1/2)/(a+b)^(1/2)+d^2*(-a*d+3*b*c)*tan(f*x+e)/b^2/f+1/2*
d^3*sec(f*x+e)*tan(f*x+e)/b/f
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used

$= \{4073, 3031, 2738, 214, 3855, 3852, 8, 3853\}$

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \frac{d(a^2d^2-3abcd+3b^2c^2)\,\text{arctanh}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^3f\sqrt{a-b}\sqrt{a+b}} + \frac{d^2(3bc-ad)\tan(e+fx)}{b^2f} + \frac{d^3\text{arctanh}(\sin(e+fx))}{2bf} + \frac{d^3\tan(e+fx)\sec(e+fx)}{2bf}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d^3*ArcTanh[Sin[e + f*x]])/(2*b*f) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*
ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcTanh[(Sqrt[a - b]*Tan[
(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*f) + (d^2*(3*b*c -
 a*d)*Tan[e + f*x])/(b^2*f) + (d^3*Sec[e + f*x]*Tan[e + f*x])/(2*b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(d+c\cos(e+fx))^3\sec^3(e+fx)}{b+a\cos(e+fx)}\,dx \\
&= \int \left(\frac{(bc-ad)^3}{b^3(b+a\cos(e+fx))}+\frac{d(3b^2c^2-3abcd+a^2d^2)\sec(e+fx)}{b^3} + \frac{d^2(3bc-ad)\sec^2(e+fx)}{b^2}+\frac{d^3\sec^3(e+fx)}{b}\right)\,dx \\
&= \frac{d^3\int\sec^3(e+fx)\,dx}{b}+\frac{(bc-ad)^3\int\frac{1}{b+a\cos(e+fx)}\,dx}{b^3} + \frac{(d^2(3bc-ad))\int\sec^2(e+fx)\,dx}{b^2}+\frac{(d(3b^2c^2-3abcd+a^2d^2))\int\sec(e+fx)\,dx}{b^3}
\end{aligned}$$

$$= \frac{d(3b^2c^2 - 3abcd + a^2d^2)\operatorname{arctanh}(\sin(e+fx))}{b^3f} + \frac{d^3\sec(e+fx)\tan(e+fx)}{2bf} + \frac{d^3\int\sec(e+fx)\,dx}{2b} + \frac{(2(bc-ad)^3)\operatorname{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{b^3f} - \frac{(d^2(3bc-ad))\operatorname{Subst}(\int 1\,dx, x, -\tan(e+fx))}{b^2f}$$

$$= \frac{d^3\operatorname{arctanh}(\sin(e+fx))}{2bf} + \frac{d(3b^2c^2 - 3abcd + a^2d^2)\operatorname{arctanh}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}f} + \frac{d^2(3bc-ad)\tan(e+fx)}{b^2f} + \frac{d^3\sec(e+fx)\tan(e+fx)}{2bf}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 389 vs. 2(170) = 340.

Time = 3.11 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.29

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$$

$$= \frac{\cos^2(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^3\left(\frac{8(-bc+ad)^3\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 2d(-6abcd+\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((8*(-(b*c) + a
*d)^3*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
 - 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]] + 2*d*(-6*a*b*c*d + 2*a^2*d^2 + b^2*(6*c^2 + d^2))*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^3)/(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^2 + (4*b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]) - (b^2*d^3)/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*
b*d^2*(3*b*c - a*d)*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
))/(4*b^3*f*(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x]))
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\frac{-\frac{d^3}{2b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{d\left(2a^2d^2-6abcd+6b^2c^2+b^2d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^3}+\frac{d^2(2ad-6bc+bd)}{2b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{d^3}{2b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{d(}{}}{}$
default	$\frac{-\frac{d^3}{2b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{d\left(2a^2d^2-6abcd+6b^2c^2+b^2d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^3}+\frac{d^2(2ad-6bc+bd)}{2b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{d^3}{2b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{d(}{}}{}$
risch	$-\frac{id^2\left(bd\,e^{3i(fx+e)}+2ad\,e^{2i(fx+e)}-6bc\,e^{2i(fx+e)}-bd\,e^{i(fx+e)}+2ad-6bc\right)}{f\,b^2\left(1+e^{2i(fx+e)}\right)^2}+\frac{\ln\left(e^{i(fx+e)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)a^3d^3}{\sqrt{a^2-b^2}\,f\,b^3}-$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/2*d^3/b/(tan(1/2*f*x+1/2*e)+1)^2+1/2*d*(2*a^2*d^2-6*a*b*c*d+6*b^2*c
^2+b^2*d^2)/b^3*ln(tan(1/2*f*x+1/2*e)+1)+1/2*d^2*(2*a*d-6*b*c+b*d)/b^2/(tan
(1/2*f*x+1/2*e)+1)+1/2*d^3/b/(tan(1/2*f*x+1/2*e)-1)^2-1/2*d*(2*a^2*d^2-6*a*
b*c*d+6*b^2*c^2+b^2*d^2)/b^3*ln(tan(1/2*f*x+1/2*e)-1)+1/2*d^2*(2*a*d-6*b*c+
b*d)/b^2/(tan(1/2*f*x+1/2*e)-1)-2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)/b^3/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a-b)*(a+b))
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(157) = 314$.

Time = 22.06 (sec) , antiderivative size = 779, normalized size of antiderivative = 4.58

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$$
$$=\Bigg[-\frac{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{a^2-b^2}\cos\left(fx+e\right)^2\log\left(\frac{2\,ab\cos(fx+e)-(a^2-2\,b^2)\cos(fx+e)^2-2\,\sqrt{a^2-b^2}}{a^2\cos(fx+e)^2+2\,ab\cos(}\right.}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [-1/4*(2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a^2 - b^2
)*cos(f*x + e)^2*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2
*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(
f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3
*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(sin(f*x
```

```
 + e) + 1) + (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 -
a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e) + 1) - 2*((a^2*b^2 - b
^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b^3)*d^3)*cos(f*x + e))*s
in(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2), 1/4*(4*(b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*
(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e)^2 + (6*(a^2*b
^2 - b^4)*c^2*d - 6*(a^3*b - a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*co
s(f*x + e)^2*log(sin(f*x + e) + 1) - (6*(a^2*b^2 - b^4)*c^2*d - 6*(a^3*b -
a*b^3)*c*d^2 + (2*a^4 - a^2*b^2 - b^4)*d^3)*cos(f*x + e)^2*log(-sin(f*x + e
) + 1) + 2*((a^2*b^2 - b^4)*d^3 + 2*(3*(a^2*b^2 - b^4)*c*d^2 - (a^3*b - a*b
^3)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*b^3 - b^5)*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \int \frac{(c+d\sec(e+fx))^3\sec(e+fx)}{a+b\sec(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(157) = 314$.

Time = 0.40 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.99

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx$$

$$= \frac{\frac{(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^3} - \frac{(6\,b^2c^2d-6\,abcd^2+2\,a^2d^3+b^2d^3)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^3} - \frac{4\,(b^3c^3-3\,ab^2c^2d+}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/2*((6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*log(abs(tan(1/2*f*x
+ 1/2*e) + 1))/b^3 - (6*b^2*c^2*d - 6*a*b*c*d^2 + 2*a^2*d^3 + b^2*d^3)*log(
abs(tan(1/2*f*x + 1/2*e) - 1))/b^3 - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((
a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/(sqrt(-
a^2 + b^2)*b^3) - 2*(6*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a*d^3*tan(1/2*f*x
 + 1/2*e)^3 - b*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*b*c*d^2*tan(1/2*f*x + 1/2*e)
 + 2*a*d^3*tan(1/2*f*x + 1/2*e) - b*d^3*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x
 + 1/2*e)^2 - 1)^2*b^2))/f
```

Mupad [B] (verification not implemented)

Time = 21.04 (sec) , antiderivative size = 6730, normalized size of antiderivative = 39.59

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] ((tan(e/2 + (f*x)/2)*(b*d^3 - 2*a*d^3 + 6*b*c*d^2))/b^2 + (tan(e/2 + (f*x)/
2)^3*(2*a*d^3 + b*d^3 - 6*b*c*d^2))/b^2)/(f*(tan(e/2 + (f*x)/2)^4 - 2*tan(e
/2 + (f*x)/2)^2 + 1)) - (atan(((((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c
^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 1
3*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c
^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b
^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*
b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2
+ 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*
b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 +
24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 + (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a
```

*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*
a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*
b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*(
8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*
c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3)*(b^2*(3*
c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2)*1i)/b^3 + (((8*tan(e/2 + (f*x)/2)*(
8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6
- 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7
*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6
*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*
b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 1
68*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4
*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4
+ 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((8*(4*b^10*c^3
 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 +
 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*
b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 + (8*t
an(e/2 + (f*x)/2)*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2)
 + a^2*d^3 - 3*a*b*c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*
d^2))/b^3)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2)*1i)/b^3)/((16*(4
*a^8*d^9 - 6*a^7*b*d^9 - 12*b^8*c^8*d - a^3*b^5*d^9 + 2*a^4*b^4*d^9 - 5*a^5
*b^3*d^9 + 6*a^6*b^2*d^9 + b^8*c^3*d^6 + 12*b^8*c^5*d^4 - 2*b^8*c^6*d^3 + 3
6*b^8*c^7*d^2 - 3*a*b^7*c^2*d^7 - 2*a*b^7*c^3*d^6 - 48*a*b^7*c^4*d^5 - 12*a
*b^7*c^5*d^4 - 178*a*b^7*c^6*d^3 + 12*a*b^7*c^7*d^2 + 3*a^2*b^6*c*d^8 - 6*a
^3*b^5*c*d^8 + 27*a^4*b^4*c*d^8 - 36*a^5*b^3*c*d^8 + 48*a^6*b^2*c*d^8 + 6*a
^2*b^6*c^2*d^7 + 77*a^2*b^6*c^3*d^6 + 66*a^2*b^6*c^4*d^5 + 384*a^2*b^6*c^5*
d^4 + 104*a^2*b^6*c^6*d^3 - 48*a^2*b^6*c^7*d^2 - 63*a^3*b^5*c^2*d^7 - 112*a
^3*b^5*c^3*d^6 - 474*a^3*b^5*c^4*d^5 - 324*a^3*b^5*c^5*d^4 + 76*a^3*b^5*c^6
*d^3 + 90*a^4*b^4*c^2*d^7 + 364*a^4*b^4*c^3*d^6 + 432*a^4*b^4*c^4*d^5 - 60*
a^4*b^4*c^5*d^4 - 174*a^5*b^3*c^2*d^7 - 324*a^5*b^3*c^3*d^6 + 24*a^5*b^3*c^
4*d^5 + 144*a^6*b^2*c^2*d^7 - 4*a^6*b^2*c^3*d^6 + 12*a*b^7*c^8*d - 36*a^7*b
*c*d^8))/b^6 - (((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4
*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 -
16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^
6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^
2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a
^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^
2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 15
2*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d -
 48*a^6*b*c*d^5))/b^4 + (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b
^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*
a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a
*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*(8*a*b^8 - 16*a^2
*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^7)*(b^
2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3)*(b^2*(3*c^2*d + d^3/2) +

```
a^2*d^3 - 3*a*b*c*d^2))/b^3 + (((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c
^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 1
3*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c
^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b
^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*
b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2
+ 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*
b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 +
24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a
*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*
a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*
b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 + (8*tan(e/2 + (f*x)/2)*(
8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*
c*d^2))/b^7)*(b^2*(3*c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3)*(b^2*(3*
c^2*d + d^3/2) + a^2*d^3 - 3*a*b*c*d^2))/b^3))*(b^2*(3*c^2*d + d^3/2) + a^2
*d^3 - 3*a*b*c*d^2)*2i)/(b^3*f) - (atan(((((a + b)*(a - b))^(1/2)*(a*d - b*
c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6
+ 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*
d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 +
 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d
 + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*
d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296
*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c
^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c
*d^5))/b^4 + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(4*b^10*c^3 + 2*b^1
0*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b
^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*
d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 +
 (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(8*a*b^8 - 16*a^2*b^7 + 8*a
^3*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3))*1i)/(b^5 - a^2*b^3) + (((
a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b
^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6
 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b
^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a
^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*
a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*
d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*
a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^
5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((a + b)*(a - b))^(1/2)*(a*d -
 b*c)^3*((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*
c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*
a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a
*b^9*c^2*d))/b^6 + (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c
)^3*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*
b^3))*1i)/(b^5 - a^2*b^3))/((16*(4*a^8*d^9 - 6*a^7*b*d^9 - 12*b^8*c^8*d - a
```

^3*b^5*d^9 + 2*a^4*b^4*d^9 - 5*a^5*b^3*d^9 + 6*a^6*b^2*d^9 + b^8*c^3*d^6 + 12*b^8*c^5*d^4 - 2*b^8*c^6*d^3 + 36*b^8*c^7*d^2 - 3*a*b^7*c^2*d^7 - 2*a*b^7*c^3*d^6 - 48*a*b^7*c^4*d^5 - 12*a*b^7*c^5*d^4 - 178*a*b^7*c^6*d^3 + 12*a*b^7*c^7*d^2 + 3*a^2*b^6*c*d^8 - 6*a^3*b^5*c*d^8 + 27*a^4*b^4*c*d^8 - 36*a^5*b^3*c*d^8 + 48*a^6*b^2*c*d^8 + 6*a^2*b^6*c^2*d^7 + 77*a^2*b^6*c^3*d^6 + 66*a^2*b^6*c^4*d^5 + 384*a^2*b^6*c^5*d^4 + 104*a^2*b^6*c^6*d^3 - 48*a^2*b^6*c^7*d^2 - 63*a^3*b^5*c^2*d^7 - 112*a^3*b^5*c^3*d^6 - 474*a^3*b^5*c^4*d^5 - 324*a^3*b^5*c^5*d^4 + 76*a^3*b^5*c^6*d^3 + 90*a^4*b^4*c^2*d^7 + 364*a^4*b^4*c^3*d^6 + 432*a^4*b^4*c^4*d^5 - 60*a^4*b^4*c^5*d^4 - 174*a^5*b^3*c^2*d^7 - 324*a^5*b^3*c^3*d^6 + 24*a^5*b^3*c^4*d^5 + 144*a^6*b^2*c^2*d^7 - 4*a^6*b^2*c^3*d^6 + 12*a*b^7*c^8*d - 36*a^7*b*c*d^8))/b^6 - (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 - (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3)))/(b^5 - a^2*b^3) + (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*tan(e/2 + (f*x)/2)*(8*a^7*d^6 - 4*b^7*c^6 - b^7*d^6 + 4*a*b^6*c^6 + 3*a*b^6*d^6 - 16*a^6*b*d^6 - 7*a^2*b^5*d^6 + 13*a^3*b^4*d^6 - 16*a^4*b^3*d^6 + 16*a^5*b^2*d^6 - 12*b^7*c^2*d^4 - 36*b^7*c^4*d^2 + 36*a*b^6*c^2*d^4 + 72*a*b^6*c^3*d^3 + 108*a*b^6*c^4*d^2 - 36*a^2*b^5*c*d^5 - 24*a^2*b^5*c^5*d + 60*a^3*b^4*c*d^5 - 84*a^4*b^3*c*d^5 + 96*a^5*b^2*c*d^5 - 96*a^2*b^5*c^2*d^4 - 216*a^2*b^5*c^3*d^3 - 168*a^2*b^5*c^4*d^2 + 192*a^3*b^4*c^2*d^4 + 296*a^3*b^4*c^3*d^3 + 96*a^3*b^4*c^4*d^2 - 240*a^4*b^3*c^2*d^4 - 152*a^4*b^3*c^3*d^3 + 120*a^5*b^2*c^2*d^4 + 12*a*b^6*c*d^5 + 24*a*b^6*c^5*d - 48*a^6*b*c*d^5))/b^4 - (((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*((8*(4*b^10*c^3 + 2*b^10*d^3 - 8*a*b^9*c^3 - 2*a*b^9*d^3 + 12*b^10*c^2*d + 4*a^2*b^8*c^3 + 2*a^2*b^8*d^3 - 6*a^3*b^7*d^3 + 4*a^4*b^6*d^3 + 24*a^2*b^8*c*d^2 + 12*a^2*b^8*c^2*d - 12*a^3*b^7*c*d^2 - 12*a*b^9*c*d^2 - 24*a*b^9*c^2*d))/b^6 + (8*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*(8*a*b^8 - 16*a^2*b^7 + 8*a^3*b^6))/(b^4*(b^5 - a^2*b^3))))/(b^5 - a^2*b^3)))/(b^5 - a^2*b^3)))*((a + b)*(a - b))^(1/2)*(a*d - b*c)^3*2i)/(f*(b^5 - a^2*b^3))

3.254 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 103

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \frac{d(2bc-ad)\text{arctanh}(\sin(e+fx))}{b^2f} + \frac{2(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}f} + \frac{d^2\tan(e+fx)}{bf}$$

```
[Out] d*(-a*d+2*b*c)*arctanh(sin(f*x+e))/b^2/f+2*(-a*d+b*c)^2*arctanh((a-b)^(1/2)
*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/b^2/f/(a-b)^(1/2)/(a+b)^(1/2)+d^2*tan(f*x+
e)/b/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4073, 3031, 2738, 214, 3855, 3852, 8}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \frac{d(2bc-ad)\text{arctanh}(\sin(e+fx))}{b^2f} + \frac{2(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{b^2f\sqrt{a-b}\sqrt{a+b}} + \frac{d^2\tan(e+fx)}{bf}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d*(2*b*c - a*d)*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[
(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*f
) + (d^2*Tan[e + f*x])/(b*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d+c\cos(e+fx))^2\sec^2(e+fx)}{b+a\cos(e+fx)}\,dx \\
&= \int \left(\frac{(bc-ad)^2}{b^2(b+a\cos(e+fx))}+\frac{d(2bc-ad)\sec(e+fx)}{b^2}+\frac{d^2\sec^2(e+fx)}{b}\right)dx \\
&= \frac{d^2\int\sec^2(e+fx)\,dx}{b}+\frac{(bc-ad)^2\int\frac{1}{b+a\cos(e+fx)}\,dx}{b^2}+\frac{(d(2bc-ad))\int\sec(e+fx)\,dx}{b^2} \\
&= \frac{d(2bc-ad)\text{arctanh}(\sin(e+fx))}{b^2f}-\frac{d^2\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{bf} \\
&\quad+\frac{(2(bc-ad)^2)\,\text{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{b^2f} \\
&= \frac{d(2bc-ad)\text{arctanh}(\sin(e+fx))}{b^2f}+\frac{2(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}f}+\frac{d^2\tan(e+fx)}{bf}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.51 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$
\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx
$$
$$
=\frac{-\frac{2(bc-ad)^2\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+d\left(-\left((2bc-ad)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\text{co}\right.\right.\right.\right.}{b^2f}
$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x]),x]
```

```
[Out] ((-2*(b*c - a*d)^2*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + d*(-((2*b*c - a*d)*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + b*d*Tan[e + f*x]))/(b^2*f)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d(ad-2bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2d^2+2abcd-b^2c^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}}{f}$
default	$\frac{-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}-\frac{d(ad-2bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2d^2+2abcd-b^2c^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b^2\sqrt{(a-b)(a+b)}}-\frac{d^2}{b\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right.}}{f}$
risch	$\frac{2id^2}{fb\left(1+e^{2i(fx+e)}\right)}+\frac{d^2\ln\left(e^{i(fx+e)}-i\right)a}{b^2f}-\frac{2d\ln\left(e^{i(fx+e)}-i\right)c}{bf}+\frac{\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)a^2d^2}{\sqrt{a^2-b^2}\,f\,b^2}-\frac{2\ln\left(e^{i(\right.}}{}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-d^2/b/(tan(1/2*f*x+1/2*e)+1)-d*(a*d-2*b*c)/b^2*ln(tan(1/2*f*x+1/2*e)+
1)-2/b^2*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan
(1/2*f*x+1/2*e)/((a-b)*(a+b))^(1/2))-d^2/b/(tan(1/2*f*x+1/2*e)-1)+d*(a*d-2*
b*c)/b^2*ln(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(94) = 188$.

Time = 3.30 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.03

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx$$
$$=\left[\frac{2\,(a^2b-b^3)d^2\sin\left(fx+e\right)+(b^2c^2-2\,abcd+a^2d^2)\sqrt{a^2-b^2}\cos\left(fx+e\right)\log\left(\frac{2\,ab\cos(fx+e)-(a^2-2\,b^2)\cos(fx+e}{a^2\cos(fx-}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sq
rt(a^2 - b^2)*cos(f*x + e)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x
+ e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)
/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*b - b^3)*c*d -
(a^3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^3)*c*
d - (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2 - b^4
)*f*cos(f*x + e)), 1/2*(2*(a^2*b - b^3)*d^2*sin(f*x + e) + 2*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
 + a)/((a^2 - b^2)*sin(f*x + e)))*cos(f*x + e) + (2*(a^2*b - b^3)*c*d - (a^
3 - a*b^2)*d^2)*cos(f*x + e)*log(sin(f*x + e) + 1) - (2*(a^2*b - b^3)*c*d -
 (a^3 - a*b^2)*d^2)*cos(f*x + e)*log(-sin(f*x + e) + 1))/((a^2*b^2 - b^4)*f
*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \int \frac{(c+d\sec(e+fx))^2\sec(e+fx)}{a+b\sec(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e)),x)
[Out] Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="maxi
ma")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(94) = 188$.

Time = 0.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.89

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx =$$

$$-\frac{\frac{2\,d^2\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)b} - \frac{(2\,bcd-ad^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right|\right)}{b^2} + \frac{(2\,bcd-ad^2)\log\left(\left|\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-1\right|\right)}{b^2} + \frac{2\,(b^2c^2-2\,abcd+a^2d^2)\left(\pi\left\lfloor \cdots\right.\right.}{\cdots}}{f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e)),x, algorithm="giac
")
[Out] -(2*d^2*tan(1/2*f*x + 1/2*e)/((tan(1/2*f*x + 1/2*e)^2 - 1)*b) - (2*b*c*d -
a*d^2)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^2 + (2*b*c*d - a*d^2)*log(abs(t
an(1/2*f*x + 1/2*e) - 1))/b^2 + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) -
b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^2))/f
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 3559, normalized size of antiderivative = 34.55

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{a+b\sec(e+fx)}\,dx = \text{Too large to display}$$

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + b/cos(e + f*x))),x)
```

```
[Out] - (2*d^2*tan(e/2 + (f*x)/2))/(b*f*(tan(e/2 + (f*x)/2)^2 - 1)) - (atan(((((a
 + b)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c
^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c
^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3
*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d
^3))/b^2 + (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2
 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*
a^2*b^5*c*d))/b^3 - (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b
*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)
^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2*1i)/(b^4 - a^2*b^2) + (((a + b)*(a - b))
^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d
^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^
2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*
a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 - (((
a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2
 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/
b^3 + (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^2*(2*a*b^6
 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)^2)/(b^4 - a^2
*b^2))*(a*d - b*c)^2*1i)/(b^4 - a^2*b^2))/((64*(a^4*b*d^6 - a^5*d^6 - 2*b^5
*c^5*d + 4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + a*b^4*c^4*d^2 - 6*a^3*b^2*c*d^5
 - a^4*b*c^2*d^4 + 13*a^2*b^3*c^2*d^4 + 8*a^2*b^3*c^3*d^3 - 5*a^2*b^3*c^4*d
^2 - 12*a^3*b^2*c^2*d^4 + 4*a^3*b^2*c^3*d^3 + 2*a*b^4*c^5*d + 6*a^4*b*c*d^5
))/b^3 - (((a + b)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*
c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2
 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3
 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d
- 8*a^4*b*c*d^3))/b^2 + (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^
2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a
*b^6*c*d + 2*a^2*b^5*c*d))/b^3 - (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(
1/2)*(a*d - b*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2))
)*(a*d - b*c)^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2)/(b^4 - a^2*b^2) + (((a + b
)*(a - b))^(1/2)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 -
 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d
^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2
*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))
/b^2 - (((a + b)*(a - b))^(1/2)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a
```

^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3 + (32*tan(e/2 + (f*x)/2)*((a + b)*(a - b))^(1/2)*(a*d - b*c)^2*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/(b^2*(b^4 - a^2*b^2)))*(a*d - b*c)^2)/(b^4 - a^2*b^2))*(a*d - b*c)^2)/(b^4 - a^2*b^2)))*((a + b)*(a - b))^(1/2)*(a*d - b*c)^2*2i)/(f*(b^4 - a^2*b^2)) - (d*atan(((d*(a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 + (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3 - (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/b^4))/b^2)*1i)/b^2 + (d*(a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 - (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3 + (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/b^4))/b^2)*1i)/b^2)/((64*(a^4*b*d^6 - a^5*d^6 - 2*b^5*c^5*d + 4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + a*b^4*c^4*d^2 - 6*a^3*b^2*c*d^5 - a^4*b*c^2*d^4 + 13*a^2*b^3*c^2*d^4 + 8*a^2*b^3*c^3*d^3 - 5*a^2*b^3*c^4*d^2 - 12*a^3*b^2*c^2*d^4 + 4*a^3*b^2*c^3*d^3 + 2*a*b^4*c^5*d + 6*a^4*b*c*d^5))/b^3 - (d*(a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 + (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3 - (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/b^4))/b^2))/b^2 + (d*(a*d - 2*b*c)*((32*tan(e/2 + (f*x)/2)*(2*a^5*d^4 - b^5*c^4 + a*b^4*c^4 - 4*a^4*b*d^4 - a^2*b^3*d^4 + 3*a^3*b^2*d^4 - 4*b^5*c^2*d^2 + 12*a*b^4*c^2*d^2 - 12*a^2*b^3*c*d^3 - 4*a^2*b^3*c^3*d + 16*a^3*b^2*c*d^3 - 18*a^2*b^3*c^2*d^2 + 10*a^3*b^2*c^2*d^2 + 4*a*b^4*c*d^3 + 4*a*b^4*c^3*d - 8*a^4*b*c*d^3))/b^2 - (d*(a*d - 2*b*c)*((32*(b^7*c^2 - 2*a*b^6*c^2 - a*b^6*d^2 + a^2*b^5*c^2 + 2*a^2*b^5*d^2 - a^3*b^4*d^2 + 2*b^7*c*d - 4*a*b^6*c*d + 2*a^2*b^5*c*d))/b^3 + (32*d*tan(e/2 + (f*x)/2)*(a*d - 2*b*c)*(2*a*b^6 - 4*a^2*b^5 + 2*a^3*b^4))/b^4))/b^2))/b^2))*(a*d - 2*b*c)*2i)/(b^2*f)

3.255 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \frac{d\text{arctanh}(\sin(e+fx))}{bf} + \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}f}$$

```
[Out] d*arctanh(sin(f*x+e))/b/f+2*(-a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*
e)/(a+b)^(1/2))/b/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4083, 3855, 3916, 2738, 214}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{bf\sqrt{a-b}\sqrt{a+b}} + \frac{d\text{arctanh}(\sin(e+fx))}{bf}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

```
[Out] (d*ArcTanh[Sin[e + f*x]])/(b*f) + (2*(b*c - a*d)*ArcTanh[(Sqrt[a - b]*Tan[(
e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\text{integral} = \frac{d\int \sec(e+fx)\,dx}{b} + \frac{(bc-ad)\int \frac{\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{b}$$

$$= \frac{d\text{arctanh}(\sin(e+fx))}{bf} + \frac{(bc-ad)\int \frac{1}{1+\frac{a\cos(e+fx)}{b}}\,dx}{b^2}$$

$$= \frac{d\text{arctanh}(\sin(e+fx))}{bf} + \frac{(2(bc-ad))\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{b^2f}$$

$$= \frac{d\text{arctanh}(\sin(e+fx))}{bf} + \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}f}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$$
$$= \frac{\frac{2(-bc+ad)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + d\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\right.\right.}{bf}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x]),x]
```

```
[Out] ((2*(-(b*c) + a*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + d*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]]))/(b*f)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{2(ad-bc)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b}}{f}$
default	$\frac{-\frac{2(ad-bc)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{b\sqrt{(a-b)(a+b)}}+\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b}-\frac{d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b}}{f}$
risch	$\frac{\ln\left(e^{i(fx+e)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)ad}{\sqrt{a^2-b^2}\,fb}-\frac{\ln\left(e^{i(fx+e)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)c}{\sqrt{a^2-b^2}\,f}-\frac{\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)ad}{\sqrt{a^2-b^2}\,fb}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(a*d-b*c)/b/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((
a-b)*(a+b))^(1/2))+d/b*ln(tan(1/2*f*x+1/2*e)+1)-d/b*ln(tan(1/2*f*x+1/2*e)-1
))
```

Fricas [A] (verification not implemented)

none

Time = 0.57 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.07

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \left[\frac{(a^2-b^2)d\log\left(\sin\left(fx+e\right)+1\right)-(a^2-b^2)d\log\left(-\sin\left(fx+e\right)+1\right)-\sqrt{a^2-b^2}(bc-ad)\log\left(\frac{2\,ab\cos(fx}{}\right.}{2\left(a^2b-b^3\right)f}\right.$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x + e)
 + 1) - sqrt(a^2 - b^2)*(b*c - a*d)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)
*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a
^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/((a^2*b - b^3)*
f), 1/2*((a^2 - b^2)*d*log(sin(f*x + e) + 1) - (a^2 - b^2)*d*log(-sin(f*x +
 e) + 1) + 2*sqrt(-a^2 + b^2)*(b*c - a*d)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f
*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/((a^2*b - b^3)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \int \frac{(c+d\sec\left(e+fx\right))\sec\left(e+fx\right)}{a+b\sec\left(e+fx\right)}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x)
```

```
[Out] Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$$
$$= \frac{\frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{b} - \frac{d\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{b} - \frac{2\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)(bc-ad}{\sqrt{-a^2+b^2}b}}{f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] (d*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b - d*log(abs(tan(1/2*f*x + 1/2*e) -
1))/b - 2*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(b*c - a*d)/(
sqrt(-a^2 + b^2)*b))/f
```

Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 571, normalized size of antiderivative = 7.51

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{a+b\sec(e+fx)}\,dx$$
$$= \frac{b^2\,c\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(a^2-b^2\right)}^{3/2}}$$
$$- \frac{a^2\,c\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(a^2-b^2\right)}^{3/2}} - \frac{2\,b\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,\left(a^2-b^2\right)}$$
$$+ \frac{c\,\ln\left(\frac{a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\sqrt{\left(a+b\right)\,\left(a-b\right)}}{f\,\left(a^2-b^2\right)}$$
$$- \frac{a\,b\,d\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{f\,{\left(a^2-b^2\right)}^{3/2}}$$
$$+ \frac{2\,a^2\,d\,\mathrm{atanh}\left(\frac{\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{b\,f\,\left(a^2-b^2\right)} + \frac{a^3\,d\,\ln\left(\frac{b\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)-a\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)}{b\,f\,{\left(a^2-b^2\right)}^{3/2}}$$
$$- \frac{a\,d\,\ln\left(\frac{a\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+b\,\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)+\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)}\right)\sqrt{\left(a+b\right)\,\left(a-b\right)}}{b\,f\,\left(a^2-b^2\right)}$$

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))),x)

[Out] (b^2*c*log((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2
)*(a^2 - b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) - (a^2*c*lo
g((b*sin(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 -
b^2)^(1/2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) - (2*b*d*atanh(sin(e
/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)) + (c*log((a*cos(e/2 + (f
*x)/2) + b*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e
/2 + (f*x)/2))*((a + b)*(a - b))^(1/2))/(f*(a^2 - b^2)) - (a*b*d*log((b*sin
(e/2 + (f*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 - b^2)^(1/
2))/cos(e/2 + (f*x)/2)))/(f*(a^2 - b^2)^(3/2)) + (2*a^2*d*atanh(sin(e/2 + (
f*x)/2)/cos(e/2 + (f*x)/2)))/(b*f*(a^2 - b^2)) + (a^3*d*log((b*sin(e/2 + (f
*x)/2) - a*sin(e/2 + (f*x)/2) + cos(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e
/2 + (f*x)/2)))/(b*f*(a^2 - b^2)^(3/2)) - (a*d*log((a*cos(e/2 + (f*x)/2) +
b*cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2))/cos(e/2 + (f*x
)/2))*((a + b)*(a - b))^(1/2))/(b*f*(a^2 - b^2))
```

3.256 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)f} - \frac{2d\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}(bc-ad)f}$$

```
[Out] 2*b*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(-a*d+b*c)/f/(a-b)^
(1/2)/(a+b)^(1/2)-2*d*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1/2))/(
-a*d+b*c)/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4073, 3080, 2738, 214}

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)} - \frac{2d\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-ad)}$$

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*b*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*(b*c - a*d)*f) - (2*d*ArcTanh[(Sqrt[c - d]*Tan[(e + f*x)/2])/Sqrt[c
 + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}\,dx \\
&= \frac{b\int \frac{1}{b+a\cos(e+fx)}\,dx}{bc-ad} - \frac{d\int \frac{1}{d+c\cos(e+fx)}\,dx}{bc-ad} \\
&= \frac{(2b)\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} \\
&\quad - \frac{(2d)\text{Subst}\left(\int \frac{1}{c+d+(-c+d)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)f} \\
&= \frac{2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)f} - \frac{2d\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}(bc-ad)f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = -\frac{2b\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}(bc-ad)f} - \frac{2d\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{(-bc+ad)\sqrt{c^2-d^2}f}$$

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])),x]
[Out] (-2*b*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt[a^2 - b^2
]*(b*c - a*d)*f) - (2*d*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]
])/((-(b*c) + a*d)*Sqrt[c^2 - d^2]*f)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{2d\,\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(ad-bc)\sqrt{(c+d)(c-d)}}-\frac{2b\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(ad-bc)\sqrt{(a-b)(a+b)}}}{f}$
default	$\frac{\frac{2d\,\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{(ad-bc)\sqrt{(c+d)(c-d)}}-\frac{2b\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(ad-bc)\sqrt{(a-b)(a+b)}}}{f}$
risch	$\frac{b\ln\left(e^{i(fx+e)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(ad-bc)f}-\frac{b\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(ad-bc)f}+\frac{d\ln\left(e^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)}{\sqrt{c^2-d^2}\,(ad-bc)f}-$

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)
[Out] 1/f*(2*d/(a*d-b*c)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c
+d)*(c-d))^(1/2))-2*b/(a*d-b*c)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f
*x+1/2*e)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 1.86 (sec) , antiderivative size = 1040, normalized size of antiderivative = 8.60

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((a^2 - b^2)*sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2
)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*
c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + (b*c^2 - b*d^
2)*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 -
 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*co
s(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b
^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -1/2*((a^2 - b^2)*
sqrt(c^2 - d^2)*d*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 +
2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos
(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) - 2*(b*c^2 - b*d^2)*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))
))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 -
 a*b^2)*d^3)*f), -1/2*(2*(a^2 - b^2)*sqrt(-c^2 + d^2)*d*arctan(-sqrt(-c^2 +
 d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + (b*c^2 - b*d^2)*sq
rt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sq
rt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x
 + e)^2 + 2*a*b*cos(f*x + e) + b^2)))/(((a^2*b - b^3)*c^3 - (a^3 - a*b^2)*c
^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f), -((a^2 - b^2)*sqrt(-c^2
 + d^2)*d*arctan(-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*
x + e))) - (b*c^2 - b*d^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))))/(((a^2*b - b^3)*c^3 - (a^3 - a*
b^2)*c^2*d - (a^2*b - b^3)*c*d^2 + (a^3 - a*b^2)*d^3)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec(e+fx))\,(c+d\sec(e+fx))}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(103) = 206$.

Time = 0.40 (sec) , antiderivative size = 522, normalized size of antiderivative = 4.31

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \frac{\left(\sqrt{-c^2+d^2}b(c-2\,d)|c-d|+\sqrt{-c^2+d^2}ad|c-d|+\sqrt{-c^2+d^2}|-bc+ad||c-d|\right)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\sqrt{\frac{1}{2}}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-\frac{2\,ac-2\,bd+\sqrt{-4\,(ac+bc+ad+bd)(ac-bc-ad+bd)}}{ac-bc-ad+bd}}}\right.\right.}{(bc-ad)^2(c^2-2\,cd+d^2)+(c^3-2\,c^2d+cd^2)a|-bc+ad|-(c^2d-2\,cd^2+d^3)b|-bc+ad|}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e)),x, algorithm="giac")
```

```
[Out] ((sqrt(-c^2 + d^2)*b*(c - 2*d)*abs(c - d) + sqrt(-c^2 + d^2)*a*d*abs(c - d)
 + sqrt(-c^2 + d^2)*abs(-b*c + a*d)*abs(c - d))*(pi*floor(1/2*(f*x + e)/pi
+ 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d + sqr
t(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2))/(a
*c - b*c - a*d + b*d))))/((b*c - a*d)^2*(c^2 - 2*c*d + d^2) + (c^3 - 2*c^2*
d + c*d^2)*a*abs(-b*c + a*d) - (c^2*d - 2*c*d^2 + d^3)*b*abs(-b*c + a*d)) +
 (sqrt(-a^2 + b^2)*b*c*abs(a - b) + sqrt(-a^2 + b^2)*(a - 2*b)*d*abs(a - b)
 - sqrt(-a^2 + b^2)*abs(-b*c + a*d)*abs(a - b))*(pi*floor(1/2*(f*x + e)/pi
+ 1/2) + arctan(2*sqrt(1/2)*tan(1/2*f*x + 1/2*e)/sqrt(-(2*a*c - 2*b*d - sqr
t(-4*(a*c + b*c + a*d + b*d)*(a*c - b*c - a*d + b*d) + 4*(a*c - b*d)^2))/(a
*c - b*c - a*d + b*d))))/((a^2 - 2*a*b + b^2)*(b*c - a*d)^2 - (a^3 - 2*a^2*
b + a*b^2)*c*abs(-b*c + a*d) + (a^2*b - 2*a*b^2 + b^3)*d*abs(-b*c + a*d)))/
f
```

Mupad [B] (verification not implemented)

Time = 15.93 (sec) , antiderivative size = 2665, normalized size of antiderivative = 22.02

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))*(c + d/cos(e + f*x))),x)
[Out] (b*c^2*atan((b^5*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^5*d^2*tan(
e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i + b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2
)^(3/2)*2i + b^5*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a^2*b^3*c^2*
tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^3*b^2*c^2*tan(e/2 + (f*x)/2)*(a
^2 - b^2)^(1/2)*1i - a^2*b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*3i +
a^3*b^2*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - b^3*c*d*tan(e/2 + (f*
x)/2)*(a^2 - b^2)^(3/2)*2i - b^5*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2
i + a*b^2*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i + a*b^4*c^2*tan(e/2 +
 (f*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(
1/2)*1i + a^2*b^3*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a^3*b^2*c*d
*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a*b^2*c*d*tan(e/2 + (f*x)/2)*(a^
2 - b^2)^(3/2)*2i - a*b^4*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i)/(a^6
*d^2 - b^6*c^2 + 2*a^2*b^4*c^2 - a^4*b^2*c^2 + a^2*b^4*d^2 - 2*a^4*b^2*d^2)
)*(a^2 - b^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3
*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) - (b*d^2*atan((b^5*c^2*tan
(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^5*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^
2)^(1/2)*1i + b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i + b^5*d^2*tan
(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i - a^2*b^3*c^2*tan(e/2 + (f*x)/2)*(a^2
- b^2)^(1/2)*1i - a^3*b^2*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i - a^2
*b^3*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*d^2*tan(e/2 + (f
*x)/2)*(a^2 - b^2)^(1/2)*1i - b^3*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*
2i - b^5*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a*b^2*c^2*tan(e/2 +
(f*x)/2)*(a^2 - b^2)^(3/2)*2i + a*b^4*c^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1
/2)*1i + a^4*b*d^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*1i + a^2*b^3*c*d*ta
n(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i + a^3*b^2*c*d*tan(e/2 + (f*x)/2)*(a^2
 - b^2)^(1/2)*2i - a*b^2*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(3/2)*2i - a*b^
4*c*d*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*2i)/(a^6*d^2 - b^6*c^2 + 2*a^2*b
^4*c^2 - a^4*b^2*c^2 + a^2*b^4*d^2 - 2*a^4*b^2*d^2))*(a^2 - b^2)^(1/2)*2i)/
(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b
^2*c^2*d - a^2*b*c*d^2)) + (a^2*d*atan((a^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d
^2)^(1/2)*1i - b^2*c^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*d^3*ta
n(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i + b^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d
^2)^(1/2)*2i - a^2*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a^2*c^
3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - b^2*c^2*d^3*tan(e/2 + (f*x)
/2)*(c^2 - d^2)^(1/2)*3i + b^2*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)
*1i - a*b*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*d^5*tan(e/2 + (
```

f*x)/2)*(c^2 - d^2)^(1/2)*2i + a^2*c*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i + a^2*c*d^4*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*c^4*d*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + a*b*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i + a*b*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a*b*c*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*c*d^4*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i)/(a^2*d^6 - b^2*c^6 - 2*a^2*c^2*d^4 + a^2*c^4*d^2 - b^2*c^2*d^4 + 2*b^2*c^4*d^2))*(c^2 - d^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2)) - (b^2*d*atan((a^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - b^2*c^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i + b^2*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a^2*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a^2*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - b^2*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*3i + b^2*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i - a*b*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*d^5*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i + a^2*c*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i + a^2*c*d^4*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + b^2*c^4*d*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*1i + a*b*c^2*d^3*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i + a*b*c^3*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i - a*b*c*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(3/2)*2i - a*b*c*d^4*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*2i)/(a^2*d^6 - b^2*c^6 - 2*a^2*c^2*d^4 + a^2*c^4*d^2 - b^2*c^2*d^4 + 2*b^2*c^4*d^2))*(c^2 - d^2)^(1/2)*2i)/(f*(a^3*d^3 - b^3*c^3 + a^2*b*c^3 - a*b^2*d^3 - a^3*c^2*d + b^3*c*d^2 + a*b^2*c^2*d - a^2*b*c*d^2))

3.257 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx &= \frac{2b^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)^2f} - \frac{2d(2bc^2-acd-bd^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2f} \\ &\quad + \frac{d^2\sin(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(d+c\cos(e+fx))}\end{aligned}$$

```
[Out] -2*d*(-a*c*d+2*b*c^2-b*d^2)*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(1
/2))/(c-d)^(3/2)/(c+d)^(3/2)/(-a*d+b*c)^2/f+d^2*sin(f*x+e)/(-a*d+b*c)/(c^2-
d^2)/f/(d+c*cos(f*x+e))+2*b^2*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^
(1/2))/(-a*d+b*c)^2/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4073, 3135, 3080, 2738, 214}

$$\begin{aligned}\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx &= \frac{2b^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f\sqrt{a-b}\sqrt{a+b}(bc-ad)^2} - \frac{2d(-acd+2bc^2-bd^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2} \\ &\quad + \frac{d^2\sin(e+fx)}{f\,(c^2-d^2)\,(bc-ad)(c\cos(e+fx)+d)}\end{aligned}$$

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (2*b^2*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(b*c - a*d)^2*f) - (2*d*(2*b*c^2 - a*c*d - b*d^2)*ArcTanh[(Sqrt[c
 - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/((c - d)^(3/2)*(c + d)^(3/2)*(b*c - a
*d)^2*f) + (d^2*Sin[e + f*x])/((b*c - a*d)*(c^2 - d^2)*f*(d + c*Cos[e + f*x
]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
```

```
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))^2}\,dx \\
&= \frac{d^2\sin(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(d+c\cos(e+fx))} + \frac{\int \frac{-bcd-(acd-b(c^2-d^2))\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}\,dx}{(bc-ad)\,(c^2-d^2)} \\
&= \frac{d^2\sin(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(d+c\cos(e+fx))} + \frac{b^2\int \frac{1}{b+a\cos(e+fx)}\,dx}{(bc-ad)^2} \\
&\quad + \frac{(d(acd-b(2c^2-d^2)))\int \frac{1}{d+c\cos(e+fx)}\,dx}{(bc-ad)^2\,(c^2-d^2)} \\
&= \frac{d^2\sin(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(d+c\cos(e+fx))} \\
&\quad + \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)^2 f} \\
&\quad + \frac{(2d(acd-b(2c^2-d^2)))\,\text{Subst}\left(\int \frac{1}{c+d+(-c+d)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)^2\,(c^2-d^2)\,f} \\
&= \frac{2b^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(bc-ad)^2 f} - \frac{2d(2bc^2-acd-bd^2)\,\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{(c-d)^{3/2}(c+d)^{3/2}(bc-ad)^2 f} \\
&\quad + \frac{d^2\sin(e+fx)}{(bc-ad)\,(c^2-d^2)\,f(d+c\cos(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \frac{-2b^2(c^2-d^2)^{3/2}\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(d+c\cos(e+fx)) - \sqrt{a^2-b^2}d\Big(-2(2bc^2-acd-bd^2)\,\text{arct}}{\sqrt{a^2-b^2}(c-d)(c+d)(bc-ad)^2\sqrt{c^2-d^2}f}$$

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])*(c + d*Sec[e + f*x])^2),x]
```

```
[Out] (-2*b^2*(c^2 - d^2)^(3/2)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^
2]]*(d + c*Cos[e + f*x]) - Sqrt[a^2 - b^2]*d*(-2*(2*b*c^2 - a*c*d - b*d^2)*
```

```
ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(d + c*Cos[e + f*x]) +
 d*(-(b*c) + a*d)*Sqrt[c^2 - d^2]*Sin[e + f*x]))/(Sqrt[a^2 - b^2]*(c - d)*(
c + d)*(b*c - a*d)^2*Sqrt[c^2 - d^2]*f*(d + c*Cos[e + f*x]))
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(ad-bc)^2\sqrt{(a-b)(a+b)}} - \frac{2d\left(-\frac{d(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c^2-d^2)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d-c-d\right)} - \frac{(acd-2b\,c^2+b\,d^2)\operatorname{arctanh}\left(\frac{(c-d)\tan}{\sqrt{(c+}}\right.}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}\right.}{(ad-bc)^2}}{f}$
default	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(ad-bc)^2\sqrt{(a-b)(a+b)}} - \frac{2d\left(-\frac{d(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{(c^2-d^2)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 c-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 d-c-d\right)} - \frac{(acd-2b\,c^2+b\,d^2)\operatorname{arctanh}\left(\frac{(c-d)\tan}{\sqrt{(c+}}\right.}{(c+d)(c-d)\sqrt{(c+d)(c-d)}}\right.}{(ad-bc)^2}}{f}$
risch	$\frac{2id^2\left(d\,\mathrm{e}^{i(fx+e)}+c\right)}{c(c^2-d^2)(-ad+bc)f\left(\mathrm{e}^{2i(fx+e)}c+2d\,\mathrm{e}^{i(fx+e)}+c\right)} + \frac{d^2\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-id^2+\sqrt{c^2-d^2}\,d}{\sqrt{c^2-d^2}\,c}\right)ac}{\sqrt{c^2-d^2}\,(ad-bc)^2(c+d)(c-d)f} - \frac{2d\ln\left(\mathrm{e}^{i(fx+e)}+\frac{ic^2-i}{\sqrt{}}\right.}{\sqrt{c^2-d^2}\,(ad-bc)^2}$

```
[In] int(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(2*b^2/(a*d-b*c)^2/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)
/((a-b)*(a+b))^(1/2))-2*d/(a*d-b*c)^2*(-d*(a*d-b*c)/(c^2-d^2)*tan(1/2*f*x+1
/2*e)/(tan(1/2*f*x+1/2*e)^2*c-tan(1/2*f*x+1/2*e)^2*d-c-d)-(a*c*d-2*b*c^2+b*
d^2)/(c+d)/(c-d)/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d
)*(c-d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(169) = 338$.

Time = 96.83 (sec) , antiderivative size = 2863, normalized size of antiderivative = 15.31

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2
*c*d^4)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^
2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2
*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) + (2*(a^2*b -
```

b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a^2*b - b^3)*c
^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x + e))*sqrt(c^2
- d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 - 2*sqrt(c^2
- d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2
 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^3 - a*b^2)*c^
2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))/(((a^2*b^2 -
 b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^5*d^2 + 4
*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(a^3*b - a
*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*b^2 - b^4)*c^
6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^3 + 4*(a^
3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*b - a*b^3
)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), 1/2*(2*(b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*
d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4)*cos(f*x + e))*sqrt(-a^2 + b^2)*
arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) +
 (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a
^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x +
e))*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2
- 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*c
os(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b - b^3)*c^3*d^2 - (a^
3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))
/(((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4
)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 -
 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*
b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^
4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(
a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), -1/2*(2*(2*(a^2*b - b^3)*c^
2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2*b - b^3)*d^4 + (2*(a^2*b - b^3)*c^3*d -
(a^3 - a*b^2)*c^2*d^2 - (a^2*b - b^3)*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)
arctan(-sqrt(-c^2 + d^2)(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e)))
- (b^2*c^4*d - 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d
^4)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*c
os(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2
 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - 2*((a^2*b - b^3)
*c^3*d^2 - (a^3 - a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)
*sin(f*x + e))/(((a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a
^2*b^2 + 2*b^4)*c^5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 +
b^4)*c^3*d^4 - 2*(a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x
 + e) + ((a^2*b^2 - b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b
^2 + 2*b^4)*c^4*d^3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)
*c^2*d^5 - 2*(a^3*b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f), ((b^2*c^4*d -
 2*b^2*c^2*d^3 + b^2*d^5 + (b^2*c^5 - 2*b^2*c^3*d^2 + b^2*c*d^4)*cos(f*x +
e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 -
b^2)*sin(f*x + e))) - (2*(a^2*b - b^3)*c^2*d^2 - (a^3 - a*b^2)*c*d^3 - (a^2
*b - b^3)*d^4 + (2*(a^2*b - b^3)*c^3*d - (a^3 - a*b^2)*c^2*d^2 - (a^2*b - b
^3)*c*d^3)*cos(f*x + e))*sqrt(-c^2 + d^2)*arctan(-sqrt(-c^2 + d^2)*(d*cos(f

```
*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) + ((a^2*b - b^3)*c^3*d^2 - (a^3 -
a*b^2)*c^2*d^3 - (a^2*b - b^3)*c*d^4 + (a^3 - a*b^2)*d^5)*sin(f*x + e))/(((
a^2*b^2 - b^4)*c^7 - 2*(a^3*b - a*b^3)*c^6*d + (a^4 - 3*a^2*b^2 + 2*b^4)*c^
5*d^2 + 4*(a^3*b - a*b^3)*c^4*d^3 - (2*a^4 - 3*a^2*b^2 + b^4)*c^3*d^4 - 2*(
a^3*b - a*b^3)*c^2*d^5 + (a^4 - a^2*b^2)*c*d^6)*f*cos(f*x + e) + ((a^2*b^2
- b^4)*c^6*d - 2*(a^3*b - a*b^3)*c^5*d^2 + (a^4 - 3*a^2*b^2 + 2*b^4)*c^4*d^
3 + 4*(a^3*b - a*b^3)*c^3*d^4 - (2*a^4 - 3*a^2*b^2 + b^4)*c^2*d^5 - 2*(a^3*
b - a*b^3)*c*d^6 + (a^4 - a^2*b^2)*d^7)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \int \frac{\sec\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)\left(c+d\sec\left(e+fx\right)\right)^2}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**2,x)
```

```
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))*(c + d*sec(e + f*x))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.77

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor \frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)b^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{-a^2+b^2}}+\frac{d^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(bc^3-ac^2d-bcd^2+ad^3)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-d\tan\left(\frac{1}{2}fx\right.\right.}}{f}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*
x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*b^2/((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(-a^2 + b^2)) + d^2*tan(1/2*f*x + 1/2*e)/((b*c^3 - a*
c^2*d - b*c*d^2 + a*d^3)*(c*tan(1/2*f*x + 1/2*e)^2 - d*tan(1/2*f*x + 1/2*e)
^2 - c - d)) - (2*b*c^2*d - a*c*d^2 - b*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1
/2)*sgn(2*c - 2*d) + arctan((c*tan(1/2*f*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e
))/sqrt(-c^2 + d^2)))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 - b^2*c^2*d^2 +
 2*a*b*c*d^3 - a^2*d^4)*sqrt(-c^2 + d^2)))/f
```

Mupad [B] (verification not implemented)

Time = 27.24 (sec) , antiderivative size = 20827, normalized size of antiderivative = 111.37

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))(c+d\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))*(c + d/cos(e + f*x))^2),x)
```

```
[Out] (2*d^2*tan(e/2 + (f*x)/2))/(f*(c + d)*(c + d - tan(e/2 + (f*x)/2)^2*(c - d)
)*(a*d^2 + b*c^2 - a*c*d - b*c*d)) - (d*atan(((d*((32*tan(e/2 + (f*x)/2)*(b
^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d +
3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 +
 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*
a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^
2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 -
 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2
*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a
^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*
c^2*d^3 + 2*a*b*c^3*d^2) + (d*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7
```

*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*
d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2
 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 1
1*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a
^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*
b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^
2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2
 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^
4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*
a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^
3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^
2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*
c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*
d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b
^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*
c^5*d - 3*a^2*b*c*d^5) + (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/
2)*(b*d^2 - 2*b*c^2 + a*c*d)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2
*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d
^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7
*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 +
2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*
b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3
*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*
b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*
c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*
a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d
^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b
^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 -
 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c
^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a
^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^
3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^
5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c
^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^
8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3
*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 -
6*a*b*c^5*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^
2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6
 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^
5 - 6*a*b*c^5*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*1
i)/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c
^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*
c^3*d^5 - 6*a*b*c^5*d^3) + (d*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6
 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3
*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13

*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a
^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*
a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3
 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2
*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^
2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3
d^2) - (d((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*
d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 +
 a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6
 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8
*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^
5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b
*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*
a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d
^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*
b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 + 4
*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^7*
d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5*b
^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6 +
b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 -
 b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*
b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d
^5) - (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2
 + a*c*d)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*
b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8
 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^
4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a
*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*
b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4
*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b
*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^
5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 2
0*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2
*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3
*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6
 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3
*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14
*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*
d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2
*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^
4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^8 - b^2*c^8 - 3*a^2
*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^
2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))*((
c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3
*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 +

3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))
((c + d)^3(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*1i)/(a^2*d^8 - b^2*c
^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*
d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5
d^3))/((64(b^5*d^5 - a*b^4*d^5 - b^5*c*d^4 + 2*b^5*c^4*d - 3*b^5*c^2*d^3
+ 2*b^5*c^3*d^2 + 2*a*b^4*c^2*d^3 - 5*a*b^4*c^3*d^2 - 2*a^2*b^3*c*d^4 + 2*a
^2*b^3*c^2*d^3 + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + 3*a*b^4*c*d^4 - 2*a*
b^4*c^4*d))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*
c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3
*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*
b^2*c^5*d - 3*a^2*b*c*d^5) + (d*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^
6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a
^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 +
13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6
*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 1
2*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d
^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a
^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*
c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c
^3*d^2) + (d*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^
8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6
 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^
6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 -
 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*
a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6
*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 1
6*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2
*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^
3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 +
 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^
7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5
*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6
+ b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3
 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^
2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c
*d^5) + (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c
^2 + a*c*d)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^
2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d
^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*
c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8
*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*
a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a
^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6
*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*
b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 +

20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c ^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a ^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d ^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b ^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^ 8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b ^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c* d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^8 - b^2*c^8 - 3*a ^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3* b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))* ((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^ 8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d ^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5* d^3) - (d*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^ 4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d ^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a *b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4 *b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12 *a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2 *c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) - (d*((32*(2*a *b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^ 4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c ^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a ^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6* b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5 *c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23 *a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3* d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3 *b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 3 3*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2 *d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5* b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b ^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b* c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d^5) - (32*d*tan(e/2 + (f*x)/2)*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*(2*a*b^6*c ^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4 *c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4 *a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2)*(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d))/(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)))*((c + d)^3*(c - d)^3)^(1/2)*(b*d^2 - 2*b*c^2 + a*c*d)*2i)/(f*(a^2*d^8 - b^2*c^8 - 3*a^2*c^2*d^6 + 3*a^2*c^4*d^4 - a^2*c^6*d^2 + b^2*c^2*d^6 - 3*b^2*c^4*d^4 + 3*b^2*c^6*d^2 - 2*a*b*c*d^7 + 2*a*b*c^7*d + 6*a*b*c^3*d^5 - 6*a*b*c^5*d^3)) - (b^2*atan(((b^2*(a^2 - b^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) + (b^2*(a^2 - b^2)^(1/2)*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b

^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31
*a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d
^3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b
^2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b
*c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3
*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*
b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2
*c^5*d - 3*a^2*b*c*d^5) + (32*b^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*(2*a
*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a
^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d
^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*
c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 +
4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 -
6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8
*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*
a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*
a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*
d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*
b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 -
24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c
^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a
^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d
^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*
c^9*d + 2*a^6*b*c*d^9))/((a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2
*a*b^3*c*d - 2*a^3*b*c*d)*(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*
c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4
*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2))))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a
^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))*1i)/(a^4*d^2 - b^4*c^2 + a^2*b^2*c
^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d) + (b^2*(a^2 - b^2)^(1/2)*((32
*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*
c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2
*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 -
11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4
*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d
^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d
^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c
^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4
+ 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) - (b^2*(a^2 - b^2)^(1/2)*((3
2*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^
9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 +
b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*
d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d
- 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 -
5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a
^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^

4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^
4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 +
11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d
^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b
^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 1
0*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*
c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 +
3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*
a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d^5) - (32*b^2*
tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c
*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a
^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5
+ 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^
7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^
5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9
+ 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 -
12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 -
8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d
^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b
^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 -
2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c
^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a
^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^
7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^
2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^4*d^2
- b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)*(a^2*d^5
- b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^
3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2
))))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b
*c*d))*1i)/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2
*a^3*b*c*d))/((64*(b^5*d^5 - a*b^4*d^5 - b^5*c*d^4 + 2*b^5*c^4*d - 3*b^5*c^
2*d^3 + 2*b^5*c^3*d^2 + 2*a*b^4*c^2*d^3 - 5*a*b^4*c^3*d^2 - 2*a^2*b^3*c*d^4
+ 2*a^2*b^3*c^2*d^3 + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + 3*a*b^4*c*d^4
- 2*a*b^4*c^4*d))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4
- a^3*c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d
^3 - 3*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2
- 3*a*b^2*c^5*d - 3*a^2*b*c*d^5) + (b^2*(a^2 - b^2)^(1/2)*((32*tan(e/2 + (f
x)/2)(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5
*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*
c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4
*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^
3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*
c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*
c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^
2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d

- 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) + (b^2*(a^2 - b^2)^(1/2)*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 + 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d - 3*a^2*b*c*d^5) + (32*b^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*(2*a*b^6*c^10 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 - 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 + 36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2*a^6*b*c*d^9))/((a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)*(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2))))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d) - (b^2*(a^2 - b^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(b^5*c^6 + 2*b^5*d^6 - a*b^4*c^6 - 4*a*b^4*d^6 - 2*b^5*c*d^5 - 2*b^5*c^5*d + 3*a^2*b^3*d^6 - a^3*b^2*d^6 - a^5*c^2*d^4 - 5*b^5*c^2*d^4 + 4*b^5*c^3*d^3 + 3*b^5*c^4*d^2 + 13*a*b^4*c^2*d^4 - 8*a*b^4*c^3*d^3 - 11*a*b^4*c^4*d^2 - 6*a^2*b^3*c*d^5 + 6*a^3*b^2*c*d^5 + 3*a^4*b*c^2*d^4 + 4*a^4*b*c^3*d^3 - 11*a^2*b^3*c^2*d^4 + 12*a^2*b^3*c^3*d^3 + 12*a^2*b^3*c^4*d^2 + a^3*b^2*c^2*d^4 - 12*a^3*b^2*c^3*d^3 - 4*a^3*b^2*c^4*d^2 + 4*a*b^4*c*d^5 + 2*a*b^4*c^5*d - 2*a^4*b*c*d^5))/(a^2*d^5 - b^

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2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 - a^2*c^3*d^2 + b^2*c^2*d^3 + b
^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*b*c^2*d^3 + 2*a*b*c^3*d^2) - (
b^2*(a^2 - b^2)^(1/2)*((32*(2*a*b^6*c^9 - b^7*c^9 + a^6*b*d^9 + a^7*c*d^8 +
 2*b^7*c^8*d - a^2*b^5*c^9 + a^4*b^3*d^9 - 2*a^5*b^2*d^9 - a^7*c^2*d^7 - a^
7*c^3*d^6 + a^7*c^4*d^5 + b^7*c^4*d^5 - 3*b^7*c^6*d^3 + b^7*c^7*d^2 - 4*a*b
^6*c^3*d^6 - 2*a*b^6*c^4*d^5 + 13*a*b^6*c^5*d^4 + a*b^6*c^6*d^3 - 11*a*b^6*
c^7*d^2 - 8*a^2*b^5*c^8*d - 4*a^3*b^4*c*d^8 + 5*a^3*b^4*c^8*d + 8*a^4*b^3*c
*d^8 - 3*a^5*b^2*c*d^8 - 5*a^6*b*c^2*d^7 + 7*a^6*b*c^3*d^6 + 4*a^6*b*c^4*d^
5 - 5*a^6*b*c^5*d^4 + 6*a^2*b^5*c^2*d^7 + 8*a^2*b^5*c^3*d^6 - 21*a^2*b^5*c^
4*d^5 - 16*a^2*b^5*c^5*d^4 + 23*a^2*b^5*c^6*d^3 + 9*a^2*b^5*c^7*d^2 - 12*a^
3*b^4*c^2*d^7 + 14*a^3*b^4*c^3*d^6 + 34*a^3*b^4*c^4*d^5 - 21*a^3*b^4*c^5*d^
4 - 27*a^3*b^4*c^6*d^3 + 11*a^3*b^4*c^7*d^2 - a^4*b^3*c^2*d^7 - 31*a^4*b^3*
c^3*d^6 + 4*a^4*b^3*c^4*d^5 + 33*a^4*b^3*c^5*d^4 - 4*a^4*b^3*c^6*d^3 - 10*a
^4*b^3*c^7*d^2 + 13*a^5*b^2*c^2*d^7 + 7*a^5*b^2*c^3*d^6 - 21*a^5*b^2*c^4*d^
5 - 4*a^5*b^2*c^5*d^4 + 10*a^5*b^2*c^6*d^3 + a*b^6*c^8*d - 2*a^6*b*c*d^8))/
(a^3*d^6 + b^3*c^6 + a^3*c*d^5 + b^3*c^5*d - a^3*c^2*d^4 - a^3*c^3*d^3 - b^
3*c^3*d^3 - b^3*c^4*d^2 + 3*a*b^2*c^2*d^4 + 3*a*b^2*c^3*d^3 - 3*a*b^2*c^4*d
^2 - 3*a^2*b*c^2*d^4 + 3*a^2*b*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d -
3*a^2*b*c*d^5) - (32*b^2*tan(e/2 + (f*x)/2)*(a^2 - b^2)^(1/2)*(2*a*b^6*c^10
 + 2*a^6*b*d^10 - 2*a^7*c*d^9 - 2*b^7*c^9*d - 4*a^2*b^5*c^10 + 2*a^3*b^4*c^
10 + 2*a^4*b^3*d^10 - 4*a^5*b^2*d^10 + 2*a^7*c^2*d^8 + 4*a^7*c^3*d^7 - 4*a^
7*c^4*d^6 - 2*a^7*c^5*d^5 + 2*a^7*c^6*d^4 + 2*b^7*c^4*d^6 - 2*b^7*c^5*d^5 -
 4*b^7*c^6*d^4 + 4*b^7*c^7*d^3 + 2*b^7*c^8*d^2 - 8*a*b^6*c^3*d^7 + 4*a*b^6*
c^4*d^6 + 18*a*b^6*c^5*d^5 - 6*a*b^6*c^6*d^4 - 12*a*b^6*c^7*d^3 - 6*a^2*b^5
*c^9*d - 8*a^3*b^4*c*d^9 + 14*a^3*b^4*c^9*d + 14*a^4*b^3*c*d^9 - 8*a^4*b^3*
c^9*d - 6*a^5*b^2*c*d^9 - 12*a^6*b*c^3*d^7 - 6*a^6*b*c^4*d^6 + 18*a^6*b*c^5
*d^5 + 4*a^6*b*c^6*d^4 - 8*a^6*b*c^7*d^3 + 12*a^2*b^5*c^2*d^8 + 4*a^2*b^5*c
^3*d^7 - 30*a^2*b^5*c^4*d^6 - 14*a^2*b^5*c^5*d^5 + 20*a^2*b^5*c^6*d^4 + 16*
a^2*b^5*c^7*d^3 + 2*a^2*b^5*c^8*d^2 - 16*a^3*b^4*c^2*d^8 + 20*a^3*b^4*c^3*d
^7 + 36*a^3*b^4*c^4*d^6 - 2*a^3*b^4*c^5*d^5 - 22*a^3*b^4*c^6*d^4 - 24*a^3*b
^4*c^7*d^3 - 24*a^4*b^3*c^3*d^7 - 22*a^4*b^3*c^4*d^6 - 2*a^4*b^3*c^5*d^5 +
36*a^4*b^3*c^6*d^4 + 20*a^4*b^3*c^7*d^3 - 16*a^4*b^3*c^8*d^2 + 2*a^5*b^2*c^
2*d^8 + 16*a^5*b^2*c^3*d^7 + 20*a^5*b^2*c^4*d^6 - 14*a^5*b^2*c^5*d^5 - 30*a
^5*b^2*c^6*d^4 + 4*a^5*b^2*c^7*d^3 + 12*a^5*b^2*c^8*d^2 + 2*a*b^6*c^9*d + 2
*a^6*b*c*d^9))/((a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*
d - 2*a^3*b*c*d)*(a^2*d^5 - b^2*c^5 + a^2*c*d^4 - b^2*c^4*d - a^2*c^2*d^3 -
 a^2*c^3*d^2 + b^2*c^2*d^3 + b^2*c^3*d^2 - 2*a*b*c*d^4 + 2*a*b*c^4*d - 2*a*
b*c^2*d^3 + 2*a*b*c^3*d^2))))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2*d^
2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))/(a^4*d^2 - b^4*c^2 + a^2*b^2*c^2 - a^2*b^2
*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d)))*(a^2 - b^2)^(1/2)*2i)/(f*(a^4*d^2 - b^4
*c^2 + a^2*b^2*c^2 - a^2*b^2*d^2 + 2*a*b^3*c*d - 2*a^3*b*c*d))
```

3.258 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 379

$$\begin{aligned}
&\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx \\
&= \frac{d^4(5bc-2ad)\text{arctanh}(\sin(e+fx))}{2b^3f} \\
&\quad + \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,\text{arctanh}(\sin(e+fx))}{b^5f} \\
&\quad + \frac{2(bc-ad)^5\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b^3(a+b)^{3/2}f} \\
&\quad + \frac{2(bc-ad)^4(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^5\sqrt{a+b}f} - \frac{(bc-ad)^5\sin(e+fx)}{b^4(a^2-b^2)\,f(b+a\cos(e+fx))} \\
&\quad + \frac{d^5\tan(e+fx)}{b^2f} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\tan(e+fx)}{b^4f} \\
&\quad + \frac{d^4(5bc-2ad)\sec(e+fx)\tan(e+fx)}{2b^3f} + \frac{d^5\tan^3(e+fx)}{3b^2f}
\end{aligned}$$

```
[Out] 1/2*d^4*(-2*a*d+5*b*c)*arctanh(sin(f*x+e))/b^3/f+d^2*(-4*a^3*d^3+15*a^2*b*c
*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*arctanh(sin(f*x+e))/b^5/f+2*(-a*d+b*c)^5*ar
ctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/b^3/(a+b)^(
3/2)/f-(-a*d+b*c)^5*sin(f*x+e)/b^4/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c
)^4*(4*a*d+b*c)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/b^5/f
/(a-b)^(1/2)/(a+b)^(1/2)+d^5*tan(f*x+e)/b^2/f+d^3*(3*a^2*d^2-10*a*b*c*d+10*
b^2*c^2)*tan(f*x+e)/b^4/f+1/2*d^4*(-2*a*d+5*b*c)*sec(f*x+e)*tan(f*x+e)/b^3/
f+1/3*d^5*tan(f*x+e)^3/b^2/f
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {4073, 3031, 2743, 12, 2738, 214, 3855, 3852, 8, 3853}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \frac{d^3(3a^2d^2-10abcd+10b^2c^2)\tan(e+fx)}{b^4f} - \frac{(bc-ad)^5\sin(e+fx)}{b^4f\,(a^2-b^2)\,(a\cos(e+fx)+b)} + \frac{d^2(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)\,\text{arctanh}(\sin(e+fx))}{b^5f} + \frac{2(bc-ad)^4(4ad+bc)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^5f\sqrt{a-b}\sqrt{a+b}} + \frac{d^4(5bc-2ad)\text{arctanh}(\sin(e+fx))}{2b^3f} + \frac{2(bc-ad)^5\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^3f(a-b)^{3/2}(a+b)^{3/2}} + \frac{d^4(5bc-2ad)\tan(e+fx)\sec(e+fx)}{2b^3f} + \frac{d^5\tan^3(e+fx)}{3b^2f} + \frac{d^5\tan(e+fx)}{b^2f}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^4*(5*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(2*b^3*f) + (d^2*(10*b^3*c^3 -
20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*ArcTanh[Sin[e + f*x]])/(b^5*f)
 + (2*(b*c - a*d)^5*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a
*(a - b)^(3/2)*b^3*(a + b)^(3/2)*f) + (2*(b*c - a*d)^4*(b*c + 4*a*d)*ArcTan
h[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^5*Sqrt[a +
b]*f) - ((b*c - a*d)^5*Sin[e + f*x])/(b^4*(a^2 - b^2)*f*(b + a*Cos[e + f*x]
)) + (d^5*Tan[e + f*x])/(b^2*f) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2
)*Tan[e + f*x])/(b^4*f) + (d^4*(5*b*c - 2*a*d)*Sec[e + f*x]*Tan[e + f*x])/(
2*b^3*f) + (d^5*Tan[e + f*x]^3)/(3*b^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{(d+c\cos(e+fx))^5\sec^4(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(\frac{(-bc+ad)^5}{ab^4(b+a\cos(e+fx))^2} + \frac{(-bc+ad)^4(bc+4ad)}{ab^5(b+a\cos(e+fx))} + \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\sec(e+fx)}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\sec^2(e+fx)}{b^4} + \frac{d^4(5bc-2ad)\sec^3(e+fx)}{b^3} + \frac{d^5\sec^4(e+fx)}{b^2}\right)dx$$

$$= \frac{d^5\int\sec^4(e+fx)\,dx}{b^2} + \frac{(d^4(5bc-2ad))\int\sec^3(e+fx)\,dx}{b^3} - \frac{(bc-ad)^5\int\frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab^4} + \frac{((bc-ad)^4(bc+4ad))\int\frac{1}{b+a\cos(e+fx)}\,dx}{ab^5} + \frac{(d^3(10b^2c^2-10abcd+3a^2d^2))\int\sec^2(e+fx)\,dx}{b^4} + \frac{(d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3))\int\sec(e+fx)\,dx}{b^5}$$

$$= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\operatorname{arctanh}(\sin(e+fx))}{b^5f} - \frac{(bc-ad)^5\sin(e+fx)}{b^4(a^2-b^2)f(b+a\cos(e+fx))} + \frac{d^4(5bc-2ad)\sec(e+fx)\tan(e+fx)}{2b^3f} + \frac{(d^4(5bc-2ad))\int\sec(e+fx)\,dx}{2b^3} + \frac{(bc-ad)^5\int\frac{b}{b+a\cos(e+fx)}\,dx}{ab^4(a^2-b^2)} - \frac{d^5\text{Subst}\left(\int(1+x^2)\,dx,x,-\tan(e+fx)\right)}{b^2f} + \frac{(2(bc-ad)^4(bc+4ad))\text{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{ab^5f} - \frac{(d^3(10b^2c^2-10abcd+3a^2d^2))\text{Subst}(\int 1\,dx,x,-\tan(e+fx))}{b^4f}$$

$$
\begin{aligned}
&=\frac{d^4(5bc-2ad)\operatorname{arctanh}(\sin(e+fx))}{2b^3f} \\
&+\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\operatorname{arctanh}(\sin(e+fx))}{b^5f} \\
&+\frac{2(bc-ad)^4(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^5\sqrt{a+b}f} \\
&-\frac{(bc-ad)^5\sin(e+fx)}{b^4(a^2-b^2)f(b+a\cos(e+fx))}+\frac{d^5\tan(e+fx)}{b^2f} \\
&+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\tan(e+fx)}{b^4f} \\
&+\frac{d^4(5bc-2ad)\sec(e+fx)\tan(e+fx)}{2b^3f} \\
&+\frac{d^5\tan^3(e+fx)}{3b^2f}+\frac{(bc-ad)^5\int\frac{1}{b+a\cos(e+fx)}\,dx}{ab^3(a^2-b^2)}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{d^4(5bc-2ad)\operatorname{arctanh}(\sin(e+fx))}{2b^3f} \\
&+\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\operatorname{arctanh}(\sin(e+fx))}{b^5f} \\
&+\frac{2(bc-ad)^4(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^5\sqrt{a+b}f} \\
&-\frac{(bc-ad)^5\sin(e+fx)}{b^4(a^2-b^2)f(b+a\cos(e+fx))}+\frac{d^5\tan(e+fx)}{b^2f} \\
&+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\tan(e+fx)}{b^4f} \\
&+\frac{d^4(5bc-2ad)\sec(e+fx)\tan(e+fx)}{2b^3f}+\frac{d^5\tan^3(e+fx)}{3b^2f} \\
&+\frac{(2(bc-ad)^5)\operatorname{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{ab^3(a^2-b^2)f}
\end{aligned}
$$

$$= \frac{d^4(5bc-2ad)\text{arctanh}(\sin(e+fx))}{2b^3f} + \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,\text{arctanh}(\sin(e+fx))}{b^5f} + \frac{2(bc-ad)^5\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b^3(a+b)^{3/2}f} + \frac{2(bc-ad)^4(bc+4ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^5\sqrt{a+b}f} - \frac{(bc-ad)^5\sin(e+fx)}{b^4(a^2-b^2)f(b+a\cos(e+fx))} + \frac{d^5\tan(e+fx)}{b^2f} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\tan(e+fx)}{b^4f} + \frac{d^4(5bc-2ad)\sec(e+fx)\tan(e+fx)}{2b^3f} + \frac{d^5\tan^3(e+fx)}{3b^2f}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 784 vs. 2(379) = 758.

Time = 10.09 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.07

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \frac{(b+a\cos(e+fx))(c+d\sec(e+fx))^5\left(-\frac{24(bc-ad)^4(abc+4a^2d-5b^2d)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)\cos^3(e+fx)(b+a\cos}{(a^2-b^2)^{3/2}}\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^5)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^5*((-24*(b*c - a*d)^4*(a*b*c + 4
*a^2*d - 5*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]]*Cos[
e + f*x]^3*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) + 6*d^2*(-30*a^2*b*c*d^2
 + 8*a^3*d^3 - 5*b^3*c*(4*c^2 + d^2) + 2*a*b^2*d*(20*c^2 + d^2))*Cos[e + f*
x]^3*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 6*d^2*
(30*a^2*b*c*d^2 - 8*a^3*d^3 + 5*b^3*c*(4*c^2 + d^2) - 2*a*b^2*d*(20*c^2 + d
^2))*Cos[e + f*x]^3*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*
x)/2]] + (b*(-60*a^2*b^3*c^2*d^3 + 60*b^5*c^2*d^3 + 45*a^3*b^2*c*d^4 - 45*a
*b^4*c*d^4 - 12*a^4*b*d^5 + 4*a^2*b^3*d^5 + 8*b^5*d^5 + (135*a^4*b*c*d^4 -
36*a^5*d^5 + 30*a^2*b^3*c*d^2*(3*c^2 - 4*d^2) + a^3*b^2*d^3*(-180*c^2 + 29*
d^2) + a*b^4*d*(-45*c^4 + 90*c^2*d^2 - 2*d^4) + b^5*(9*c^5 + 30*c*d^4))*Cos
[e + f*x] + b*(-a^2 + b^2)*d^3*(-45*a*b*c*d + 12*a^2*d^2 + 4*b^2*(15*c^2 +
d^2))*Cos[2*(e + f*x)] + 3*b^5*c^5*Cos[3*(e + f*x)] - 15*a*b^4*c^4*d*Cos[3*
```

```
(e + f*x)] + 30*a^2*b^3*c^3*d^2*Cos[3*(e + f*x)] - 60*a^3*b^2*c^2*d^3*Cos[3
*(e + f*x)] + 30*a*b^4*c^2*d^3*Cos[3*(e + f*x)] + 45*a^4*b*c*d^4*Cos[3*(e +
 f*x)] - 30*a^2*b^3*c*d^4*Cos[3*(e + f*x)] - 12*a^5*d^5*Cos[3*(e + f*x)] +
7*a^3*b^2*d^5*Cos[3*(e + f*x)] + 2*a*b^4*d^5*Cos[3*(e + f*x)])*Sin[e + f*x]
)/(-a^2 + b^2)))/(12*b^5*f*(d + c*Cos[e + f*x])^5*(a + b*Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 689, normalized size of antiderivative = 1.82

method	result
derivativedivides	$-\frac{d^5}{3b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{d^2\left(8a^3d^3-30a^2bc\,d^2+40a\,b^2c^2d+2a\,b^2d^3-20b^3c^3-5c\,d^2b^3\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^5}-\frac{d^3\left(6a^2d^2-20abcd+\right.}{2b^4\left(\right.}$
default	$-\frac{d^5}{3b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{d^2\left(8a^3d^3-30a^2bc\,d^2+40a\,b^2c^2d+2a\,b^2d^3-20b^3c^3-5c\,d^2b^3\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^5}-\frac{d^3\left(6a^2d^2-20abcd+\right.}{2b^4\left(\right.}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-1/3*d^5/b^2/(tan(1/2*f*x+1/2*e)+1)^3-1/2*d^2*(8*a^3*d^3-30*a^2*b*c*d^
2+40*a*b^2*c^2*d+2*a*b^2*d^3-20*b^3*c^3-5*b^3*c*d^2)/b^5*ln(tan(1/2*f*x+1/2
*e)+1)-1/2*d^3*(6*a^2*d^2-20*a*b*c*d+2*a*b*d^2+20*b^2*c^2-5*b^2*c*d+2*b^2*d
^2)/b^4/(tan(1/2*f*x+1/2*e)+1)+1/2*d^4*(2*a*d-5*b*c+b*d)/b^3/(tan(1/2*f*x+1
/2*e)+1)^2-1/3*d^5/b^2/(tan(1/2*f*x+1/2*e)-1)^3+1/2*d^2*(8*a^3*d^3-30*a^2*b
*c*d^2+40*a*b^2*c^2*d+2*a*b^2*d^3-20*b^3*c^3-5*b^3*c*d^2)/b^5*ln(tan(1/2*f*
x+1/2*e)-1)-1/2*d^3*(6*a^2*d^2-20*a*b*c*d+2*a*b*d^2+20*b^2*c^2-5*b^2*c*d+2*
b^2*d^2)/b^4/(tan(1/2*f*x+1/2*e)-1)-1/2*d^4*(2*a*d-5*b*c+b*d)/b^3/(tan(1/2*
f*x+1/2*e)-1)^2-2/b^5*(b*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b
^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x
+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(4*a^6*d^5-15*a^5*b*c*d^4+20*a^4*b^
2*c^2*d^3-5*a^4*b^2*d^5-10*a^3*b^3*c^3*d^2+20*a^3*b^3*c*d^4-30*a^2*b^4*c^2*
d^3+a*b^5*c^5+20*a*b^5*c^3*d^2-5*b^6*c^4*d)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)
*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a-b)*(a+b))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c+d\sec(e+fx))^5\sec(e+fx)}{(a+b\sec(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**5/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**5*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(355) = 710$.

Time = 0.45 (sec) , antiderivative size = 857, normalized size of antiderivative = 2.26

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^5/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*(a*b^5*c^5 - 5*b^6*c^4*d - 10*a^3*b^3*c^3*d^2 + 20*a*b^5*c^3*d^2 +
 20*a^4*b^2*c^2*d^3 - 30*a^2*b^4*c^2*d^3 - 15*a^5*b*c*d^4 + 20*a^3*b^3*c*d^
4 + 4*a^6*d^5 - 5*a^4*b^2*d^5)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 +
 b^2)))/((a^2*b^5 - b^7)*sqrt(-a^2 + b^2)) - 12*(b^5*c^5*tan(1/2*f*x + 1/2*
e) - 5*a*b^4*c^4*d*tan(1/2*f*x + 1/2*e) + 10*a^2*b^3*c^3*d^2*tan(1/2*f*x +
1/2*e) - 10*a^3*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) + 5*a^4*b*c*d^4*tan(1/2*f*
x + 1/2*e) - a^5*d^5*tan(1/2*f*x + 1/2*e))/((a^2*b^4 - b^6)*(a*tan(1/2*f*x
+ 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)) - 3*(20*b^3*c^3*d^2 - 40*a*
b^2*c^2*d^3 + 30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*log(a
bs(tan(1/2*f*x + 1/2*e) + 1))/b^5 + 3*(20*b^3*c^3*d^2 - 40*a*b^2*c^2*d^3 +
30*a^2*b*c*d^4 + 5*b^3*c*d^4 - 8*a^3*d^5 - 2*a*b^2*d^5)*log(abs(tan(1/2*f*x
 + 1/2*e) - 1))/b^5 + 2*(60*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^5 - 60*a*b*c*d
^4*tan(1/2*f*x + 1/2*e)^5 - 15*b^2*c*d^4*tan(1/2*f*x + 1/2*e)^5 + 18*a^2*d^
5*tan(1/2*f*x + 1/2*e)^5 + 6*a*b*d^5*tan(1/2*f*x + 1/2*e)^5 + 6*b^2*d^5*tan
(1/2*f*x + 1/2*e)^5 - 120*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 120*a*b*c*d^
4*tan(1/2*f*x + 1/2*e)^3 - 36*a^2*d^5*tan(1/2*f*x + 1/2*e)^3 - 4*b^2*d^5*ta
n(1/2*f*x + 1/2*e)^3 + 60*b^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 60*a*b*c*d^4*t
an(1/2*f*x + 1/2*e) + 15*b^2*c*d^4*tan(1/2*f*x + 1/2*e) + 18*a^2*d^5*tan(1/
2*f*x + 1/2*e) - 6*a*b*d^5*tan(1/2*f*x + 1/2*e) + 6*b^2*d^5*tan(1/2*f*x + 1
/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*b^4))/f
```

Mupad [B] (verification not implemented)

Time = 28.86 (sec) , antiderivative size = 17256, normalized size of antiderivative = 45.53

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^5}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d/cos(e + f*x))^5/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (atan(((((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^1
0*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^
10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 -
 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4
+ 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^
4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*
a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9
+ 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3
*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 +
 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 16
0*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*
b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7
```

*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2 + (f*x)/2)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*1i)/b^5 + (((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a

*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (((8
*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*
c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 -
16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b
^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 2
0*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^
4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^
16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*
d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80
*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12)
 - (8*tan(e/2 + (f*x)/2)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*
c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^
13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9
- a^3*b^8)))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 1
0*c^3*d^2) - 15*a^2*b*c*d^4))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5
- b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*1i)/b^5)/((16*(256*a^14*
d^15 - 128*a^13*b*d^15 + 20*a^6*b^8*d^15 - 20*a^7*b^7*d^15 + 124*a^8*b^6*d^
15 - 24*a^9*b^5*d^15 + 48*a^10*b^4*d^15 + 192*a^11*b^3*d^15 - 448*a^12*b^2*
d^15 + 125*b^14*c^6*d^9 + 1000*b^14*c^8*d^7 - 250*b^14*c^9*d^6 + 2000*b^14*
c^10*d^5 - 1000*b^14*c^11*d^4 - 600*a*b^13*c^5*d^10 - 125*a*b^13*c^6*d^9 -
6425*a*b^13*c^7*d^8 + 1100*a*b^13*c^8*d^7 - 16200*a*b^13*c^9*d^6 + 8100*a*b
^13*c^10*d^5 - 400*a*b^13*c^11*d^4 + 400*a*b^13*c^12*d^3 - 180*a^5*b^9*c*d^
14 + 180*a^6*b^8*c*d^14 - 1320*a^7*b^7*c*d^14 + 270*a^8*b^6*c*d^14 - 900*a^
9*b^5*c*d^14 - 2160*a^10*b^4*c*d^14 + 5280*a^11*b^3*c*d^14 + 1440*a^12*b^2*
c*d^14 + 1170*a^2*b^12*c^4*d^11 + 600*a^2*b^12*c^5*d^10 + 17795*a^2*b^12*c^
6*d^9 - 1375*a^2*b^12*c^7*d^8 + 57480*a^2*b^12*c^8*d^7 - 29740*a^2*b^12*c^9
*d^6 - 400*a^2*b^12*c^10*d^5 - 2010*a^2*b^12*c^11*d^4 - 40*a^2*b^12*c^13*d^
2 - 1180*a^3*b^11*c^3*d^12 - 1170*a^3*b^11*c^4*d^11 - 27754*a^3*b^11*c^5*d^
10 - 995*a^3*b^11*c^6*d^9 - 117635*a^3*b^11*c^7*d^8 + 66680*a^3*b^11*c^8*d^
7 + 17400*a^3*b^11*c^9*d^6 + 2604*a^3*b^11*c^10*d^5 + 400*a^3*b^11*c^11*d^4
 + 80*a^3*b^11*c^12*d^3 + 645*a^4*b^10*c^2*d^13 + 1180*a^4*b^10*c^3*d^12 +
26690*a^4*b^10*c^4*d^11 + 4654*a^4*b^10*c^5*d^10 + 153580*a^4*b^10*c^6*d^9
- 103805*a^4*b^10*c^7*d^8 - 79760*a^4*b^10*c^8*d^7 + 5840*a^4*b^10*c^9*d^6
- 1600*a^4*b^10*c^10*d^5 + 340*a^4*b^10*c^11*d^4 - 645*a^5*b^9*c^2*d^13 - 1
6245*a^5*b^9*c^3*d^12 - 5690*a^5*b^9*c^4*d^11 - 133278*a^5*b^9*c^5*d^10 + 1
19980*a^5*b^9*c^6*d^9 + 188520*a^5*b^9*c^7*d^8 - 28880*a^5*b^9*c^8*d^7 - 12
00*a^5*b^9*c^9*d^6 - 1584*a^5*b^9*c^10*d^5 + 6135*a^6*b^8*c^2*d^13 + 3645*a
^6*b^8*c^3*d^12 + 77460*a^6*b^8*c^4*d^11 - 105562*a^6*b^8*c^5*d^10 - 279820
*a^6*b^8*c^6*d^9 + 57980*a^6*b^8*c^7*d^8 + 21280*a^6*b^8*c^8*d^7 + 2800*a^6
*b^8*c^9*d^6 - 1335*a^7*b^7*c^2*d^13 - 29515*a^7*b^7*c^3*d^12 + 69980*a^7*b
^7*c^4*d^11 + 279768*a^7*b^7*c^5*d^10 - 74940*a^7*b^7*c^6*d^9 - 64460*a^7*b
^7*c^7*d^8 - 2720*a^7*b^7*c^8*d^7 + 6960*a^8*b^6*c^2*d^13 - 33645*a^8*b^6*c
^3*d^12 - 192920*a^8*b^6*c^4*d^11 + 69104*a^8*b^6*c^5*d^10 + 108320*a^8*b^6
*c^6*d^9 + 1540*a^8*b^6*c^7*d^8 + 10980*a^9*b^5*c^2*d^13 + 91160*a^9*b^5*c^
3*d^12 - 46520*a^9*b^5*c^4*d^11 - 118136*a^9*b^5*c^5*d^10 - 480*a^9*b^5*c^6

*d^9 - 28380*a^10*b^4*c^2*d^13 + 22430*a^10*b^4*c^3*d^12 + 87600*a^10*b^4*c
^4*d^11 + 64*a^10*b^4*c^5*d^10 - 7320*a^11*b^3*c^2*d^13 - 44220*a^11*b^3*c^
3*d^12 + 14640*a^12*b^2*c^2*d^13 - 2880*a^13*b*c*d^14))/(a*b^14 + b^15 - a^
2*b^13 - a^3*b^12) + (((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^
10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 -
48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*
a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400
*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 -
400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11
*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*
a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9
+ 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*
b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^1
0*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d
^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 1
60*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4
*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8
*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6
+ 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^
6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6
*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d
^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5
440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^
9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d -
960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^
5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b
^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d
^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 +
80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^
4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b
^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 +
40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4
*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2
*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2
+ (f*x)/2)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10
*c^3*d^2) - 15*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b
^12 + 8*a^5*b^11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*
(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) -
15*a^2*b*c*d^4))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d
^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5 - (((8*tan(e/2 + (f*x)/2)*(128*a
^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*
d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10
+ 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8
+ 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^
8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b

^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d ^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2 *b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^1 0*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2* d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^ 4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8* c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160* a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^ 6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d ^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a ^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c *d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3* b^8) - (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4* b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10 *d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^1 6*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^ 4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^ 3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c ^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (8*tan(e/2 + (f*x)/2)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/(b^5*(a*b^10 + b^11 - a^2*b^9 - a^3*b^8)))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c *d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5)*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2*b*c*d^4))/b^5))*(b^2*(a*d^5 + 20*a*c^2*d^3) + 4*a^3*d^5 - b^3*((5*c*d^4)/2 + 10*c^3*d^2) - 15*a^2 *b*c*d^4)*2i)/(b^5*f) - ((tan(e/2 + (f*x)/2)^5*(18*b^5*c^5 - 72*a^5*d^5 + 2 *b^5*d^5 + 16*a*b^4*d^5 + 12*a^4*b*d^5 - 15*b^5*c*d^4 - 14*a^2*b^3*d^5 + 38 *a^3*b^2*d^5 - 60*b^5*c^2*d^3 + 180*a*b^4*c^2*d^3 - 165*a^2*b^3*c*d^4 - 45* a^3*b^2*c*d^4 + 60*a^2*b^3*c^2*d^3 + 180*a^2*b^3*c^3*d^2 - 360*a^3*b^2*c^2* d^3 + 45*a*b^4*c*d^4 - 90*a*b^4*c^4*d + 270*a^4*b*c*d^4))/(3*(a*b^4 - b^5)* (a + b)) - (tan(e/2 + (f*x)/2)^7*(2*b^5*c^5 - 8*a^5*d^5 - 2*b^5*d^5 + 4*a^4 *b*d^5 + 5*b^5*c*d^4 - 2*a^2*b^3*d^5 + 6*a^3*b^2*d^5 - 20*b^5*c^2*d^3 + 20* a*b^4*c^2*d^3 - 25*a^2*b^3*c*d^4 - 15*a^3*b^2*c*d^4 + 20*a^2*b^3*c^2*d^3 + 20*a^2*b^3*c^3*d^2 - 40*a^3*b^2*c^2*d^3 + 15*a*b^4*c*d^4 - 10*a*b^4*c^4*d + 30*a^4*b*c*d^4))/((a*b^4 - b^5)*(a + b)) + (tan(e/2 + (f*x)/2)*(2*b^5*c^5 - 8*a^5*d^5 + 2*b^5*d^5 - 4*a^4*b*d^5 + 5*b^5*c*d^4 + 2*a^2*b^3*d^5 + 6*a^3 *b^2*d^5 + 20*b^5*c^2*d^3 + 20*a*b^4*c^2*d^3 - 25*a^2*b^3*c*d^4 + 15*a^3*b^ 2*c*d^4 - 20*a^2*b^3*c^2*d^3 + 20*a^2*b^3*c^3*d^2 - 40*a^3*b^2*c^2*d^3 - 15

*a*b^4*c*d^4 - 10*a*b^4*c^4*d + 30*a^4*b*c*d^4))/((a*b^4 - b^5)*(a + b)) +
(tan(e/2 + (f*x)/2)^3*(72*a^5*d^5 - 18*b^5*c^5 + 2*b^5*d^5 - 16*a*b^4*d^5 +
12*a^4*b*d^5 + 15*b^5*c*d^4 - 14*a^2*b^3*d^5 - 38*a^3*b^2*d^5 - 60*b^5*c^2
*d^3 - 180*a*b^4*c^2*d^3 + 165*a^2*b^3*c*d^4 - 45*a^3*b^2*c*d^4 + 60*a^2*b^
3*c^2*d^3 - 180*a^2*b^3*c^3*d^2 + 360*a^3*b^2*c^2*d^3 + 45*a*b^4*c*d^4 + 90
*a*b^4*c^4*d - 270*a^4*b*c*d^4))/(3*b^4*(a + b)*(a - b)))/(f*(a + b + tan(e
/2 + (f*x)/2)^8*(a - b) - tan(e/2 + (f*x)/2)^2*(4*a + 2*b) - tan(e/2 + (f*x
)/2)^6*(4*a - 2*b) + 6*a*tan(e/2 + (f*x)/2)^4)) + (atan((((a*d - b*c)^4*((8
*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*
a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6
*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b
^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*
c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 160
0*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*
d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b
^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 96
0*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b
^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*
c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6
 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055
*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b
^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d
^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 -
800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*
b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^
6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6
 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 560
0*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10
*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^
10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^1
8*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5
+ 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*
a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a
*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 -
 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c
*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120
*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^
13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/
(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^4
((a + b)^3(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c)*(8*a*b^15 - 8*a^2*
b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/((a*b^10 + b^1
1 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(a*d - b*
c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2
*b^9 + 3*a^4*b^7 - a^6*b^5))*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d
 + a*b*c)*1i)/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5) + ((a*d - b*c)^4*((8

*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/((a*b^10 + b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c)*1i)/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))/((16*(256*a^14*d^15 - 128*a^13*b*d^15 + 20*a^6*b^8*d^15 - 20*a^7*b^7*d^15 + 124*a^8*b^6*d^15 - 24*a^9*b^5*d^15 + 48*a^10*b^4*d^15 + 192*a^11*b^3*d^15 - 448*a^12*b^2*d^15 + 125*b^14*c^6*d^9 + 1000*b^14*c^8*d^7 - 250*b^14*c^9*d^6 + 2000*b^14*c^10*d^5 - 1000*b^14*c^11*d^4 - 600*a*b^13*c^5*d^10 - 125*a*b^13*c^6*d^9 - 6425*a*b^13*c^7*d^8 + 1100*a*b^13*c^8*d^7 - 16200*a*b^13*c^9*d^6 + 8100*a*b^13*c^10*d^5 - 400*a*b^13*c^11*d^4 + 400*a*b^13*c^12*d^3 - 180*a^5*b^9*c*d^14 + 180*a^6*b^8*c*d^14 - 1320*a^7*b^7*c*d^14 + 270*a^8*b^6*c*d^14 - 900*a^9*b^5*c*d^14 - 2160*a^10*b^4*c*d^14 + 5280*a^11*b^3*c*d^14 + 1440*a^12*b^2*c*d^

14 + 1170*a^2*b^12*c^4*d^11 + 600*a^2*b^12*c^5*d^10 + 17795*a^2*b^12*c^6*d^
9 - 1375*a^2*b^12*c^7*d^8 + 57480*a^2*b^12*c^8*d^7 - 29740*a^2*b^12*c^9*d^6
 - 400*a^2*b^12*c^10*d^5 - 2010*a^2*b^12*c^11*d^4 - 40*a^2*b^12*c^13*d^2 -
1180*a^3*b^11*c^3*d^12 - 1170*a^3*b^11*c^4*d^11 - 27754*a^3*b^11*c^5*d^10 -
 995*a^3*b^11*c^6*d^9 - 117635*a^3*b^11*c^7*d^8 + 66680*a^3*b^11*c^8*d^7 +
17400*a^3*b^11*c^9*d^6 + 2604*a^3*b^11*c^10*d^5 + 400*a^3*b^11*c^11*d^4 + 8
0*a^3*b^11*c^12*d^3 + 645*a^4*b^10*c^2*d^13 + 1180*a^4*b^10*c^3*d^12 + 2669
0*a^4*b^10*c^4*d^11 + 4654*a^4*b^10*c^5*d^10 + 153580*a^4*b^10*c^6*d^9 - 10
3805*a^4*b^10*c^7*d^8 - 79760*a^4*b^10*c^8*d^7 + 5840*a^4*b^10*c^9*d^6 - 16
00*a^4*b^10*c^10*d^5 + 340*a^4*b^10*c^11*d^4 - 645*a^5*b^9*c^2*d^13 - 16245
*a^5*b^9*c^3*d^12 - 5690*a^5*b^9*c^4*d^11 - 133278*a^5*b^9*c^5*d^10 + 11998
0*a^5*b^9*c^6*d^9 + 188520*a^5*b^9*c^7*d^8 - 28880*a^5*b^9*c^8*d^7 - 1200*a
^5*b^9*c^9*d^6 - 1584*a^5*b^9*c^10*d^5 + 6135*a^6*b^8*c^2*d^13 + 3645*a^6*b
^8*c^3*d^12 + 77460*a^6*b^8*c^4*d^11 - 105562*a^6*b^8*c^5*d^10 - 279820*a^6
*b^8*c^6*d^9 + 57980*a^6*b^8*c^7*d^8 + 21280*a^6*b^8*c^8*d^7 + 2800*a^6*b^8
*c^9*d^6 - 1335*a^7*b^7*c^2*d^13 - 29515*a^7*b^7*c^3*d^12 + 69980*a^7*b^7*c
^4*d^11 + 279768*a^7*b^7*c^5*d^10 - 74940*a^7*b^7*c^6*d^9 - 64460*a^7*b^7*c
^7*d^8 - 2720*a^7*b^7*c^8*d^7 + 6960*a^8*b^6*c^2*d^13 - 33645*a^8*b^6*c^3*d
^12 - 192920*a^8*b^6*c^4*d^11 + 69104*a^8*b^6*c^5*d^10 + 108320*a^8*b^6*c^6
*d^9 + 1540*a^8*b^6*c^7*d^8 + 10980*a^9*b^5*c^2*d^13 + 91160*a^9*b^5*c^3*d^
12 - 46520*a^9*b^5*c^4*d^11 - 118136*a^9*b^5*c^5*d^10 - 480*a^9*b^5*c^6*d^9
 - 28380*a^10*b^4*c^2*d^13 + 22430*a^10*b^4*c^3*d^12 + 87600*a^10*b^4*c^4*d
^11 + 64*a^10*b^4*c^5*d^10 - 7320*a^11*b^3*c^2*d^13 - 44220*a^11*b^3*c^3*d^
12 + 14640*a^12*b^2*c^2*d^13 - 2880*a^13*b*c*d^14))/(a*b^14 + b^15 - a^2*b^
13 - a^3*b^12) + ((a*d - b*c)^4*((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128
*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*
b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d
^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4
*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11
*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 -
800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*
d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*
b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8
 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 24
00*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3
*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c
^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7
 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a
^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^
7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^
3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 32
00*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7
*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c
^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8
 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^

11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) + (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 + 200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) + (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^11 - 8*a^6*b^10))/((a*b^10 + b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5) - ((a*d - b*c)^4*((8*tan(e/2 + (f*x)/2)*(128*a^12*d^10 - 128*a^11*b*d^10 + 4*a^2*b^10*c^10 + 4*a^2*b^10*d^10 - 8*a^3*b^9*d^10 + 28*a^4*b^8*d^10 - 48*a^5*b^7*d^10 + 28*a^6*b^6*d^10 - 8*a^7*b^5*d^10 + 8*a^8*b^4*d^10 + 192*a^9*b^3*d^10 - 192*a^10*b^2*d^10 + 25*b^12*c^2*d^8 + 200*b^12*c^4*d^6 + 400*b^12*c^6*d^4 + 100*b^12*c^8*d^2 - 50*a*b^11*c^2*d^8 - 480*a*b^11*c^3*d^7 - 400*a*b^11*c^4*d^6 - 1600*a*b^11*c^5*d^5 - 800*a*b^11*c^6*d^4 - 800*a*b^11*c^7*d^3 + 40*a^2*b^10*c*d^9 - 180*a^3*b^9*c*d^9 + 320*a^4*b^8*c*d^9 - 260*a^5*b^7*c*d^9 + 200*a^6*b^6*c*d^9 - 140*a^7*b^5*c*d^9 - 1520*a^8*b^4*c*d^9 + 1520*a^9*b^3*c*d^9 + 960*a^10*b^2*c*d^9 + 435*a^2*b^10*c^2*d^8 + 960*a^2*b^10*c^3*d^7 + 2600*a^2*b^10*c^4*d^6 + 3200*a^2*b^10*c^5*d^5 + 2400*a^2*b^10*c^6*d^4 + 160*a^2*b^10*c^8*d^2 - 820*a^3*b^9*c^2*d^8 - 2240*a^3*b^9*c^3*d^7 - 4800*a^3*b^9*c^4*d^6 - 4000*a^3*b^9*c^5*d^5 + 1600*a^3*b^9*c^6*d^4 + 160*a^3*b^9*c^7*d^3 + 1055*a^4*b^8*c^2*d^8 + 3520*a^4*b^8*c^3*d^7 + 4000*a^4*b^8*c^4*d^6 - 6400*a^4*b^8*c^5*d^5 - 2640*a^4*b^8*c^6*d^4 - 80*a^4*b^8*c^8*d^2 - 1290*a^5*b^7*c^2*d^8 - 2400*a^5*b^7*c^3*d^7 + 10800*a^5*b^7*c^4*d^6 + 7760*a^5*b^7*c^5*d^5 - 800*a^5*b^7*c^6*d^4 + 160*a^5*b^7*c^7*d^3 + 825*a^6*b^6*c^2*d^8 - 9920*a^6*b^6*c^3*d^7 - 11560*a^6*b^6*c^4*d^6 + 3200*a^6*b^6*c^5*d^5 + 680*a^6*b^6*c^6*d^4 + 5240*a^7*b^5*c^2*d^8 + 10080*a^7*b^5*c^3*d^7 - 5600*a^7*b^5*c^4*d^6 - 3168*a^7*b^5*c^5*d^5 - 5240*a^8*b^4*c^2*d^8 + 5440*a^8*b^4*c^3*d^7 + 5600*a^8*b^4*c^4*d^6 - 3080*a^9*b^3*c^2*d^8 - 5440*a^9*b^3*c^3*d^7 + 3080*a^10*b^2*c^2*d^8 - 20*a*b^11*c*d^9 - 40*a*b^11*c^9*d - 960*a^11*b*c*d^9))/(a*b^10 + b^11 - a^2*b^9 - a^3*b^8) - (((8*(4*a*b^17*c^5 + 4*a*b^17*d^5 - 10*b^18*c*d^4 - 20*b^18*c^4*d - 4*a^2*b^16*c^5 - 4*a^3*b^15*c^5 + 4*a^4*b^14*c^5 + 4*a^3*b^15*d^5 - 20*a^4*b^14*d^5 - 16*a^5*b^13*d^5 + 36*a^6*b^12*d^5 + 8*a^7*b^11*d^5 - 16*a^8*b^10*d^5 - 40*b^18*c^3*d^2 + 80*a*b^17*c^2*d^3 + 80*a*b^17*c^3*d^2 - 30*a^2*b^16*c*d^4 + 20*a^2*b^16*c^4*d + 80*a^3*b^15*c*d^4 - 20*a^3*b^15*c^4*d + 70*a^4*b^14*c*d^4 - 140*a^5*b^13*c*d^4 - 30*a^6*b^12*c*d^4 + 60*a^7*b^11*c*d^4 - 120*a^2*b^16*c^2*d^3 + 40*a^2*b^16*c^3*d^2 - 120*a^3*b^15*c^2*d^3 - 120*a^3*b^15*c^3*d^2 +

```
200*a^4*b^14*c^2*d^3 + 40*a^5*b^13*c^2*d^3 + 40*a^5*b^13*c^3*d^2 - 80*a^6*b
^12*c^2*d^3 + 20*a*b^17*c^4*d))/(a*b^14 + b^15 - a^2*b^13 - a^3*b^12) - (8*
tan(e/2 + (f*x)/2)*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b
^2*d + a*b*c)*(8*a*b^15 - 8*a^2*b^14 - 16*a^3*b^13 + 16*a^4*b^12 + 8*a^5*b^
11 - 8*a^6*b^10))/((a*b^10 + b^11 - a^2*b^9 - a^3*b^8)*(b^11 - 3*a^2*b^9 +
3*a^4*b^7 - a^6*b^5)))*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d -
 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))*((a + b)^3*(a
- b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a*b*c))/(b^11 - 3*a^2*b^9 + 3*a^4*b^7 -
a^6*b^5)))*(a*d - b*c)^4*((a + b)^3*(a - b)^3)^(1/2)*(4*a^2*d - 5*b^2*d + a
*b*c)*2i)/(f*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5))
```

3.259 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 297

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \frac{d^4\text{arctanh}(\sin(e+fx))}{2b^2f} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,\text{arctanh}(\sin(e+fx))}{b^4f} + \frac{2(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b^2(a+b)^{3/2}f} + \frac{2(bc-ad)^3(bc+3ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^4\sqrt{a+b}f} - \frac{(bc-ad)^4\sin(e+fx)}{b^3(a^2-b^2)\,f(b+a\cos(e+fx))} + \frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f} + \frac{d^4\sec(e+fx)\tan(e+fx)}{2b^2f}$$

```
[Out] 1/2*d^4*arctanh(sin(f*x+e))/b^2/f+d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*arcta
nh(sin(f*x+e))/b^4/f+2*(-a*d+b*c)^4*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/
(a+b)^(1/2))/a/(a-b)^(3/2)/b^2/(a+b)^(3/2)/f-(-a*d+b*c)^4*sin(f*x+e)/b^3/(a
^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c)^3*(3*a*d+b*c)*arctanh((a-b)^(1/2)*t
an(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/b^4/f/(a-b)^(1/2)/(a+b)^(1/2)+2*d^3*(-a*d+
2*b*c)*tan(f*x+e)/b^3/f+1/2*d^4*sec(f*x+e)*tan(f*x+e)/b^2/f
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {4073, 3031, 2743, 12, 2738, 214, 3855, 3852, 8, 3853}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \frac{d^2(3a^2d^2-8abcd+6b^2c^2)\,\text{arctanh}(\sin(e+fx))}{b^4f} - \frac{(bc-ad)^4\sin(e+fx)}{b^3f\,(a^2-b^2)\,(a\cos(e+fx)+b)} + \frac{2(bc-ad)^3(3ad+bc)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^4f\sqrt{a-b}\sqrt{a+b}} + \frac{2(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f(a-b)^{3/2}(a+b)^{3/2}} + \frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f} + \frac{d^4\text{arctanh}(\sin(e+fx))}{2b^2f} + \frac{d^4\tan(e+fx)\sec(e+fx)}{2b^2f}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^4*ArcTanh[Sin[e + f*x]])/(2*b^2*f) + (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2
*d^2)*ArcTanh[Sin[e + f*x]])/(b^4*f) + (2*(b*c - a*d)^4*ArcTanh[(Sqrt[a - b
]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b^2*(a + b)^(3/2)*f) + (
2*(b*c - a*d)^3*(b*c + 3*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a
 + b]])/(a*Sqrt[a - b]*b^4*Sqrt[a + b]*f) - ((b*c - a*d)^4*Sin[e + f*x])/(b
^3*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (2*d^3*(2*b*c - a*d)*Tan[e + f*x])
/(b^3*f) + (d^4*Sec[e + f*x]*Tan[e + f*x])/(2*b^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
```

```
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \frac{(d+c\cos(e+fx))^4\sec^3(e+fx)}{(b+a\cos(e+fx))^2}\,dx$$

$$= \int \left(-\frac{(-bc+ad)^4}{ab^3(b+a\cos(e+fx))^2} - \frac{(-bc+ad)^3(bc+3ad)}{ab^4(b+a\cos(e+fx))} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\sec(e+fx)}{b^4} + \frac{2d^3(2bc-ad)\sec^2(e+fx)}{b^3} + \frac{d^4\sec^3(e+fx)}{b^2}\right)dx$$

$$= \frac{d^4\int\sec^3(e+fx)\,dx}{b^2} - \frac{(bc-ad)^4\int\frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab^3} + \frac{(2d^3(2bc-ad))\int\sec^2(e+fx)\,dx}{b^3} + \frac{((bc-ad)^3(bc+3ad))\int\frac{1}{b+a\cos(e+fx)}\,dx}{ab^4} + \frac{(d^2(6b^2c^2-8abcd+3a^2d^2))\int\sec(e+fx)\,dx}{b^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\operatorname{arctanh}(\sin(e+fx))}{b^4f} - \frac{(bc-ad)^4\sin(e+fx)}{b^3(a^2-b^2)f(b+a\cos(e+fx))} + \frac{d^4\sec(e+fx)\tan(e+fx)}{2b^2f} + \frac{d^4\int\sec(e+fx)\,dx}{2b^2} + \frac{(bc-ad)^4\int\frac{b}{b+a\cos(e+fx)}\,dx}{ab^3(a^2-b^2)} - \frac{(2d^3(2bc-ad))\operatorname{Subst}(\int 1\,dx,x,-\tan(e+fx))}{b^3f} + \frac{(2(bc-ad)^3(bc+3ad))\operatorname{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{ab^4f}$$

$$= \frac{d^4\operatorname{arctanh}(\sin(e+fx))}{2b^2f} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\operatorname{arctanh}(\sin(e+fx))}{b^4f} + \frac{2(bc-ad)^3(bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^4\sqrt{a+b}f} - \frac{(bc-ad)^4\sin(e+fx)}{b^3(a^2-b^2)f(b+a\cos(e+fx))} + \frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f} + \frac{d^4\sec(e+fx)\tan(e+fx)}{2b^2f} + \frac{(bc-ad)^4\int\frac{1}{b+a\cos(e+fx)}\,dx}{ab^2(a^2-b^2)}$$

$$= \frac{d^4\text{arctanh}(\sin(e+fx))}{2b^2f} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,\text{arctanh}(\sin(e+fx))}{b^4f} + \frac{2(bc-ad)^3(bc+3ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^4\sqrt{a+b}f} - \frac{(bc-ad)^4\sin(e+fx)}{b^3(a^2-b^2)\,f(b+a\cos(e+fx))} + \frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f} + \frac{d^4\sec(e+fx)\tan(e+fx)}{2b^2f} + \frac{(2(bc-ad)^4)\,\text{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{ab^2(a^2-b^2)\,f}$$

$$= \frac{d^4\text{arctanh}(\sin(e+fx))}{2b^2f} + \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,\text{arctanh}(\sin(e+fx))}{b^4f} + \frac{2(bc-ad)^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b^2(a+b)^{3/2}f} + \frac{2(bc-ad)^3(bc+3ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^4\sqrt{a+b}f} - \frac{(bc-ad)^4\sin(e+fx)}{b^3(a^2-b^2)\,f(b+a\cos(e+fx))} + \frac{2d^3(2bc-ad)\tan(e+fx)}{b^3f} + \frac{d^4\sec(e+fx)\tan(e+fx)}{2b^2f}$$

Mathematica [A] (verified)

Time = 5.40 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.72

$$\int\frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx$$

$$= \frac{\cos^2(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^4\left(\frac{8(-bc+ad)^3(abc+3a^2d-4b^2d)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(b+}{(a^2-b^2)^{3/2}}\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^4)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]^2*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^4*((8*(-(b*c) + a
*d)^3*(a*b*c + 3*a^2*d - 4*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[
a^2 - b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) - 2*d^2*(-16*a*b*c*d +
6*a^2*d^2 + b^2*(12*c^2 + d^2))*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]] + 2*d^2*(-16*a*b*c*d + 6*a^2*d^2 + b^2*(12*c^2 + d^2))*(
b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b^2*d^4*(b
```

```
+ a*Cos[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (8*b*d^3*(2*b*
c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]) - (b^2*d^4*(b + a*Cos[e + f*x]))/(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])^2 + (8*b*d^3*(2*b*c - a*d)*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(C
os[(e + f*x)/2] + Sin[(e + f*x)/2]) + (4*b*(b*c - a*d)^4*Sin[e + f*x])/((-a
 + b)*(a + b))))/(4*b^4*f*(d + c*Cos[e + f*x])^4*(a + b*Sec[e + f*x])^2)
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{-\frac{d^4}{2b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{d^2\left(6a^2d^2-16abcd+12b^2c^2+b^2d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^4}+\frac{d^3(4ad-8bc+bd)}{2b^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2b\left(a^4d^4-4a^3bc\,d^3+6a^2\right.}{\left(a^2-b^2\right)\left(\tan\left(\frac{f}{2}\right.\right.}}{}$
default	$\frac{-\frac{d^4}{2b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}+\frac{d^2\left(6a^2d^2-16abcd+12b^2c^2+b^2d^2\right)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{2b^4}+\frac{d^3(4ad-8bc+bd)}{2b^3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}+\frac{2b\left(a^4d^4-4a^3bc\,d^3+6a^2\right.}{\left(a^2-b^2\right)\left(\tan\left(\frac{f}{2}\right.\right.}}{}$
risch	Expression too large to display

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-1/2*d^4/b^2/(tan(1/2*f*x+1/2*e)+1)^2+1/2*d^2*(6*a^2*d^2-16*a*b*c*d+12
*b^2*c^2+b^2*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)+1)+1/2*d^3*(4*a*d-8*b*c+b*d)/b^
3/(tan(1/2*f*x+1/2*e)+1)+2/b^4*(b*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-
4*a*b^3*c^3*d+b^4*c^4)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a
-tan(1/2*f*x+1/2*e)^2*b-a-b)-(3*a^5*d^4-8*a^4*b*c*d^3+6*a^3*b^2*c^2*d^2-4*a
^3*b^2*d^4+12*a^2*b^3*c*d^3-a*b^4*c^4-12*a*b^4*c^2*d^2+4*b^5*c^3*d)/(a-b)/(
a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a-b)*(a+b))^(1/
2)))+1/2*d^4/b^2/(tan(1/2*f*x+1/2*e)-1)^2-1/2*d^2*(6*a^2*d^2-16*a*b*c*d+12*
b^2*c^2+b^2*d^2)/b^4*ln(tan(1/2*f*x+1/2*e)-1)+1/2*d^3*(4*a*d-8*b*c+b*d)/b^3
/(tan(1/2*f*x+1/2*e)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(275) = 550$.

Time = 170.55 (sec) , antiderivative size = 1925, normalized size of antiderivative = 6.48

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(2*((a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*(a^4*b^2 - 2*a^2*b^4)*c^2*d^2 +
4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*a^4*b^2)*d^4)*cos(f*x + e)^3 + (
a*b^5*c^4 - 4*b^6*c^3*d - 6*(a^3*b^3 - 2*a*b^5)*c^2*d^2 + 4*(2*a^4*b^2 - 3*
a^2*b^4)*c*d^3 - (3*a^5*b - 4*a^3*b^3)*d^4)*cos(f*x + e)^2)*sqrt(a^2 - b^2)
*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)
*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a
*b*cos(f*x + e) + b^2)) - ((12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^2*d^2 - 16*(
a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 11*a^5*b^2 + 4*a^3*b^4 + a*b^
6)*d^4)*cos(f*x + e)^3 + (12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 - 16*(a^5*
b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d
^4)*cos(f*x + e)^2)*log(sin(f*x + e) + 1) + ((12*(a^5*b^2 - 2*a^3*b^4 + a*b
^6)*c^2*d^2 - 16*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 11*a^5*b^2
+ 4*a^3*b^4 + a*b^6)*d^4)*cos(f*x + e)^3 + (12*(a^4*b^3 - 2*a^2*b^5 + b^7)*
c^2*d^2 - 16*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 +
4*a^2*b^5 + b^7)*d^4)*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) - 2*((a^4*b^3
- 2*a^2*b^5 + b^7)*d^4 - 2*((a^2*b^5 - b^7)*c^4 - 4*(a^3*b^4 - a*b^6)*c^3*d
 + 6*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 4*(2*a^5*b^2 - 3*a^3*b^4 + a*b^6)*c*d^3
+ (3*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*d^4)*cos(f*x + e)^2 + (8*(a^4*b^3 - 2*a
^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^4)*cos(f*x + e))*si
n(f*x + e))/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*f*cos(f*x + e)^3 + (a^4*b^5 - 2*
a^2*b^7 + b^9)*f*cos(f*x + e)^2), 1/4*(4*((a^2*b^4*c^4 - 4*a*b^5*c^3*d - 6*
(a^4*b^2 - 2*a^2*b^4)*c^2*d^2 + 4*(2*a^5*b - 3*a^3*b^3)*c*d^3 - (3*a^6 - 4*
a^4*b^2)*d^4)*cos(f*x + e)^3 + (a*b^5*c^4 - 4*b^6*c^3*d - 6*(a^3*b^3 - 2*a*
b^5)*c^2*d^2 + 4*(2*a^4*b^2 - 3*a^2*b^4)*c*d^3 - (3*a^5*b - 4*a^3*b^3)*d^4)
*cos(f*x + e)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
+ a)/((a^2 - b^2)*sin(f*x + e))) + ((12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^2*d
^2 - 16*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 11*a^5*b^2 + 4*a^3*b
^4 + a*b^6)*d^4)*cos(f*x + e)^3 + (12*(a^4*b^3 - 2*a^2*b^5 + b^7)*c^2*d^2 -
 16*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5
 + b^7)*d^4)*cos(f*x + e)^2)*log(sin(f*x + e) + 1) - ((12*(a^5*b^2 - 2*a^3*
b^4 + a*b^6)*c^2*d^2 - 16*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 + (6*a^7 - 11
*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d^4)*cos(f*x + e)^3 + (12*(a^4*b^3 - 2*a^2*b^
5 + b^7)*c^2*d^2 - 16*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 + (6*a^6*b - 11*a
^4*b^3 + 4*a^2*b^5 + b^7)*d^4)*cos(f*x + e)^2)*log(-sin(f*x + e) + 1) + 2*(
```

```
(a^4*b^3 - 2*a^2*b^5 + b^7)*d^4 - 2*((a^2*b^5 - b^7)*c^4 - 4*(a^3*b^4 - a*b
^6)*c^3*d + 6*(a^4*b^3 - a^2*b^5)*c^2*d^2 - 4*(2*a^5*b^2 - 3*a^3*b^4 + a*b^
6)*c*d^3 + (3*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*d^4)*cos(f*x + e)^2 + (8*(a^4*
b^3 - 2*a^2*b^5 + b^7)*c*d^3 - 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d^4)*cos(f*x
 + e))*sin(f*x + e))/((a^5*b^4 - 2*a^3*b^6 + a*b^8)*f*cos(f*x + e)^3 + (a^4
*b^5 - 2*a^2*b^7 + b^9)*f*cos(f*x + e)^2)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c+d\sec{(e+fx)})^4\sec{(e+fx)}}{(a+b\sec{(e+fx)})^2}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**4/(a+b*sec(f*x+e))**2,x)
[Out] Integral((c + d*sec(e + f*x))**4*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(275) = 550$.

Time = 0.39 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.86

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = -\frac{\frac{4\,(ab^4c^4-4\,b^5c^3d-6\,a^3b^2c^2d^2+12\,ab^4c^2d^2+8\,a^4bcd^3-12\,a^2b^3cd^3-3\,a^5d^4+4\,a^3b^2d^4)\left(\pi\left\lfloor\frac{fx+e}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-b\tan}{\sqrt{-a^2+b^2}}\right.\right.}{(a^2b^4-b^6)\sqrt{-a^2+b^2}}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^4/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(a*b^4*c^4 - 4*b^5*c^3*d - 6*a^3*b^2*c^2*d^2 + 12*a*b^4*c^2*d^2 + 8
*a^4*b*c*d^3 - 12*a^2*b^3*c*d^3 - 3*a^5*d^4 + 4*a^3*b^2*d^4)*(pi*floor(1/2*
(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan
(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) -
4*(b^4*c^4*tan(1/2*f*x + 1/2*e) - 4*a*b^3*c^3*d*tan(1/2*f*x + 1/2*e) + 6*a^
2*b^2*c^2*d^2*tan(1/2*f*x + 1/2*e) - 4*a^3*b*c*d^3*tan(1/2*f*x + 1/2*e) + a
^4*d^4*tan(1/2*f*x + 1/2*e))/((a^2*b^3 - b^5)*(a*tan(1/2*f*x + 1/2*e)^2 - b
*tan(1/2*f*x + 1/2*e)^2 - a - b)) - (12*b^2*c^2*d^2 - 16*a*b*c*d^3 + 6*a^2*
d^4 + b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^4 + (12*b^2*c^2*d^2 - 1
6*a*b*c*d^3 + 6*a^2*d^4 + b^2*d^4)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/b^4 +
 2*(8*b*c*d^3*tan(1/2*f*x + 1/2*e)^3 - 4*a*d^4*tan(1/2*f*x + 1/2*e)^3 - b*d
^4*tan(1/2*f*x + 1/2*e)^3 - 8*b*c*d^3*tan(1/2*f*x + 1/2*e) + 4*a*d^4*tan(1/
2*f*x + 1/2*e) - b*d^4*tan(1/2*f*x + 1/2*e))/((tan(1/2*f*x + 1/2*e)^2 - 1)^
2*b^3))/f
```

Mupad [B] (verification not implemented)

Time = 26.19 (sec) , antiderivative size = 12483, normalized size of antiderivative = 42.03

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^4}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d/cos(e + f*x))^4/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (atan((((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 +
 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^
4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2
*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b
^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*
a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2
*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*
b^9) - (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c
*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^
6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*c^2*d^2) +
 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*
d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*
b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^
8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2
- 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*
d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^
5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a
^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6
*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96
```

*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6
*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5
 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^
6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3
*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*
d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d
^4 - 8*a*b*c*d^3)*1i)/b^4 - (((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3
*d + 4*a^2*b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16
*a^3*b^12*d^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*
b^8*d^4 + 24*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*
b^13*c^3*d + 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16
*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*
d^2 - 24*a^5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b
^12 - a^2*b^10 - a^3*b^9) + (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2)
+ 3*a^2*d^4 - 8*a*b*c*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^
10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*
(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 + (8*tan(e/2 + (f*x)/2)
*(72*a^10*d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*
a^2*b^8*d^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4
*d^8 + 120*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d
^4 + 64*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4
*d^4 - 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b
^6*c*d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*
a^8*b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4
*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576
*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*
c^3*d^5 - 944*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 +
 1824*a^5*b^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6
*b^4*c^2*d^6 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*
d^6 - 768*a^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9
*c^7*d - 384*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 +
 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3)*1i)/b^4)/((16*(108*a^11*d^12 - 54*a^
10*b*d^12 + 4*a^3*b^8*d^12 - 4*a^4*b^7*d^12 + 41*a^5*b^6*d^12 - 9*a^6*b^5*d
^12 + 63*a^7*b^4*d^12 + 81*a^8*b^3*d^12 - 216*a^9*b^2*d^12 - 4*b^11*c^3*d^9
 - 96*b^11*c^5*d^7 + 32*b^11*c^6*d^6 - 576*b^11*c^7*d^5 + 384*b^11*c^8*d^4
+ 12*a*b^10*c^2*d^10 + 4*a*b^10*c^3*d^9 + 417*a*b^10*c^4*d^8 - 96*a*b^10*c^
5*d^7 + 3288*a*b^10*c^6*d^6 - 2256*a*b^10*c^7*d^5 + 144*a*b^10*c^8*d^4 - 19
2*a*b^10*c^9*d^3 - 12*a^2*b^9*c*d^11 + 12*a^3*b^8*c*d^11 - 252*a^4*b^7*c*d^
11 + 60*a^5*b^6*c*d^11 - 744*a^6*b^5*c*d^11 - 648*a^7*b^4*c*d^11 + 1872*a^8
*b^3*c*d^11 + 432*a^9*b^2*c*d^11 - 12*a^2*b^9*c^2*d^10 - 716*a^2*b^9*c^3*d^
9 + 63*a^2*b^9*c^4*d^8 - 7872*a^2*b^9*c^5*d^7 + 5784*a^2*b^9*c^6*d^6 + 192*
a^2*b^9*c^7*d^5 + 690*a^2*b^9*c^8*d^4 + 24*a^2*b^9*c^10*d^2 + 606*a^3*b^8*c
^2*d^10 + 76*a^3*b^8*c^3*d^9 + 10203*a^3*b^8*c^4*d^8 - 8592*a^3*b^8*c^5*d^7
 - 3752*a^3*b^8*c^6*d^6 - 480*a^3*b^8*c^7*d^5 - 144*a^3*b^8*c^8*d^4 - 32*a^
3*b^8*c^9*d^3 - 126*a^4*b^7*c^2*d^10 - 7680*a^4*b^7*c^3*d^9 + 8277*a^4*b^7*

c^4*d^8 + 11232*a^4*b^7*c^5*d^7 - 1552*a^4*b^7*c^6*d^6 + 384*a^4*b^7*c^7*d^
5 - 132*a^4*b^7*c^8*d^4 + 3318*a^5*b^6*c^2*d^10 - 5424*a^5*b^6*c^3*d^9 - 16
488*a^5*b^6*c^4*d^8 + 4128*a^5*b^6*c^5*d^7 + 464*a^5*b^6*c^6*d^6 + 384*a^5*
b^6*c^7*d^5 + 2394*a^6*b^5*c^2*d^10 + 13904*a^6*b^5*c^3*d^9 - 4860*a^6*b^5*
c^4*d^8 - 3264*a^6*b^5*c^5*d^7 - 400*a^6*b^5*c^6*d^6 - 6888*a^7*b^4*c^2*d^1
0 + 3472*a^7*b^4*c^3*d^9 + 5868*a^7*b^4*c^4*d^8 + 192*a^7*b^4*c^5*d^7 - 158
4*a^8*b^3*c^2*d^10 - 5504*a^8*b^3*c^3*d^9 - 36*a^8*b^3*c^4*d^8 + 2952*a^9*b
^2*c^2*d^10 - 864*a^10*b*c*d^11))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + ((
(((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^
12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^4*b^11*d^
4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48
*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3
 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c
*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32
*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8
*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3)*(8*
a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(
b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^
4 - 8*a*b*c*d^3))/b^4 - (8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*a
*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8 +
 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*a
^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b^
9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64*
a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7 -
 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c^
2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 70
4*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*
c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4
 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a^
5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^3
*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 80
0*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a*
b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*a*
b*c*d^3))/b^4 + (((((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b
^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^
4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24
*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d +
 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*
d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5
*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^
10 - a^3*b^9) + (8*tan(e/2 + (f*x)/2)*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4
 - 8*a*b*c*d^3)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b
^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(b^2*(d^4/2 + 6*c
^2*d^2) + 3*a^2*d^4 - 8*a*b*c*d^3))/b^4 + (8*tan(e/2 + (f*x)/2)*(72*a^10*d^
8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8

- 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a
^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10
*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a
*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 1
60*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^
7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^
2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4
*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 94
4*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^
5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6
 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^
7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384
*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6))*(b^2*(d^4/2 + 6*c^2*d^2)
+ 3*a^2*d^4 - 8*a*b*c*d^3))/b^4))*(b^2*(d^4/2 + 6*c^2*d^2) + 3*a^2*d^4 - 8*
a*b*c*d^3)*2i)/(b^4*f) - ((tan(e/2 + (f*x)/2)^5*(6*a^4*d^4 + 2*b^4*c^4 + b^
4*d^4 + 3*a*b^3*d^4 - 3*a^3*b*d^4 - 8*b^4*c*d^3 - 5*a^2*b^2*d^4 + 8*a^2*b^2
*c*d^3 + 12*a^2*b^2*c^2*d^2 + 8*a*b^3*c*d^3 - 8*a*b^3*c^3*d - 16*a^3*b*c*d^
3))/((a*b^3 - b^4)*(a + b)) + (tan(e/2 + (f*x)/2)*(6*a^4*d^4 + 2*b^4*c^4 +
b^4*d^4 - 3*a*b^3*d^4 + 3*a^3*b*d^4 + 8*b^4*c*d^3 - 5*a^2*b^2*d^4 - 8*a^2*b
^2*c*d^3 + 12*a^2*b^2*c^2*d^2 + 8*a*b^3*c*d^3 - 8*a*b^3*c^3*d - 16*a^3*b*c*
d^3))/(b^3*(a + b)*(a - b)) - (2*tan(e/2 + (f*x)/2)^3*(6*a^4*d^4 + 2*b^4*c^
4 - b^4*d^4 - 3*a^2*b^2*d^4 + 12*a^2*b^2*c^2*d^2 + 8*a*b^3*c*d^3 - 8*a*b^3*
c^3*d - 16*a^3*b*c*d^3))/(b*(a*b^2 - b^3)*(a + b)))/(f*(a + b - tan(e/2 + (
f*x)/2)^2*(3*a + b) - tan(e/2 + (f*x)/2)^6*(a - b) + tan(e/2 + (f*x)/2)^4*(
3*a - b))) - (atan((((a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(e/2
+ (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8
*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 +
17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*
b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288
*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 +
 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*
d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a
^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3
*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 153
6*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5
*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3
- 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^
7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7
- 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + ((
(8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^12
*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^4*b^11*d^4
+ 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48*a
*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3 +
 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d
^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32*a

*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (8*t
an(e/2 + (f*x)/2)*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^
2*d + a*b*c)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9
 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4
*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^
2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d
+ a*b*c)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + ((a*d - b*c)^3*((a
+ b)^3*(a - b)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*
a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8
+ 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*
a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b
^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64
*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7
- 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c
^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 7
04*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7
*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4
 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a
^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^
3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 8
00*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a
*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15
*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4
- 16*a^3*b^12*d^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*
a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*
a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3
- 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*
c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11
 + b^12 - a^2*b^10 - a^3*b^9) - (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^3*((a + b
)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c)*(8*a*b^13 - 8*a^2*b^12 - 1
6*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7
- a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b
)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4
*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c)*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*
b^6 - a^6*b^4))/((16*(108*a^11*d^12 - 54*a^10*b*d^12 + 4*a^3*b^8*d^12 - 4*a
^4*b^7*d^12 + 41*a^5*b^6*d^12 - 9*a^6*b^5*d^12 + 63*a^7*b^4*d^12 + 81*a^8*b
^3*d^12 - 216*a^9*b^2*d^12 - 4*b^11*c^3*d^9 - 96*b^11*c^5*d^7 + 32*b^11*c^6
*d^6 - 576*b^11*c^7*d^5 + 384*b^11*c^8*d^4 + 12*a*b^10*c^2*d^10 + 4*a*b^10*
c^3*d^9 + 417*a*b^10*c^4*d^8 - 96*a*b^10*c^5*d^7 + 3288*a*b^10*c^6*d^6 - 22
56*a*b^10*c^7*d^5 + 144*a*b^10*c^8*d^4 - 192*a*b^10*c^9*d^3 - 12*a^2*b^9*c*
d^11 + 12*a^3*b^8*c*d^11 - 252*a^4*b^7*c*d^11 + 60*a^5*b^6*c*d^11 - 744*a^6
*b^5*c*d^11 - 648*a^7*b^4*c*d^11 + 1872*a^8*b^3*c*d^11 + 432*a^9*b^2*c*d^11
 - 12*a^2*b^9*c^2*d^10 - 716*a^2*b^9*c^3*d^9 + 63*a^2*b^9*c^4*d^8 - 7872*a^
2*b^9*c^5*d^7 + 5784*a^2*b^9*c^6*d^6 + 192*a^2*b^9*c^7*d^5 + 690*a^2*b^9*c^
8*d^4 + 24*a^2*b^9*c^10*d^2 + 606*a^3*b^8*c^2*d^10 + 76*a^3*b^8*c^3*d^9 + 1

0203*a^3*b^8*c^4*d^8 - 8592*a^3*b^8*c^5*d^7 - 3752*a^3*b^8*c^6*d^6 - 480*a^
3*b^8*c^7*d^5 - 144*a^3*b^8*c^8*d^4 - 32*a^3*b^8*c^9*d^3 - 126*a^4*b^7*c^2*
d^10 - 7680*a^4*b^7*c^3*d^9 + 8277*a^4*b^7*c^4*d^8 + 11232*a^4*b^7*c^5*d^7
- 1552*a^4*b^7*c^6*d^6 + 384*a^4*b^7*c^7*d^5 - 132*a^4*b^7*c^8*d^4 + 3318*a
^5*b^6*c^2*d^10 - 5424*a^5*b^6*c^3*d^9 - 16488*a^5*b^6*c^4*d^8 + 4128*a^5*b
^6*c^5*d^7 + 464*a^5*b^6*c^6*d^6 + 384*a^5*b^6*c^7*d^5 + 2394*a^6*b^5*c^2*d
^10 + 13904*a^6*b^5*c^3*d^9 - 4860*a^6*b^5*c^4*d^8 - 3264*a^6*b^5*c^5*d^7 -
400*a^6*b^5*c^6*d^6 - 6888*a^7*b^4*c^2*d^10 + 3472*a^7*b^4*c^3*d^9 + 5868*
a^7*b^4*c^4*d^8 + 192*a^7*b^4*c^5*d^7 - 1584*a^8*b^3*c^2*d^10 - 5504*a^8*b^
3*c^3*d^9 - 36*a^8*b^3*c^4*d^8 + 2952*a^9*b^2*c^2*d^10 - 864*a^10*b*c*d^11)
)/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + ((a*d - b*c)^3*((a + b)^3*(a - b)^
3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(72*a^10*d^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*
a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8 - 20*a^3*b^7*d^8 + 23*a^4*b^6*d^
8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 2
4*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^10*c^6*d^2 - 48*a*b^9*c^2*d^6 - 38
4*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7
- 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 - 160*a^5*b^5*c*d^7 - 704*a^6*b^4*c
*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d^7 + 376*a^2*b^8*c^2*d^6 + 768*a^
2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*
d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*
a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c
^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4
+ 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^3*d^5 + 264*a^6
*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*
d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a*b^8 + b^9 - a^
2*b^7 - a^3*b^6) + (((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*
b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d
^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 2
4*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d
+ 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c
*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^
5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b
^10 - a^3*b^9) + (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^
(1/2)*(3*a^2*d - 4*b^2*d + a*b*c)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16
*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^1
0 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^
(1/2)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)
)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) - (
(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(e/2 + (f*x)/2)*(72*a^10*d
^8 + b^10*d^8 - 2*a*b^9*d^8 - 72*a^9*b*d^8 + 4*a^2*b^8*c^8 + 11*a^2*b^8*d^8
- 20*a^3*b^7*d^8 + 23*a^4*b^6*d^8 - 26*a^5*b^5*d^8 + 17*a^6*b^4*d^8 + 120*
a^7*b^3*d^8 - 120*a^8*b^2*d^8 + 24*b^10*c^2*d^6 + 144*b^10*c^4*d^4 + 64*b^1
0*c^6*d^2 - 48*a*b^9*c^2*d^6 - 384*a*b^9*c^3*d^5 - 288*a*b^9*c^4*d^4 - 384*
a*b^9*c^5*d^3 + 64*a^2*b^8*c*d^7 - 160*a^3*b^7*c*d^7 + 256*a^4*b^6*c*d^7 -
160*a^5*b^5*c*d^7 - 704*a^6*b^4*c*d^7 + 704*a^7*b^3*c*d^7 + 384*a^8*b^2*c*d

^7 + 376*a^2*b^8*c^2*d^6 + 768*a^2*b^8*c^3*d^5 + 816*a^2*b^8*c^4*d^4 + 96*a^2*b^8*c^6*d^2 - 704*a^3*b^7*c^2*d^6 - 896*a^3*b^7*c^3*d^5 + 576*a^3*b^7*c^4*d^4 + 96*a^3*b^7*c^5*d^3 + 536*a^4*b^6*c^2*d^6 - 1536*a^4*b^6*c^3*d^5 - 944*a^4*b^6*c^4*d^4 - 48*a^4*b^6*c^6*d^2 + 1552*a^5*b^5*c^2*d^6 + 1824*a^5*b^5*c^3*d^5 - 288*a^5*b^5*c^4*d^4 + 64*a^5*b^5*c^5*d^3 - 1624*a^6*b^4*c^2*d^6 + 768*a^6*b^4*c^3*d^5 + 264*a^6*b^4*c^4*d^4 - 800*a^7*b^3*c^2*d^6 - 768*a^7*b^3*c^3*d^5 + 800*a^8*b^2*c^2*d^6 - 32*a*b^9*c*d^7 - 32*a*b^9*c^7*d - 384*a^9*b*c*d^7))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (((8*(2*b^15*d^4 - 4*a*b^14*c^4 + 16*b^15*c^3*d + 4*a^2*b^13*c^4 + 4*a^3*b^12*c^4 - 4*a^4*b^11*c^4 + 6*a^2*b^13*d^4 - 16*a^3*b^12*d^4 - 14*a^4*b^11*d^4 + 28*a^5*b^10*d^4 + 6*a^6*b^9*d^4 - 12*a^7*b^8*d^4 + 24*b^15*c^2*d^2 - 48*a*b^14*c^2*d^2 + 48*a^2*b^13*c*d^3 - 16*a^2*b^13*c^3*d + 48*a^3*b^12*c*d^3 + 16*a^3*b^12*c^3*d - 80*a^4*b^11*c*d^3 - 16*a^5*b^10*c*d^3 + 32*a^6*b^9*c*d^3 - 24*a^2*b^13*c^2*d^2 + 72*a^3*b^12*c^2*d^2 - 24*a^5*b^10*c^2*d^2 - 32*a*b^14*c*d^3 - 16*a*b^14*c^3*d))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (8*tan(e/2 + (f*x)/2)*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/((a*b^8 + b^9 - a^2*b^7 - a^3*b^6)*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*(3*a^2*d - 4*b^2*d + a*b*c))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(a*d - b*c)^3*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*d - 4*b^2*d + a*b*c)*2i)/(f*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))

3.260 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 228

$$\begin{aligned}\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx &= \frac{d^2(3bc-2ad)\text{arctanh}(\sin(e+fx))}{b^3f} \\ &+ \frac{2(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b(a+b)^{3/2}f} \\ &+ \frac{2(bc-ad)^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^3\sqrt{a+b}f} \\ &- \frac{(bc-ad)^3\sin(e+fx)}{b^2(a^2-b^2)f(b+a\cos(e+fx))} \\ &+ \frac{d^3\tan(e+fx)}{b^2f}\end{aligned}$$

```
[Out] d^2*(-2*a*d+3*b*c)*arctanh(sin(f*x+e))/b^3/f+2*(-a*d+b*c)^3*arctanh((a-b)^(
1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/b/(a+b)^(3/2)/f-(-a*d+b*
c)^3*sin(f*x+e)/b^2/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a*d+b*c)^2*(2*a*d+b*c)
*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/a/b^3/f/(a-b)^(1/2)/(a
+b)^(1/2)+d^3*tan(f*x+e)/b^2/f
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4073, 3031, 2743, 12, 2738, 214, 3855, 3852, 8}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = -\frac{(bc-ad)^3\sin(e+fx)}{b^2f\,(a^2-b^2)\,(a\cos(e+fx)+b)} + \frac{d^2(3bc-2ad)\text{arctanh}(\sin(e+fx))}{b^3f} + \frac{2(bc-ad)^2(2ad+bc)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^3f\sqrt{a-b}\sqrt{a+b}} + \frac{2(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{abf(a-b)^{3/2}(a+b)^{3/2}} + \frac{d^3\tan(e+fx)}{b^2f}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*ArcTanh[Sin[e + f*x]])/(b^3*f) + (2*(b*c - a*d)^3*ArcT
anh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*b*(a + b)
^(3/2)*f) + (2*(b*c - a*d)^2*(b*c + 2*a*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*
x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*b^3*Sqrt[a + b]*f) - ((b*c - a*d)^3*Sin
[e + f*x])/(b^2*(a^2 - b^2)*f*(b + a*Cos[e + f*x])) + (d^3*Tan[e + f*x])/(b
^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(d+c\cos(e+fx))^3\sec^2(e+fx)}{(b+a\cos(e+fx))^2}\,dx\\ &= \int \left(\frac{(-bc+ad)^3}{ab^2(b+a\cos(e+fx))^2}+\frac{(-bc+ad)^2(bc+2ad)}{ab^3(b+a\cos(e+fx))}+\frac{d^2(3bc-2ad)\sec(e+fx)}{b^3}\right.\\ &\qquad \left.+\frac{d^3\sec^2(e+fx)}{b^2}\right)\,dx\end{aligned}$$

$$= \frac{d^3 \int \sec^2(e+fx)\,dx}{b^2} + \frac{(d^2(3bc-2ad)) \int \sec(e+fx)\,dx}{b^3} - \frac{(bc-ad)^3 \int \frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab^2} + \frac{((bc-ad)^2(bc+2ad)) \int \frac{1}{b+a\cos(e+fx)}\,dx}{ab^3}$$

$$= \frac{d^2(3bc-2ad)\text{arctanh}(\sin(e+fx))}{b^3 f} - \frac{(bc-ad)^3 \sin(e+fx)}{b^2(a^2-b^2) f(b+a\cos(e+fx))} + \frac{(bc-ad)^3 \int \frac{b}{b+a\cos(e+fx)}\,dx}{ab^2(a^2-b^2)} - \frac{d^3\text{Subst}(\int 1\,dx, x, -\tan(e+fx))}{b^2 f} + \frac{(2(bc-ad)^2(bc+2ad))\,\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{ab^3 f}$$

$$= \frac{d^2(3bc-2ad)\text{arctanh}(\sin(e+fx))}{b^3 f} + \frac{2(bc-ad)^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^3\sqrt{a+b}f} - \frac{(bc-ad)^3 \sin(e+fx)}{b^2(a^2-b^2) f(b+a\cos(e+fx))} + \frac{d^3\tan(e+fx)}{b^2 f} + \frac{(bc-ad)^3 \int \frac{1}{b+a\cos(e+fx)}\,dx}{ab(a^2-b^2)}$$

$$= \frac{d^2(3bc-2ad)\text{arctanh}(\sin(e+fx))}{b^3 f} + \frac{2(bc-ad)^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^3\sqrt{a+b}f} - \frac{(bc-ad)^3 \sin(e+fx)}{b^2(a^2-b^2) f(b+a\cos(e+fx))} + \frac{d^3\tan(e+fx)}{b^2 f} + \frac{(2(bc-ad)^3)\,\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{ab(a^2-b^2) f}$$

$$= \frac{d^2(3bc-2ad)\text{arctanh}(\sin(e+fx))}{b^3 f} + \frac{2(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}b(a+b)^{3/2}f} + \frac{2(bc-ad)^2(bc+2ad)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^3\sqrt{a+b}f} - \frac{(bc-ad)^3 \sin(e+fx)}{b^2(a^2-b^2) f(b+a\cos(e+fx))} + \frac{d^3\tan(e+fx)}{b^2 f}$$

Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.59

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx$$
$$= \frac{\cos(e+fx)(b+a\cos(e+fx))(c+d\sec(e+fx))^3\left(-\frac{2(bc-ad)^2(abc+2a^2d-3b^2d)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)(b+a}{(a^2-b^2)^{3/2}}\right.}{}$$

[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^3)/(a + b*Sec[e + f*x])^2,x]

[Out] (Cos[e + f*x]*(b + a*Cos[e + f*x])*(c + d*Sec[e + f*x])^3*((-2*(b*c - a*d)^2*(a*b*c + 2*a^2*d - 3*b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[e + f*x]))/(a^2 - b^2)^(3/2) + d^2*(-3*b*c + 2*a*d)*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + d^2*(3*b*c - 2*a*d)*(b + a*Cos[e + f*x])*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*d^3*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (b*d^3*(b + a*Cos[e + f*x])*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (b*(b*c - a*d)^3*Sin[e + f*x])/((-a + b)*(a + b))))/(b^3*f*(d + c*Cos[e + f*x])^3*(a + b*Sec[e + f*x])^2)

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{-\frac{d^3}{b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{d^2(2ad-3bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^3}-\frac{2\left(\frac{b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\frac{\left(2a^4d^3-3a^3bc\,d\right.}{}\right.}{b}}{f}$
default	$\frac{-\frac{d^3}{b^2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}+\frac{d^2(2ad-3bc)\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^3}-\frac{2\left(\frac{b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\frac{\left(2a^4d^3-3a^3bc\,d\right.}{}\right.}{b}}{f}$
risch	Expression too large to display

[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOSE)

[Out] 1/f*(-d^3/b^2/(tan(1/2*f*x+1/2*e)-1)+d^2*(2*a*d-3*b*c)/b^3*ln(tan(1/2*f*x+1/2*e)-1)-2/b^3*(b*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a^2-b^2)*t

```
an(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(2*a^
4*d^3-3*a^3*b*c*d^2-3*a^2*b^2*d^3+a*b^3*c^3+6*a*b^3*c*d^2-3*b^4*c^2*d)/(a-b
)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a-b)*(a+b))^
(1/2)))-d^3/b^2/(tan(1/2*f*x+1/2*e)+1)-d^2*(2*a*d-3*b*c)/b^3*ln(tan(1/2*f*x
+1/2*e)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(210) = 420$.

Time = 39.31 (sec) , antiderivative size = 1326, normalized size of antiderivative = 5.82

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5
- 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 -
2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2)*l
og((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(
b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b
*cos(f*x + e) + b^2)) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*
a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d
^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f*x + e) + 1)
 - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^
3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3
*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a^4*b^2 - 2*a
^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*c^2*d + 3*(a
^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*
sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2 + (a^4*b^4 -
2*a^2*b^6 + b^8)*f*cos(f*x + e)), 1/2*(2*((a^2*b^3*c^3 - 3*a*b^4*c^2*d - 3*
(a^4*b - 2*a^2*b^3)*c*d^2 + (2*a^5 - 3*a^3*b^2)*d^3)*cos(f*x + e)^2 + (a*b^
4*c^3 - 3*b^5*c^2*d - 3*(a^3*b^2 - 2*a*b^4)*c*d^2 + (2*a^4*b - 3*a^2*b^3)*d
^3)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e)
 + a)/((a^2 - b^2)*sin(f*x + e))) + ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 -
 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^
4 + b^6)*c*d^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(sin(f
*x + e) + 1) - ((3*(a^5*b - 2*a^3*b^3 + a*b^5)*c*d^2 - 2*(a^6 - 2*a^4*b^2 +
 a^2*b^4)*d^3)*cos(f*x + e)^2 + (3*(a^4*b^2 - 2*a^2*b^4 + b^6)*c*d^2 - 2*(a
^5*b - 2*a^3*b^3 + a*b^5)*d^3)*cos(f*x + e))*log(-sin(f*x + e) + 1) + 2*((a
^4*b^2 - 2*a^2*b^4 + b^6)*d^3 - ((a^2*b^4 - b^6)*c^3 - 3*(a^3*b^3 - a*b^5)*
c^2*d + 3*(a^4*b^2 - a^2*b^4)*c*d^2 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*d^3)*co
s(f*x + e))*sin(f*x + e))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*f*cos(f*x + e)^2 +
 (a^4*b^4 - 2*a^2*b^6 + b^8)*f*cos(f*x + e))]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c+d\sec(e+fx))^3\sec(e+fx)}{(a+b\sec(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**3/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))**3*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(210) = 420$.

Time = 0.38 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.36

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx =$$

$$-\frac{\frac{2\left(ab^3c^3-3b^4c^2d-3a^3bcd^2+6ab^3cd^2+2a^4d^3-3a^2b^2d^3\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{-a^2+b^2}} - \frac{2\left(b^3c\right.}{}}{}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^3/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(a*b^3*c^3 - 3*b^4*c^2*d - 3*a^3*b*c*d^2 + 6*a*b^3*c*d^2 + 2*a^4*d^3 -
3*a^2*b^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a
*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^
3 - b^5)*sqrt(-a^2 + b^2)) - 2*(b^3*c^3*tan(1/2*f*x + 1/2*e)^3 - 3*a*b^2*c^
```

```
2*d*tan(1/2*f*x + 1/2*e)^3 + 3*a^2*b*c*d^2*tan(1/2*f*x + 1/2*e)^3 - 2*a^3*d
^3*tan(1/2*f*x + 1/2*e)^3 + a^2*b*d^3*tan(1/2*f*x + 1/2*e)^3 + a*b^2*d^3*ta
n(1/2*f*x + 1/2*e)^3 - b^3*d^3*tan(1/2*f*x + 1/2*e)^3 - b^3*c^3*tan(1/2*f*x
 + 1/2*e) + 3*a*b^2*c^2*d*tan(1/2*f*x + 1/2*e) - 3*a^2*b*c*d^2*tan(1/2*f*x
+ 1/2*e) + 2*a^3*d^3*tan(1/2*f*x + 1/2*e) + a^2*b*d^3*tan(1/2*f*x + 1/2*e)
- a*b^2*d^3*tan(1/2*f*x + 1/2*e) - b^3*d^3*tan(1/2*f*x + 1/2*e))/((a*tan(1/
2*f*x + 1/2*e)^4 - b*tan(1/2*f*x + 1/2*e)^4 - 2*a*tan(1/2*f*x + 1/2*e)^2 +
a + b)*(a^2*b^2 - b^4)) - (3*b*c*d^2 - 2*a*d^3)*log(abs(tan(1/2*f*x + 1/2*e
) + 1))/b^3 + (3*b*c*d^2 - 2*a*d^3)*log(abs(tan(1/2*f*x + 1/2*e) - 1))/b^3)
/f
```

Mupad [B] (verification not implemented)

Time = 22.92 (sec) , antiderivative size = 7958, normalized size of antiderivative = 34.90

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^3}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d/cos(e + f*x))^3/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (d^2*atan(((d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*
c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a
^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^
3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3
*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^
3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2
 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c
*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) +
 (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b
^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^
4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2
*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 +
 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*d^2*tan(e/2 + (f*
x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^
5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c
))/b^3)*(2*a*d - 3*b*c)*1i)/b^3 + (d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 -
 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6
+ 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^
7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4
*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*
a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*
d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b
^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7
 - a^2*b^5 - a^3*b^4) - (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2
 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3
```

- 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10
*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c
*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)
- (32*d^2*tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3
*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^
3*b^4)))*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c)*1i)/b^3)/((64*(8*a^8*d^9 - 4
*a^7*b*d^9 + 12*a^4*b^4*d^9 + 6*a^5*b^3*d^9 - 20*a^6*b^2*d^9 + 27*b^8*c^4*d
^5 - 27*b^8*c^5*d^4 - 90*a*b^7*c^3*d^6 + 99*a*b^7*c^4*d^5 - 9*a*b^7*c^5*d^4
+ 18*a*b^7*c^6*d^3 - 60*a^3*b^5*c*d^8 - 39*a^4*b^4*c*d^8 + 96*a^5*b^3*c*d^
8 + 24*a^6*b^2*c*d^8 + 111*a^2*b^6*c^2*d^7 - 144*a^2*b^6*c^3*d^6 - 15*a^2*b
^6*c^4*d^5 - 39*a^2*b^6*c^5*d^4 - 3*a^2*b^6*c^7*d^2 + 105*a^3*b^5*c^2*d^7 +
113*a^3*b^5*c^3*d^6 + 3*a^3*b^5*c^4*d^5 + 9*a^3*b^5*c^5*d^4 + 2*a^3*b^5*c^
6*d^3 - 165*a^4*b^4*c^2*d^7 + 55*a^4*b^4*c^3*d^6 - 12*a^4*b^4*c^4*d^5 + 9*a
^4*b^4*c^5*d^4 - 57*a^5*b^3*c^2*d^7 - 23*a^5*b^3*c^3*d^6 - 12*a^5*b^3*c^4*d
^5 + 54*a^6*b^2*c^2*d^7 + 4*a^6*b^2*c^3*d^6 - 36*a^7*b*c*d^8))/(a*b^8 + b^9
- a^2*b^7 - a^3*b^6) + (d^2*((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d
^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b
^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4
- 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5
+ 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^
4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^
4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4
- 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5
- a^3*b^4) + (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*
c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b
^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3
*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a
*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*d^2*
tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a
^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(
2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c))/b^3 - (d^2*((32*tan(e/2 + (f*x)/2)*(8
*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5*a^
4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*d^2
- 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c*d^
5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*c^2
*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57*a^
4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d^3
+ 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))/(a
*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (d^2*((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*
b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^
2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 +
3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3
*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 -
a^3*b^6) - (32*d^2*tan(e/2 + (f*x)/2)*(2*a*d - 3*b*c)*(2*a*b^11 - 2*a^2*b^
10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^

2*b^5 - a^3*b^4)))*(2*a*d - 3*b*c))/b^3)*(2*a*d - 3*b*c))/b^3))*(2*a*d - 3*
b*c)*2i)/(b^3*f) - ((2*tan(e/2 + (f*x)/2)*(b^3*c^3 - 2*a^3*d^3 + b^3*d^3 +
a*b^2*d^3 - a^2*b*d^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(b^2*(a + b)*(a - b
)) - (2*tan(e/2 + (f*x)/2)^3*(b^3*c^3 - 2*a^3*d^3 - b^3*d^3 + a*b^2*d^3 + a
^2*b*d^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2))/(b^2*(a + b)*(a - b)))/(f*(a + b
 + tan(e/2 + (f*x)/2)^4*(a - b) - 2*a*tan(e/2 + (f*x)/2)^2)) + (atan(((((32
*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6
- 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c
^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c
*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*
c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a
^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^
4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d
 - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (((32*(a*b^11*c^3 +
 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 +
a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3
- 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3
*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8
 + b^9 - a^2*b^7 - a^3*b^6) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)^2*((a + b)
^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*
a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3
b^4)(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^3*(a
 - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -
a^6*b^3))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*
b*c)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (((32*tan(e/2 + (f*x)/2)
*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 + 5
*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4*
d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*c
*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6*
c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 57
*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*d
^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5))
/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*b
^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^2
*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 + 3
*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3*
a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 -
a^3*b^6) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)
*(2*a^2*d - 3*b^2*d + a*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8
 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2*
b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a
^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(a*d - b*
c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c)*1i)/(b^9 - 3*a
^2*b^7 + 3*a^4*b^5 - a^6*b^3))/((64*(8*a^8*d^9 - 4*a^7*b*d^9 + 12*a^4*b^4*d
^9 + 6*a^5*b^3*d^9 - 20*a^6*b^2*d^9 + 27*b^8*c^4*d^5 - 27*b^8*c^5*d^4 - 90*

a*b^7*c^3*d^6 + 99*a*b^7*c^4*d^5 - 9*a*b^7*c^5*d^4 + 18*a*b^7*c^6*d^3 - 60*
a^3*b^5*c*d^8 - 39*a^4*b^4*c*d^8 + 96*a^5*b^3*c*d^8 + 24*a^6*b^2*c*d^8 + 11
1*a^2*b^6*c^2*d^7 - 144*a^2*b^6*c^3*d^6 - 15*a^2*b^6*c^4*d^5 - 39*a^2*b^6*c
^5*d^4 - 3*a^2*b^6*c^7*d^2 + 105*a^3*b^5*c^2*d^7 + 113*a^3*b^5*c^3*d^6 + 3*
a^3*b^5*c^4*d^5 + 9*a^3*b^5*c^5*d^4 + 2*a^3*b^5*c^6*d^3 - 165*a^4*b^4*c^2*d
^7 + 55*a^4*b^4*c^3*d^6 - 12*a^4*b^4*c^4*d^5 + 9*a^4*b^4*c^5*d^4 - 57*a^5*b
^3*c^2*d^7 - 23*a^5*b^3*c^3*d^6 - 12*a^5*b^3*c^4*d^5 + 54*a^6*b^2*c^2*d^7 +
4*a^6*b^2*c^3*d^6 - 36*a^7*b*c*d^8))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (
((32*tan(e/2 + (f*x)/2)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*
d^6 - 8*a^3*b^5*d^6 + 5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b
^8*c^2*d^4 + 9*b^8*c^4*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b
^6*c*d^5 - 24*a^3*b^5*c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*
b^2*c*d^5 + 45*a^2*b^6*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 +
12*a^3*b^5*c^3*d^3 - 57*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^
2*d^4 + 4*a^5*b^3*c^3*d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c
^5*d - 24*a^7*b*c*d^5))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (((32*(a*b^11*c
^3 + 2*a*b^11*d^3 - 3*b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^
3 + a^4*b^8*c^3 - 3*a^2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*
d^3 - 2*a^6*b^6*d^3 + 3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2
- 3*a^3*b^9*c^2*d + 3*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a
*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)^2*((a
+ b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c)*(2*a*b^11 - 2*a^2*b^10
- 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 -
a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^
3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^
5 - a^6*b^3))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d
+ a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) - (((32*tan(e/2 + (f*x)/2
)*(8*a^8*d^6 - 8*a^7*b*d^6 + a^2*b^6*c^6 + 4*a^2*b^6*d^6 - 8*a^3*b^5*d^6 +
5*a^4*b^4*d^6 + 16*a^5*b^3*d^6 - 16*a^6*b^2*d^6 + 9*b^8*c^2*d^4 + 9*b^8*c^4
*d^2 - 18*a*b^7*c^2*d^4 - 36*a*b^7*c^3*d^3 + 24*a^2*b^6*c*d^5 - 24*a^3*b^5*
c*d^5 - 48*a^4*b^4*c*d^5 + 54*a^5*b^3*c*d^5 + 24*a^6*b^2*c*d^5 + 45*a^2*b^6
*c^2*d^4 + 12*a^2*b^6*c^4*d^2 + 36*a^3*b^5*c^2*d^4 + 12*a^3*b^5*c^3*d^3 - 5
7*a^4*b^4*c^2*d^4 - 6*a^4*b^4*c^4*d^2 - 18*a^5*b^3*c^2*d^4 + 4*a^5*b^3*c^3*
d^3 + 18*a^6*b^2*c^2*d^4 - 12*a*b^7*c*d^5 - 6*a*b^7*c^5*d - 24*a^7*b*c*d^5)
)/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (((32*(a*b^11*c^3 + 2*a*b^11*d^3 - 3*
b^12*c*d^2 - 3*b^12*c^2*d - a^2*b^10*c^3 - a^3*b^9*c^3 + a^4*b^8*c^3 - 3*a^
2*b^10*d^3 - 3*a^3*b^9*d^3 + 5*a^4*b^8*d^3 + a^5*b^7*d^3 - 2*a^6*b^6*d^3 +
3*a^2*b^10*c*d^2 + 3*a^2*b^10*c^2*d - 9*a^3*b^9*c*d^2 - 3*a^3*b^9*c^2*d + 3
*a^5*b^7*c*d^2 + 6*a*b^11*c*d^2 + 3*a*b^11*c^2*d))/(a*b^8 + b^9 - a^2*b^7 -
a^3*b^6) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2
)*(2*a^2*d - 3*b^2*d + a*b*c)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^
8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b^5 - a^3*b^4)*(b^9 - 3*a^2
*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*
a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))*(a*d - b
c)^2((a + b)^3*(a - b)^3)^(1/2)*(2*a^2*d - 3*b^2*d + a*b*c))/(b^9 - 3*a^2

```
*b^7 + 3*a^4*b^5 - a^6*b^3)))*(a*d - b*c)^2*((a + b)^3*(a - b)^3)^(1/2)*(2*
a^2*d - 3*b^2*d + a*b*c)*2i)/(f*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))
```

3.261 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 198

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \frac{d^2\operatorname{arctanh}(\sin(e+fx))}{b^2f} + \frac{2(bc-ad)^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}f} + \frac{2(b^2c^2-a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^2\sqrt{a+b}f} - \frac{(bc-ad)^2\sin(e+fx)}{b(a^2-b^2)f(b+a\cos(e+fx))}$$

```
[Out] d^2*arctanh(sin(f*x+e))/b^2/f+2*(-a*d+b*c)^2*arctanh((a-b)^(1/2)*tan(1/2*f*
x+1/2*e)/(a+b)^(1/2))/a/(a-b)^(3/2)/(a+b)^(3/2)/f-(-a*d+b*c)^2*sin(f*x+e)/b
/(a^2-b^2)/f/(b+a*cos(f*x+e))+2*(-a^2*d^2+b^2*c^2)*arctanh((a-b)^(1/2)*tan(
1/2*f*x+1/2*e)/(a+b)^(1/2))/a/b^2/f/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used

$= \{4073, 3031, 2743, 12, 2738, 214, 3855\}$

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \frac{2(b^2c^2-a^2d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{ab^2f\sqrt{a-b}\sqrt{a+b}} - \frac{(bc-ad)^2\sin(e+fx)}{bf\,(a^2-b^2)\,(a\cos(e+fx)+b)} + \frac{2(bc-ad)^2\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{af(a-b)^{3/2}(a+b)^{3/2}} + \frac{d^2\mathrm{arctanh}(\sin(e+fx))}{b^2f}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (d^2*ArcTanh[Sin[e + f*x]])/(b^2*f) + (2*(b*c - a*d)^2*ArcTanh[(Sqrt[a - b]
*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)*f) + (2*(b^
2*c^2 - a^2*d^2)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/(a*Sq
rt[a - b]*b^2*Sqrt[a + b]*f) - ((b*c - a*d)^2*Sin[e + f*x])/(b*(a^2 - b^2)*
f*(b + a*Cos[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3031

```
Int[((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*sin[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[b*c - a*d, 0] && (Int
egersQ[m, n] || IntegersQ[m, p] || IntegersQ[n, p]) && NeQ[p, 2]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(d+c\cos(e+fx))^2\sec(e+fx)}{(b+a\cos(e+fx))^2}\,dx \\
&= \int \left(-\frac{(-bc+ad)^2}{ab(b+a\cos(e+fx))^2}+\frac{b^2c^2-a^2d^2}{ab^2(b+a\cos(e+fx))}+\frac{d^2\sec(e+fx)}{b^2}\right)dx \\
&= \frac{d^2\int\sec(e+fx)\,dx}{b^2}-\frac{(bc-ad)^2\int\frac{1}{(b+a\cos(e+fx))^2}\,dx}{ab}+\frac{(b^2c^2-a^2d^2)\int\frac{1}{b+a\cos(e+fx)}\,dx}{ab^2} \\
&= \frac{d^2\text{arctanh}(\sin(e+fx))}{b^2f}-\frac{(bc-ad)^2\sin(e+fx)}{b\,(a^2-b^2)\,f(b+a\cos(e+fx))} \\
&\quad+\frac{(bc-ad)^2\int\frac{b}{b+a\cos(e+fx)}\,dx}{ab\,(a^2-b^2)} \\
&\quad+\frac{(2(b^2c^2-a^2d^2))\,\text{Subst}\left(\int\frac{1}{a+b+(-a+b)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{ab^2f} \\
&= \frac{d^2\text{arctanh}(\sin(e+fx))}{b^2f}+\frac{2(b^2c^2-a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,b^2\sqrt{a+b}\,f} \\
&\quad-\frac{(bc-ad)^2\sin(e+fx)}{b\,(a^2-b^2)\,f(b+a\cos(e+fx))}+\frac{(bc-ad)^2\int\frac{1}{b+a\cos(e+fx)}\,dx}{a\,(a^2-b^2)}
\end{aligned}
$$

$$= \frac{d^2\text{arctanh}(\sin(e+fx))}{b^2 f} + \frac{2(b^2c^2-a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^2\sqrt{a+b}f} - \frac{(bc-ad)^2\sin(e+fx)}{b(a^2-b^2)f(b+a\cos(e+fx))} + \frac{(2(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{a(a^2-b^2)f}$$

$$= \frac{d^2\text{arctanh}(\sin(e+fx))}{b^2 f} + \frac{2(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}f} + \frac{2(b^2c^2-a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}b^2\sqrt{a+b}f} - \frac{(bc-ad)^2\sin(e+fx)}{b(a^2-b^2)f(b+a\cos(e+fx))}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \frac{\frac{2(2b^3cd+a^3d^2-ab^2(c^2+2d^2))\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - d^2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right) + d^2\log\left(\text{c}\right.}{b^2 f}$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x])^2)/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((2*(2*b^3*c*d + a^3*d^2 - a*b^2*(c^2 + 2*d^2))*ArcTanh[((-a + b)*Tan[(e +
f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - d^2*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]] + d^2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (b*(b*c -
 a*d)^2*Sin[e + f*x])/((-a + b)*(a + b)*(b + a*Cos[e + f*x])))/(b^2*f)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^2}+\frac{d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}+\frac{\frac{2b\left(a^2d^2-2abcd+b^2c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\frac{2\left(a^3d^2-b^2c^2a-2b^2d^2a+2c\right.}{(a-b)(a}}{b^2}}{f}$
default	$\frac{-\frac{d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{b^2}+\frac{d^2\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}{b^2}+\frac{\frac{2b\left(a^2d^2-2abcd+b^2c^2\right)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\frac{2\left(a^3d^2-b^2c^2a-2b^2d^2a+2c\right.}{(a-b)(a}}{b^2}}{f}$
risch	$-\frac{2i(a^2d^2-2abcd+b^2c^2)\left(e^{i(fx+e)}b+a\right)}{(a^2-b^2)fba\left(e^{2i(fx+e)}a+2e^{i(fx+e)}b+a\right)}+\frac{\ln\left(e^{i(fx+e)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)a^3d^2}{\sqrt{a^2-b^2}\,(a+b)(a-b)fb^2}-\frac{\ln\left(e^{i(fx+e)}-\frac{ia^2-ib^2-b\sqrt{a^2-}}{\sqrt{a^2-b^2}\,a}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)f}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/f*(-d^2/b^2*ln(tan(1/2*f*x+1/2*e)-1)+d^2/b^2*ln(tan(1/2*f*x+1/2*e)+1)+2/b
^2*(b*(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x
+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(a^3*d^2-a*b^2*c^2-2*a*b^2*d^2+2*b^
3*c*d)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a
-b)*(a+b))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(180) = 360$.

Time = 5.70 (sec) , antiderivative size = 798, normalized size of antiderivative = 4.03

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$$
$$=\left[-\frac{(ab^3c^2-2\,b^4cd-(a^3b-2\,ab^3)d^2+(a^2b^2c^2-2\,ab^3cd-(a^4-2\,a^2b^2)d^2)\cos(fx+e))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\,c}{}\right.}{}\right.$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] [-1/2*((a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c^2 - 2*a*
b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*c
os(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x +
 e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x +
e) + b^2)) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b
^3 + b^5)*d^2)*log(sin(f*x + e) + 1) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f
*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) + 2*((a^2*b
^3 - b^5)*c^2 - 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x +
```

```
e))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5 +
b^7)*f), 1/2*(2*(a*b^3*c^2 - 2*b^4*c*d - (a^3*b - 2*a*b^3)*d^2 + (a^2*b^2*c
^2 - 2*a*b^3*c*d - (a^4 - 2*a^2*b^2)*d^2)*cos(f*x + e))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) + (
(a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)
*log(sin(f*x + e) + 1) - ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4
*b - 2*a^2*b^3 + b^5)*d^2)*log(-sin(f*x + e) + 1) - 2*((a^2*b^3 - b^5)*c^2
- 2*(a^3*b^2 - a*b^4)*c*d + (a^4*b - a^2*b^3)*d^2)*sin(f*x + e))/((a^5*b^2
- 2*a^3*b^4 + a*b^6)*f*cos(f*x + e) + (a^4*b^3 - 2*a^2*b^5 + b^7)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c+d\sec(e+fx))^2\sec(e+fx)}{(a+b\sec(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))**2/(a+b*sec(f*x+e))**2,x)
[Out] Integral((c + d*sec(e + f*x))**2*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="ma
xima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.35

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx$$
$$= \frac{\frac{d^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right|\right)}{b^2} - \frac{d^2\log\left(\left|\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right|\right)}{b^2} - \frac{2\left(ab^2c^2-2b^3cd-a^3d^2+2ab^2d^2\right)\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}f\right.}{}\right.\right.}{(a^2b^2-b^4)\sqrt{-a^2+b^2}}}{f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))^2/(a+b*sec(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (d^2*log(abs(tan(1/2*f*x + 1/2*e) + 1))/b^2 - d^2*log(abs(tan(1/2*f*x + 1/2
*e) - 1))/b^2 - 2*(a*b^2*c^2 - 2*b^3*c*d - a^3*d^2 + 2*a*b^2*d^2)*(pi*floor
(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) -
b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2
)) + 2*(b^2*c^2*tan(1/2*f*x + 1/2*e) - 2*a*b*c*d*tan(1/2*f*x + 1/2*e) + a^2
*d^2*tan(1/2*f*x + 1/2*e))/((a^2*b - b^3)*(a*tan(1/2*f*x + 1/2*e)^2 - b*tan
(1/2*f*x + 1/2*e)^2 - a - b)))/f
```

Mupad [B] (verification not implemented)

Time = 21.29 (sec) , antiderivative size = 4926, normalized size of antiderivative = 24.88

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))^2}{(a+b\sec(e+fx))^2}\,dx = \text{Too large to display}$$

```
[In] int((c + d/cos(e + f*x))^2/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] - (d^2*atan(((d^2*((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^
4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d
^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^
2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (d^
2*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4
*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*
c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (
32*d^2*tan(e/2 + (f*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*
a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2)*1i)/b^
2 + (d^2*((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5
*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^
6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b
^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (d^2*((32*(a
*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2
+ a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a
^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*d^2*ta
n(e/2 + (f*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 -
 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2)*1i)/b^2)/((64*(
a^5*d^6 + 2*a*b^4*d^6 - a^4*b*d^6 - 2*b^5*c*d^5 + 2*a^2*b^3*d^6 - 3*a^3*b^2
*d^6 + 4*b^5*c^2*d^4 + a*b^4*c^2*d^4 - 4*a*b^4*c^3*d^3 + 2*a^2*b^3*c*d^5 +
2*a^3*b^2*c*d^5 - a^4*b*c^2*d^4 + 3*a^2*b^3*c^2*d^4 + a^2*b^3*c^4*d^2 - a^3
*b^2*c^2*d^4 - 6*a*b^4*c*d^5))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (d^2*((3
2*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2
*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 +
4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4
*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + (d^2*((32*(a*b^8*c^2 - b
```

^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^
2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d -
 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*d^2*tan(e/2 + (f*x
)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))
/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2))/b^2 + (d^2*((32*tan(e/2 +
(f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3
*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*
d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d
))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (d^2*((32*(a*b^8*c^2 - b^9*d^2 + 2*a
*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^
6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c
*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*d^2*tan(e/2 + (f*x)/2)*(2*a*b^
9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4
 + b^5 - a^2*b^3 - a^3*b^2))))/b^2))/b^2))*2i)/(b^2*f) - (2*tan(e/2 + (f*x)
/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(f*(a + b)*(a*b - b^2)*(a + b - tan(e/
2 + (f*x)/2)^2*(a - b))) - (atan(((((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6
*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*
d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 -
 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3
 - a^3*b^2) + ((a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*c^2 - b^
9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2
 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d -
2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*tan(e/2 + (f*x)/2)*
(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*(2*a*b^9
- 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5
 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2*d - 2*
b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a*d - b*c)*((a +
b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*1i)/(b^8 - 3*a^2*b^6 + 3*a^
4*b^4 - a^6*b^2) + (((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*
d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2
*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*
d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - (
(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8
*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^
2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))
/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((a
 + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*(2*a*b^9 - 2*a^2*b^8 - 4
*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^
3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2*d - 2*b^2*d + a*b*c))
/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*(a*d - b*c)*((a + b)^3*(a - b)^3)
^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2
))/((64*(a^5*d^6 + 2*a*b^4*d^6 - a^4*b*d^6 - 2*b^5*c*d^5 + 2*a^2*b^3*d^6 -
3*a^3*b^2*d^6 + 4*b^5*c^2*d^4 + a*b^4*c^2*d^4 - 4*a*b^4*c^3*d^3 + 2*a^2*b^3
*c*d^5 + 2*a^3*b^2*c*d^5 - a^4*b*c^2*d^4 + 3*a^2*b^3*c^2*d^4 + a^2*b^3*c^4*
d^2 - a^3*b^2*c^2*d^4 - 6*a*b^4*c*d^5))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) -

```
(((32*tan(e/2 + (f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4
+ a^2*b^4*c^4 + 3*a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d
^2 + 4*a^3*b^3*c*d^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^
3 - 4*a*b^5*c^3*d))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) + ((a*d - b*c)*((a +
b)^3*(a - b)^3)^(1/2)*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2
 - a^3*b^6*c^2 + a^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 -
2*b^9*c*d + 2*a*b^8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^
2*b^4 - a^3*b^3) + (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((a + b)^3*(a - b)^3)
^(1/2)*(a^2*d - 2*b^2*d + a*b*c)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b
^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^
2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6
+ 3*a^4*b^4 - a^6*b^2))*(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*
b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (((32*tan(e/2 + (
f*x)/2)*(2*a^6*d^4 + b^6*d^4 - 2*a*b^5*d^4 - 2*a^5*b*d^4 + a^2*b^4*c^4 + 3*
a^2*b^4*d^4 + 4*a^3*b^3*d^4 - 5*a^4*b^2*d^4 + 4*b^6*c^2*d^2 + 4*a^3*b^3*c*d
^3 + 4*a^2*b^4*c^2*d^2 - 2*a^4*b^2*c^2*d^2 - 8*a*b^5*c*d^3 - 4*a*b^5*c^3*d)
)/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2) - ((a*d - b*c)*((a + b)^3*(a - b)^3)^(1
/2)*((32*(a*b^8*c^2 - b^9*d^2 + 2*a*b^8*d^2 - a^2*b^7*c^2 - a^3*b^6*c^2 + a
^4*b^5*c^2 + a^2*b^7*d^2 - 3*a^3*b^6*d^2 + a^5*b^4*d^2 - 2*b^9*c*d + 2*a*b^
8*c*d + 2*a^2*b^7*c*d - 2*a^3*b^6*c*d))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) -
 (32*tan(e/2 + (f*x)/2)*(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*
b^2*d + a*b*c)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2
*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4
- a^6*b^2)))*(a^2*d - 2*b^2*d + a*b*c))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*
b^2))*(a*d - b*c)*((a + b)^3*(a - b)^3)^(1/2)*(a^2*d - 2*b^2*d + a*b*c))/(b
^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*(a*d - b*c)*((a + b)^3*(a - b)^3)^(
1/2)*(a^2*d - 2*b^2*d + a*b*c)*2i)/(f*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^
2))
```

3.262 $\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 100

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}f} - \frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}$$

```
[Out] 2*(a*c-b*d)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/2))/(a-b)^(3/2)
/(a+b)^(3/2)/f-(-a*d+b*c)*tan(f*x+e)/(a^2-b^2)/f/(a+b*sec(f*x+e))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4088, 12, 3916, 2738, 214}

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}} - \frac{(bc-ad)\tan(e+fx)}{f\,(a^2-b^2)\,(a+b\sec(e+fx))}$$

```
[In] Int[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] (2*(a*c - b*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*f) - ((b*c - a*d)*Tan[e + f*x])/((a^2 - b^2)*f*(a + b
*Sec[e + f*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}+\frac{\int\frac{(-ac+bd)\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{-a^2+b^2}\\
&= -\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}+\frac{(ac-bd)\int\frac{\sec(e+fx)}{a+b\sec(e+fx)}\,dx}{a^2-b^2}\\
&= -\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}+\frac{(ac-bd)\int\frac{1}{1+\frac{a\cos(e+fx)}{b}}\,dx}{b\,(a^2-b^2)}\\
&= -\frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))}+\frac{(2(ac-bd))\text{Subst}\left(\int\frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(e+fx)\right)\right)}{b\,(a^2-b^2)\,f}
\end{aligned}$$

$$
= \frac{2(ac-bd)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}f} - \frac{(bc-ad)\tan(e+fx)}{(a^2-b^2)\, f(a+b\sec(e+fx))}
$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$
\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \frac{-\frac{2(ac-bd)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + \frac{(-bc+ad)\sin(e+fx)}{(a-b)(a+b)(b+a\cos(e+fx))}}{f}
$$

```
[In] Integrate[(Sec[e + f*x]*(c + d*Sec[e + f*x]))/(a + b*Sec[e + f*x])^2,x]
```

```
[Out] ((-2*(a*c - b*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2) + ((-(b*c) + a*d)*Sin[e + f*x])/((a - b)*(a + b)*(b + a*Cos[e
 + f*x])))/f
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{-\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 b-a-b\right)}+\frac{2(ac-bd)\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{f}$
default	$\frac{-\frac{2(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2 b-a-b\right)}+\frac{2(ac-bd)\,\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}}{f}$
risch	$\frac{2i(ad-bc)\left(e^{i(fx+e)}b+a\right)}{a(a^2-b^2)f\left(e^{2i(fx+e)}a+2\,e^{i(fx+e)}b+a\right)} + \frac{\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)ac}{\sqrt{a^2-b^2}\,(a+b)(a-b)f} - \frac{\ln\left(e^{i(fx+e)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)f}$

```
[In] int(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-2*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(
1/2*f*x+1/2*e)^2*b-a-b)+2*(a*c-b*d)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh
((a-b)*tan(1/2*f*x+1/2*e)/((a-b)*(a+b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.94

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx$$
$$= \left[\frac{(abc-b^2d+(a^2c-abd)\cos(fx+e))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(fx+e)-(a^2-2\,b^2)\cos(fx+e)^2+2\,\sqrt{a^2-b^2}(b\cos(fx+e)+a)\sin(}{a^2\cos(fx+e)^2+2\,ab\cos(fx+e)+b^2}\right.}{2\,((a^5-2\,a^3b^2+ab^4)f\cos(fx+e)+(a^4b-2\,a^2b^3+}\right.$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((a*b*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2
*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(f*x + e)^2 + 2*sqrt(a^2 - b^2)*(b*cos
(f*x + e) + a)*sin(f*x + e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(
f*x + e) + b^2)) - 2*((a^2*b - b^3)*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^
5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*f), ((a*b
*c - b^2*d + (a^2*c - a*b*d)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) - ((a^2*b - b^3)
*c - (a^3 - a*b^2)*d)*sin(f*x + e))/((a^5 - 2*a^3*b^2 + a*b^4)*f*cos(f*x +
e) + (a^4*b - 2*a^2*b^3 + b^5)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \int \frac{(c+d\sec(e+fx))\sec(e+fx)}{(a+b\sec(e+fx))^2}\,dx$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**2,x)
```

```
[Out] Integral((c + d*sec(e + f*x))*sec(e + f*x)/(a + b*sec(e + f*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.73

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor \mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-a^2+b^2}}\right)\right)(ac-bd)}{(a^2-b^2)\sqrt{-a^2+b^2}} - \frac{bc\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-ad\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-a-b\right)(a^2-b^2)}\right)}{f}$$

```
[In] integrate(sec(f*x+e)*(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*
x + 1/2*e) - b*tan(1/2*f*x + 1/2*e))/sqrt(-a^2 + b^2)))*(a*c - b*d)/((a^2 -
 b^2)*sqrt(-a^2 + b^2)) - (b*c*tan(1/2*f*x + 1/2*e) - a*d*tan(1/2*f*x + 1/2
*e))/((a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e)^2 - a - b)*(a^2 -
b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 13.75 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{\sec(e+fx)(c+d\sec(e+fx))}{(a+b\sec(e+fx))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\sqrt{a-b}}{\sqrt{a+b}}\right)\,(a\,c-b\,d)}{f\,(a+b)^{3/2}\,(a-b)^{3/2}} + \frac{2\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)\,(a\,d-b\,c)}{f\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(b-a\right)\,\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+a+b\right)}$$

```
[In] int((c + d/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^2),x)
```

```
[Out] (2*atanh((tan(e/2 + (f*x)/2)*(a - b)^(1/2))/(a + b)^(1/2))*(a*c - b*d))/(f*
(a + b)^(3/2)*(a - b)^(3/2)) + (2*tan(e/2 + (f*x)/2)*(a*d - b*c))/(f*(a + b
)*(a - b)*(a + b - tan(e/2 + (f*x)/2)^2*(a - b)))
```

3.263 $\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 186

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \frac{2b(abc-2a^2d+b^2d)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2f} + \frac{2d^2\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}(bc-ad)^2f} - \frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))}$$

```
[Out] 2*b*(-2*a^2*d+a*b*c+b^2*d)*arctanh((a-b)^(1/2)*tan(1/2*f*x+1/2*e)/(a+b)^(1/
2))/(a-b)^(3/2)/(a+b)^(3/2)/(-a*d+b*c)^2/f-b^2*sin(f*x+e)/(a^2-b^2)/(-a*d+b
*c)/f/(b+a*cos(f*x+e))+2*d^2*arctanh((c-d)^(1/2)*tan(1/2*f*x+1/2*e)/(c+d)^(
1/2))/(-a*d+b*c)^2/f/(c-d)^(1/2)/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {4073, 3135, 3080, 2738, 214}

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \frac{2b(-2a^2d+abc+b^2d)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{f(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2} - \frac{b^2\sin(e+fx)}{f(a^2-b^2)(bc-ad)(a\cos(e+fx)+b)} + \frac{2d^2\operatorname{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{f\sqrt{c-d}\sqrt{c+d}(bc-ad)^2}$$

```
[In] Int[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[(Sqrt[a - b]*Tan[(e + f*x)/2])/Sqrt[
a + b]])/((a - b)^(3/2)*(a + b)^(3/2)*(b*c - a*d)^2*f) + (2*d^2*ArcTanh[(Sq
rt[c - d]*Tan[(e + f*x)/2])/Sqrt[c + d]])/(Sqrt[c - d]*Sqrt[c + d]*(b*c - a
*d)^2*f) - (b^2*Sin[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*(b + a*Cos[e + f*x
]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 4073

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_), x_Symbol] :> Dist[1
```

```
/g^(m + n), Int[(g*Csc[e + f*x])^(m + n + p)*(b + a*Sin[e + f*x])^m*(d + c*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
a*d, 0] && IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cos^2(e+fx)}{(b+a\cos(e+fx))^2(d+c\cos(e+fx))}\,dx \\
&= -\frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))} - \frac{\int \frac{-abd-(abc-a^2d+b^2d)\cos(e+fx)}{(b+a\cos(e+fx))(d+c\cos(e+fx))}\,dx}{(a^2-b^2)(bc-ad)} \\
&= -\frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))} + \frac{d^2\int \frac{1}{d+c\cos(e+fx)}\,dx}{(bc-ad)^2} \\
&\quad + \frac{(b(abc-2a^2d+b^2d))\int \frac{1}{b+a\cos(e+fx)}\,dx}{(a^2-b^2)(bc-ad)^2} \\
&= -\frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))} \\
&\quad + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{c+d+(-c+d)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(bc-ad)^2f} \\
&\quad + \frac{(2b(abc-2a^2d+b^2d))\,\text{Subst}\left(\int \frac{1}{a+b+(-a+b)x^2}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{(a^2-b^2)(bc-ad)^2f} \\
&= \frac{2b(abc-2a^2d+b^2d)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}(bc-ad)^2f} \\
&\quad + \frac{2d^2\text{arctanh}\left(\frac{\sqrt{c-d}\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}(bc-ad)^2f} - \frac{b^2\sin(e+fx)}{(a^2-b^2)(bc-ad)f(b+a\cos(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.95

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \frac{\frac{2b(abc-2a^2d+b^2d)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2(a^2-b^2)d^2\text{arctanh}\left(\frac{(-c+d)\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{b^2(bc-ad)\sin(e+fx)}{b+a\cos(e+fx)}}{(-a+b)(a+b)(bc-ad)^2f}$$

```
[In] Integrate[Sec[e + f*x]/((a + b*Sec[e + f*x])^2*(c + d*Sec[e + f*x])),x]
```

[Out] ((2*b*(a*b*c - 2*a^2*d + b^2*d)*ArcTanh[((-a + b)*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (2*(a^2 - b^2)*d^2*ArcTanh[((-c + d)*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + (b^2*(b*c - a*d)*Sin[e + f*x])/(b + a*Cos[e + f*x]))/((-a + b)*(a + b)*(b*c - a*d)^2*f)

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{2b\left(-\frac{b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2d-abc-d\,b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{(ad-bc)^2}+\frac{2d^2\operatorname{arctanh}\left(\frac{(c-d}{\sqrt{}}\right.}{(ad-bc)^2\sqrt{(}}}{f}$
default	$\frac{\frac{2b\left(-\frac{b(ad-bc)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\left(a^2-b^2\right)\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2a-\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^2b-a-b\right)}-\frac{\left(2a^2d-abc-d\,b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}\right)}{(ad-bc)^2}+\frac{2d^2\operatorname{arctanh}\left(\frac{(c-d}{\sqrt{}}\right.}{(ad-bc)^2\sqrt{(}}}{f}$
risch	$\frac{2ib^2\left(e^{i(fx+e)}b+a\right)}{a(a^2-b^2)(ad-bc)f\left(e^{2i(fx+e)}a+2\,e^{i(fx+e)}b+a\right)}+\frac{2b\ln\left(e^{i(fx+e)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{\sqrt{a^2-b^2}\,a}\right)a^2d}{\sqrt{a^2-b^2}\,(ad-bc)^2(a+b)(a-b)f}-\frac{b^2\ln\left(e^{i(fx+e)}-\frac{ia^2-}{}\right.}{\sqrt{a^2-b^2}\,(ad-bc)^2}$

[In] int(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/f*(2*b/(a*d-b*c)^2*(-b*(a*d-b*c)/(a^2-b^2)*tan(1/2*f*x+1/2*e)/(tan(1/2*f*x+1/2*e)^2*a-tan(1/2*f*x+1/2*e)^2*b-a-b)-(2*a^2*d-a*b*c-b^2*d)/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctanh((a-b)*tan(1/2*f*x+1/2*e)/((a-b)*(a+b))^(1/2)))+2*d^2/(a*d-b*c)^2/((c+d)*(c-d))^(1/2)*arctanh((c-d)*tan(1/2*f*x+1/2*e)/((c+d)*(c-d))^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(168) = 336.

Time = 60.79 (sec) , antiderivative size = 2852, normalized size of antiderivative = 15.33

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="fricas")

[Out] [-1/2*((a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3 + (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3)*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2

```
- 2*b^2)*cos(f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x +
e) + 2*a^2 - b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - ((a^5
- 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt
(c^2 - d^2)*log((2*c*d*cos(f*x + e) - (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt
(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x +
 e)^2 + 2*c*d*cos(f*x + e) + d^2)) + 2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*
b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d^3)*sin(f*x + e))/(
((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d
+ (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^
2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f*cos(f*x + e) + ((a^4*b^3
- 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3
 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f), 1/2*(2*((a^5 - 2*a^3*b^2 + a*b^4)
*d^2*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(-c^2 + d^2)*arctan(
-sqrt(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - (a*b^3
*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3 + (a^2
*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3)
*cos(f*x + e))*sqrt(a^2 - b^2)*log((2*a*b*cos(f*x + e) - (a^2 - 2*b^2)*cos(
f*x + e)^2 - 2*sqrt(a^2 - b^2)*(b*cos(f*x + e) + a)*sin(f*x + e) + 2*a^2 -
b^2)/(a^2*cos(f*x + e)^2 + 2*a*b*cos(f*x + e) + b^2)) - 2*((a^2*b^3 - b^5)*
c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a*b^4)*d
^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2*a^4*b^
3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 + 2*(a^6
*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f*cos(f*
x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)
*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2 - 2*a^3
*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f), 1/2*(2*(a*b^3*
c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4)*d^3 + (a^2*
b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b - a*b^3)*d^3)*
cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(f*x + e) + a
)/((a^2 - b^2)*sin(f*x + e))) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2*cos(f*x + e)
 + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(c^2 - d^2)*log((2*c*d*cos(f*x + e) -
 (c^2 - 2*d^2)*cos(f*x + e)^2 + 2*sqrt(c^2 - d^2)*(d*cos(f*x + e) + c)*sin(
f*x + e) + 2*c^2 - d^2)/(c^2*cos(f*x + e)^2 + 2*c*d*cos(f*x + e) + d^2)) -
2*((a^2*b^3 - b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 +
(a^3*b^2 - a*b^4)*d^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 -
2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6)*c^2*d^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3
*b^4)*d^4)*f*cos(f*x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 -
 2*a^3*b^4 + a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 +
 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)
*f), ((a*b^3*c^3 - a*b^3*c*d^2 - (2*a^2*b^2 - b^4)*c^2*d + (2*a^2*b^2 - b^4
)*d^3 + (a^2*b^2*c^3 - a^2*b^2*c*d^2 - (2*a^3*b - a*b^3)*c^2*d + (2*a^3*b -
 a*b^3)*d^3)*cos(f*x + e))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos
(f*x + e) + a)/((a^2 - b^2)*sin(f*x + e))) + ((a^5 - 2*a^3*b^2 + a*b^4)*d^2
```

```
*cos(f*x + e) + (a^4*b - 2*a^2*b^3 + b^5)*d^2)*sqrt(-c^2 + d^2)*arctan(-sqr
t(-c^2 + d^2)*(d*cos(f*x + e) + c)/((c^2 - d^2)*sin(f*x + e))) - ((a^2*b^3
- b^5)*c^3 - (a^3*b^2 - a*b^4)*c^2*d - (a^2*b^3 - b^5)*c*d^2 + (a^3*b^2 - a
*b^4)*d^3)*sin(f*x + e))/(((a^5*b^2 - 2*a^3*b^4 + a*b^6)*c^4 - 2*(a^6*b - 2
*a^4*b^3 + a^2*b^5)*c^3*d + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*c^2*d^2 +
 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*c*d^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d^4)*f
*cos(f*x + e) + ((a^4*b^3 - 2*a^2*b^5 + b^7)*c^4 - 2*(a^5*b^2 - 2*a^3*b^4 +
 a*b^6)*c^3*d + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*c^2*d^2 + 2*(a^5*b^2
- 2*a^3*b^4 + a*b^6)*c*d^3 - (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^4)*f)]
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2\,(c+d\sec(e+fx))}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))**2/(c+d*sec(f*x+e)),x)
[Out] Integral(sec(e + f*x)/((a + b*sec(e + f*x))**2*(c + d*sec(e + f*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="maxi
ma")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.77

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \frac{2\left(\frac{\left(\pi\left\lfloor\frac{fx+e}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2c+2d)+\arctan\left(-\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{\sqrt{-c^2+d^2}}\right)\right)d^2}{(b^2c^2-2abcd+a^2d^2)\sqrt{-c^2+d^2}}+\frac{b^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}{(a^2bc-b^3c-a^3d+ab^2d)\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-b\tan\left(\frac{1}{2}fx-\right.\right.}}{f}$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^2/(c+d*sec(f*x+e)),x, algorithm="giac
")
```

```
[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*f
*x + 1/2*e) - d*tan(1/2*f*x + 1/2*e))/sqrt(-c^2 + d^2)))*d^2/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(-c^2 + d^2)) + b^2*tan(1/2*f*x + 1/2*e)/((a^2*b*c -
 b^3*c - a^3*d + a*b^2*d)*(a*tan(1/2*f*x + 1/2*e)^2 - b*tan(1/2*f*x + 1/2*e
)^2 - a - b)) - (a*b^2*c - 2*a^2*b*d + b^3*d)*(pi*floor(1/2*(f*x + e)/pi +
1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*f*x + 1/2*e) - b*tan(1/2*f*x + 1/2*
e))/sqrt(-a^2 + b^2)))/((a^2*b^2*c^2 - b^4*c^2 - 2*a^3*b*c*d + 2*a*b^3*c*d
+ a^4*d^2 - a^2*b^2*d^2)*sqrt(-a^2 + b^2)))/f
```

Mupad [B] (verification not implemented)

Time = 27.48 (sec) , antiderivative size = 20827, normalized size of antiderivative = 111.97

$$\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^2(c+d\sec(e+fx))}\,dx = \text{Too large to display}$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^2*(c + d/cos(e + f*x))),x)
```

```
[Out] (d^2*atan(((d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*
d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5
*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2
- 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d -
8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 +
 a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2
*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/
(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^
2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3
*b^2*c*d) + (d^2*(c^2 - d^2)^(1/2)*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7
+ 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b
```

^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) + (32*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)*(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d))))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d) + (d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) - (d^2*(c^2 - d^2)^(1/2)

((32(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) - (32*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)*(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d))))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))*1i)/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d))/((64*(b^5*d^5 - a*b^4*d^5 + 2*a^4*b*d^5 - b^5*c*d^4 - 3*a^2*b^3*d^5 + 2*a^3*b^2*d^5 - 2*a*b^4*c^2*d^3 + 2*a^2*b^3*c*d^4 - 5*a^3*b^2*c*d^4 + 2*a^2*b^3*c^2*d^3 - a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 + 3*a*b^4*c*d^4 - 2*a^4*b*c*d^4))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) + (d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4

*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) + (d^2*(c^2 - d^2)^(1/2)*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) + (32*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)*(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d))))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)

```
- (d^2*(c^2 - d^2)^(1/2)*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a
*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*
d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5
*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3
*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*
c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*
a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2
- b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2
 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)
 - (d^2*(c^2 - d^2)^(1/2)*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c
*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 -
3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4
*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^
3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*
b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^
2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*
c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*
a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d
^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b
^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 -
27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2
*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^
6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3
- a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3
*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*
d - 3*a^5*b*c*d^2) - (32*d^2*tan(e/2 + (f*x)/2)*(c^2 - d^2)^(1/2)*(2*a^10*c
*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7
*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^
5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5
+ 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a
*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4
*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*
b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*
c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*
b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 -
24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c
^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a
^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^
5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b
^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 +
2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d
 + 2*a^9*b*c*d^6))/((a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*
c*d^3 + 2*a*b*c^3*d)*(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c
^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d +
2*a^2*b^3*c*d - 2*a^3*b^2*c*d))))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2*c^
```

2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))/(a^2*d^4 - b^2*c^4 - a^2*c^2*d^2 + b^2
*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)))*(c^2 - d^2)^(1/2)*2i)/(f*(a^2*d^4 -
b^2*c^4 - a^2*c^2*d^2 + b^2*c^2*d^2 - 2*a*b*c*d^3 + 2*a*b*c^3*d)) + (2*b^2
*tan(e/2 + (f*x)/2))/(f*(a + b)*(a + b - tan(e/2 + (f*x)/2)^2*(a - b))*(a^2
*d + b^2*c - a*b*c - a*b*d)) + (b*atan(((b*((32*tan(e/2 + (f*x)/2)*(a^6*d^5
+ 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^
4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6
*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4
*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*
c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^
4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b
*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2
*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*
d - 2*a^3*b^2*c*d) + (b*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d
^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*
a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a
*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*
b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^
4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*
d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^
4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^
3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4
+ 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4
*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27
*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d
^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6)
)/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 -
a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c
*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d
- 3*a^5*b*c*d^2) + (32*b*tan(e/2 + (f*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(b^
2*d - 2*a^2*d + a*b*c)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c
^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^
6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 +
2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*
c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*
c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c
*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*
d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^
3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7
*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 3
0*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^
5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5
*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3
- 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^
3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12

*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^5*d^2 - b^5*c^2 - a*
b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2
 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)*(a^8*d^2 - b^
8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b
^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*
b^3*c*d)))*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2
- b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a
^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*
a^5*b^3*c*d))*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*1i)/(a^
8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2
 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*
d + 6*a^5*b^3*c*d) + (b*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*
b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d
^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*
c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*
c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c
^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a
^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 -
 b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2
- a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)
- (b*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^
2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b
^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*
b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b
^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4
*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d
^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*
c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a
^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^
3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b
^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 -
4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3
*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^
3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b
^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^
2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) -
(32*b*tan(e/2 + (f*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b
c)(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^
7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b
^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*
a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c
^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*
c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c
^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6
*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*

d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*1i)/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))/((64*(b^5*d^5 - a*b^4*d^5 + 2*a^4*b*d^5 - b^5*c*d^4 - 3*a^2*b^3*d^5 + 2*a^3*b^2*d^5 - 2*a*b^4*c^2*d^3 + 2*a^2*b^3*c*d^4 - 5*a^3*b^2*c*d^4 + 2*a^2*b^3*c^2*d^3 - a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 + 3*a*b^4*c*d^4 - 2*a^4*b*c*d^4))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) + (b*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 + 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) + (b*((32*(a*b^8*c^7 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 + 10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c

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^3*d^4 - 10*a^7*b^2*c^4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*
c^3 + a*b^5*c^3 + a^5*b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4
*b^2*d^3 + 3*a^2*b^4*c*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*
c^2*d - 3*a^4*b^2*c*d^2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2)
+ (32*b*tan(e/2 + (f*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a
*b*c)*(2*a^10*c*d^6 - 2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*
c^7 + 4*a^3*b^7*c^7 - 4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4
*b^6*d^7 - 2*a^5*b^5*d^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 -
4*a^10*c^2*d^5 + 2*a^10*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9
*c^3*d^4 + 14*a*b^9*c^4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^
7*c^6*d + 4*a^4*b^6*c*d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5
*c^6*d - 6*a^6*b^4*c*d^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c
^6*d - 6*a^9*b*c^2*d^5 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^
2*d^5 - 16*a^2*b^8*c^3*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3
*b^7*c^3*d^4 - 24*a^3*b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5
 + 36*a^4*b^6*c^3*d^4 - 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^
5*c^2*d^5 - 2*a^5*b^5*c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20
*a^6*b^4*c^2*d^5 - 22*a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5
*d^2 + 16*a^7*b^3*c^2*d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7
*b^3*c^5*d^2 + 2*a^8*b^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2
+ 2*a*b^9*c^6*d + 2*a^9*b*c*d^6))/((a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d
^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d -
2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d)*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6
*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^
2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*((a +
b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2
*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^
6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*((a
 + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*
a^2*b^6*c^2 - 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3
*a^6*b^2*d^2 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d) -
 (b*((32*tan(e/2 + (f*x)/2)*(a^6*d^5 + 2*b^6*d^5 - 2*a*b^5*d^5 - 2*a^5*b*d^
5 - a^6*c*d^4 - 4*b^6*c*d^4 - a^2*b^4*c^5 - 5*a^2*b^4*d^5 + 4*a^3*b^3*d^5 +
 3*a^4*b^2*d^5 + 3*b^6*c^2*d^3 - b^6*c^3*d^2 - 6*a*b^5*c^2*d^3 + 6*a*b^5*c^
3*d^2 + 13*a^2*b^4*c*d^4 + 3*a^2*b^4*c^4*d - 8*a^3*b^3*c*d^4 + 4*a^3*b^3*c^
4*d - 11*a^4*b^2*c*d^4 - 11*a^2*b^4*c^2*d^3 + a^2*b^4*c^3*d^2 + 12*a^3*b^3*
c^2*d^3 - 12*a^3*b^3*c^3*d^2 + 12*a^4*b^2*c^2*d^3 - 4*a^4*b^2*c^3*d^2 + 4*a
*b^5*c*d^4 - 2*a*b^5*c^4*d + 2*a^5*b*c*d^4))/(a^5*d^2 - b^5*c^2 - a*b^4*c^2
 + a^4*b*d^2 + a^2*b^3*c^2 + a^3*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*
b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^3*c*d - 2*a^3*b^2*c*d) - (b*((32*(a*b^8*c^7
 - a^9*d^7 + 2*a^8*b*d^7 + 2*a^9*c*d^6 + b^9*c^6*d - a^2*b^7*c^7 - a^3*b^6*
c^7 + a^4*b^5*c^7 + a^4*b^5*d^7 - 3*a^6*b^3*d^7 + a^7*b^2*d^7 - a^9*c^2*d^5
 + b^9*c^4*d^3 - 2*b^9*c^5*d^2 - 4*a*b^8*c^3*d^4 + 8*a*b^8*c^4*d^3 - 3*a*b^
8*c^5*d^2 - 5*a^2*b^7*c^6*d - 4*a^3*b^6*c*d^6 + 7*a^3*b^6*c^6*d - 2*a^4*b^5
*c*d^6 + 4*a^4*b^5*c^6*d + 13*a^5*b^4*c*d^6 - 5*a^5*b^4*c^6*d + a^6*b^3*c*d
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^6 - 11*a^7*b^2*c*d^6 - 8*a^8*b*c^2*d^5 + 5*a^8*b*c^3*d^4 + 6*a^2*b^7*c^2*d
^5 - 12*a^2*b^7*c^3*d^4 - a^2*b^7*c^4*d^3 + 13*a^2*b^7*c^5*d^2 + 8*a^3*b^6*
c^2*d^5 + 14*a^3*b^6*c^3*d^4 - 31*a^3*b^6*c^4*d^3 + 7*a^3*b^6*c^5*d^2 - 21*
a^4*b^5*c^2*d^5 + 34*a^4*b^5*c^3*d^4 + 4*a^4*b^5*c^4*d^3 - 21*a^4*b^5*c^5*d
^2 - 16*a^5*b^4*c^2*d^5 - 21*a^5*b^4*c^3*d^4 + 33*a^5*b^4*c^4*d^3 - 4*a^5*b
^4*c^5*d^2 + 23*a^6*b^3*c^2*d^5 - 27*a^6*b^3*c^3*d^4 - 4*a^6*b^3*c^4*d^3 +
10*a^6*b^3*c^5*d^2 + 9*a^7*b^2*c^2*d^5 + 11*a^7*b^2*c^3*d^4 - 10*a^7*b^2*c^
4*d^3 - 2*a*b^8*c^6*d + a^8*b*c*d^6))/(a^6*d^3 + b^6*c^3 + a*b^5*c^3 + a^5*
b*d^3 - a^2*b^4*c^3 - a^3*b^3*c^3 - a^3*b^3*d^3 - a^4*b^2*d^3 + 3*a^2*b^4*c
*d^2 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 + 3*a^3*b^3*c^2*d - 3*a^4*b^2*c*d^
2 + 3*a^4*b^2*c^2*d - 3*a*b^5*c^2*d - 3*a^5*b*c*d^2) - (32*b*tan(e/2 + (f*x
)/2)*((a + b)^3*(a - b)^3)^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*(2*a^10*c*d^6 -
2*a^9*b*d^7 - 2*a*b^9*c^7 + 2*b^10*c^6*d + 2*a^2*b^8*c^7 + 4*a^3*b^7*c^7 -
4*a^4*b^6*c^7 - 2*a^5*b^5*c^7 + 2*a^6*b^4*c^7 + 2*a^4*b^6*d^7 - 2*a^5*b^5*d
^7 - 4*a^6*b^4*d^7 + 4*a^7*b^3*d^7 + 2*a^8*b^2*d^7 - 4*a^10*c^2*d^5 + 2*a^1
0*c^3*d^4 + 2*b^10*c^4*d^3 - 4*b^10*c^5*d^2 - 8*a*b^9*c^3*d^4 + 14*a*b^9*c^
4*d^3 - 6*a*b^9*c^5*d^2 - 8*a^3*b^7*c*d^6 - 12*a^3*b^7*c^6*d + 4*a^4*b^6*c*
d^6 - 6*a^4*b^6*c^6*d + 18*a^5*b^5*c*d^6 + 18*a^5*b^5*c^6*d - 6*a^6*b^4*c*d
^6 + 4*a^6*b^4*c^6*d - 12*a^7*b^3*c*d^6 - 8*a^7*b^3*c^6*d - 6*a^9*b*c^2*d^5
 + 14*a^9*b*c^3*d^4 - 8*a^9*b*c^4*d^3 + 12*a^2*b^8*c^2*d^5 - 16*a^2*b^8*c^3
*d^4 + 2*a^2*b^8*c^5*d^2 + 4*a^3*b^7*c^2*d^5 + 20*a^3*b^7*c^3*d^4 - 24*a^3*
b^7*c^4*d^3 + 16*a^3*b^7*c^5*d^2 - 30*a^4*b^6*c^2*d^5 + 36*a^4*b^6*c^3*d^4
- 22*a^4*b^6*c^4*d^3 + 20*a^4*b^6*c^5*d^2 - 14*a^5*b^5*c^2*d^5 - 2*a^5*b^5*
c^3*d^4 - 2*a^5*b^5*c^4*d^3 - 14*a^5*b^5*c^5*d^2 + 20*a^6*b^4*c^2*d^5 - 22*
a^6*b^4*c^3*d^4 + 36*a^6*b^4*c^4*d^3 - 30*a^6*b^4*c^5*d^2 + 16*a^7*b^3*c^2*
d^5 - 24*a^7*b^3*c^3*d^4 + 20*a^7*b^3*c^4*d^3 + 4*a^7*b^3*c^5*d^2 + 2*a^8*b
^2*c^2*d^5 - 16*a^8*b^2*c^4*d^3 + 12*a^8*b^2*c^5*d^2 + 2*a*b^9*c^6*d + 2*a^
9*b*c*d^6))/((a^5*d^2 - b^5*c^2 - a*b^4*c^2 + a^4*b*d^2 + a^2*b^3*c^2 + a^3
*b^2*c^2 - a^2*b^3*d^2 - a^3*b^2*d^2 + 2*a*b^4*c*d - 2*a^4*b*c*d + 2*a^2*b^
3*c*d - 2*a^3*b^2*c*d)*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c^2 +
 a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*d -
2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*((a + b)^3*(a - b)^3)^(1/2)*
(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^4*c
^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7*c*
d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))*((a + b)^3*(a - b)^3)^(1/
2)*(b^2*d - 2*a^2*d + a*b*c))/(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2 - 3*a^4*b^
4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2 + 2*a*b^7
*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d)))*((a + b)^3*(a - b)^3)
^(1/2)*(b^2*d - 2*a^2*d + a*b*c)*2i)/(f*(a^8*d^2 - b^8*c^2 + 3*a^2*b^6*c^2
- 3*a^4*b^4*c^2 + a^6*b^2*c^2 - a^2*b^6*d^2 + 3*a^4*b^4*d^2 - 3*a^6*b^2*d^2
 + 2*a*b^7*c*d - 2*a^7*b*c*d - 6*a^3*b^5*c*d + 6*a^5*b^3*c*d))
```

3.264 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df} - \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/d/f-2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/
f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{4054, 3917, 4058\}$

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

$$= \frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}$$
$$-\frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{df(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4054

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d, Int[Csc[e + f*
x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Csc[e + f*x]/
(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df} - \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 4.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$

$$= \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)\right.}{(c-d)(c+d)f(b+a\cos(e+}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/
((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A] (verified)

Time = 9.15 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

method	result
default	$-\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)ac+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)ad-\text{EllipticF}\left(\cot(fx+e)\right.\right.}{}$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/f/(c+d)/(c-d)*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+
b))^(1/2))*a*c+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*d-Ell
ipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c-EllipticF(cot(f*x+e)-
csc(f*x+e),((a-b)/(a+b))^(1/2))*b*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d
```

```
)/(c+d),((a-b)/(a+b))^(1/2))*a*d+2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(
c+d),((a-b)/(a+b))^(1/2))*b*c)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+b\sec(e+fx)}\sec(e+fx)}{c+d\sec(e+fx)}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}\sec(fx+e)}{d\sec(fx+e)+c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{d\sec\left(fx+e\right)+c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)
```

3.265 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 196

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(1+\dots}{(c-d)(a+\dots}}}{d\sqrt{\frac{a+b}{c+d}}f}$$

```
[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec
(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*
x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)
*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/d/f/((a+b)/(c+d))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4067}

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+\dots}{c+\dots}}\dots}{\sqrt{\dots}}\right.\right.}{df\sqrt{\frac{a+b}{c+d}}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],x]
```

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(d*Sqrt[(a + b)/(c + d)]*f)
```

Rule 4067

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

integral

$$= \frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d}, \arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(1+s}{(c-d)(a+bs}}}{d\sqrt{\frac{a+b}{c+d}}f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 37.86 (sec) , antiderivative size = 44664, normalized size of antiderivative = 227.88

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/Sqrt[c + d*Sec[e + f*x]],
x]
```

```
[Out] Result too large to show
```

Maple [A] (verified)

Time = 12.84 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.64

method	result
default	$\frac{2\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\,\sqrt{a+b\sec(fx+e)}\,\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\left(\mathrm{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\frac{(a}{(a}}\right.\right.}{f\sqrt{}}$

[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/f/((a-b)/(a+b))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*a-EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*b+2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2))*b)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a+b\sec(e+fx)}\sec(e+fx)}{\sqrt{c+d\sec(e+fx)}}\,dx$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/sqrt(c + d*sec(e + f*x)), x)

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{b\sec\left(fx+e\right)+a}\sec\left(fx+e\right)}{\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/sqrt(d*sec(f*x + e) + c), x
)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)),x)

[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))^(1/2)), x
)

3.266 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 192

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\text{se}}{(a-b)(c+d\,\text{se}}}}{\sqrt{c+d}(bc-ad)f}$$

```
[Out] 2*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*
sec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(
1/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*
(1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4069}

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}}{\sqrt{a+b}}\right.\right.}{f\sqrt{c+d}(bc-ad)}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e
+ f*x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b
```

```
)*(c + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f
*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x]
)))]*(c + d*Sec[e + f*x]))/(Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

integral

$$= \frac{2\sqrt{a+b}\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(1+\sec}{(a-b)(c+d\sec(}}}{\sqrt{c+d}(bc-ad)f}$$

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.21

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{4\sqrt{\frac{(c+d)\cot^2\left(\frac{1}{2}(e+fx)\right)}{c-d}}\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{-bc+ad}}\csc(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a+b)(d+c\cos(e+fx))\csc^2\left(\frac{1}{2}(e\right.}{-bc+ad}}}{\sqrt{2}}\right.\right.}{(a+b)f\sqrt{\frac{(c+d)(b+a\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)}{bc-ad}}\sqrt{c+d\sec(e+}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (4*Sqrt[((c + d)*Cot[(e + f*x)/2]^2)/(c - d)]*Sqrt[((a + b)*(d + c*Cos[e +
f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]*Csc[e + f*x]*EllipticF[ArcSin[Sqr
t[((a + b)*(d + c*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(-(b*c) + a*d)]/Sqrt[2]
], (2*(b*c - a*d))/((a + b)*(c - d))]*Sqrt[a + b*Sec[e + f*x]]*Sin[(e + f*x
)/2]^2)/((a + b)*f*Sqrt[((c + d)*(b + a*Cos[e + f*x])*Csc[(e + f*x)/2]^2)/(
b*c - a*d)]*Sqrt[c + d*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 10.71 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99

method	result
default	$\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a+b\sec(fx+e)}\,\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\frac{(a+}{(a-}}\right.}{f\sqrt{\frac{a-b}{a+b}}\,(d+c\cos(fx+e))(b+a\cos(fx+e))}$

[In] int(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/f/((a-b)/(a+b))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))

Fricas [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}\sqrt{d\sec\left(fx+e\right)+c}}\,dx$$

[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)*sec(f*x + e)/(b*d*sec(f*x + e)^2 + a*c + (b*c + a*d)*sec(f*x + e)), x)

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}\sqrt{c+d\sec\left(e+fx\right)}}\,dx$$

[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))), x)

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)),
 x)
```

3.267 $\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 110

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2+3\sec(e+fx)}}{\sqrt{5}\sqrt{-4+5\sec(e+fx)}}\right),45\right)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{1+\sec(e+fx)}{4-5\sec(e+fx)}}}{f}$$

```
[Out] 2*cot(f*x+e)*EllipticF(1/5*(2+3*sec(f*x+e))^(1/2)*5^(1/2)/(-4+5*sec(f*x+e))
^(1/2),3*5^(1/2))*(4-5*sec(f*x+e))*((1-sec(f*x+e))/(4-5*sec(f*x+e)))^(1/2)*
((1+sec(f*x+e))/(4-5*sec(f*x+e)))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4069}

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \frac{2\cot(e+fx)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{\sec(e+fx)+1}{4-5\sec(e+fx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3\sec(e+fx)+2}}{\sqrt{5}\sqrt{5\sec(e+fx)-4}}\right),45\right)}{f}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]]),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[2 + 3*Sec[e + f*x]]/(Sqrt[5]*Sqrt[-4
+ 5*Sec[e + f*x]])], 45]*(4 - 5*Sec[e + f*x])*Sqrt[(1 - Sec[e + f*x])/(4 -
5*Sec[e + f*x])]*Sqrt[(1 + Sec[e + f*x])/(4 - 5*Sec[e + f*x])])/f
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

integral

$$= \frac{2\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2+3\sec(e+fx)}}{\sqrt{5}\sqrt{-4+5\sec(e+fx)}}\right),45\right)(4-5\sec(e+fx))\sqrt{\frac{1-\sec(e+fx)}{4-5\sec(e+fx)}}\sqrt{\frac{1+\sec(e+fx)}{4-5\sec(e+fx)}}}{f}$$

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.60

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = -\frac{4\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{-\left((3+2\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\sqrt{-\left((-5+4\cos(e+fx))\csc^2\left(\frac{1}{2}(e+\right.\right.}}{3\sqrt{5}f\sqrt{2+3\sec(e+fx)}\sqrt{-}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[2 + 3*Sec[e + f*x]]*Sqrt[-4 + 5*Sec[e + f*x]])
,x]
```

```
[Out] (-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44/45
]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[2 + 3*Sec[e + f*x]]*Sq
rt[-4 + 5*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

method	result
default	$-\frac{i\sqrt{5}\,\mathrm{EllipticF}\left(\frac{i\sqrt{5}\,(\cot(fx+e)-\csc(fx+e))}{5},3\sqrt{5}\right)\sqrt{2+3\sec(fx+e)}\,\sqrt{-4+5\sec(fx+e)}\,\sqrt{-\frac{2(4\cos(fx+e)-5)}{\cos(fx+e)+1}}\,\sqrt{10}\,\sqrt{\frac{2\cos(fx+e)+3}{\cos(fx+e)+1}}\,(\mathrm{c}}{5f\left(8\cos(fx+e)^{2}+2\cos(fx+e)-15\right)}$

```
[In] int(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -1/5*I/f*5^(1/2)*EllipticF(1/5*I*5^(1/2)*(cot(f*x+e)-csc(f*x+e)),3*5^(1/2))
*(2+3*sec(f*x+e))^(1/2)*(-4+5*sec(f*x+e))^(1/2)*(-2*(4*cos(f*x+e)-5)/(cos(f
*x+e)+1))^(1/2)*10^(1/2)*((2*cos(f*x+e)+3)/(cos(f*x+e)+1))^(1/2)/(8*cos(f*x
+e)^2+2*cos(f*x+e)-15)*(cos(f*x+e)^2+cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{5\,\sec\left(fx+e\right)-4}\sqrt{3\,\sec\left(fx+e\right)+2}}\,dx$$

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)*sec(f*x + e)/(15
*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{3\sec\left(e+fx\right)+2}\sqrt{5\sec\left(e+fx\right)-4}}\,dx$$

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))**(1/2)/(-4+5*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(3*sec(e + f*x) + 2)*sqrt(5*sec(e + f*x) - 4)),
x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{5\sec(fx+e)-4}\sqrt{3\sec(fx+e)+2}}\,dx$$

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="maxima")
[Out] integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)),
 x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{5\sec(fx+e)-4}\sqrt{3\sec(fx+e)+2}}\,dx$$

```
[In] integrate(sec(f*x+e)/(2+3*sec(f*x+e))^(1/2)/(-4+5*sec(f*x+e))^(1/2),x, algo
rithm="giac")
[Out] integrate(sec(f*x + e)/(sqrt(5*sec(f*x + e) - 4)*sqrt(3*sec(f*x + e) + 2)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{2+3\sec(e+fx)}\sqrt{-4+5\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+fx)\sqrt{\frac{3}{\cos(e+fx)}+2}\sqrt{\frac{5}{\cos(e+fx)}-4}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(5/cos(e + f*x) - 4)^(1/2)),
x)
[Out] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(5/cos(e + f*x) - 4)^(1/2)),
 x)
```

3.268 $\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 125

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \frac{2i\cot(e+fx)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{2+3\sec(e+fx)}}\right),\frac{1}{45}\right)\sqrt{\frac{1-\sec(e+fx)}{2+3\sec(e+fx)}}\sqrt{\frac{1+\sec(e+fx)}{2+3\sec(e+fx)}}(2+3\sec(e+fx))}{3\sqrt{5}f}$$

```
[Out] 2/15*I*cot(f*x+e)*EllipticF(I*5^(1/2)*(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e
))^(1/2),1/15*5^(1/2))*(2+3*sec(f*x+e))*((1-sec(f*x+e))/(2+3*sec(f*x+e)))^(
1/2)*((1+sec(f*x+e))/(2+3*sec(f*x+e)))^(1/2)/f*5^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {4069}

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \frac{2i\cot(e+fx)\sqrt{\frac{1-\sec(e+fx)}{3\sec(e+fx)+2}}\sqrt{\frac{\sec(e+fx)+1}{3\sec(e+fx)+2}}(3\sec(e+fx)+2)\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{3\sec(e+fx)+2}}\right),\frac{1}{45}\right)}{3\sqrt{5}f}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),x]
```

```
[Out] (((2*I)/3)*Cot[e + f*x]*EllipticF[I*ArcSinh[(Sqrt[5]*Sqrt[4 - 5*Sec[e + f*x
]])/Sqrt[2 + 3*Sec[e + f*x]]], 1/45]*Sqrt[(1 - Sec[e + f*x])/(2 + 3*Sec[e +
```

```
f*x])]*Sqrt[(1 + Sec[e + f*x])/(2 + 3*Sec[e + f*x])]*(2 + 3*Sec[e + f*x]))
/(Sqrt[5]*f)
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

integral

$$= \frac{2i\cot(e+fx)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{5}\sqrt{4-5\sec(e+fx)}}{\sqrt{2+3\sec(e+fx)}}\right),\frac{1}{45}\right)\sqrt{\frac{1-\sec(e+fx)}{2+3\sec(e+fx)}}\sqrt{\frac{1+\sec(e+fx)}{2+3\sec(e+fx)}}(2+3\sec(e+fx))}{3\sqrt{5}f}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx =$$

$$-\frac{4\sqrt{-\cot^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{-\left((3+2\cos(e+fx))\csc^2\left(\frac{1}{2}(e+fx)\right)\right)}\sqrt{-\left((-5+4\cos(e+fx))\csc^2\left(\frac{1}{2}(e+\right.\right.}}{3\sqrt{5}f\sqrt{4-5\sec(e+fx)}\sqrt{}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[4 - 5*Sec[e + f*x]]*Sqrt[2 + 3*Sec[e + f*x]]),
x]
```

```
[Out] (-4*Sqrt[-Cot[(e + f*x)/2]^2]*Sqrt[-((3 + 2*Cos[e + f*x])*Csc[(e + f*x)/2]^
2)]*Sqrt[-((-5 + 4*Cos[e + f*x])*Csc[(e + f*x)/2]^2)]*Csc[e + f*x]*Elliptic
F[ArcSin[Sqrt[5/22]*Sqrt[(-5 + 4*Cos[e + f*x])/(-1 + Cos[e + f*x])]], 44/45
]*Sec[e + f*x]*Sin[(e + f*x)/2]^4)/(3*Sqrt[5]*f*Sqrt[4 - 5*Sec[e + f*x]]*Sq
rt[2 + 3*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 7.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

method	result
default	$-\frac{i\sqrt{2+3\sec(fx+e)}\,\sqrt{4-5\sec(fx+e)}\,\sqrt{-\frac{2(4\cos(fx+e)-5)}{\cos(fx+e)+1}}\,\sqrt{10}\,\sqrt{\frac{2\cos(fx+e)+3}{\cos(fx+e)+1}}\,\mathrm{EllipticF}\left(3i(-\cot(fx+e)+\csc(fx+e)),\frac{\sqrt{5}}{15}\right)\left(\cos(\right.}{15f\left(8\cos(fx+e)^2+2\cos(fx+e)-15\right)}$

[In] int(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/15*I/f*(2+3*sec(f*x+e))^(1/2)*(4-5*sec(f*x+e))^(1/2)*(-2*(4*cos(f*x+e)-5)/(cos(f*x+e)+1))^(1/2)*10^(1/2)*((2*cos(f*x+e)+3)/(cos(f*x+e)+1))^(1/2)*EllipticF(3*I*(-cot(f*x+e)+csc(f*x+e)),1/15*5^(1/2))/(8*cos(f*x+e)^2+2*cos(f*x+e)-15)*(cos(f*x+e)^2+cos(f*x+e))

Fricas [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{3\sec\left(fx+e\right)+2}\sqrt{-5\sec\left(fx+e\right)+4}}\,dx$$

[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4)*sec(f*x + e)/(15*sec(f*x + e)^2 - 2*sec(f*x + e) - 8), x)

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{\sec\left(e+fx\right)}{\sqrt{4-5\sec\left(e+fx\right)}\sqrt{3\sec\left(e+fx\right)+2}}\,dx$$

[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))**(1/2)/(2+3*sec(f*x+e))**(1/2),x)

[Out] Integral(sec(e + f*x)/(sqrt(4 - 5*sec(e + f*x))*sqrt(3*sec(e + f*x) + 2)), x)

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{3\sec(fx+e)+2}\sqrt{-5\sec(fx+e)+4}}\,dx$$

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4))
, x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)}{\sqrt{3\sec(fx+e)+2}\sqrt{-5\sec(fx+e)+4}}\,dx$$

```
[In] integrate(sec(f*x+e)/(4-5*sec(f*x+e))^(1/2)/(2+3*sec(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(3*sec(f*x + e) + 2)*sqrt(-5*sec(f*x + e) + 4))
, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{4-5\sec(e+fx)}\sqrt{2+3\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{\frac{3}{\cos(e+f\,x)}+2}\,\sqrt{4-\frac{5}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(4 - 5/cos(e + f*x))^(1/2)),
x)
```

```
[Out] int(1/(cos(e + f*x)*(3/cos(e + f*x) + 2)^(1/2)*(4 - 5/cos(e + f*x))^(1/2)),
 x)
```

3.269 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 396

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$
$$= \frac{2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right),\frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(1\cdots}{(c-d)(a+\cdots}}}{bd\sqrt{\frac{a+b}{c+d}}f}$$
$$-\frac{2a\sqrt{a+b}\cot(e+fx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right),\frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)\cdots}{(a-b)(c\cdots}}}{b\sqrt{c+d}(bc-ad)f}$$

```
[Out] 2*cot(f*x+e)*EllipticPi(((a+b)/(c+d))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec
(f*x+e))^(1/2),b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*(a+b*sec(f*
x+e))*(-(-a*d+b*c)*(1-sec(f*x+e))/(c+d)/(a+b*sec(f*x+e)))^(1/2)*((-a*d+b*c)
*(1+sec(f*x+e))/(c-d)/(a+b*sec(f*x+e)))^(1/2)/b/d/f/((a+b)/(c+d))^(1/2)-2*a
*cot(f*x+e)*EllipticF((c+d)^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*s
ec(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*(c+d*sec(f*x+e))*(a+b)^(1
/2)*((-a*d+b*c)*(1-sec(f*x+e))/(a+b)/(c+d*sec(f*x+e)))^(1/2)*(-(-a*d+b*c)*(
1+sec(f*x+e))/(a-b)/(c+d*sec(f*x+e)))^(1/2)/b/(-a*d+b*c)/f/(c+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {4070, 4069, 4067}

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

$$= \frac{2\cot(e+fx)(a+b\sec(e+fx))\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{\frac{(bc-ad)(\sec(e+fx)+1)}{(c-d)(a+b\sec(e+fx))}}\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d},\arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}}{\sqrt{a}}\right.\right.}{bdf\sqrt{\frac{a+b}{c+d}}}$$

$$-\frac{2a\sqrt{a+b}\cot(e+fx)(c+d\sec(e+fx))\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{-\frac{(bc-ad)(\sec(e+fx)+1)}{(a-b)(c+d\sec(e+fx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}}{\sqrt{a}}\right.\right.}{bf\sqrt{c+d}(bc-ad)}$$

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticPi[(b*(c + d))/((a + b)*d), ArcSin[(Sqrt[(a + b)/(c
 + d)]*Sqrt[c + d*Sec[e + f*x]])/Sqrt[a + b*Sec[e + f*x]]], ((a - b)*(c + d
))/((a + b)*(c - d))]*Sqrt[-(((b*c - a*d)*(1 - Sec[e + f*x]))/((c + d)*(a +
 b*Sec[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sec[e + f*x]))/((c - d)*(a + b*S
ec[e + f*x]))]*(a + b*Sec[e + f*x]))/(b*d*Sqrt[(a + b)/(c + d)]*f) - (2*a*S
qrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sec[e + f*
x]])/(Sqrt[a + b]*Sqrt[c + d*Sec[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c
 + d))]*Sqrt[((b*c - a*d)*(1 - Sec[e + f*x]))/((a + b)*(c + d*Sec[e + f*x])
)]*Sqrt[-(((b*c - a*d)*(1 + Sec[e + f*x]))/((a - b)*(c + d*Sec[e + f*x])))]
*(c + d*Sec[e + f*x]))/(b*Sqrt[c + d]*(b*c - a*d)*f)
```

Rule 4067

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/Sq
rt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)], x_Symbol] :> Simp[-2*((a + b*Csc[
e + f*x])/(d*f*Sqrt[(a + b)/(c + d)]*Cot[e + f*x]))*Sqrt[(-(b*c - a*d))*((1
 - Csc[e + f*x])/((c + d)*(a + b*Csc[e + f*x])))]*Sqrt[(b*c - a*d)*((1 + Cs
c[e + f*x])/((c - d)*(a + b*Csc[e + f*x])))]*EllipticPi[b*((c + d)/(d*(a +
b))), ArcSin[Sqrt[(a + b)/(c + d)]*(Sqrt[c + d*Csc[e + f*x]]/Sqrt[a + b*Csc
[e + f*x]])], (a - b)*((c + d)/((a + b)*(c - d)))], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4069

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*Sqr
t[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Simp[-2*((c + d*Csc[
e + f*x])/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cot[e + f*x]))*Sqrt[(b*c -
a*d)*((1 - Csc[e + f*x])/((a + b)*(c + d*Csc[e + f*x])))]*Sqrt[(-(b*c - a*d
))*((1 + Csc[e + f*x])/((a - b)*(c + d*Csc[e + f*x])))]*EllipticF[ArcSin[Rt
```

```
[(c + d)/(a + b), 2]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])],
(a + b)*((c - d)/((a - b)*(c + d)))], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4070

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*S
qrt[csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)]), x_Symbol] :> Dist[-a/b, Int[Csc
[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[c + d*Csc[e + f*x]]), x], x] + Dis
t[1/b, Int[Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/Sqrt[c + d*Csc[e + f*x]])
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{\sqrt{c+d\sec(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx}{b}$$

$$= \frac{2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b(c+d)}{(a+b)d}, \arcsin\left(\frac{\sqrt{\frac{a+b}{c+d}}\sqrt{c+d\sec(e+fx)}}{\sqrt{a+b\sec(e+fx)}}\right), \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)\sqrt{-\frac{(bc-ad)(1-\sec(e+fx))}{(c+d)(a+b\sec(e+fx))}}\sqrt{}}{bd\sqrt{\frac{a+b}{c+d}}f}$$

$$- \frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c+d}\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}\sqrt{c+d\sec(e+fx)}}\right), \frac{(a+b)(c-d)}{(a-b)(c+d)}\right)\sqrt{\frac{(bc-ad)(1-\sec(e+fx))}{(a+b)(c+d\sec(e+fx))}}\sqrt{}}{b\sqrt{c+d}(bc-ad)f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.53 (sec) , antiderivative size = 39359, normalized size of antiderivative = 99.39

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Result too large to show}$$

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]]
),x]

[Out] Result too large to show
```

Maple [A] (verified)

Time = 15.93 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.66

method	result
default	$-\frac{2\sqrt{c+d\sec(fx+e)}\,\sqrt{a+b\sec(fx+e)}\,\left(\text{EllipticF}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx+e)+\csc(fx+e)),\sqrt{\frac{(a+b)(c-d)}{(a-b)(c+d)}}\right)-2\,\text{EllipticPi}\left(\sqrt{\frac{a-b}{a+b}}\,(-\cot(fx\right.\right.}{f\sqrt{\frac{a-b}{a+b}}\,(d+c\cos(fx+e))(b+a\cos(}$

[In] int(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/f/((a-b)/(a+b))^(1/2)*(c+d*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(EllipticF(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))-2*EllipticPi(((a-b)/(a+b))^(1/2)*(-cot(f*x+e)+csc(f*x+e)),(a+b)/(a-b),((c-d)/(c+d))^(1/2)/((a-b)/(a+b))^(1/2)))*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))/(b+a*cos(f*x+e))*(cos(f*x+e)^2+cos(f*x+e))

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx$$

[In] integrate(sec(f*x+e)**2/(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e))**(1/2),x)

[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sqrt(c + d*sec(e + f*x))), x)

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{b\sec(fx+e)+a}\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e) + c)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))^(1/2)
), x)
```

3.270 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 170

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \frac{2dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{bf\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2(bc-ad)g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{b(a+b)f\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2*d*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2,2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f
*x+e))^(1/2)/b/f/(c+d*sec(f*x+e))^(1/2)+2*(-a*d+b*c)*g*(cos(1/2*f*x+1/2*e)^
2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*a/(a+b),2^(1/2)
*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x+e))^(1/2)/b/(a+
b)/f/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4056, 3944, 2886, 2884, 4060}

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+\right.}{bf(a+b)\sqrt{c+d\sec(e+fx)}}$$
$$+\frac{2dg\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{bf\sqrt{c+d\sec(e+fx)}}$$

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e + f*x]),
x]
```

```
[Out] (2*d*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/
(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(b*c - a
*d)*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x
)/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(b*(a + b)*f*Sqrt[c + d*Sec[e + f
*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4056

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx}{b} - \frac{(-bc+ad)\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx}{b} \\
&= \frac{\left(dg\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+fx)}}\,dx}{b\sqrt{c+d\sec(e+fx)}} \\
&\quad - \frac{\left((-bc+ad)g\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(b+a\cos(e+fx))\sqrt{d+c\cos(e+fx)}}\,dx}{b\sqrt{c+d\sec(e+fx)}} \\
&= \frac{\left(dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{b\sqrt{c+d\sec(e+fx)}} \\
&\quad - \frac{\left((-bc+ad)g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(b+a\cos(e+fx))\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{b\sqrt{c+d\sec(e+fx)}} \\
&= \frac{2dg\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{bf\sqrt{c+d\sec(e+fx)}} \\
&\quad + \frac{2(bc-ad)g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{b(a+b)f\sqrt{c+d\sec(e+fx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.31

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = -\frac{2ig\sqrt{-\frac{c(-1+\cos(e+fx))}{c+d}}\sqrt{\frac{c(1+\cos(e+fx))}{c-d}}\cot(e+fx)\left(\text{EllipticPi}\left(1-\frac{c}{d},i\,\text{arcsinh}\left(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e+fx)}\right),\right.\right.}{b\sqrt{\frac{1}{c-d}}f\sqrt{d}}$$

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[c + d*Sec[e + f*x]])/(a + b*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x
]))/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]
```

```
*Sqrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(a*(-c + d))/(-(
b*c) + a*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c +
d)/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(b*Sqrt[(c - d)
^(-1)]*f*Sqrt[d + c*Cos[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.49 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.62

method	result
default	$-\frac{2ig\sqrt{g\sec(fx+e)}\,\cos(fx+e)\sqrt{c+d\sec(fx+e)}\,\sqrt{\frac{d+c\cos(fx+e)}{(c+d)(\cos(fx+e)+1)}}\left(\operatorname{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{c-d}{c+d}}\right)abc-\operatorname{Elliptic}\right.}{}$

```
[In] int((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*I*g/f/b/(a-b)/(a+b)*(g*sec(f*x+e))^(1/2)*cos(f*x+e)*(c+d*sec(f*x+e))^(1/
2)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(EllipticF(I*(cot(f*x+e)
-csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*a*b*c-EllipticF(I*(cot(f*x+e)-csc(f*x+e)
),(-(c-d)/(c+d))^(1/2))*a*b*d+EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(
c+d))^(1/2))*b^2*c-EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d))^(1/2)
)*b^2*d-2*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1,I*((c-d)/(c+d))^(1/2))*a^
2*d+2*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1,I*((c-d)/(c+d))^(1/2))*b^2*d+
2*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))*
a^2*d-2*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-(a-b)/(a+b),I*((c-d)/(c+d))^(
1/2))*a*b*c)/(d+c*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}\sqrt{c+d\sec{(e+fx)}}}{a+b\sec{(e+fx)}}\,dx$$

```
[In] integrate((g*sec(f*x+e))**(3/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*sec(f*x+e)),x)
[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(c + d*sec(e + f*x))/(a + b*sec(e + f*
x)), x)
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}+c}(g\sec{(fx+e)})^{\frac{3}{2}}}{b\sec{(fx+e)}+a}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e) +
 a), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}+c}(g\sec{(fx+e)})^{\frac{3}{2}}}{b\sec{(fx+e)}+a}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*sec(f*x+e)),x, a
lgorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e) + c)*(g*sec(f*x + e))^(3/2)/(b*sec(f*x + e) +
 a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{a+b\sec(e+fx)}\,dx = \int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}\,\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{a+\frac{b}{\cos(e+f\,x)}}\,dx$$

```
[In] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(a + b/cos(e + f*x)
),x)
[Out] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(a + b/cos(e + f*x)
), x)
```

3.271 $\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 83

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+}}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+
1/2*e),2*a/(a+b),2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g
*sec(f*x+e))^(1/2)/(a+b)/f/(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4060, 2886, 2884}

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\right.}{f(a+b)\sqrt{c+d\sec(e+fx)}}$$

```
[In] Int[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f*x]]),
x]
```

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(g\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(b+a\cos(e+fx))\sqrt{d+c\cos(e+fx)}}\,dx}{\sqrt{c+d\sec(e+fx)}}\\
&= \frac{\left(g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(b+a\cos(e+fx))\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{\sqrt{c+d\sec(e+fx)}}\\
&= \frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{(a+b)f\sqrt{c+d\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \frac{2g\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2a}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{(a+b)f\sqrt{c+d\sec(e+fx)}}$$

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/((a + b*Sec[e + f*x])*Sqrt[c + d*Sec[e + f
*x]]),x]
```

```
[Out] (2*g*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*a)/(a + b), (e + f*x)
/2, (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/((a + b)*f*Sqrt[c + d*Sec[e + f*x]
])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

method	result
default	$-\frac{2ig\cos(fx+e)\sqrt{c+d\sec(fx+e)}\left(2a\,\text{EllipticPi}\left(i(-\cot(fx+e)+\csc(fx+e)),-\frac{a-b}{a+b},i\sqrt{\frac{c-d}{c+d}}\right)-a\,\text{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+\right.\right.}{f(a-b)(a+b)(d+c\cos(fx+e))\sqrt{\cos}}$

```
[In] int((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*I*g/f/(a-b)/(a+b)*cos(f*x+e)*(c+d*sec(f*x+e))^(1/2)*(2*a*EllipticPi(I*(-
cot(f*x+e)+csc(f*x+e)),-(a-b)/(a+b),I*((c-d)/(c+d))^(1/2))-a*EllipticF(I*(-
cot(f*x+e)+csc(f*x+e)),(-(c-d)/(c+d))^(1/2))-b*EllipticF(I*(-cot(f*x+e)+csc
(f*x+e)),(-(c-d)/(c+d))^(1/2)))*(g*sec(f*x+e))^(1/2)*(1/(c+d)*(d+c*cos(f*x+
e))/(cos(f*x+e)+1))^(1/2)/(d+c*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx = \int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx$$

```
[In] integrate((g*sec(f*x+e))**(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/((a + b*sec(e + f*x))*sqrt(c + d*sec(e + f
*x))), x)
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
+ c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{(b\sec(fx+e)+a)\sqrt{d\sec(fx+e)+c}}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(a+b*sec(f*x+e))/(c+d*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/((b*sec(f*x + e) + a)*sqrt(d*sec(f*x + e)
+ c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{(a+b\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx=\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(e+f\,x)}\right)\,\sqrt{c+\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))*(c + d/cos(e + f*x))^(1/2)
), x)
```

3.272 $\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 168

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \frac{2d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{af\sqrt{c+d\sec(e+fx)}} + \frac{2(ac-bd)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{a(a+b)f\sqrt{c+d\sec(e+fx)}}$$

```
[Out] 2*d*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+
1/2*e),2,2^(1/2)*(c/(c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x
+e))^(1/2)/a/f/(c+d*sec(f*x+e))^(1/2)+2*(a*c-b*d)*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(a+b),2^(1/2)*(c/(
c+d))^(1/2))*((d+c*cos(f*x+e))/(c+d))^(1/2)*(g*sec(f*x+e))^(1/2)/a/(a+b)/f/
(c+d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

$= \{3041, 4056, 3944, 2886, 2884, 4060\}$

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \frac{2(ac-bd)\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{af(a+b)\sqrt{c+d\sec(e+fx)}} + \frac{2d\sqrt{g\sec(e+fx)}\sqrt{\frac{c\cos(e+fx)+d}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)}{af\sqrt{c+d\sec(e+fx)}}$$

```
[In] Int[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*x]),x]
```

```
[Out] (2*d*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[2, (e + f*x)/2, (2*c)/(c
 + d)]*Sqrt[g*Sec[e + f*x]])/(a*f*Sqrt[c + d*Sec[e + f*x]]) + (2*(a*c - b*d
)*Sqrt[(d + c*Cos[e + f*x])/(c + d)]*EllipticPi[(2*b)/(a + b), (e + f*x)/2,
 (2*c)/(c + d)]*Sqrt[g*Sec[e + f*x]])/(a*(a + b)*f*Sqrt[c + d*Sec[e + f*x]]
)
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3041

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
(c_))^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist
[g^m, Int[(g*Csc[e + f*x])^(p - m)*(b + a*Csc[e + f*x])^m*(c + d*Csc[e + f*
x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[m]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
```

```
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4056

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(g\sec(e+fx))^{3/2}\sqrt{c+d\sec(e+fx)}}{b+a\sec(e+fx)}\,dx}{g}\\
&= \frac{d\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{c+d\sec(e+fx)}}\,dx}{ag} + \frac{(ac-bd)\int \frac{(g\sec(e+fx))^{3/2}}{(b+a\sec(e+fx))\sqrt{c+d\sec(e+fx)}}\,dx}{ag}\\
&= \frac{\left(d\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{d+c\cos(e+fx)}}\,dx}{a\sqrt{c+d\sec(e+fx)}}\\
&\quad + \frac{\left((ac-bd)\sqrt{d+c\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(a+b\cos(e+fx))\sqrt{d+c\cos(e+fx)}}\,dx}{a\sqrt{c+d\sec(e+fx)}}\\
&= \frac{\left(d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{a\sqrt{c+d\sec(e+fx)}}\\
&\quad + \frac{\left((ac-bd)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{(a+b\cos(e+fx))\sqrt{\frac{d}{c+d}+\frac{c\cos(e+fx)}{c+d}}}\,dx}{a\sqrt{c+d\sec(e+fx)}}
\end{aligned}$$

$$= \frac{2d\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{af\sqrt{c+d\sec(e+fx)}} + \frac{2(ac-bd)\sqrt{\frac{d+c\cos(e+fx)}{c+d}}\,\text{EllipticPi}\left(\frac{2b}{a+b},\frac{1}{2}(e+fx),\frac{2c}{c+d}\right)\sqrt{g\sec(e+fx)}}{a(a+b)f\sqrt{c+d\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = -\frac{2i\sqrt{-\frac{c(-1+\cos(e+fx))}{c+d}}\sqrt{\frac{c(1+\cos(e+fx))}{c-d}}\cot(e+fx)\left(\text{EllipticPi}\left(1-\frac{c}{d},i\text{arcsinh}\left(\sqrt{\frac{1}{c-d}}\sqrt{d+c\cos(e+fx)}\right)\right.\right.}{a\sqrt{\frac{1}{c-d}}f\sqrt{}}$$

[In] Integrate[(Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a + b*Cos[e + f*x]),x]

[Out] ((-2*I)*Sqrt[-((c*(-1 + Cos[e + f*x]))/(c + d))]*Sqrt[(c*(1 + Cos[e + f*x]))/(c - d)]*Cot[e + f*x]*(EllipticPi[1 - c/d, I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)] - EllipticPi[(b*(-c + d))/(-(a*c) + b*d), I*ArcSinh[Sqrt[(c - d)^(-1)]*Sqrt[d + c*Cos[e + f*x]]], (-c + d)/(c + d)])*Sqrt[g*Sec[e + f*x]]*Sqrt[c + d*Sec[e + f*x]])/(a*Sqrt[(c - d)^(-1)]*f*Sqrt[d + c*Cos[e + f*x]])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.24 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.63

method	result
default	$\frac{2i\sqrt{g\sec(fx+e)}\cos(fx+e)\sqrt{c+d\sec(fx+e)}\left(\text{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{c-d}{c+d}}\right)a^2c-\text{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)\right.\right.}{}$

[In] int((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+cos(f*x+e)*b),x,method=_RETURNVERBOSE)

[Out] 2*I/f/a/(a-b)/(a+b)*(g*sec(f*x+e))^(1/2)*cos(f*x+e)*(c+d*sec(f*x+e))^(1/2)*(EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*a^2*c-EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*a^2*d+EllipticF(I*(cot(f*x+

```
e)-csc(f*x+e)),(-(c-d)/(c+d))^(1/2))*a*b*c-EllipticF(I*(cot(f*x+e)-csc(f*x+
e)),(-(c-d)/(c+d))^(1/2))*a*b*d+2*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1,I
*((c-d)/(c+d))^(1/2))*a^2*d-2*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1,I*((c
-d)/(c+d))^(1/2))*b^2*d-2*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),(a-b)/(a+b),
I*((c-d)/(c+d))^(1/2))*a*b*c+2*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),(a-b)/(
a+b),I*((c-d)/(c+d))^(1/2))*b^2*d)*(1/(c+d)*(d+c*cos(f*x+e))/(cos(f*x+e)+1)
)^(1/2)/(d+c*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \int \frac{\sqrt{g\sec{(e+fx)}}\sqrt{c+d\sec{(e+fx)}}}{a+b\cos{(e+fx)}}\,dx$$

```
[In] integrate((g*sec(f*x+e))**(1/2)*(c+d*sec(f*x+e))**(1/2)/(a+b*cos(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*sec(e + f*x))*sqrt(c + d*sec(e + f*x))/(a + b*cos(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}+c}\sqrt{g\sec{(fx+e)}}}{b\cos{(fx+e)}+a}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) + a
), x)
```

Giac [F]

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \int \frac{\sqrt{d\sec(fx+e)+c}\sqrt{g\sec(fx+e)}}{b\cos(fx+e)+a}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(1/2)*(c+d*sec(f*x+e))^(1/2)/(a+b*cos(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(f*x + e) + c)*sqrt(g*sec(f*x + e))/(b*cos(f*x + e) + a
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{g\sec(e+fx)}\sqrt{c+d\sec(e+fx)}}{a+b\cos(e+fx)}\,dx = \int \frac{\sqrt{c+\frac{d}{\cos(e+f\,x)}}\,\sqrt{\frac{g}{\cos(e+f\,x)}}}{a+b\,\cos(e+f\,x)}\,dx$$

```
[In] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(1/2))/(a + b*cos(e + f*x)
),x)
```

```
[Out] int(((c + d/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(1/2))/(a + b*cos(e + f*x)
), x)
```

3.273 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 95

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

```
[Out] EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*(1/(1+sec(f*x+e)))
^(1/2)*(a+b*sec(f*x+e))^(1/2)/c/f/((a+b*sec(f*x+e))/(a+b)/(1+sec(f*x+e)))^(
1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {4053}

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

```
[Out] (EllipticE[ArcSin[Tan[e + f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(
1 + Sec[e + f*x])^(-1)]*Sqrt[a + b*Sec[e + f*x]])/(c*f*Sqrt[(a + b*Sec[e +
f*x])/((a + b)*(1 + Sec[e + f*x]))])
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 264 vs. $2(95) = 190$.

Time = 6.44 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.78

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$$
$$= \frac{\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\sqrt{a+b\sec(e+fx)}\left(\frac{2\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\sec^4\left(\frac{1}{2}(e+fx)\right)\sqrt{1+\sec(\cdots}}{\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}}\cdots\right.}{4cf(1+\mathrm{s}\cdots}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),x]
```

```
[Out] (Cos[(e + f*x)/2]^2*Sqrt[Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]]*((2*Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/
(a + b)]*Sec[(e + f*x)/2]^4*Sqrt[1 + Sec[e + f*x]])/(((1 + Cos[e + f*x])^(-
1))^(3/2)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]) + (Sec[(
e + f*x)/2]^5*Sqrt[1 + Sec[e + f*x]]*(-Sin[(e + f*x)/2] + Sin[(3*(e + f*x))
/2]))/((1 + Cos[e + f*x])^(-1))^(3/2) - 8*Sqrt[Sec[e + f*x]]*(Sin[e + f*x]
- Tan[(e + f*x)/2])))/(4*c*f*(1 + Sec[e + f*x]))
```

Maple [A] (verified)

Time = 7.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{(-a-b)(\cos(fx+e)+1)\sqrt{\frac{b+a\cos(fx+e)}{(a+b)(\cos(fx+e)+1)}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\operatorname{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{a+b\sec(fx+e)}}{cf(b+a\cos(fx+e))}$	123

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/c/f*(-a-b)*(cos(f*x+e)+1)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(
a+b))^(1/2))*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}\sec(fx+e)}{c\sec(fx+e)+c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{\int \frac{\sqrt{a+b\sec(e+fx)}\sec(e+fx)}{\sec(e+fx)+1}\,dx}{c}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(sec(e + f*x) + 1), x)/c
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}\sec(fx+e)}{c\sec(fx+e)+c}\,dx$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}\sec(fx+e)}{c\sec(fx+e)+c}\,dx$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(c*sec(f*x + e) + c), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos\left(e+f\,x\right)\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + c/cos(e + f*x))),x)

[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + c/cos(e + f*x))), x)

3.274 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 295

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{(a-b)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} + \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}$$

```
[Out] -g*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/f/(c+c*cos(f*x+e))/(a+b
*sec(f*x+e))^(1/2)+g*(b+a*cos(f*x+e))*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*
f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*(g*sec(f*x
+e))^(1/2)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e))^(1/2)+(a-b)*
g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2
*e),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^
(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)+2*b*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2
*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.282, Rules used = {4056, 3944, 2886, 2884, 4060, 2847, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = -\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}} + \frac{g(a-b)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}} + \frac{g\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e + f*x]),
x]
```

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/(c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e + f*x]])
+ ((a - b)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2*b*g*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a + b)]*
Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a*Cos[e + f*
x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/(f*(c + c*Cos[e + f*x])*Sqrt[a + b*S
ec[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
```

```
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4056

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left((-a+b)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx\right)+\frac{b\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{c}\\
&= -\frac{\left((-a+b)g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}(c+c\cos(e+fx))}\,dx}{\sqrt{a+b\sec(e+fx)}}\\
&\quad+\frac{\left(bg\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{c\sqrt{a+b\sec(e+fx)}}\\
&= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}\\
&\quad+\frac{\left(a(-a+b)g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{-\frac{c}{2}-\frac{1}{2}c\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{(a-b)c^2\sqrt{a+b\sec(e+fx)}}\\
&\quad+\frac{\left(bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{c\sqrt{a+b\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} - \frac{\left((-a+b)g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{b+a\cos(e+fx)}}\,dx}{2c\sqrt{a+b\sec(e+fx)}} - \frac{\left((-a+b)g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\sqrt{b+a\cos(e+fx)}\,dx}{2(a-b)c\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} - \frac{\left((-a+b)g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\right)\int\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}\,dx}{2(a-b)c\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} - \frac{\left((-a+b)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{2c\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)\middle|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{(a-b)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} + \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}$$

Mathematica [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx$$

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
f*x]),x]
```

```
[Out] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + c*Sec[e +
f*x]), x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.93

method	result
default	$\frac{ig\sqrt{a+b\sec(fx+e)}\,\sqrt{g\sec(fx+e)}\,\cos(fx+e)\left(2a\,\mathrm{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)-2b\,\mathrm{EllipticF}\left(i(\cot(fx+e)-\csc(fx+\right.\right.}{}$

```
[In] int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] I*g/c/f*(a+b*sec(f*x+e))^(1/2)*(g*sec(f*x+e))^(1/2)*cos(f*x+e)*(2*a*Ellipti
cF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-2*b*EllipticF(I*(cot(f*x
+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-a*EllipticE(I*(cot(f*x+e)-csc(f*x+e))
,(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))
^(1/2))+4*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))*b)
*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/(b+a*cos(f*x+e))/(1/(cos(f
*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(g \sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \frac{\int \frac{(g\sec(e+fx))^{\frac{3}{2}}\sqrt{a+b\sec(e+fx)}}{\sec(e+fx)+1}\,dx}{c}$$

```
[In] integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+c*sec(f*x+e)),x)
[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(a + b*sec(e + f*x))/(sec(e + f*x) + 1
), x)/c
```

Maxima [F]

$$\int \frac{(g \sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}\,(g\sec(fx+e))^{\frac{3}{2}}}{c\sec(fx+e)+c}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e) +
 c), x)
```

Giac [F]

$$\int \frac{(g \sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}\,(g\sec(fx+e))^{\frac{3}{2}}}{c\sec(fx+e)+c}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+c*sec(f*x+e)),x, a
lgorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(c*sec(f*x + e) +
 c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g \sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{c+\frac{c}{\cos(e+f\,x)}}\,dx$$

```
[In] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + c/cos(e + f*x)
),x)
[Out] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + c/cos(e + f*x)
), x)
```

3.275 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 209

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)cf} + \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{(a-b)cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

```
[Out] -2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1
/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(
1/2)/(a-b)/c/f+EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*(1/
(1+sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sec(f*x+e))/(a
+b)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used

$= \{4057, 3917, 4053\}$

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$
$$= \frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$
$$- \frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{cf(a-b)}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[e + f*x]))/(a - b))])/((a - b)*c*f) + (EllipticE[ArcSin[Tan[e + f*x
]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[
a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1 + S
ec[e + f*x]))])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 4057

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[
Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[d/(b*c - a*d), Int[Csc
[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ[c^
2 - d^2, 0])
```

Rubi steps

$$\text{integral} = -\frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{(a-b)c} - \frac{c\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx}{-ac+bc}$$

$$= -\frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)cf}$$
$$+\frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{(a-b)cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

Mathematica [A] (warning: unable to verify)

Time = 13.34 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.79

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$
$$= \frac{\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(b+a\cos(e+fx))\sec^2(e+fx)\left(\frac{2\sin(e+fx)}{-a+b}-\frac{2\tan\left(\frac{1}{2}(e+fx)\right)}{-a+b}\right)}{f\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}$$
$$-\frac{2\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^{\frac{3}{2}}(e+fx)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}\left((a-b)E\left(\arcsin\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right)\right|\right.}{\left(\frac{a-b}{a+b}\right)^{3/2}(a+b)f\sqrt{\cos(e+fx)\sec^4}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*Sec[e + f*x]^2*((2*Sin[e + f*x])
/(-a + b) - (2*Tan[(e + f*x)/2])/(-a + b)))/(f*Sqrt[a + b*Sec[e + f*x]]*(c
+ c*Sec[e + f*x])) - (2*Cos[e/2 + (f*x)/2]^2*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e
 + f*x)/2]^2*Sec[e + f*x]]*((a - b)*EllipticE[ArcSin[Sqrt[(a - b)/(a + b)]*
Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[((b + a*Cos[e + f*x])*Sec[(e + f*x
)/2]^2)/(a + b)] + Sqrt[2]*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[e + f*x]/(1 + Cos
[e + f*x])]*(b + a*Cos[e + f*x])*Tan[(e + f*x)/2])*(-1 + Tan[(e + f*x)/2]^2
))/(((a - b)/(a + b))^(3/2)*(a + b)*f*Sqrt[Cos[e + f*x]*Sec[(e + f*x)/2]^4]
*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))
```

Maple [A] (verified)

Time = 5.33 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

method	result
default	$\frac{(\cos(fx+e)+1)\left(2\,\mathrm{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)b-a\,\mathrm{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)-b\,\mathrm{EllipticE}\left(\cot(fx+e)\right.\right.}{cf(a-b)(b+a\cos(fx+e))}$

[In] int(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/c/f/(a-b)*(cos(f*x+e)+1)*(2*EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b-a*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))-b*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2)))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))

Fricas [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(b*c*sec(f*x + e)^2 + (a + b)*c*sec(f*x + e) + a*c), x)

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\sec(e+fx)+\sqrt{a+b\sec(e+fx)}}\,dx}{c}$$

[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)

[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a + b*sec(e + f*x))), x)/c

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)+c)}\,dx$$

[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)}{\sqrt{b\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)+c)}\,dx$$

[In] integrate(sec(f*x+e)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))),x)

[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))), x)

3.276 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 214

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)bcf} - \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{(a-b)cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

```
[Out] 2*a*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^
(1/2)/(a-b)/b/c/f-EllipticE(tan(f*x+e)/(1+sec(f*x+e)),((a-b)/(a+b))^(1/2))*
(1/(1+sec(f*x+e)))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(a-b)/c/f/((a+b*sec(f*x+e))
/(a+b)/(1+sec(f*x+e)))^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used

$= \{4061, 3917, 4053\}$

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$
$$= \frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bcf(a-b)}$$
$$- \frac{\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}E\left(\arcsin\left(\frac{\tan(e+fx)}{\sec(e+fx)+1}\right)|\frac{a-b}{a+b}\right)}{cf(a-b)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}}$$

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[e + f*x]))/(a - b))])/((a - b)*b*c*f) - (EllipticE[ArcSin[Tan[e +
f*x]/(1 + Sec[e + f*x])], (a - b)/(a + b)]*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sq
rt[a + b*Sec[e + f*x]])/((a - b)*c*f*Sqrt[(a + b*Sec[e + f*x])/((a + b)*(1
+ Sec[e + f*x]))])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4053

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x])))]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rule 4061

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[-a/(b*c - a*d), I
nt[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[c/(b*c - a*d), Int[
Csc[e + f*x]*(Sqrt[a + b*Csc[e + f*x]]/(c + d*Csc[e + f*x])), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && (EqQ[a^2 - b^2, 0] || EqQ
[c^2 - d^2, 0])
```

Rubi steps

$$\text{integral} = \frac{a\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{(a-b)c} + \frac{c\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+c\sec(e+fx)}\,dx}{-ac+bc}$$

$$= \frac{2a\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right), \frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)bcf}$$

$$- \frac{E\left(\arcsin\left(\frac{\tan(e+fx)}{1+\sec(e+fx)}\right)\middle|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}{(a-b)cf\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}$$

Mathematica [A] (verified)

Time = 5.48 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$

$$= \frac{4\cos^4\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a+b)E\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|\frac{a-b}{a+b}\right) - 2a\,\text{EllipticF}\left(\arcsin\left(\tan\right.\right.\right.}{(-a+b)cf\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}(1+\cos(e+fx))^2\sqrt{a+b\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^4*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x])
)]*((a + b)*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*a*Elli
pticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/((-a + b)*c*f*Sqrt[Cos[e
 + f*x]/(1 + Cos[e + f*x])]*(1 + Cos[e + f*x])^2*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 8.77 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.90

method	result
default	$-\frac{(\cos(fx+e)+1)\left(-a\,\text{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)-b\,\text{EllipticE}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)+2\,\text{EllipticF}\left(\cot(fx+\right.\right.}{cf(a-b)(b+a\cos(fx+e))}$

```
[In] int(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/c/f/(a-b)*(cos(f*x+e)+1)*(-a*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b
))^(1/2))-b*EllipticE(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))+2*Elliptic
F(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a)*(cos(f*x+e)/(cos(f*x+e)+1))
```

```
^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/
2)/(b+a*cos(f*x+e))
```

Fricas [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}+a}(c\sec{(fx+e)}+c)}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)^2/(b*c*sec(f*x + e)^2 + (a +
 b)*c*sec(f*x + e) + a*c), x)
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\int \frac{\sec^2{(e+fx)}}{\sqrt{a+b\sec{(e+fx)}}\sec{(e+fx)}+\sqrt{a+b\sec{(e+fx)}}}\,dx}{c}$$

```
[In] integrate(sec(f*x+e)**2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + sqrt(a +
b*sec(e + f*x))), x)/c
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec{(fx+e)}^2}{\sqrt{b\sec{(fx+e)}+a}(c\sec{(fx+e)}+c)}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x
)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}(c\sec\left(fx+e\right)+c)}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e) + c)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))), x)
```

3.277 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 229

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}$$
$$+\frac{g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}$$
$$-\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}$$

```
[Out] -g*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/(a-b)/f/(c+c*cos(f*x+e)
)/(a+b*sec(f*x+e))^(1/2)+g*(b+a*cos(f*x+e))*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*(g*s
ec(f*x+e))^(1/2)/(a-b)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e))^
(1/2)+g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f
*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*
x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used

$= \{4060, 2847, 2831, 2742, 2740, 2734, 2732\}$

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = -\frac{g\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}} + \frac{g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}} + \frac{g\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),
x]
```

```
[Out] (g*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e
+ f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec[e +
 f*x]]) + (g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/2, (2*a
)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) - (g*(b + a
*Cos[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e +
 f*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{b+a\cos(e+fx)}(c+c\cos(e+fx))}\,dx}{\sqrt{a+b\sec(e+fx)}}\\
&= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}\\
&\quad-\frac{\left(ag\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\frac{-\frac{c}{2}-\frac{1}{2}c\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{(a-b)c^2\sqrt{a+b\sec(e+fx)}}\\
&= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}\\
&\quad+\frac{\left(g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{b+a\cos(e+fx)}}\,dx}{2c\sqrt{a+b\sec(e+fx)}}\\
&\quad+\frac{\left(g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\sqrt{b+a\cos(e+fx)}\,dx}{2(a-b)c\sqrt{a+b\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\right)\int\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}\,dx}{2(a-b)c\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{\left(g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{2c\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{g(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)\middle|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.39 (sec) , antiderivative size = 1019, normalized size of antiderivative = 4.45

$$\int\frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)(b+a\cos(e+fx))(g\sec(e+fx))^{3/2}\left(\frac{2\csc(e)}{(-a+b)f}\right.}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+f}$$

$$+\frac{\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{\csc(e)\left(b-a\sqrt{1+\cot^2(e)}\sin(e)\sin(fx-\arctan(\cot(e)))\right)}{a\sqrt{1+\cot^2(e)}\left(1+\frac{b\csc(e)}{a\sqrt{1+\cot^2(e)}}\right)},\frac{\csc(e)\left(b-a\sqrt{1+\cot^2(e)}\sin(e)\sin(fx-\arctan(\cot(e)))\right)}{a\sqrt{1+\cot^2(e)}\left(-1+\frac{b\csc(e)}{a\sqrt{1+\cot^2(e)}}\right)}\right)}{}$$

$$+\frac{a\cos^2\left(\frac{e}{2}+\frac{fx}{2}\right)\sqrt{b+a\cos(e+fx)}\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)(g\sec(e+fx))^{3/2}\left(\frac{\text{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},-\frac{\sec(e)\left(b+a\cos(e)\cos(\right.}{a\sqrt{1+\tan^2(}}\right.}{\sqrt{1+\tan^2(e)}\sqrt{\frac{a\sqrt{1+\tan^2(e)}-a\cos(fx+\arctan(\tan}{b\sec(e)+a\sqrt{1+\tan^2(}}}}\right.}{2(-}$$

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])),x]
```

[Out] (Cos[e/2 + (f*x)/2]^2*(b + a*Cos[e + f*x])*(g*Sec[e + f*x])^(3/2)*((2*Csc[e
])/((-a + b)*f) + (2*Sec[e/2]*Sec[e/2 + (f*x)/2]*Sin[(f*x)/2])/((-a + b)*f)
))/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])) + (AppellF1[1/2, 1/2, 1/
2, 3/2, (Csc[e]*(b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]])
)/(a*Sqrt[1 + Cot[e]^2]*(1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2]))), (Csc[e]*(
b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]))/(a*Sqrt[1 + Cot
[e]^2]*(-1 + (b*Csc[e])/(a*Sqrt[1 + Cot[e]^2])))]*Cos[e/2 + (f*x)/2]^2*Sqrt
[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(g*Sec[e + f*x])^(3/2)*Sec[f*x - Arc
Tan[Cot[e]]]*Sqrt[(a*Sqrt[1 + Cot[e]^2] - a*Sqrt[1 + Cot[e]^2]*Sin[f*x - Ar
cTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] - b*Csc[e])]*Sqrt[(a*Sqrt[1 + Cot[e]^2
] + a*Sqrt[1 + Cot[e]^2]*Sin[f*x - ArcTan[Cot[e]]])/(a*Sqrt[1 + Cot[e]^2] +
 b*Csc[e])]*Sqrt[b - a*Sqrt[1 + Cot[e]^2]*Sin[e]*Sin[f*x - ArcTan[Cot[e]]]]
)/((-a + b)*f*Sqrt[1 + Cot[e]^2]*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*
x])) + (a*Cos[e/2 + (f*x)/2]^2*Sqrt[b + a*Cos[e + f*x]]*Csc[e/2]*Sec[e/2]*(
g*Sec[e + f*x])^(3/2)*((AppellF1[-1/2, -1/2, -1/2, 1/2, -((Sec[e]*(b + a*Co
s[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^2]*(
1 - (b*Sec[e])/(a*Sqrt[1 + Tan[e]^2])))), -((Sec[e]*(b + a*Cos[e]*Cos[f*x +
 ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a*Sqrt[1 + Tan[e]^2]*(-1 - (b*Sec[e]
)/(a*Sqrt[1 + Tan[e]^2]))))]*Sin[f*x + ArcTan[Tan[e]]]*Tan[e])/(Sqrt[1 + Ta
n[e]^2]*Sqrt[(a*Sqrt[1 + Tan[e]^2] - a*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + T
an[e]^2])/(b*Sec[e] + a*Sqrt[1 + Tan[e]^2])]*Sqrt[(a*Sqrt[1 + Tan[e]^2] + a
*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2])/(-(b*Sec[e]) + a*Sqrt[1 + Ta
n[e]^2])]*Sqrt[b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]])
- ((Sin[f*x + ArcTan[Tan[e]]]*Tan[e])/Sqrt[1 + Tan[e]^2] + (2*a*Cos[e]*(b +
 a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]))/(a^2*Cos[e]^2 + a^
2*Sin[e]^2))/Sqrt[b + a*Cos[e]*Cos[f*x + ArcTan[Tan[e]]]*Sqrt[1 + Tan[e]^2]
]))/(2*(-a + b)*f*Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x]))

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.70 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.90

method	result
default	$-\frac{i g \cos(fx+e)\sqrt{g \sec(fx+e)}\,\sqrt{a+b\sec(fx+e)}\,\left(2\,\mathrm{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)a-a\,\mathrm{EllipticE}\left(i(-\cot(fx+e)+\csc\right.\right.}{cf(a-b)(b+a\cos(fx+e))\sqrt{\frac{1}{\cos(fx+e)+}}}$

[In] int((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)

[Out] -I*g/c/f/(a-b)*cos(f*x+e)*(g*sec(f*x+e))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(2*El
lipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*a-a*EllipticE(I*(-
cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-cot(f*x+e)+csc
(f*x+e)),(-(a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(

```
1/2)/(b+a*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.26

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx =$$
$$-\frac{6\,ag\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{g}{\cos(fx+e)}}\cos\left(fx+e\right)\sin\left(fx+e\right)+\sqrt{2}(i\,(3\,a-2\,b)g\cos\left(fx+e\right)+i\,(3\,a-2\,b)g)\sqrt{a}}{}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] -1/6*(6*a*g*sqrt((a*cos(f*x + e) + b)/cos(f*x + e))*sqrt(g/cos(f*x + e))*co
s(f*x + e)*sin(f*x + e) + sqrt(2)*(I*(3*a - 2*b)*g*cos(f*x + e) + I*(3*a -
2*b)*g)*sqrt(a*g)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) + 3*I*a*sin(f*x + e) + 2*b)/a) + sqr
t(2)*(-I*(3*a - 2*b)*g*cos(f*x + e) - I*(3*a - 2*b)*g)*sqrt(a*g)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(f*x + e) - 3*I*a*sin(f*x + e) + 2*b)/a) - 3*sqrt(2)*(I*a*g*cos(f*x + e) +
 I*a*g)*sqrt(a*g)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) + 3*I*a*sin(f*x + e) + 2*b)/a)) - 3*sqrt(
2)*(-I*a*g*cos(f*x + e) - I*a*g)*sqrt(a*g)*weierstrassZeta(-4/3*(3*a^2 - 4*
b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(f*x + e) - 3*I*a*sin(f*x
+ e) + 2*b)/a)))/((a^2 - a*b)*c*f*cos(f*x + e) + (a^2 - a*b)*c*f)
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{\int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}}{\sqrt{a+b\sec{(e+fx)}}\sec{(e+fx)}+\sqrt{a+b\sec{(e+fx)}}}\,dx}{c}$$

```
[In] integrate((g*sec(f*x+e))**(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*sec(e + f*x) + s
qrt(a + b*sec(e + f*x))), x)/c
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
), x)
```

3.278 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 312

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = -\frac{g^2(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)\middle|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}} - \frac{g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} + \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}} + \frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}$$

```
[Out] g^2*(b+a*cos(f*x+e))*sin(f*x+e)*(g*sec(f*x+e))^(1/2)/(a-b)/f/(c+c*cos(f*x+e
))/(a+b*sec(f*x+e))^(1/2)-g^2*(b+a*cos(f*x+e))*(cos(1/2*f*x+1/2*e)^2)^(1/2)
/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*(
g*sec(f*x+e))^(1/2)/(a-b)/c/f/((b+a*cos(f*x+e))/(a+b))^(1/2)/(a+b*sec(f*x+e
))^(1/2)-g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(
1/2*f*x+1/2*e),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*s
ec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e))^(1/2)+2*g^2*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2,2^(1/2)*(a/(a+b))^(1
/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/c/f/(a+b*sec(f*x+e
))^(1/2)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.282$, Rules used = {4064, 3944, 2886, 2884, 4060, 2847, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \frac{g^2\sin(e+fx)\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)}{f(a-b)(c\cos(e+fx)+c)\sqrt{a+b\sec(e+fx)}} - \frac{g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}} - \frac{g^2\sqrt{g\sec(e+fx)}(a\cos(e+fx)+b)E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)}{cf(a-b)\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\sqrt{a+b\sec(e+fx)}} + \frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{cf\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f*x])),
x]
```

```
[Out] -((g^2*(b + a*Cos[e + f*x])*EllipticE[(e + f*x)/2, (2*a)/(a + b)]*Sqrt[g*Se
c[e + f*x]])/((a - b)*c*f*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*Sqrt[a + b*Sec
[e + f*x]])) - (g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticF[(e + f*x)/
2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (2
*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/(a
 + b)]*Sqrt[g*Sec[e + f*x]])/(c*f*Sqrt[a + b*Sec[e + f*x]]) + (g^2*(b + a*C
os[e + f*x])*Sqrt[g*Sec[e + f*x]]*Sin[e + f*x])/((a - b)*f*(c + c*Cos[e + f
*x])*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
```

```
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4064

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c*(g/d),
 Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx\right)+\frac{g\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{c}$$

$$= -\frac{\left(g^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}(c+c\cos(e+fx))}\,dx}{\sqrt{a+b\sec(e+fx)}} + \frac{\left(g^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{c\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}} + \frac{\left(ag^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{-\frac{c}{2}-\frac{1}{2}c\cos(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{(a-b)c^2\sqrt{a+b\sec(e+fx)}} + \frac{\left(g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{c\sqrt{a+b\sec(e+fx)}}$$

$$=\frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}-\frac{\left(g^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{b+a\cos(e+fx)}}\,dx}{2c\sqrt{a+b\sec(e+fx)}}-\frac{\left(g^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\sqrt{b+a\cos(e+fx)}\,dx}{2(a-b)c\sqrt{a+b\sec(e+fx)}}$$

$$=\frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}-\frac{\left(g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\right)\int\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}\,dx}{2(a-b)c\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}-\frac{\left(g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{2c\sqrt{a+b\sec(e+fx)}}$$

$$=-\frac{g^2(b+a\cos(e+fx))E\left(\frac{1}{2}(e+fx)|\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(a-b)cf\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{a+b\sec(e+fx)}}-\frac{g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}+\frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{cf\sqrt{a+b\sec(e+fx)}}+\frac{g^2(b+a\cos(e+fx))\sqrt{g\sec(e+fx)}\sin(e+fx)}{(a-b)f(c+c\cos(e+fx))\sqrt{a+b\sec(e+fx)}}$$

Mathematica [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx$$

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])),x]
[Out] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + c*Sec[e + f
*x])), x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.04

method	result
default	$\frac{i\left(4\,\mathrm{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)a-2b\,\mathrm{EllipticF}\left(i(-\cot(fx+e)+\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)-a\,\mathrm{EllipticE}\left(i(-\cot(fx+e$

```
[In] int((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
[Out] I/c/f/(a-b)*(4*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*a
-2*b*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-a*EllipticE
(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-b*EllipticE(I*(-cot(f*x+e
)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-4*EllipticPi(I*(-cot(f*x+e)+csc(f*x+e))
,-1,I*((a-b)/(a+b))^(1/2))*a+4*b*EllipticPi(I*(-cot(f*x+e)+csc(f*x+e)),-1,I
*((a-b)/(a+b))^(1/2)))*(a+b*sec(f*x+e))^(1/2)*(g*sec(f*x+e))^(1/2)*(1/(a+b)
*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*g^2/(1/(cos(f*x+e)+1))^(1/2)/(b+a*c
os(f*x+e))*cos(f*x+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))**(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)+a}(c\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+c*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(c*sec(f*x + e)
+ c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+c\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{c}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
),x)
[Out] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + c/cos(e + f*x))
), x)
```

3.279 $\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 213

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df} - \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/d/f-2*(-a*d+b*c)*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/
f/(a+b*sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used

$= \{4054, 3917, 4058\}$

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$$
$$= \frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{df}$$
$$- \frac{2(bc-ad)\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{df(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(d*f) - (2*(b*c - a*d)*EllipticPi[(2*d)/(c + d
), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e
 + f*x])/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[
-Tan[e + f*x]^2])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4054

```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d, Int[Csc[e + f*
x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/d, Int[Csc[e + f*x]/
(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{df} - \frac{2(bc-ad)\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((a-b)(c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)\right.}{(c-d)(c+d)f(b+a\cos(e+}$$

```
[In] Integrate[(Sec[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*(c + d)*EllipticF[ArcSin[
Tan[(e + f*x)/2]], (a - b)/(a + b)] + 2*(b*c - a*d)*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/
((c - d)*(c + d)*f*(b + a*Cos[e + f*x]))
```

Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

method	result
default	$-\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)ac+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)ad-\text{EllipticF}\left(\cot(fx+e)\right.\right.}{}$

```
[In] int(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/f/(c+d)/(c-d)*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+
b))^(1/2))*a*c+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*a*d-Ell
ipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*b*c-EllipticF(cot(f*x+e)-
csc(f*x+e),((a-b)/(a+b))^(1/2))*b*d-2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d
```

```
)/(c+d),((a-b)/(a+b))^(1/2))*a*d+2*EllipticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(
c+d),((a-b)/(a+b))^(1/2))*b*c)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+b\sec{(e+fx)}}\sec{(e+fx)}}{c+d\sec{(e+fx)}}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*sec(e + f*x)/(c + d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}\sec{(fx+e)}}{d\sec{(fx+e)}+c}\,dx$$

```
[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec(fx+e)+a}\sec(fx+e)}{d\sec(fx+e)+c}\,dx$$

[In] integrate(sec(f*x+e)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*sec(f*x + e)/(d*sec(f*x + e) + c), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}}{\cos(e+f\,x)\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

[In] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))),x)

[Out] int((a + b/cos(e + f*x))^(1/2)/(cos(e + f*x)*(c + d/cos(e + f*x))), x)

3.280 $\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 170

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}} - \frac{2(bc-ad)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\operatorname{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{d(c+d)f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*b*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f
*x+e))^(1/2)/d/f/(a+b*sec(f*x+e))^(1/2)-2*(-a*d+b*c)*g*(cos(1/2*f*x+1/2*e)^
2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*c/(c+d),2^(1/2)
*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/d/(c+
d)/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4056, 3944, 2886, 2884, 4060}

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \frac{2bg\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}} - \frac{2g(bc-ad)\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\operatorname{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e + f*x]),
x]
```

```
[Out] (2*b*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*(b*c - a
*d)*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x
)/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f
*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4056

```
Int[((csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)])/(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Dist[b/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a
*d)/d, Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e +
f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && N
eQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(bc-ad)\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{\left(bg\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left((bc-ad)g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))}\,dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{\left(bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left((bc-ad)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}(d+c\cos(e+fx))}\,dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2bg\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}} - \frac{2(bc-ad)g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{d(c+d)f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.56 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.31

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = -\frac{2ig\sqrt{-\frac{a(-1+\cos(e+fx))}{a+b}}\sqrt{\frac{a(1+\cos(e+fx))}{a-b}}\cot(e+fx)\left(\text{EllipticPi}\left(1-\frac{a}{b},i\,\text{arcsinh}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(e+fx)}\right)\right.\right.}{\sqrt{\frac{1}{a-b}}df\sqrt{b}}$$

```
[In] Integrate[((g*Sec[e + f*x])^(3/2)*Sqrt[a + b*Sec[e + f*x]])/(c + d*Sec[e +
f*x]),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*x
]))/(a - b)]*Cot[e + f*x]*(EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]
```

```
*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)] - EllipticPi[((a - b)*c)/(-(b
*c) + a*d), I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b
)/(a + b)])*Sqrt[g*Sec[e + f*x]]*Sqrt[a + b*Sec[e + f*x]])/(Sqrt[(a - b)^(-
1)]*d*f*Sqrt[b + a*Cos[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.22 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.62

method	result
default	$-\frac{2ig\left(\operatorname{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)acd+\operatorname{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)a\,d^2-\operatorname{EllipticF}\left(i(\cot(fx+e)\right.\right.}{}$

```
[In] int((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] -2*I*g/f/d/(c+d)/(c-d)*(EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^
(1/2))*a*c*d+EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*a*d^
2-EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*b*c*d-EllipticF
(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*b*d^2-2*EllipticPi(I*(cot(
f*x+e)-csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))*b*c^2+2*EllipticPi(I*(cot(f*x+
e)-csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))*b*d^2-2*EllipticPi(I*(cot(f*x+e)-c
sc(f*x+e)),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2))*a*c*d+2*EllipticPi(I*(cot(f*
x+e)-csc(f*x+e)),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2))*b*c^2)*cos(f*x+e)*(a+b
*sec(f*x+e))^(1/2)*(g*sec(f*x+e))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+
e)+1))^(1/2)/(b+a*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{(g\sec{(e+fx)})^{\frac{3}{2}}\sqrt{a+b\sec{(e+fx)}}}{c+d\sec{(e+fx)}}\,dx$$

[In] integrate((g*sec(f*x+e))**(3/2)*(a+b*sec(f*x+e))**(1/2)/(c+d*sec(f*x+e)),x)

[Out] Integral((g*sec(e + f*x))**(3/2)*sqrt(a + b*sec(e + f*x))/(c + d*sec(e + f*x)), x)

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}(g\sec{(fx+e)})^{\frac{3}{2}}}{d\sec{(fx+e)}+c}\,dx$$

[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="maxima")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) + c), x)

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{b\sec{(fx+e)}+a}(g\sec{(fx+e)})^{\frac{3}{2}}}{d\sec{(fx+e)}+c}\,dx$$

[In] integrate((g*sec(f*x+e))^(3/2)*(a+b*sec(f*x+e))^(1/2)/(c+d*sec(f*x+e)),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e) + a)*(g*sec(f*x + e))^(3/2)/(d*sec(f*x + e) + c), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}\sqrt{a+b\sec(e+fx)}}{c+d\sec(e+fx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,{\left(\frac{g}{\cos(e+f\,x)}\right)}^{3/2}}{c+\frac{d}{\cos(e+f\,x)}}\,dx$$

[In] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)),x)

[Out] int(((a + b/cos(e + f*x))^(1/2)*(g/cos(e + f*x))^(3/2))/(c + d/cos(e + f*x)), x)

3.281 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[Out] 2*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)*(b/(a+b))^(
1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/(c+d)/f/(a+b*sec(f*x+e))^(1
/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {4058}

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{f(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*EllipticPi[(2*d)/(c + d), ArcSin[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/
(a + b)]*Sqrt[(a + b*Sec[e + f*x])/(a + b)]*Tan[e + f*x])/((c + d)*f*Sqrt[a
 + b*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2\,\text{EllipticPi}\left(\frac{2d}{c+d}, \arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right), \frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 8.83 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.83

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right), \frac{a-b}{a+b}\right) - 2d\,\text{EllipticPi}\left(\frac{c-d}{c+d}, \arcsin\left(\tan\left(\frac{1}{2}(\right.\right.\right.\right.}{(c-d)(c+d)f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{a}}$$

```
[In] Integrate[Sec[e + f*x]/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*Ellipti
cF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - 2*d*EllipticPi[(c - d)/(c +
 d), ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)])*Sqrt[Cos[e + f*x]*Sec[(e +
 f*x)/2]^2]*Sqrt[Sec[e + f*x]]*Sqrt[1 + Sec[e + f*x]])/((c - d)*(c + d)*f*S
qrt[Sec[(e + f*x)/2]^2]*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(97) = 194.

Time = 7.64 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.99

method	result
default	$-\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e), \sqrt{\frac{a-b}{a+b}}\right)c+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e), \sqrt{\frac{a-b}{a+b}}\right)d-2\,\text{EllipticPi}\left(\cot(fx+\right.\right.}{f(c-d)(c+d)(b+a\cos(fx+e))}$

```
[In] int(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/f/(c-d)/(c+d)*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+
b))^(1/2))*c+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*d-2*Ellip
ticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2))*d)*(cos(f*x+e)
/(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b
*sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$$

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)
```

Maxima [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Giac [F]

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate(sec(f*x+e)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sec(f*x + e)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\cos(e+f\,x)\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.282 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 209

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bdf} - \frac{2c\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[Out] 2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1
/2)/b/d/f-2*c*EllipticPi(1/2*(1-sec(f*x+e))^(1/2)*2^(1/2),2*d/(c+d),2^(1/2)
*(b/(a+b))^(1/2))*((a+b*sec(f*x+e))/(a+b))^(1/2)*tan(f*x+e)/d/(c+d)/f/(a+b*
sec(f*x+e))^(1/2)/(-tan(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used

$= \{4062, 3917, 4058\}$

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$$
$$= \frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bdf}$$
$$- \frac{2c\tan(e+fx)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)}{df(c+d)\sqrt{-\tan^2(e+fx)}\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[e + f*x]))/(a - b))])/(b*d*f) - (2*c*EllipticPi[(2*d)/(c + d), ArcSi
n[Sqrt[1 - Sec[e + f*x]]/Sqrt[2]], (2*b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/(a + b)]*Tan[e + f*x])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]]*Sqrt[-Tan[e +
 f*x]^2])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4058

```
Int[csc[(e_.) + (f_.)*(x_)]/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(cs
c[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Simp[-2*(Cot[e + f*x]/(f
*(c + d)*Sqrt[a + b*Csc[e + f*x]]*Sqrt[-Cot[e + f*x]^2]))*Sqrt[(a + b*Csc[e
 + f*x])/(a + b)]*EllipticPi[2*(d/(c + d)), ArcSin[Sqrt[1 - Csc[e + f*x]]/S
qrt[2]], 2*(b/(a + b))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 4062

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(
csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[1/d, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c/d, Int[Csc[e + f*x]/(Sqrt[a
+ b*Csc[e + f*x]]*(c + d*Csc[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{c\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{2\sqrt{a+b}\cot(e+fx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{bdf} - \frac{2c\,\text{EllipticPi}\left(\frac{2d}{c+d},\arcsin\left(\frac{\sqrt{1-\sec(e+fx)}}{\sqrt{2}}\right),\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{a+b}}\tan(e+fx)}{d(c+d)f\sqrt{a+b\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 4.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = -\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((c+d)\,\text{EllipticF}\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-2c\ldots\right.}{(c-d)(c+d)f\sqrt{a+b\sec(e+fx)}}$$

```
[In] Integrate[Sec[e + f*x]^2/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((c + d)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] - 2*c*EllipticPi[(c - d)/(c + d), ArcSin[Tan[(
e + f*x)/2]], (a - b)/(a + b)])*Sec[e + f*x])/((c - d)*(c + d)*f*Sqrt[a + b
*Sec[e + f*x]])
```

Maple [A] (verified)

Time = 8.54 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.97

method	result
default	$\frac{2(\cos(fx+e)+1)\left(\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)c+\text{EllipticF}\left(\cot(fx+e)-\csc(fx+e),\sqrt{\frac{a-b}{a+b}}\right)d-2c\,\text{EllipticPi}\left(\cot(fx+e)-\ldots\right.\right.}{f(c+d)(c-d)(b+a\cos(fx+e))}$

```
[In] int(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/f/(c+d)/(c-d)*(cos(f*x+e)+1)*(EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b
))^(1/2))*c+EllipticF(cot(f*x+e)-csc(f*x+e),((a-b)/(a+b))^(1/2))*d-2*c*Elli
pticPi(cot(f*x+e)-csc(f*x+e),(c-d)/(c+d),((a-b)/(a+b))^(1/2)))*(cos(f*x+e)/
```

```
(cos(f*x+e)+1))^(1/2)*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*
sec(f*x+e))^(1/2)/(b+a*cos(f*x+e))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$$

```
[In] integrate(sec(f*x+e)**2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x
)
```

Maxima [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

Giac [F]

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\sec(fx+e)^2}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate(sec(f*x+e)^2/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e) + c)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{1}{\cos(e+f\,x)^2\,\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))), x)
```

3.283 $\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 83

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+
1/2*e),2*c/(c+d),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g
*sec(f*x+e))^(1/2)/(c+d)/f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4060, 2886, 2884}

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\right.}{f(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(g\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))}\,dx}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{\left(g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}(d+c\cos(e+fx))}\,dx}{\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int\frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+f}}{(c+d)f\sqrt{a+b\sec(e+fx)}}$$

```
[In] Integrate[(g*Sec[e + f*x])^(3/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] (2*g*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)
/2, (2*a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/((c + d)*f*Sqrt[a + b*Sec[e + f*x]
])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.69

method	result
default	$-\frac{2ig\sqrt{a+b\sec(fx+e)}\,\cos(fx+e)\sqrt{g\sec(fx+e)}\,\left(2c\,\mathrm{EllipticPi}\left(i(-\cot(fx+e)+\csc(fx+e)),-\frac{c-d}{c+d},i\sqrt{\frac{a-b}{a+b}}\right)-c\,\mathrm{EllipticF}\left(i(-\cot(f\right.\right.}{f(c-d)(c+d)(b+a\cos(fx+e))\sqrt{\cos}}$

[In] int((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2*I*g/f/(c-d)/(c+d)*(a+b*sec(f*x+e))^(1/2)*cos(f*x+e)*(g*sec(f*x+e))^(1/2)*(2*c*EllipticPi(I*(-cot(f*x+e)+csc(f*x+e)),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2))-c*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2))-d*EllipticF(I*(-cot(f*x+e)+csc(f*x+e)),(-(a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)/(b+a*cos(f*x+e))/(1/(cos(f*x+e)+1))^(1/2)

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{(g\sec(e+fx))^{\frac{3}{2}}}{\sqrt{a+b\sec(e+fx)}\,(c+d\sec(e+fx))}\,dx$$

[In] integrate((g*sec(f*x+e))**(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)

[Out] Integral((g*sec(e + f*x))**(3/2)/(sqrt(a + b*sec(e + f*x))*(c + d*sec(e + f*x))), x)

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{3}{2}}}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(3/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(3/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{3/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(3/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

3.284 $\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 166

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}} - \frac{2cg^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{d(c+d)f\sqrt{a+b\sec(e+fx)}}$$

```
[Out] 2*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f
*x+e))^(1/2)/d/f/(a+b*sec(f*x+e))^(1/2)-2*c*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*c/(c+d),2^(1/2)*(a/(a+
b))^(1/2))*((b+a*cos(f*x+e))/(a+b))^(1/2)*(g*sec(f*x+e))^(1/2)/d/(c+d)/f/(a
+b*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {4064, 3944, 2886, 2884, 4060}

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \frac{2g^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df\sqrt{a+b\sec(e+fx)}} - \frac{2cg^2\sqrt{g\sec(e+fx)}\sqrt{\frac{a\cos(e+fx)+b}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)}{df(c+d)\sqrt{a+b\sec(e+fx)}}$$

```
[In] Int[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f*x])),
x]
```

```
[Out] (2*g^2*Sqrt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[2, (e + f*x)/2, (2*a)/
(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*f*Sqrt[a + b*Sec[e + f*x]]) - (2*c*g^2*Sq
rt[(b + a*Cos[e + f*x])/(a + b)]*EllipticPi[(2*c)/(c + d), (e + f*x)/2, (2*
a)/(a + b)]*Sqrt[g*Sec[e + f*x]])/(d*(c + d)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4060

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(3/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g*Sqr
t[g*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/Sqrt[a + b*Csc[e + f*x]]), Int[
1/(Sqrt[b + a*Sin[e + f*x]]*(d + c*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c
, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4064

```
Int[(csc[(e_.) + (f_.)*(x_)]*(g_.))^(5/2)/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))), x_Symbol] :> Dist[g/d,
Int[(g*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[c*(g/d),
 Int[(g*Csc[e + f*x])^(3/2)/(Sqrt[a + b*Csc[e + f*x]]*(c + d*Csc[e + f*x]))
, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2
 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{g\int\frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}}\,dx}{d} - \frac{(cg)\int\frac{(g\sec(e+fx))^{3/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx}{d}$$

$$= \frac{\left(g^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\frac{\sec(e+fx)}{\sqrt{b+a\cos(e+fx)}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left(cg^2\sqrt{b+a\cos(e+fx)}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{b+a\cos(e+fx)}(d+c\cos(e+fx))}\,dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{\left(g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int\frac{\sec(e+fx)}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}}\,dx}{d\sqrt{a+b\sec(e+fx)}} - \frac{\left(cg^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\sqrt{g\sec(e+fx)}\right)\int\frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(e+fx)}{a+b}}(d+c\cos(e+fx))}\,dx}{d\sqrt{a+b\sec(e+fx)}}$$

$$= \frac{2g^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(2,\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{df\sqrt{a+b\sec(e+fx)}} - \frac{2cg^2\sqrt{\frac{b+a\cos(e+fx)}{a+b}}\,\text{EllipticPi}\left(\frac{2c}{c+d},\frac{1}{2}(e+fx),\frac{2a}{a+b}\right)\sqrt{g\sec(e+fx)}}{d(c+d)f\sqrt{a+b\sec(e+fx)}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.48

$$\int\frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx =$$

$$-\frac{2ig\sqrt{-\frac{a(-1+\cos(e+fx))}{a+b}}\sqrt{\frac{a(1+\cos(e+fx))}{a-b}}\sqrt{b+a\cos(e+fx)}\cot(e+fx)\left((-bc+ad)\,\text{EllipticPi}\left(1-\frac{a}{b},i\text{arcsi}\right.\right.}{\sqrt{\frac{1}{a-b}}bd(}$$

```
[In] Integrate[(g*Sec[e + f*x])^(5/2)/(Sqrt[a + b*Sec[e + f*x]]*(c + d*Sec[e + f
*x])),x]
```

```
[Out] ((-2*I)*g*Sqrt[-((a*(-1 + Cos[e + f*x]))/(a + b))]*Sqrt[(a*(1 + Cos[e + f*x
]))/(a - b)]*Sqrt[b + a*Cos[e + f*x]]*Cot[e + f*x]*((-(b*c) + a*d)*Elliptic
Pi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b
)/(a + b)] + b*c*EllipticPi[((a - b)*c)/(-(b*c) + a*d), I*ArcSinh[Sqrt[(a -
 b)^(-1)]*Sqrt[b + a*Cos[e + f*x]]], (-a + b)/(a + b)])*(g*Sec[e + f*x])^(3
/2))/(Sqrt[(a - b)^(-1)]*b*d*(-(b*c) + a*d)*f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.42 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.89

method	result
default	$\frac{2i\left(\text{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)dc+\text{EllipticF}\left(i(\cot(fx+e)-\csc(fx+e)),\sqrt{-\frac{a-b}{a+b}}\right)d^2+2c^2\,\text{EllipticPi}\left(i(\cot(fx+e)-\right.\right.}{}$

[In] int((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)

[Out] 2*I/f/d/(c+d)/(c-d)*(EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/
2))*d*c+EllipticF(I*(cot(f*x+e)-csc(f*x+e)),(-(a-b)/(a+b))^(1/2))*d^2+2*c^2
EllipticPi(I(cot(f*x+e)-csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))-2*EllipticP
i(I*(cot(f*x+e)-csc(f*x+e)),-1,I*((a-b)/(a+b))^(1/2))*d^2-2*c^2*EllipticPi(
I*(cot(f*x+e)-csc(f*x+e)),-(c-d)/(c+d),I*((a-b)/(a+b))^(1/2)))*(1/(a+b)*(b+
a*cos(f*x+e))/(cos(f*x+e)+1))^(1/2)*(a+b*sec(f*x+e))^(1/2)*(g*sec(f*x+e))^(
1/2)*g^2/(1/(cos(f*x+e)+1))^(1/2)/(b+a*cos(f*x+e))*cos(f*x+e)

Fricas [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \text{Timed out}$$

[In] integrate((g*sec(f*x+e))**(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)

[Out] Timed out

Maxima [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Giac [F]

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{(g\sec(fx+e))^{\frac{5}{2}}}{\sqrt{b\sec(fx+e)+a}(d\sec(fx+e)+c)}\,dx$$

```
[In] integrate((g*sec(f*x+e))^(5/2)/(c+d*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^(5/2)/(sqrt(b*sec(f*x + e) + a)*(d*sec(f*x + e)
+ c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g\sec(e+fx))^{5/2}}{\sqrt{a+b\sec(e+fx)}(c+d\sec(e+fx))}\,dx = \int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^{5/2}}{\sqrt{a+\frac{b}{\cos(e+f\,x)}}\,\left(c+\frac{d}{\cos(e+f\,x)}\right)}\,dx$$

```
[In] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
),x)
```

```
[Out] int((g/cos(e + f*x))^(5/2)/((a + b/cos(e + f*x))^(1/2)*(c + d/cos(e + f*x))
), x)
```

3.285 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f} - \frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f} + \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}$$

```
[Out] 1/20*cot(1/2*f*x+1/2*e)^5/c^7/f-1/14*cot(1/2*f*x+1/2*e)^7/c^7/f+1/36*cot(1/
2*f*x+1/2*e)^9/c^7/f
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {12, 276}

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f} - \frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f} + \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}$$

```
[In] Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] Cot[(e + f*x)/2]^5/(20*c^7*f) - Cot[(e + f*x)/2]^7/(14*c^7*f) + Cot[(e + f*
x)/2]^9/(36*c^7*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int -\frac{(1-x^2)^2}{8c^7x^{10}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^{10}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{4c^7f}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{10}}-\frac{2}{x^8}+\frac{1}{x^6}\right)\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{4c^7f}\\
&= \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{20c^7f}-\frac{\cot^7\left(\frac{1}{2}(e+fx)\right)}{14c^7f}+\frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{36c^7f}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 151 vs. 2(67) = 134.

Time = 4.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx$$
$$= \frac{\csc\left(\frac{e}{2}\right)\csc^9\left(\frac{1}{2}(e+fx)\right)\left(-971082\sin\left(\frac{fx}{2}\right)-718830\sin\left(e+\frac{fx}{2}\right)+467208\sin\left(e+\frac{3fx}{2}\right)+659400\sin\left(2e\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^7,x]
```

```
[Out] (Csc[e/2]*Csc[(e + f*x)/2]^9*(-971082*Sin[(f*x)/2] - 718830*Sin[e + (f*x)/2
] + 467208*Sin[e + (3*f*x)/2] + 659400*Sin[2*e + (3*f*x)/2] - 303192*Sin[2*
e + (5*f*x)/2] - 179640*Sin[3*e + (5*f*x)/2] + 30753*Sin[3*e + (7*f*x)/2] +
 89955*Sin[4*e + (7*f*x)/2] - 13427*Sin[4*e + (9*f*x)/2] + 15*Sin[5*e + (9*
f*x)/2]))/(23063040*c^7*f)
```

Maple [A] (verified)

Time = 11.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}}{4f\,c^7}$
default	$\frac{\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^5}-\frac{2}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^7}+\frac{1}{9\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^9}}{4f\,c^7}$
risch	$\frac{2i\left(315\,e^{8i(fx+e)}-630\,e^{7i(fx+e)}+2310\,e^{6i(fx+e)}-2520\,e^{5i(fx+e)}+3402\,e^{4i(fx+e)}-1638\,e^{3i(fx+e)}+1062\,e^{2i(fx+e)}-108\,e^{i}\right.}{315f\,c^7\left(e^{i(fx+e)}-1\right)^9}$

```
[In] int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/f/c^7*(1/5/tan(1/2*f*x+1/2*e)^5-2/7/tan(1/2*f*x+1/2*e)^7+1/9/tan(1/2*f*
x+1/2*e)^9)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(58) = 116$.

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{47\cos(fx+e)^5+127\cos(fx+e)^4+101\cos(fx+e)^3+11\cos(fx+e)^2-8\cos(fx+e)+2}{315\left(c^7f\cos(fx+e)^4-4\,c^7f\cos(fx+e)^3+6\,c^7f\cos(fx+e)^2-4\,c^7f\cos(fx+e)+c^7f\right)\sin(fx+e)}$$

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="fricas")
```

```
[Out] 1/315*(47*cos(f*x + e)^5 + 127*cos(f*x + e)^4 + 101*cos(f*x + e)^3 + 11*cos
(f*x + e)^2 - 8*cos(f*x + e) + 2)/((c^7*f*cos(f*x + e)^4 - 4*c^7*f*cos(f*x
+ e)^3 + 6*c^7*f*cos(f*x + e)^2 - 4*c^7*f*cos(f*x + e) + c^7*f)*sin(f*x + e
))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = -\frac{\int \frac{\tan^4(e+fx)\sec(e+fx)}{\sec^7(e+fx)-7\sec^6(e+fx)+21\sec^5(e+fx)-35\sec^4(e+fx)+35\sec^3(e+fx)-21\sec^2(e+fx)+7\sec(e+fx)-1}\,dx}{c^7}$$

```
[In] integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**7,x)
```

```
[Out] -Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**7 - 7*sec(e + f*x)**6
 + 21*sec(e + f*x)**5 - 35*sec(e + f*x)**4 + 35*sec(e + f*x)**3 - 21*sec(e
+ f*x)**2 + 7*sec(e + f*x) - 1), x)/c**7
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = -\frac{\left(\frac{90\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{63\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 35\right)(\cos(fx+e)+1)^9}{1260\,c^7 f \sin(fx+e)^9}$$

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="maxima")
```

```
[Out] -1/1260*(90*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 63*sin(f*x + e)^4/(cos(f*
x + e) + 1)^4 - 35)*(cos(f*x + e) + 1)^9/(c^7*f*sin(f*x + e)^9)
```

Giac [A] (verification not implemented)

none

Time = 1.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{63\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 - 90\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 + 35}{1260\,c^7 f \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^9}$$

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^7,x, algorithm="giac")
```

```
[Out] 1/1260*(63*tan(1/2*f*x + 1/2*e)^4 - 90*tan(1/2*f*x + 1/2*e)^2 + 35)/(c^7*f*
tan(1/2*f*x + 1/2*e)^9)
```

Mupad [B] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^7}\,dx = \frac{63\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^4 - 90\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 + 35}{1260\,c^7\,f\tan\left(\frac{e}{2}+\frac{f\,x}{2}\right)^9}$$

```
[In] int(tan(e + f*x)^4/(cos(e + f*x)*(c - c/cos(e + f*x))^7),x)
```

```
[Out] (63*tan(e/2 + (f*x)/2)^4 - 90*tan(e/2 + (f*x)/2)^2 + 35)/(1260*c^7*f*tan(e/
2 + (f*x)/2)^9)
```

3.286 $\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 89

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f} - \frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f} + \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f} - \frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}$$

```
[Out] 1/40*cot(1/2*f*x+1/2*e)^5/c^8/f-3/56*cot(1/2*f*x+1/2*e)^7/c^8/f+1/24*cot(1/
2*f*x+1/2*e)^9/c^8/f-1/88*cot(1/2*f*x+1/2*e)^11/c^8/f
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {12, 276}

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = -\frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f} + \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f} - \frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f} + \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f}$$

```
[In] Int[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

```
[Out] Cot[(e + f*x)/2]^5/(40*c^8*f) - (3*Cot[(e + f*x)/2]^7)/(56*c^8*f) + Cot[(e
+ f*x)/2]^9/(24*c^8*f) - Cot[(e + f*x)/2]^11/(88*c^8*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\text{Subst}\left(\int \frac{(1-x^2)^3}{16c^8x^{12}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{f} \\
&= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^{12}}\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{8c^8f} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^{12}} - \frac{3}{x^{10}} + \frac{3}{x^8} - \frac{1}{x^6}\right)\,dx, x, \tan\left(\frac{1}{2}(e+fx)\right)\right)}{8c^8f} \\
&= \frac{\cot^5\left(\frac{1}{2}(e+fx)\right)}{40c^8f} - \frac{3\cot^7\left(\frac{1}{2}(e+fx)\right)}{56c^8f} + \frac{\cot^9\left(\frac{1}{2}(e+fx)\right)}{24c^8f} - \frac{\cot^{11}\left(\frac{1}{2}(e+fx)\right)}{88c^8f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.02 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.97

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = -\frac{\csc\left(\frac{e}{2}\right)\csc^{11}\left(\frac{1}{2}(e+fx)\right)\left(425964\sin\left(\frac{fx}{2}\right) + 486024\sin\left(e+\frac{fx}{2}\right) - 351450\sin\left(e+\frac{3fx}{2}\right) - 299970\sin\left(2\right.\right.}{}$$

```
[In] Integrate[(Sec[e + f*x]*Tan[e + f*x]^4)/(c - c*Sec[e + f*x])^8,x]
```

```
[Out] -1/15375360*(Csc[e/2]*Csc[(e + f*x)/2]^11*(425964*Sin[(f*x)/2] + 486024*Sin
[e + (f*x)/2] - 351450*Sin[e + (3*f*x)/2] - 299970*Sin[2*e + (3*f*x)/2] + 1
45695*Sin[2*e + (5*f*x)/2] + 180015*Sin[3*e + (5*f*x)/2] - 63580*Sin[3*e +
(7*f*x)/2] - 44990*Sin[4*e + (7*f*x)/2] + 6710*Sin[4*e + (9*f*x)/2] + 15004
*Sin[5*e + (9*f*x)/2] - 1975*Sin[5*e + (11*f*x)/2] + Sin[6*e + (11*f*x)/2])
)/(c^8*f)
```

Maple [A] (verified)

Time = 17.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}}{8f\,c^{8}}$
default	$\frac{-\frac{3}{7\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{7}}-\frac{1}{11\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{11}}+\frac{1}{5\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{5}}+\frac{1}{3\tan\left(\frac{fx}{2}+\frac{e}{2}\right)^{9}}}{8f\,c^{8}}$
risch	$\frac{2i\left(1155\,\mathrm{e}^{10i(fx+e)}-3465\,\mathrm{e}^{9i(fx+e)}+13860\,\mathrm{e}^{8i(fx+e)}-23100\,\mathrm{e}^{7i(fx+e)}+37422\,\mathrm{e}^{6i(fx+e)}-32802\,\mathrm{e}^{5i(fx+e)}+27060\,\mathrm{e}^{4i(fx+e)}\right.}{1155f\,c^{8}\left(\mathrm{e}^{i(fx+e)}-1\right)^{11}}$

```
[In] int(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/f/c^8*(-3/7/tan(1/2*f*x+1/2*e)^7-1/11/tan(1/2*f*x+1/2*e)^11+1/5/tan(1/2
*f*x+1/2*e)^5+1/3/tan(1/2*f*x+1/2*e)^9)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.64

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{152\,\cos\left(fx+e\right)^{6}+395\,\cos\left(fx+e\right)^{5}+289\,\cos\left(fx+e\right)^{4}+15\,\cos\left(fx+e\right)^{3}-19\,\cos\left(fx+e\right)^{2}+10}{1155\left(c^{8}f\cos\left(fx+e\right)^{5}-5\,c^{8}f\cos\left(fx+e\right)^{4}+10\,c^{8}f\cos\left(fx+e\right)^{3}-10\,c^{8}f\cos\left(fx+e\right)^{2}+5\,c^{8}f\cos\left(fx\right.\right.}$$

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="fricas")
```

```
[Out] 1/1155*(152*cos(f*x + e)^6 + 395*cos(f*x + e)^5 + 289*cos(f*x + e)^4 + 15*c
os(f*x + e)^3 - 19*cos(f*x + e)^2 + 10*cos(f*x + e) - 2)/((c^8*f*cos(f*x +
e)^5 - 5*c^8*f*cos(f*x + e)^4 + 10*c^8*f*cos(f*x + e)^3 - 10*c^8*f*cos(f*x
+ e)^2 + 5*c^8*f*cos(f*x + e) - c^8*f)*sin(f*x + e))
```

Sympy [F]

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{\int \frac{\tan^{4}(e+fx)\sec(e+fx)}{\sec^{8}(e+fx)-8\sec^{7}(e+fx)+28\sec^{6}(e+fx)-56\sec^{5}(e+fx)+70\sec^{4}(e+fx)-56\sec^{3}(e+fx)+28\sec^{2}(e+fx)-8\sec(e+fx)+1}\,dx}{c^{8}}$$

```
[In] integrate(sec(f*x+e)*tan(f*x+e)**4/(c-c*sec(f*x+e))**8,x)
```

```
[Out] Integral(tan(e + f*x)**4*sec(e + f*x)/(sec(e + f*x)**8 - 8*sec(e + f*x)**7
+ 28*sec(e + f*x)**6 - 56*sec(e + f*x)**5 + 70*sec(e + f*x)**4 - 56*sec(e +
 f*x)**3 + 28*sec(e + f*x)**2 - 8*sec(e + f*x) + 1), x)/c**8
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{\left(\frac{385\,\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{495\,\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{231\,\sin(fx+e)^6}{(\cos(fx+e)+1)^6} - 105\right)(\cos(fx+e)+1)^{11}}{9240\,c^8 f \sin(fx+e)^{11}}$$

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="maxima")
```

```
[Out] 1/9240*(385*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 495*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 231*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 105)*(cos(f*x +
e) + 1)^11/(c^8*f*sin(f*x + e)^11)
```

Giac [A] (verification not implemented)

none

Time = 1.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{231\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6 - 495\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4 + 385\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2 - 105}{9240\,c^8 f \tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{11}}$$

```
[In] integrate(sec(f*x+e)*tan(f*x+e)^4/(c-c*sec(f*x+e))^8,x, algorithm="giac")
```

```
[Out] 1/9240*(231*tan(1/2*f*x + 1/2*e)^6 - 495*tan(1/2*f*x + 1/2*e)^4 + 385*tan(1
/2*f*x + 1/2*e)^2 - 105)/(c^8*f*tan(1/2*f*x + 1/2*e)^11)
```

Mupad [B] (verification not implemented)

Time = 14.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec(e+fx)\tan^4(e+fx)}{(c-c\sec(e+fx))^8}\,dx = \frac{\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6}{5}-\frac{3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4}{7}+\frac{\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2}{3}-\frac{1}{11}}{8\,c^8\,f\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^{11}}$$

```
[In] int(tan(e + f*x)^4/(cos(e + f*x)*(c - c/cos(e + f*x))^8),x)
```

```
[Out] (tan(e/2 + (f*x)/2)^2/3 - (3*tan(e/2 + (f*x)/2)^4)/7 + tan(e/2 + (f*x)/2)^6
/5 - 1/11)/(8*c^8*f*tan(e/2 + (f*x)/2)^11)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                          convert(ExpnType_result,string)," vs. order ",
                          convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```